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High-Efficiency Computational Methods for Exact Solutions of Coupled Nonlinear Systems: Applications to Plasma and Optics

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Abstract. This investigation delves into the coupled nonlinear Schrödinger-Korteweg-de Vries (CNLS-KdV) framework, a quintessential paradigm for examining the intricate interplay between dispersive and nonlinear wave mechanics in multi-component environments. Mathematically, the CNLS-KdV configuration merges the nonlinear Schrödinger formulation, governing modulation instabilities and wave dispersion phenomena, with the Korteweg-de Vries formulation, characterizing soliton genesis and wave profile steepening. From a physical standpoint, this theoretical construct proves invaluable across diverse domains, including optical pulse interactions in fiber-optic media, surface wave mechanics in shallow aquatic systems, and electrostatic wave propagation through plasma media.
Via systematic implementation of the Khater III methodology alongside the enhanced Kudryashov approach, we establish precise closed-form solutions that reveal complex soliton architectures and their collisional behavior. These mathematical expressions illuminate fundamental energy transfer mechanisms and stability characteristics in coupled wave configurations, with particular emphasis on coherent soliton interactions and multi-wave patterns that have remained insufficiently characterized in existing literature. The employed theoretical methods exhibit remarkable computational effectiveness and mathematical rigor, facilitating comprehensive exploration of nonlinear evolution equations within a mathematically tractable paradigm.
These discoveries strengthen theoretical comprehension of coupled nonlinear frameworks by broadening the arsenal of exact solution strategies for nonlinear partial differential equations. This advancement enriches predictive modeling approaches essential for engineering and scientific implementations. In particular, the outcomes enhance quantitative perspectives on soliton behavior in fluid mechanics, plasma physics, and nonlinear optics, with ramifications for optimizing fiber-optic communication technologies and plasma-based energy infrastructures. The research emphasizes the CNLS-KdV framework's significance as a cornerstone model for investigating universal nonlinear wave phenomena spanning multiple disciplines.

2020 Mathematics Subject Classifications: 35Q55, 35Q53, 35C08, 35B35
Key Words and Phrases: Coupled Schrödinger-Korteweg de Vries system, nonlinear wave propagation, Khater III method, enhanced Kudryashov method

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1. Introduction

Nonlinear partial differential equations provide a versatile mathematical framework for representing intricate physical phenomena across multiple scientific domains [1**?** –7]. Among these, the nonlinear Schrödinger formulation (NLS) and the Korteweg–de Vries (KdV) formulation emerge as pivotal constructs [8]. The NLS formulation excels at characterizing optical soliton propagation and wave-envelope evolution in nonlinear media, whereas the KdV formulation adeptly models shallow-water wave packets and dispersive shock structures [9]. Fusing these two formulations into a coupled CNLS–KdV system yields a sophisticated paradigm for probing pulse-soliton interactions and coherent-structure dynamics [10].

The CNLS–KdV system's versatility spans applications from hydrodynamic wave coupling to advanced optical communications [11]. By accommodating both short-pulse and long-wave components, this coupled framework delivers a unified perspective on multi-scale wave phenomena [12]. Despite extensive explorations of the standalone NLS and KdV models, comprehensive analyses of their coupled counterpart remain scarce [13].

Prior investigations have largely presented isolated exact solutions for either the NLS or KdV equations, focusing on single-wave behaviors or specialized scenarios [14]. In contrast, the full CNLS–KdV system poses substantial analytical challenges yet promises richer insights into nonlinear wave propagation [15]. This complexity motivates the development of advanced mathematical approaches to unlock the system's complete solution space and deepen our understanding of coupled wave mechanics [16].

