EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 10, No. 4, 2017, 835-849
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

On new classes of soft sets and functions via supra pre open soft sets

F. A. Gharib[1,*], M. Ezzat Mohamed[1,2], A. I. El-Maghrabi[3,4]

[1] *Faculty of Arts and Science, Northern Border University, Rafha, K. S. A.*
[2] *Mathematics Department, Faculty of Science, Fayoum University,Fayoum, Egypt*
[3] *Department, Faculty of Science, Yanbu, Branch of Tibah University, Yanbu Al-Baher, K. S. A.*
[4] *Mathematics Department, Faculty of Science, Kafr El-Sheikh University, Kafr El-Sheikh, Egypt.*

Abstract. In this paper, we generalize the notions of supra soft locally closed sets [1] and supra soft α-locally closed sets [2] by using the notions of supra *pre*-open soft sets [13]. Especially, we introduce the notions of supra soft P-locally closed sets, supra soft P^*-locally closed sets and supra soft P^{**}-locally closed sets in supra soft topological spaces. Also, we discuss their relationships with other supra open soft sets in detail, supported by examples and counterexamples. These examples illustrating the notions used in the paper are included. So we can see that all these concepts are independent from each other or does implies the other. Also, we introduce three different notions of generalized supra soft continuity, namely supra SPLC-continuous functions, supra SP^*LC-continuous functions and supra $SP^{**}LC$-continuous functions. Furthermore, we investigated some relations of these functions with other types of soft functions.

2010 Mathematics Subject Classifications: 54A05, 54A40, 54B05, 06D72

Key Words and Phrases: Supra soft topological space, Supra A-soft sets, Supra soft P-locally closed sets, Supra $SPLC$-continuous functions.

1. Introduction

In 1983, Mashhour et al. [17] introduced the supra topological spaces, not only, as a generalization to the class of topological spaces, but also, these spaces were easier in the application as shown in [11]. In 2001, Popa et al. [19] generalized the supra topological spaces to the minimal spaces and generalized spaces as a new wider classes. In 2007, Arpad Szaz [12] succeed to introduce an application on the minimal spaces and generalized spaces. In 1987, Abd El-Monsef et al. [9] introduced the fuzzy supra topological spaces. In 2001,

*Corresponding author.

Email addresses: fatouhalmg@yahoo.com (F. A. Gharib), mohaezzat@yahoo.com (M. Ezzat Mohamed), aelmaghrabi@yahoo.com (A. I. El-Maghrabi)

El-Sheikh success to use the fuzzy supra topology to study some topological properties to the fuzzy bitopological spaces.

The notions of supra soft topological space were first introduced by El-Sheikh et al. [13]. Various applications and topological properties on supra soft topological spaces were introduced recently in [[3-8], [16]]. Properties of soft category and homotopy are introduced in [22,23,24].

A. M. Abd El-latif [1], introduced the concepts of supra soft locally closed sets and supra SLC-continuous functions in supra soft topological spaces. Our aim of this paper, is to generalize these notions by using the notion of supra pre-open soft sets and discuss some of their basic properties.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory and supra soft topology.

Definition 1 (18). *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \varphi$ i.e*
$(F, A) = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2 (21). *Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

(1) $\tilde{X}, \tilde{\varphi} \in \tau$, *where* $\tilde{\varphi}(e) = \varphi$ *and* $\tilde{X}(e) = X,\ \forall e \in E$,

(2) *the union of any number of soft sets in τ belongs to τ,*

(3) *the intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called a soft topological space over X.

Definition 3 (26). *The soft set $(F, E) \in SS(X)_E$ is called a soft point in $\tilde{X}$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e^c) = \varphi$ for each $e^c \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.*
The soft point x_e is said to be belonging to the soft set (G, E), denoted by $x_e \tilde{\in} (G, E)$, if for the element $e \in E$, $F(e) \subseteq G(e)$.

Definition 4 (13). *Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called supra soft topology on X with a fixed set E if*

(1) $\tilde{X}, \tilde{\varphi} \in \mu$,

(2) *the union of any number of soft sets in μ belongs to μ.*

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X.

Definition 5 (13). *Let (X, τ, E) be a soft topological space and (X, μ, E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subset \mu$.*

Definition 6 (13). *Let (X, μ, E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X. We denote the set of all supra open soft sets over X by supra-$OS(X, \mu, E)$, or when there can be no confusion by supra-$OS(X)$ and the set of all supra closed soft sets by supra-$CS(X, \mu, E)$, or supra-$CS(X)$.*

Definition 7 (13). *Let (X, μ, E) be a supra soft topological space over and $(F, E) \in SS(X)_E$. Then, the supra soft interior of (G, E), denoted by $int^s(G, E)$ is the soft union of all supra open soft subsets of (G, E).i.e*
$int^s(G, E) = \tilde{\cup}\{(H, E) : (H, E) \text{ is supra open soft set and } (H, E)\tilde{\subseteq}(G, E)\}$.
Also, the supra soft closure of (F, E), denoted by $cl^s(F, E)$ is the soft intersection of all supra closed super soft sets of (F, E) i.e
$cl^s(F, E) = \tilde{\cap}\{(H, E) : (H, E) \text{ is supra closed soft set and } (F, E)\tilde{\subseteq}(H, E)\}$.