The mathematical structure of the CNLS-KdV system is given by [17–19]

$$\begin{cases} -\dfrac{\partial^2 \mathcal{S}}{\partial x^2} + i\,\dfrac{\partial \mathcal{S}}{\partial t} - \mathcal{S}\,\mathcal{V} = 0, \\ \\ \dfrac{\partial^3 \mathcal{V}}{\partial x^3} - \dfrac{\partial\,|\mathcal{S}|^2}{\partial\,x} + \dfrac{\partial \mathcal{V}}{\partial t} + 6\,\mathcal{V}\,\dfrac{\partial \mathcal{V}}{\partial x} = 0, \end{cases} \tag{1}$$

where,

- $\mathcal{S}(x,t)$: This represents the complex envelope of the short wave. In physical terms, it could describe [20]:
  - The amplitude of a short wavelength pulse in an optical fiber
  - A wave packet in a dispersive medium
  - The envelope of a modulated wave in plasma
- $\mathcal{V}(x,t)$: This represents the long wave component. Physically, it could describe [21]:
  - A long wavelength disturbance in a fluid

- A low-frequency wave in plasma
- A background field interacting with the short wave

The CNLS-KdV system models the interaction between short waves (described by the Nonlinear Schrödinger equation) and long waves (described by the Korteweg-de Vries equation) in nonlinear media [22]. This coupling is crucial for understanding complex wave phenomena in various physical systems, such as fluid dynamics, plasma physics, and nonlinear optics. The system terms' physical meaning can be explained as following [23, 24]:

(i) First equation (Nonlinear Schrödinger part):

$$-\frac{\partial^2 \mathcal{S}}{\partial x^2} + i\,\frac{\partial \mathcal{S}}{\partial t} - \mathcal{S}\,\mathcal{V} = 0$$

- $-\frac{\partial^2 \mathcal{S}}{\partial x^2}$: Represents the dispersion of the short wave
- $i\,\frac{\partial \mathcal{S}}{\partial t}$: Describes the time evolution of the short wave
- $-\mathcal{S}\,\mathcal{V}$: Models the interaction between the short and long waves

(ii) Second equation (Korteweg-de Vries part):

$$\frac{\partial^3 \mathcal{V}}{\partial x^3} - \frac{\partial |\mathcal{S}|^2}{\partial x} + \frac{\partial \mathcal{V}}{\partial t} + 6\,\mathcal{V}\,\frac{\partial \mathcal{V}}{\partial x} = 0$$

- $\frac{\partial^3 \mathcal{V}}{\partial x^3}$: Represents the dispersion of the long wave
- $-\frac{\partial |\mathcal{S}|^2}{\partial x}$: Describes the forcing effect of the short wave on the long wave
- $\frac{\partial \mathcal{V}}{\partial t}$: Represents the time evolution of the long wave
- $6\,\mathcal{V}\,\frac{\partial \mathcal{V}}{\partial x}$: Models the nonlinear steepening of the long wave

The coupling of these formulations enables analysis of intricate short–long wave interactions, including energy exchange between distinct wave modes, soliton emergence, and wave-packet evolution in nonlinear media [25, 26]. This coupled paradigm proves especially powerful when dispersive and nonlinear influences coincide, and the interplay of diverse wave classes dictates the dynamics [27, 28].

The principal aim of this investigation is to leverage the described symbolic techniques to dissect and resolve the CNLS–KdV system [29–32]. These exact methods have exhibited outstanding performance in deriving multi-soliton solutions and assessing wave stability. By harnessing these approaches, the present work aspires to unveil previously unreported soliton envelopes within the CNLS–KdV framework, thereby enriching theoretical perspectives on wave propagation in nonlinear contexts such as shallow–water environments and guided optical pulses.

In this context, we implement the next wave transformation $\mathcal{S} = \mathcal{S}(x,t) = \psi(\mathfrak{z})\ e^{i(\kappa\, t + x)}$; $\mathcal{V} = \mathcal{V}(x,t) = \varphi(\mathfrak{z})$, $\mathfrak{z} = c\,t + x$ on the above–system (Eq. (1)), where κ, c are arbitrary constants to be determined later, to convert the system into the following ordinary differential form

$$\begin{cases} -\,e^{i\,(\kappa\, t + x)}\left(-i\,(c-2)\,\psi' + \psi\,(\kappa + \varphi - 1) + \psi''\right) = 0, \\ \\ (c + 6\,\varphi)\,\varphi' + \varphi^{(3)} - 2\,\psi\,\psi' = 0. \end{cases} \tag{2}$$

Integrating the second equation in the above–system (Eq. (2)), once with respect to $\mathfrak{z}$ and zero integration constants, yields

$$\varphi(c + 3\,\varphi) + \varphi'' - \psi^2 = 0, \tag{3}$$

Separating the real and imaginary terms in the first equation in the above–system (2), gets

$$\begin{cases} \textbf{Re:} \quad \psi\,(\kappa + \varphi - 1) + \psi'' = 0, \\ \\ \textbf{Im:} \qquad\qquad -\left((c-2)\,\psi'\right) = 0. \end{cases} \tag{4}$$

For $c = 2$, the system (2) and Eq. (3), converts into the following form

$$\begin{cases} \psi\,(\kappa + \varphi - 1) + \psi'' = 0, \\ \\ \varphi'' + \varphi\,(3\,\varphi + 2) - \psi^2 = 0. \end{cases} \tag{5}$$

Using this relationship between ψ, φ that is given by $\varphi = -\kappa - \frac{\psi''}{\psi} + 1$, converts the system (5) into the following form

$$\psi''\,\left(2\,(3\,\kappa - 4)\psi^2 - 2\,\left(\psi'\right)^2\right) - \psi\,\left(\psi\,\left(\psi\,\left(\kappa\,(8 - 3\,\kappa) + \psi^2 - 5\right) + \psi^{(4)}\right) - 2\,\psi^{(3)}\,\psi'\right) + 4\,\psi\,\left(\psi''\right)^2 = 0. \tag{6}$$

This study exhibits considerable promise for enriching both theoretical frameworks and practical applications in nonlinear physics. By advancing existing computational paradigms and unveiling novel solutions, it bridges prior research gaps and offers fresh perspectives on the CNLS–KdV system's physical and mathematical characteristics. Anticipated outcomes extend across fluid dynamics, plasma physics, and nonlinear optics, underscoring the system's cross-disciplinary relevance.

To achieve these objectives, the homogeneous balance rule is applied in tandem with the auxiliary equations inherent to our chosen symbolic strategies. This combined approach ensures systematic derivation of self-reinforcing wave envelopes and stability analyses within the coupled framework.

$$\begin{cases} f'(\mathfrak{z})^2 \to \dfrac{1}{\ln^2(\Theta)}\,\left(\varsigma_1 + \Theta^{f(\mathfrak{z})}\left(\varsigma_2 + \varsigma_3\,\Theta^{f(\mathfrak{z})}\right)\right), \\ \\ f''(\mathfrak{z}) \to f(\mathfrak{z}) - 2\,\varsigma_4\,f(\mathfrak{z})^3, \end{cases} \tag{7}$$

where ς_1, ς_2, ς_3, ς_4 are arbitrary constants to be determined later, yields constructing the general solution of Eq. (6) in the next form

$$\psi(\mathfrak{Z}) = \begin{cases} \sum_{i=0}^{2\,m} a_i \left(\Theta^{f(\mathfrak{Z})}\right)^i, \\ \\ \sum_{i=1}^{n} \left(a_i\, f(\mathfrak{Z})^i + b_i \left(\frac{f'(\mathfrak{Z})}{f(\mathfrak{Z})}\right)^i \right) + a_0, \end{cases} \tag{8}$$

where $m = 1$, $n = 2$. While a_i, b_i are arbitrary constants to be determined later.

This research furnishes the nonlinear wave theory corpus with exhaustive computational solutions to the CNLS–KdV system, thereby deepening insights into coupled soliton interactions. By harnessing state-of-the-art computational methodologies, the study not only rectifies prevailing knowledge gaps but also forges novel trajectories for theoretical inquiry and applied innovation in soliton-centric domains. The anticipated outcomes promise to propel our comprehension of intricate wave structures and dynamics in nonlinear regimes, with potential reverberations across future research and technological advancements.

The paper is structured as follows:

- Section 2: Detailed exploration of new analytical solutions for the CNLS–KdV model, accompanied by rigorous stability analyses.
- Section 3: Numerical simulations featuring graphical depictions that elucidate solution evolution and physical characteristics.
- Section 4: Discussion of scientific contributions, highlighting the unique originality of the derived solutions.
- Section 5: Concluding remarks summarizing principal findings and final observations.