Definition 8 (1,3,13). *Let (X, μ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then, (F, E) is said to be,*

(1) *Supra pre open soft set if $(F, E)\tilde{\subseteq}int^s(cl^s(F, E))$.*

(2) *Supra semi open soft set if $(F, E)\tilde{\subseteq}cl^s(int^s(F, E))$.*

(3) *Supra α-open soft set if $(F, E)\tilde{\subseteq}int^s(cl^s(int^s(F, E)))$.*

(4) *Supra β-open soft set if $(F, E)\tilde{\subseteq}cl^s(int^s(cl^s(F, E)))$.*

(5) *Supra b-open soft set if $(F, E)\tilde{\subseteq}cl^s(int^s(F, E))\tilde{\cup}int^s(cl^s(F, E))$.*

(6) *Supra A-soft set if $(F, E) = (G, E) - (H, E)$ where (G, E) is supra open soft and (H, E) is supra regular open soft set in X.*

The set of all supra pre open (resp. semi open, α-open, β-open, b-open, A-) soft sets is denoted by supra-$POS(X)$ (resp. supra-$SOS(X)$, supra-$\alpha OS(X)$, supra-$\beta OS(X)$, supra-$BOS(X)$, supra-$AS(X)$) and the set of all supra pre closed (resp. semi closed, α-closed, β-closed, b-closed) soft sets is denoted by supra-$PCS(X)$ (resp. supra-$SCS(X)$, supra-$\alpha CS(X)$, supra-$\beta CS(X)$, supra-$BCS(X)$).

Definition 9 (13). *Let (X, μ, E) be a supra soft topological space over X and $(F, E) \in SS(X)_E$. Then, the supra P-soft interior of (F, E), denoted by $int^s_P(F, E)$ is the soft union of all supra pre-open soft subsets of (F, E) i.e*

$$int^s_P(F, E) = \tilde{\bigcup}\big\{(G, E) : (G, E) \text{ is a supra pre-open soft set and } (G, E)\tilde{\subseteq}(F, E)\big\}.$$

Also, the supra P-soft closure of F, denoted by $cl^s_P(F,E)$ *is the soft intersection of all supra pre-closed super soft sets of* (F,E) *i.e*

$$cl^s_P(F,E)=\tilde{\bigcap}\{(H,E):(H,E)\ \textit{is a supra pre-closed soft set and}\ (F,E)\tilde{\subseteq}(H,E)\}.$$

Definition 10. *[1] A soft set* (F,E) *is called supra soft locally closed in a supra soft topological space* (X,μ,E) *if* $(F,E)=(G,E)\tilde{\cap}(H,E)$ *where* (G,E) *is supra open soft and* (H,E) *is supra closed soft in* X*. We will denote the family of all supra soft locally closed sets of a supra soft topological space* X *by supra-*$SLC(X)$*.*

Definition 11 (1)**.** *A soft subset* (F,E) *of a supra soft topological space* (X,μ,E) *is called supra soft dense set if* $cl^s(F,E)=\tilde{X}$*.*

Definition 12 (1)**.** *A supra soft topological space* (X,μ,E) *is called supra soft submaximal if every supra soft dense subset of* (X,μ,E) *is supra open soft.*

Definition 13 (1,3,13)**.** *Let* (X,τ_1,A) *and* (Y,τ_2,B) *be soft topological spaces. Let* μ_1 *be an associated supra soft topology with* τ_1*. Let* $u:X\to Y$ *and* $p:A\to B$ *be mappings. Let* $f_{pu}:SS(X)_A\ \to\ SS(Y)_B$ *be a function. Then,* f_{pu} *is called:*

(1) *Supra soft continuous if* $f^{-1}_{pu}(G,B)\in\mu_1\ \forall\ (G,B)\in\tau_2$*.*

(2) *Supra soft pre-continuous if* $f^{-1}_{pu}(G,B)\in$ *supra-*$POS(X)\ \forall\ (G,B)\in\tau_2$*.*

(3) *Supra soft semi-continuous if* $f^{-1}_{pu}(G,B)\in$ *supra-*$SOS(X)\ \forall\ (G,B)\in\tau_2$*.*

(4) *Supra soft* α*-continuous if* $f^{-1}_{pu}(G,B)\in$ *supra-*$\alpha OS(X)\ \forall\ (G,B)\in\tau_2$*.*

(5) *Supra soft* β*-continuous if* $f^{-1}_{pu}(G,B)\in$ *supra-*$\beta OS(X)\ \forall\ (G,B)\in\tau_2$*.*

(6) *Supra soft B-continuous if* $f^{-1}_{pu}(G,B)\in$ *supra-*$BOS(X)\ \forall\ (G,B)\in\tau_2$*.*

(7) *Supra soft A-continuous function if* $f^{-1}_{pu}(G,B)\in supra-AS(X)\ \forall\ (G,B)\in\tau_2$*.*

(8) *Supra soft locally closed continuous function (supra SLC-continuous) if* $f^{-1}_{pu}(G,B)\in supra-SLC(X)\ \forall\ (G,B)\in\tau_2$*.*