2. Exploring Waveform Evolution and Behavior

Building on the previously described methodologies, this section presents a thorough investigation of both exact and numerical solutions for the CNLS–KdV model. We outline the applied mathematical approaches, emphasizing their effectiveness in revealing new system characteristics, while also scrutinizing the stability and practical relevance of the obtained soliton envelopes. This dual analysis provides a profound comprehension of the model's dynamics and its wider physical ramifications.

2.1. Wave solutions Via Khat III method

By applying the aforementioned analytical scheme, the solutions to Eq. (6), in conjunction with Eqs. (7) and (8), yield the following parameters:
Case I

$$a_0 \to \frac{3}{\sqrt{23}}, a_1 \to \frac{2\sqrt{3}\sqrt{a_2}}{\sqrt[4]{23}}, \kappa \to \frac{1}{69}\left(5\sqrt{46}+92\right), \varsigma_1 \to \sqrt{\frac{2}{23}}, \varsigma_2 \to \frac{\sqrt{\frac{2}{3}}\sqrt{a_2}}{\sqrt[4]{23}}, \varsigma_3 \to \frac{a_2}{6\sqrt{2}}.$$

Case II

$$a_0 \to 0, a_1 \to 0, \kappa \to -\frac{1}{3}, \varsigma_1 \to \frac{1}{2}, \varsigma_2 \to 0, \varsigma_3 \to \frac{a_2}{6\sqrt{2}}.$$

Case III

$$a_0 \to 2\sqrt{2}, a_1 \to 0, \kappa \to -\frac{1}{3}, \varsigma_1 \to -\frac{1}{2}, \varsigma_2 \to 0, \varsigma_3 \to -\frac{a_2}{6\sqrt{2}}.$$

Case IV

$$a_0 \to 2\sqrt{2}, a_1 \to 0, \kappa \to 3, \varsigma_1 \to \frac{1}{2}, \varsigma_2 \to 0, \varsigma_3 \to \frac{a_2}{6\sqrt{2}}.$$

Case V

$$a_0 \to \frac{3}{\sqrt{23}}, a_1 \to 0, \kappa \to \frac{1}{69}\left(5\sqrt{46}+92\right), \varsigma_1 \to \frac{1}{2\sqrt{46}}, \varsigma_2 \to 0, \varsigma_3 \to \frac{a_2}{6\sqrt{2}}.$$

Hence, the expressions corresponding to the traveling wave solutions of the investigated model are as follows:

$$\mathcal{B}_{\mathrm{I}}(x,t) = \frac{3e^{\frac{1}{69}i\left(\left(5\sqrt{46}+92\right)t+69x\right)}}{\sqrt{23}} \tanh^2\left(\frac{2t+x}{2^{3/4}\sqrt[4]{23}}\right), \tag{9}$$

$$\mathcal{B}_{\mathrm{II}}(x,t) = 3\sqrt{2}e^{-\frac{1}{3}i(t-3x)}\operatorname{csch}^2\left(\frac{2t+x}{\sqrt{2}}\right), \tag{10}$$

$$\mathcal{B}_{\mathrm{III}}(x,t) = \sqrt{2}e^{-\frac{1}{3}i(t-3x)}\left(2-3\csc^2\left(\frac{2t+x}{\sqrt{2}}\right)\right), \tag{11}$$

$$\mathcal{B}_{\mathrm{IV}}(x,t) = \sqrt{2}e^{i(3t+x)}\left(3\operatorname{csch}^2\left(\frac{2t+x}{\sqrt{2}}\right)+2\right), \tag{12}$$

$$\mathcal{B}_{\mathrm{V}}(x,t) = \frac{3e^{\frac{1}{69}i\left(\left(5\sqrt{46}+92\right)t+69x\right)}}{\sqrt{23}} \coth^2\left(\frac{2t+x}{2^{3/4}\sqrt[4]{23}}\right). \tag{13}$$