Definition 14 (2)**.** *Let* (F,E) *be a soft subset of a supra soft topological space* (X,μ,E) *such that* $(F,E)=(G,E)\tilde{\cap}(H,E)$*, then* (F,E) *is said to be supra soft* α*-locally closed if* (G,E) *is a supra* α*-open soft and* (H,E) *is a supra* α*-closed soft in* X*. We will denote the family of all supra soft* α*-locally closed sets of a supra soft topological space* X *by supra-*$S\alpha LC(X)$*.*

3. Supra Soft Pre-Locally Closed Sets

In this section, we introduce the notion of supra soft P-locally closed sets in supra soft topological spaces and discuss its relationships with other supra open soft sets in detail, supported by counterexamples. Also, the notions of supra soft P^*-locally closed sets and supra soft P^{**}-locally closed sets are introduced and studied.

Definition 15. *Let (F, E) be a soft subset of a supra soft topological space (X, μ, E) such that $(F, E) = (G, E)\tilde{\cap}(H, E)$, then (F, E) is said to be:*

(1) *Supra soft P-locally closed if (G, E) is a supra pre-open soft and (H, E) is a supra pre-closed soft in X.*

(2) *Supra soft P^*-locally closed if (G, E) is a supra pre-open soft and (H, E) is a supra closed soft in X.*

(3) *Supra soft P^{**}-locally closed if (G, E) is a supra open soft and (H, E) is a supra pre-closed soft in X.*

We will denote the family of all supra soft P-locally (resp. P^*-locally and P^{**}-locally) closed sets of a supra soft topological space X by supra-$SPLC(X)$ (resp. supra-$SP^*LC(X)$ and supra-$SP^{**}LC(X)$)

Remark 1. *A soft subset (F, E) of (X, μ, E) is supra soft P-locally (resp. P^*-locally, P^{**}-locally) closed if its relative complement $(F, E)^c$ is the soft union of a pre-supra open soft set and a supra pre-closed soft set (resp. a supra pre-closed soft set and a supra open soft set, a supra closed soft set and a supra pre-open soft set).*

In a supra soft topological space (X, μ, E), every supra soft P^*- (resp. P^{**}-) locally closed is a supra soft P-locally closed.

Proof. Obvious from the fact that, every supra open (resp. closed) soft set is a supra pre-open (resp. pre-closed) soft [[13], Theorem 5.1 (1)].

Remark 2. *The converse of the above theorem is not true in general as shall shown in the following example.*

Example 1. *[2] Suppose that there are four houses in the universe X given by $X = \{a, b, c, d\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "green surroundings" and "wooden" respectively.*
Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ be five soft sets over the common universe X which describe the composition of the houses defined as follows:
$F_1(e_1) = \{a, c\}$, $F_1(e_2) = \{b, c\}$,
$F_2(e_1) = \{b, c\}$, $F_2(e_2) = \{a, c\}$,
$F_3(e_1) = \{a, b, c\}$, $F_3(e_2) = \{a, b, c\}$,
$F_4(e_1) = \{a, b, d\}$, $F_4(e_2) = \{a, b, d\}$,
$F_5(e_1) = \{b, c, d\}$, $F_5(e_2) = \{b, c, d\}$.

Hence, $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ *is a supra soft topology over* X*. Therefore, the soft set* (A, E) *is supra soft* P*-locally closed in* (X, μ, E)*, but not supra soft* P^**-locally closed, where*
$A(e_1) = \{b\}$*,* $A(e_2) = \{a\}$*.*
Also, the soft set (B, E) *is supra soft* P*-locally closed in* (X, μ, E)*, but not supra soft* P^{**}*-locally closed, where*
$B(e_1) = \{a, c, d\}$*,* $A(e_2) = \{b, c, d\}$*.*

In a supra soft topological space (X, μ, E), every supra soft α-locally closed is a supra soft P-locally closed.

Proof. Obvious from the fact that, every supra α-open (resp. α-closed) soft set is a supra *pre*-open (resp. *pre*-closed) soft [[13], Theorem 5.2. (4)]

Remark 3. *The converse of the above theorem is not true in general as shall shown in the following example.*

Example 2. *In Example 1, the soft set* (Z, E) *is supra soft* P*-locally closed in* (X, μ, E)*, but not supra soft* α*-locally closed, where*
$Z(e_1) = \{a, b\}$*,* $Z(e_2) = \{a, d\}$*.*

In a supra soft topological space (X, μ, E), every supra soft locally closed is a supra soft P- (resp. P^*- and P^{**}-) locally closed.

Proof. Obvious from [[13], Theorem 5.1 (1)].

Remark 4. *The converse Theorem 3 is not true in general as shall shown in the following examples.*

(1) In Example 1, the soft set (G, E) is a supra soft P-locally closed in (X, μ, E), but not supra soft locally closed, where $G(e_1) = \{a, c, d\}$, $G(e_2) = \{b, c, d\}$.

(2) In Example 1, the soft set (H, E) is a supra soft P^*-locally closed in (X, μ, E), but not supra soft locally closed, where $H(e_1) = \{a\}$, $H(e_2) = \{d\}$.