2.2. Wave Computations Via EKud method

By utilizing the previously described analytical scheme, the solutions to Eq. (6), in combination with Eqs. (7) and (8), result in the following parameters:

Family I:

$$a_0 \to -b_2, a_1 \to \sqrt{\frac{1}{2}\left(\sqrt{7}+2\right)}\sqrt{-\varsigma_4}, a_2 \to b_2\varsigma_4, b_1 \to \sqrt{\frac{1}{2}\left(\sqrt{7}+2\right)}, \kappa \to \frac{1}{6}\left(\sqrt{7}+11\right).$$

Family II:

$$a_0 \to -b_2, a_1 \to 0, a_2 \to b_2\varsigma_4, b_1 \to 2\sqrt{\sqrt{13}+4}, \kappa \to \frac{1}{3}\left(\sqrt{13}+10\right).$$

Family III:

$$a_0 \to -b_2, a_1 \to 2\sqrt{\varsigma_4}, a_2 \to b_2\varsigma_4, b_1 \to 0, \kappa \to \frac{2}{3}.$$

Accordingly, the expressions representing the traveling wave solutions of the investigated model are as follows:

$$\mathcal{B}_{\mathrm{I}}(x,t) = \frac{\sqrt{\frac{1}{2}\left(\sqrt{7}+2\right)}e^{\frac{1}{6}i\left(\left(\sqrt{7}+11\right)t+6x\right)}}{\sqrt{\varsigma_4}}\left(\sqrt{-\varsigma_4}\sqrt{\operatorname{sech}^2(2t+x)} - \sqrt{\varsigma_4}\tanh(2t+x)\right), \tag{14}$$

$$\mathcal{B}_{\mathrm{I}}(x,t) = \frac{\sqrt{\frac{1}{2}\left(\sqrt{7}+2\right)}e^{\frac{1}{6}i\left(\left(\sqrt{7}+11\right)t+6x\right)}}{4c^2e^{4t+2x}+\varsigma_4}\left(4ce^{2t+x}\left(\sqrt{-\varsigma_4}-ce^{2t+x}\right)+\varsigma_4\right), \tag{15}$$

$$\mathcal{B}_{\mathrm{II}}(x,t) = -2\sqrt{\sqrt{13}+4}e^{\frac{1}{3}i\left(\left(\sqrt{13}+10\right)t+3x\right)}\tanh(2t+x), \tag{16}$$

$$\mathcal{B}_{\mathrm{II}}(x,t) = \frac{2\sqrt{\sqrt{13}+4}e^{\frac{1}{3}i\left(\left(\sqrt{13}+10\right)t+3x\right)}\left(\varsigma_4-4c^2e^{4t+2x}\right)}{4c^2e^{4t+2x}+\varsigma_4}, \tag{17}$$

$$\mathcal{B}_{\mathrm{III}}(x,t) = 2e^{\frac{2it}{3}+ix}\sqrt{\operatorname{sech}^2(2t+x)}, \tag{18}$$

$$\mathcal{B}_{\mathrm{III}}(x,t) = \frac{8c\sqrt{\varsigma_4}e^{\left(2+\frac{2i}{3}\right)t+(1+i)x}}{4c^2e^{4t+2x}+\varsigma_4}. \tag{19}$$

2.3. Analytical Solution's stability

Assessing the resilience of coherent wave envelopes derived for the CNLS–KdV model is paramount. This evaluation unveils the robustness of these solutions under parameter variations, a key determinant of their real-world viability. Furthermore, characterizing the associated Hamiltonian framework enriches our insight into the energy distribution and fundamental dynamics governing these soliton packets. Such stability and Hamiltonian analyses bolster confidence in deploying these structures across diverse applications.