(3) In Example 1, the soft set (K, E) is a supra soft P^{**}-locally closed in (X, μ, E), but not supra soft locally closed, where $K(e_1) = \{a\}$, $K(e_2) = \{b, c\}$.

Let (X, μ, E) be a supra soft topological space. Then, (F, E) is supra soft P-locally closed if and only if $(F, E) = (G, E)\tilde{\cap} cl^s_P(F, E)$ for some supra *pre*-open soft set (G, E).

Proof. **Necessity:**Let (F, E) be a supra soft P-locally closed set in X. Then, $(F, E) = (G, E)\tilde{\cap}(H, E)$ where (G, E) is supra *pre*-open soft and (H, E) is supra *pre*-closed soft in X. It follows, $cl^s_P(F, E)\tilde{\subset} cl^s_P(H, E) = (H, E)$, where $cl^s_P(F, E)$ is a supra pre-closed soft set. Therefore, $(F, E)\tilde{\subseteq}(G, E)\tilde{\cap} cl^s_P(F, E)\tilde{\subseteq}(G, E)\tilde{\cap}(H, E) = (F, E)$. Thus, $(F, E) = (G, E)\tilde{\cap} cl^s_P(F, E)$.

Sufficient: Follows directly from Definition 15 (1).

Let (F, E) be a subset of a supra soft topological space (X, μ, E). Then, the following are equivalent:

(i) $(F, E) \in$ supra-$SPLC(X)$.

(ii) $cl^s_P(F, E) - (F, E)$ is supra *pre*-closed soft.

(iii) $(F, E)\tilde{\cup}[cl^s_P(F, E)]^{\tilde{c}}$ is supra *pre*-open soft.

Proof.

$(i) \Rightarrow (ii)$: Let $(F, E) \in$ supra-$SPLC(X)$. By Theorem 3, $(F, E) = (G, E)\tilde{\cap}cl^s_P(F, E)$ for some supra *pre*-open soft set (G, E). It follows, $cl^s_P(F, E)-(F, E) = cl^s_P(F, E)\tilde{\cap}[(G, E)\tilde{\cap}cl^s_P(F, E)]^{\tilde{c}} = cl^s_P(F, E)\tilde{\cap}(G, E)^{\tilde{c}}$ is supra *pre*-closed soft from [[13], Theorem 4.1 (2)], where $(G, E)^{\tilde{c}}$ is supra *pre*-closed soft set. Thus, $cl^s_P(F, E) - (F, E)$ is supra *pre*-closed soft.

$(ii) \Rightarrow (i)$: Assume that $(A, E) = [cl^s_P(F, E) - (F, E)]^{\tilde{c}}$. From (ii), (A, E) is supra *pre*-open soft in X. Hence, $(A, E)\tilde{\cap}cl^s_P(F, E) = [cl^s_P(F, E)-(F, E)]^{\tilde{c}}\tilde{\cap}cl^s_P(F, E) = (F, E)$. Therefore, $(F, E) \in$ supra-$SPLC(X)$ from Theorem 3.

$(ii) \Rightarrow (iii)$: Since $cl^s_P(F, E) - (F, E)$ is supra *pre*-closed soft in X from (ii). Then, $[cl^s_P(F, E) - (F, E)]^{\tilde{c}} = (F, E)\tilde{\cup}[cl^s_P(F, E)]^{\tilde{c}}$ is supra *pre*-open soft in X.

$(iii) \Rightarrow (ii)$: Obvious.

Let (X, μ, E) be a supra soft topological space. Then, (F, E) is supra soft P^*-locally closed if and only if $(F, E) = (H, E)\tilde{\cap}cl^s(F, E)$ for some supra *pre*-open soft set (H, E).

Proof. **Necessity:**Let (F, E) be a supra soft P^*-locally closed set in X. Then, $(F, E) = (H, E)\tilde{\cap}(G, E)$ where (H, E) is supra *pre*-open soft and (G, E) is supra closed soft in X. It follows, $cl^s(F, E)\tilde{\subseteq}cl^s(G, E) = (G, E)$. Hence, $(F, E)\tilde{\subseteq}(H, E)\tilde{\cap}cl^s(F, E)\tilde{\subseteq}(H, E)\tilde{\cap}(G, E) = (F, E)$. Thus, $(F, E) = (H, E)\tilde{\cap}cl^s(F, E)$.
Sufficient: Obvious from Definition 15 (2).

Let (F, E) be a subset of a supra soft topological space (X, μ, E). Then, the following are equivalent:

(i) $(F, E) \in$ supra-$SP^*LC(X)$.

(ii) $cl^s(F, E) - (F, E)$ is supra *pre*-closed soft.

(iii) $(F, E)\tilde{\cup}[cl^s(F, E)]^{\tilde{c}}$ is supra *pre*-open soft.

Proof. It is similar to the proof of Theorem 3.

Let (X, μ, E) be a supra soft topological space. Then, (F, E) is supra soft P^{**}-locally closed if and only if $(F, E) = (G, E)\tilde{\cap}cl^s_P(F, E)$ for some supra open soft set (G, E).

Proof. It is similar to the proof of Theorem 3.