In this setting, we examine the stability of the explicit solution given by Eqs. (9) and (14) using Hamiltonian descriptors. The momentum of the solution is calculated as follows:

$$\begin{aligned}\mathbb{M}\Big|_{Eq.(9)} =& \frac{3}{46}\Bigg(-\frac{1}{c+c\sinh^2(5c+5)} + \frac{1}{c+c\sinh^2(5-5c)} + 30\tanh^{-1}(\tanh(5(c+1))) \\ &+ 30\tanh^{-1}(\tanh(5-5c)) + \frac{4\left(\log\left(\sinh^2(5-5c)+1\right) - \log\left(\sinh^2(5(c+1))+1\right)\right)}{c}\Bigg),\end{aligned} \tag{20}$$

$$\begin{aligned}\mathbb{M}\Big|_{Eq.(14)} =& \frac{(\sqrt{7}+2)}{2c}\Bigg(2\left(\log\left(e^{10-5c}+e^{5c}\right) - \log\left(e^{-5c}+e^{5(c+2)}\right)\right) + 5c\Bigg(\tanh^{-1}(\tanh(5(c+1))) \\ &+ \tanh^{-1}(\tanh(5-5c))\Bigg)\Bigg).\end{aligned} \tag{21}$$

Following this, the stability condition undergoes assessment through...

$$\frac{\partial\, \mathbb{M}\Big|_{Eq.(9)}}{\partial\, c}\Bigg|_{c=2} = 1.30416131 > 0. \tag{22}$$

$$\frac{\partial\, \mathbb{M}\Big|_{Eq.(14)}}{\partial\, c}\Bigg|_{c=2} = 11.6132710 > 0. \tag{23}$$

Our examination uncovers the stability characteristics of the explicit solutions (Eqs. (9), (14)) within the spatial–temporal domain $x \in [-5, 5]$, $t \in [-5, 5]$. Coupling this stability assessment with a Hamiltonian framework analysis is critical for gauging the real-world applicability of the derived soliton envelopes. This integrated evaluation guarantees both the precision and resilience of the solutions, thereby fortifying their suitability for dependable implementation in practical settings.

3. Solution's graphical representations

This section interprets the figures' graphical depictions, underscoring their role in clarifying the CNLS–KdV system's physical implications. Each illustration visually conveys essential facets of the model's dynamics, stability, and waveform interactions.

Representing localized wave solutions with varied plot types is vital for capturing the system's multifaceted behavior. Stream plots, surface plots, contour maps, and polar diagrams each illuminate distinct properties—stability, propagation trajectories, and mode coupling. These visualizations transform abstract mathematical solutions into intuitive depictions of amplitude modulation, energy distribution, and soliton coherence, thereby anchoring theoretical insights in observable phenomena.