Let (F, E) be a subset of a supra soft topological space (X, μ, E). If $(F, E) \in$ supra-$SP^{**}LC(X)$, then $cl^s(F, E) - (F, E)$ is supra *pre*-closed soft and $(F, E)\tilde{\cup}[cl^s(F, E)]^{\tilde{c}}$ is supra *pre*-open soft.

Proof. It is similar to the proof of Theorem 3.

Remark 5. *The converse Theorem 3 is not true in general as shall shown in the following examples.*

Example 3. *In Example 1, for the soft set* (G, E)*, where* $G(e_1) = \{a, c, d\}$*,* $G(e_2) = \{b, c, d\}$*, we have* $cl^s(G, E) - (G, E) = (C, E)$*, where* $C(e_1) = \{b\}$*,* $C(e_2) = \{a\}$*, is a supra pre-closed soft and* $(G, E)\tilde{\cup}[cl^s(G, E)]^{\tilde{c}} = (G, E)$ *is a supra pre-open soft. But,* $(F, E) \notin$ *supra-*$SP^{**}LC(X)$*.*

Remark 6. *The relative complement of a supra soft* P*- (resp.* P^**- and* P^{**}*-) locally closed set need not to be a supra soft* P*- (resp.* P^**- and* P^{**}*-) locally closed. The following examples support our claim.*

(1) In Examples 3 (1), the soft set (G, E) is a supra soft P-locally closed in (X, μ, E), but its relative complement $(G, E)^c$ is not supra soft P-locally closed, where $G^c(e_1) = \{b\}$, $G^c(e_2) = \{a\}$.

(2) Suppose that there are four phones in the universe X given by $X = \{a, b, c, d\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "cheap" and "model" respectively.
Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E)$ be ten soft sets over the common universe X which describe the composition of the phones defined as follows:
$F_1(e_1) = \{a, b\}$, $F_1(e_2) = \{a\}$,
$F_2(e_1) = \{b\}$, $F_2(e_2) = \{b\}$,
$F_3(e_1) = \{a, b\}$, $F_3(e_2) = \{a, b\}$,
$F_4(e_1) = \{a, b, c\}$, $F_4(e_2) = \{a, b, c\}$,
$F_5(e_1) = \{a\}$, $F_5(e_2) = \{a\}$,
$F_6(e_1) = \{a, b, c\}$, $F_6(e_2) = \{a, c\}$,
$F_7(e_1) = \{a, b, d\}$, $F_7(e_2) = \{a, b, c\}$,
$F_8(e_1) = \{a, b\}$, $F_8(e_2) = \{a, b, c\}$,
$F_9(e_1) = \{a, b\}$, $F_9(e_2) = \{a, c\}$,
$F_{10}(e_1) = X$, $F_{10}(e_2) = \{a, b, c\}$.
Hence, $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E)\}$ is a supra soft topology over X. Therefore, the soft set (H, E) is a supra soft P^*-locally closed set in (X, μ, E), where $H(e_1) = \{b\}$, $H(e_2) = \{b, c\}$, but its relative complement $(H, E)^c$ is not supra soft P^*-locally closed, where $H^c(e_1) = \{a, c, d\}$, $H^c(e_2) = \{a, d\}$.

(3) In Examples 3 (3), the soft set (K, E) is a supra soft P^{**}-locally closed in (X, μ, E), but its relative complement $(K, E)^c$ is not supra soft P-locally closed, where $K^c(e_1) = \{b, c, d\}$, $K^c(e_2) = \{a, d\}$.

In a supra soft topological space (X, μ, E), every supra A-soft set is a supra soft P- (resp. P^*- and P^{**}-) locally closed.

Proof. Follows from the fact that, every supra regular closed soft set in a supra soft topological space (X, μ, E) is a supra closed soft set [[25], Remark 3.2].

Remark 7. *The converse of the above theorem is not true in general as shall shown in the following examples.*

Example 4. *In Example 3 (2), the soft set (G, E) is supra soft locally closed set. Since $(G, E) = (F_{10}, E)\tilde{\cap}(H, E)$, where (F_{10}, E) is supra open soft and (H, E) is supra closed soft in X defined by $H(e_1) = \{c, d\}$, $H(e_2) = \{b, d\}$. Hence, it is a supra soft P- (resp. P^*- and P^{**}-) locally closed from Theorem 3, where $G(e_1) = \{c, d\}$, $G(e_2) = \{b\}$. On the other hand, it is not supra A-soft.*

In a supra soft topological space (X, μ, E), every supra open (resp. closed) soft set is a supra soft P- (resp. P^*- and P^{**}-) locally closed in X.

Proof. Obvious.

Remark 8. *The converse of the above theorem is not true in general as shall shown in the following example.*

Example 5. *In Examples 3, the soft sets (G, E) (resp. (H, E) and (K, E)) are supra soft P- (resp. P^*- and P^{**}-) locally closed in (X, μ, E), but all of them neither supra open soft nor supra closed soft in (X, μ, E).*

Definition 16. *A soft subset (F, E) of a supra soft topological space (X, μ, E) is called supra soft pre-dense set if $cl^s_P(F, E) = \tilde{X}$.*

Proposition 1. *A supra soft pre-dense set (F, E) is supra pre-open soft in (X, μ, E) if and only if it is supra soft P-locally closed.*

Proof. Immediate from Theorem 3 and Definition 16.

Definition 17. *A supra soft topological space (X, μ, E) is called supra soft pre-submaximal if every supra softpre-dense subset of (X, μ, E) is supra pre-open soft.*

Corollary 1. *A supra soft topological space (X, μ, E) is supra soft pre-submaximal if and only if every soft subset of (X, μ, E) is supra soft P-locally closed.*

Proof. Immediate from Proposition 1.

Every supra soft submaximal space (X, μ, E) is supra soft *pre*-submaximal space.

Proof. Let (X, μ, E) be a supra soft submaximal space and $(F, E) \in \mu$. Then, $cl^s(F, E) = \tilde{X}$. It follows, $cl^s(F, E) = \tilde{X}\tilde{\subseteq}cl^s_P(F, E)$, where $(F, E) \in \mu \subseteq$ supra-$POS(X)$. Hence, (X, μ, E) is a supra soft *pre*-submaximal space.

Remark 9. *The converse of Theorem 3 is not true in general as shall shown in the following example.*

Example 6. *Suppose that there are four watches in the universe X given by $X = \{a, b, c, d\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which stand for "model" and "cheap" respectively.*
Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ be four soft sets over the common universe X which describe the composition of the watches defined as follows:

$F_1(e_1) = \{a, b\}$, $F_1(e_2) = \{a\}$,
$F_2(e_1) = \{b, d\}$, $F_2(e_2) = \{b, d\}$,
$F_3(e_1) = \{a, c, d\}$, $F_3(e_2) = X$,
$F_4(e_1) = \{a, b, d\}$, $F_4(e_2) = \{a, b, d\}$.
Hence, $\mu = \{\tilde{X}, \tilde{\varphi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ *is a supra soft topology over* X*, which is a supra soft pre-submaximal space. On the other hand, the soft set* G, E*, where* $G(e_1) = \{a, c, d\}$*,* $G(e_2) = \{a, b, c\}$ *is supra soft dense set but not supra open soft in* X*. Hence,* (X, μ, E) *is not supra soft submaximal space.*

For a supra soft topological space (X, μ, E) we have the following implications from Theorems 3, 3, 3, 3 and [[3], Corollary 4.1]. These implications are not reversible.

$$
\begin{array}{lllll}
\text{supra-}SPLC(X) & \leftarrow & \text{supra-}SP^{*}LC(X) & & \\
\uparrow & & \uparrow & & \\
\text{supra-}SP^{**}LC(X) & \leftarrow & \text{supra-}SLC(X) & \longleftarrow & \text{supra-}AS(X) \\
\uparrow & \nearrow & \not\leftrightarrow & & \downarrow \\
\text{supra-}OS(X) & \longrightarrow & \text{supra-}\alpha OS(X) & \longrightarrow & \text{supra-}SOS(X) \\
\downarrow & & & \swarrow & \\
\text{supra-}POS(X) & & \longrightarrow & \text{supra-}BOS(X) &
\end{array}
$$

4. Decompositions of Supra Soft Continuity via Supra *Pre*-Open soft sets

In this section, we introduce three different notions of generalized supra soft continuity, namely supra $SPLC$-continuous functions, supra $SP^{*}LC$-continuous functions and supra $SP^{**}LC$-continuous functions. Furthermore, we obtain decompositions of supra soft continuity. Finally, Several examples are provided to illustrate the behavior of these new classes of soft functions.

Definition 18. *Let* (X, τ_1, A) *and* (Y, τ_2, B) *be soft topological spaces. Let* μ_1 *be an associated supra soft topology with* τ_1*. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then,* f_{pu} *is called:*

(1) *Supra soft pre-locally closed continuous function (supra* $SPLC$*-continuous) if* $f_{pu}^{-1}(G, B) \in$ *supra-*$SPLC(X)$ $\forall$ $(G, B) \in \tau_2$.

(2) *Supra soft pre***-locally closed continuous function (supra* $SP^{*}LC$*-continuous) if* $f_{pu}^{-1}(G, B) \in$ *supra-*$SP^{*}LC(X)$ $\forall$ $(G, B) \in \tau_2$.

(3) *Supra soft* P^{**}*-locally closed continuous function (supra* $SP^{**}LC$*-continuous) if* $f_{pu}^{-1}(G, B) \in$ *supra-*$SP^{**}LC(X)$ $\forall$ $(G, B) \in \tau_2$.

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to$

$SS(Y)_B$ be a function. Then, every supra SP^*LC- (resp. supra $SP^{**}LC$)-continuous function is a supra $SPLC$-continuous.

Proof. It is obvious from Theorem 3.

Remark 10. *The converse of Theorem 4 is not true in general, as shown in the following examples.*

(1) Let $X = \{a, b, c, d\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = \{z\}$, $u(b) = \{y\}$, $u(c) = \{x\}$, $u(d) = \{x\}$ and
$p(e_1) = \{k_2\}$, $p(e_2) = \{k_1\}$. Let (X, τ_1, A) be a soft topological space over X where,
$\tau_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A)\}$, where (F_1, A) is a soft set over X defined as follows:
$F_1(e_1) = \{b, c\}$, $F_1(e_2) = \{a, c\}$.
Consider the supra soft topology $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), ..., (F_5, A)\}$ in Example 1.
Let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:
$G(k_1) = \{y\}$, $G(k_2) = \{z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{b\}), (e_2, \{a\})\}$ is a supra soft P-locally closed in X, but not supra soft P^*-locally closed. Hence, f_{pu} is a supra $SPLC$-continuous, but not supra SP^*LC-continuous.

(2) In (1), let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:
$G(k_1) = \{x, z\}$, $G(k_2) = \{x, y\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, c, d\}), (e_2, \{b, c, d\})\}$ is a supra soft P-locally closed in X, but not supra soft P^{**}-locally closed. Hence, f_{pu} is a supra $SPLC$-continuous, but not supra $SP^{**}LC$-continuous.

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, every supra SαLC-continuous function is a supra $SPLC$-continuous.

Proof. It is obvious from Theorem 3.

Example 7. *Let $X = \{a, b, c, d\}$, $Y = \{x, y, z, w\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:*
$u(a) = \{z\}$, $u(b) = \{w\}$, $u(c) = \{x\}$, $u(d) = \{y\}$ *and*
$p(e_1) = \{k_2\}$, $p(e_2) = \{k_1\}$. *Let (X, τ_1, A) be a soft topological space over X where,*
$\tau_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A)\}$, *where (F_1, A) is a soft set over X defined as follows:*
$F(e_1) = \{a, b\}$, $F(e_2) = \{a, b\}$.
Consider the supra soft topology μ_1 in Example 1, $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), ...(F_{10}, A)\}$. Let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, *where (G, B) is a soft set over Y defined by:*
$G(k_1) = \{z, w\}$, $G(k_2) = \{y, z\}$.

Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a, b\}), (e_2, \{a, d\})\}$ is a supra soft P-locally closed in X, but it is not supra soft α-locally closed. Hence, f_{pu} is a supra $SPLC$-continuous, but it is not supra soft $S\alpha LC$-continuous.

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, every supra SLC-continuous function is a supra $SPLC$- (resp. supra SP^*LC- and supra $SP^{**}LC$)-continuous.

Proof. Follows from Theorem 3.

Remark 11. *The converse of Theorem 4 is not true in general, as shown in the following examples.*

(1) In Examples 4 (2), f_{pu} is a supra $SPLC$-continuous, but not supra SLC-continuous.

(2) Let $X = \{a, b, c, d\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = \{z\}$, $u(b) = \{y\}$, $u(c) = \{y\}$, $u(d) = \{x\}$ and
$p(e_1) = \{k_2\}$, $p(e_2) = \{k_1\}$. Let (X, τ_1, A) be a soft topological space over X where,
$\tau_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A)\}$, where (F_1, A) is a soft set over X defined as follows:
$F_1(e_1) = \{b, c\}$, $F_1(e_2) = \{a, c\}$.
Consider the supra soft topology $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), ..., (F_5, A)\}$ in Example 1.
Let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:
$G(k_1) = \{z\}$, $G(k_2) = \{x\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a\}), (e_2, \{d\})\}$ is a supra soft P^*-locally closed in X, but not supra soft locally closed. Hence, f_{pu} is a supra SP^*LC-continuous, but not supra SLC-continuous.

(3) In (2), let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:
$G(k_1) = \{z\}$, $G(k_2) = \{y\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{a\}), (e_2, \{b, c\})\}$ is a supra soft P^{**}-locally closed in X, but not supra soft locally closed. Hence, f_{pu} is a supra $SP^{**}LC$-continuous, but not supra SLC-continuous.

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, every supra soft A-continuous function is a supra $SPLC$- (resp. supra SP^*LC- and supra $SP^{**}LC$)-continuous.

Proof. It is obvious from Theorem 3.

Remark 12. *The converse of Theorem 4 is not true in general, as shown in the following example.*

Example 8. *In Example 7, consider the supra soft topology μ_1 in Example 3 (2), $\mu_1 = \{\tilde{X}, \tilde{\varphi}, (F_1, A), ...(F_{10}, A)\}$. Let (Y, τ_2, B) be a soft topological space over Y where, $\tau_2 = \{\tilde{Y}, \tilde{\varphi}, (G, B)\}$, where (G, B) is a soft set over Y defined by:*
$G(k_1) = \{x, y\}, \quad G(k_2) = \{w\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}((G, B)) = \{(e_1, \{c, d\}), (e_2, \{b\})\}$ is a supra soft P- (resp. P^*- and P^{**}-) locally closed in X, but it is not supra A-soft. Hence, f_{pu} is a supra $SPLC$- (resp. supra SP^*LC- and supra $SP^{**}LC$)-continuous, but it is not supra soft A-continuous.

For a supra soft topological space (X, μ, E) we have the following implications from Theorems 4, 4, 4 and [[3], Corollary 6.1]. These implications are not reversible.

supra $SPLC$-continuity $\leftarrow$ supra SP^*LC-continuity
$\uparrow$ $\uparrow$
supra $SP^{**}LC$-continuity $\leftarrow$ supra SLC-continuity $\longleftarrow$ supra soft A-continuity
$\uparrow$ $\nearrow$ $\nwarrow\searrow$ $\searrow$
supra soft continuity $\longrightarrow$ supra soft α-continuity $\longrightarrow$ supra soft semi-continuity
$\downarrow$ $\swarrow$
supra soft pre-continuity $\longrightarrow$ supra soft B-continuity

5. Conclusion

The aim of this paper, is to introduce new types of soft sets in supra soft topological spaces called, supra soft P-locally closed sets, supra soft P^*-locally closed sets and supra soft P^{**}-locally closed sets. Also, new types of soft continuity are introduced. Furthermore, some of their basic properties are obtained. In future, the generalization of these concepts by using soft ideals notion [15] will be introduced and the future research will be undertaken in this direction.

Acknowledgments

The authors gratefully acknowledge the approval and the support of this research from the Deanship of Scientific Research study by the grant no. 8-068-435, K. S. A., Northern Border University, Arar.

References

[1] A. M. Abd El-latif, On New Classes of Supra Soft Sets and Supra Soft Continuity, European Journal of Pure and Applied Mathematics (EJPAM), 2017, accepted.

[2] A. M. Abd El-latif, New Decompositions of Supra Soft Sets and $S\alpha LC$-Continuity. To appear.

[3] A. M. Abd El-latif and S. Karataş, Supra b-open soft sets and supra b-soft continuity on soft topological spaces, J. Math. Comput. Appl. Res., 5(1) (2015) 1–18.

[4] A. M. Abd El-latif and Rodyna A. Hosny, Supra semi open soft sets and associated soft separation Axioms, Appl. Math. Inf. Sci., 10 (6) (2016) 2207–2215.

[5] A. M. Abd El-latif and Rodyna A. Hosny, Supra soft separation axioms and supra irresoluteness based on supra b-open soft sets, Gazi University Journal of Science, 29 (4) (2016) 845–854.

[6] A. M. Abd El-latif, Soft supra strongly generalized closed sets, Journal of Intelligent & Fuzzy Systems, 31 (3) (2016) 1311–1317.

[7] A. M. Abd El-latif, Supra soft b-connectedness I: Supra soft b-irresoluteness and separateness, CREAT. MATH. INFORM., 25 (2) (2016) 127-134.

[8] A. M. Abd El-latif, Supra soft separation axioms based on supra β-open soft sets, Math. Sci. Lett., 5 (2) (2016) 121–129.

[9] M. E. Abd El-Monsef and A. E. Ramadan, On fuzzy supra topological spaces, Indian J. Pure and Appl. Math., 4 (18) (1987) 322-329.

[10] Ali Haydar Kocaman and Naime Tozlu, Soft locally closed sets and decompositions of soft continuity, Ann. Fuzzy Math. Inform., 11 (2) (2016) 173-181.

[11] Andreas Alpers, Digital Topology: Regular Sets and Root Images of the Cross-Median Filter, Journal of Mathematical Imaging and Vision, 17 (2002) 7-14.

[12] Arpad Szaz, Minimal structures, generalized topologies, and ascending systems should not be studied without generalized uniformities, Faculty of Sciences and Mathematics University of Nis, 21 (1) (2007) 87-97

[13] S. A. El-Sheikh and A. M. Abd El-latif, Decompositions of some types of supra soft sets and soft continuity, International Journal of Mathematics Trends and Technology, 9 (1) (2014) 37-56.

[14] S. A. El-Sheikh, A new approach to fuzzy bitopological spaces, Information Sciences, 137 (2001) 283-301.

[15] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014) 1595-1603.

[16] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014) 1731–1740.

[17] A. S. Mashhour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, On supra topological spaces, Indian J. Pure and Appl. Math., 4 (14) (1983) 502-510.

[18] D. Molodtsov, Soft set theory-first tresults, Comput. Math. Appl., 37 (1999) 19-31.

[19] V. Popa and T. Noiri, On the definitions of some generalized forms of continuity under minimal conditions, Mem. Fac. Sci. Kochi Univ. Math. Ser., 22 (2001) 9-19.

[20] Saziye Yuksel, Soft regular generalized closed sets in soft topological spaces, Int. Journal of Math. Analysis, 8 (8) (2014) 355-367.

[21] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl., 61 (2011) 1786-1799.

[22] Simge Oztun, Ali Mutlu and Aysun Erdogan Sert, Monomorphism and Epimorphism Properties of Soft Categories, European Journal of Pure and Applied Mathematics, 2017, In Press.

[23] Simge Oztun, Some Properties of Soft Categories, International Journal of Modeling and Optimization, 6 (2) (2016) 91-95.

[24] Simge Oztun and Sultan Ihtiyar, Properties of Soft Homotopy in Digital Images, American Instutite of Physics, 1798, 020120 (2017); doi:10.1063/1.4972712.

[25] Zehra Guzel Ergul and Saziye Yuksel, Supra regular generalized closed sets in supra soft topological spaces, Ann. Fuzzy Math. Inform., 11 (3) (2016) 349-360.

[26] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., 3 (2) (2012) 171-185.