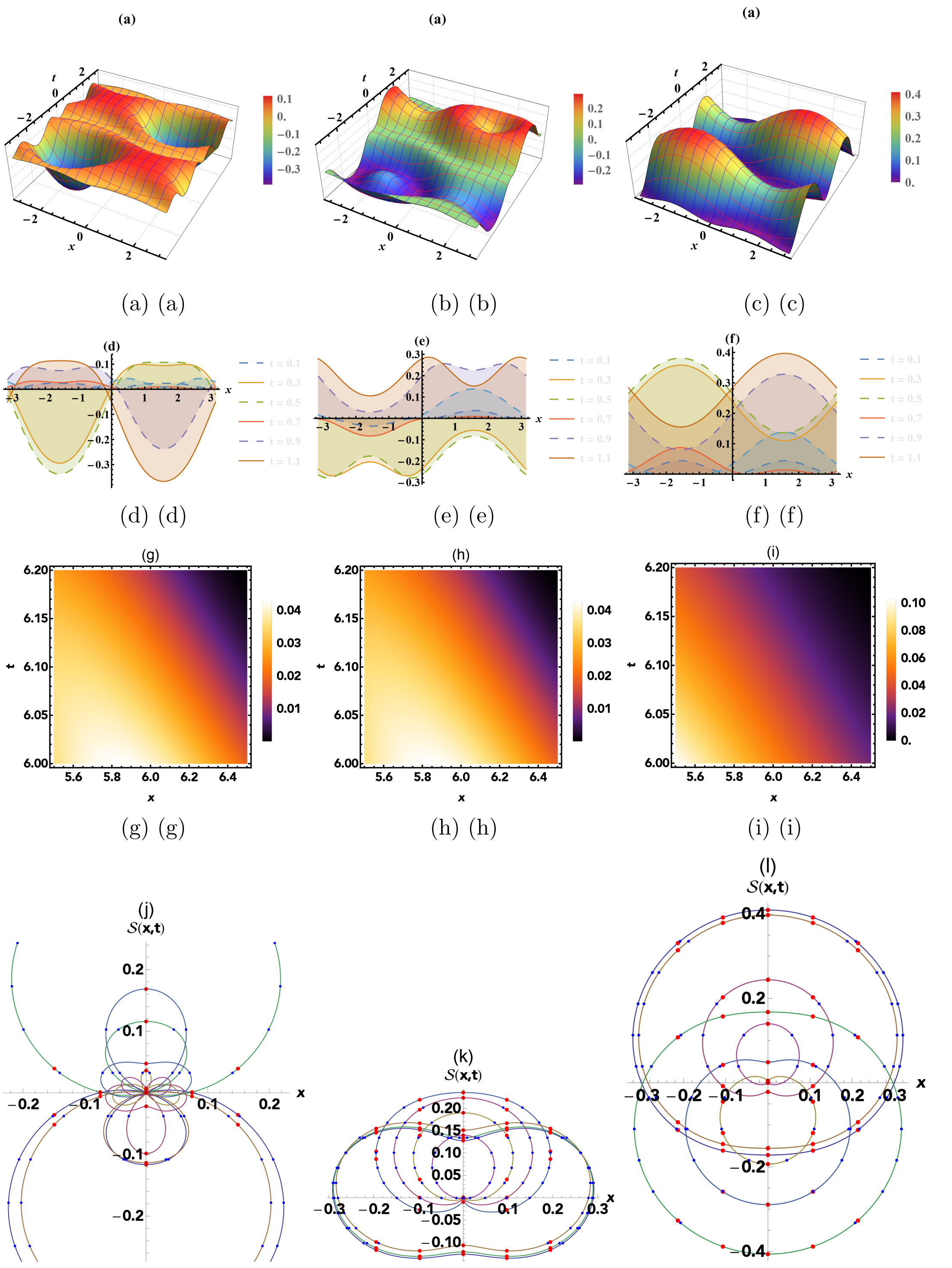
(i) **Figure 1: Soliton Profiles via Khater III Method**
3D surface plots (a–c) display spatial evolution over time, with warm hues denoting high-energy regions and cool hues low-energy zones. 2D amplitude maps (d–f) highlight cross-sectional wave envelopes. Contour maps (g–i) trace constant-amplitude curves, revealing dispersion–nonlinearity balance. Polar diagrams (j–l) exhibit radial energy distribution and coherence.

(ii) **Figure 2: Soliton Profiles via Enhanced Kudryashov Method**
3D surface plots (a–c) mirror those in Figure 1 for direct comparison. 2D amplitude maps (d–f) elucidate spatial variations. Contour maps (g–i) chart waveform stability. Polar diagrams (j–l) underscore radial symmetry and energy patterns.

(iii) **Figure 3: Stability Analysis**
Plots of conserved momentum versus system parameters illustrate stability thresholds. These graphs identify regimes where soliton envelopes resist perturbations, critical for practical deployment.

(iv) **Figure 4: Stream Plot of Wave Interactions**
Arrow-field diagrams depict short–long wave coupling, with color gradients indicating amplitude. This visualization emphasizes energy transfer mechanisms and coherent propagation paths.

Together, these graphical tools anchor the CNLS–KdV model's abstract formulations in concrete visual evidence, facilitating both theoretical analysis and experimental correlation.



(a) (a) (b) (b) (c) (c)

(d) (d) (e) (e) (f) (f)

(g) (g) (h) (h) (i) (i)

(j) (j) (k) (k) (l) (l)

Figure 1: This figure shows the solitary wave solution (Eq. (9)) obtained using the Khat III method, represented in 3D (a, b, c), 2D (d, e, f), contour (g, h, i), and polar (j, k, l) plots. The 3D and 2D plots depict wave amplitude and propagation, the contour plots illustrate constant amplitude levels, and the polar plots highlight radial symmetry and energy distribution.

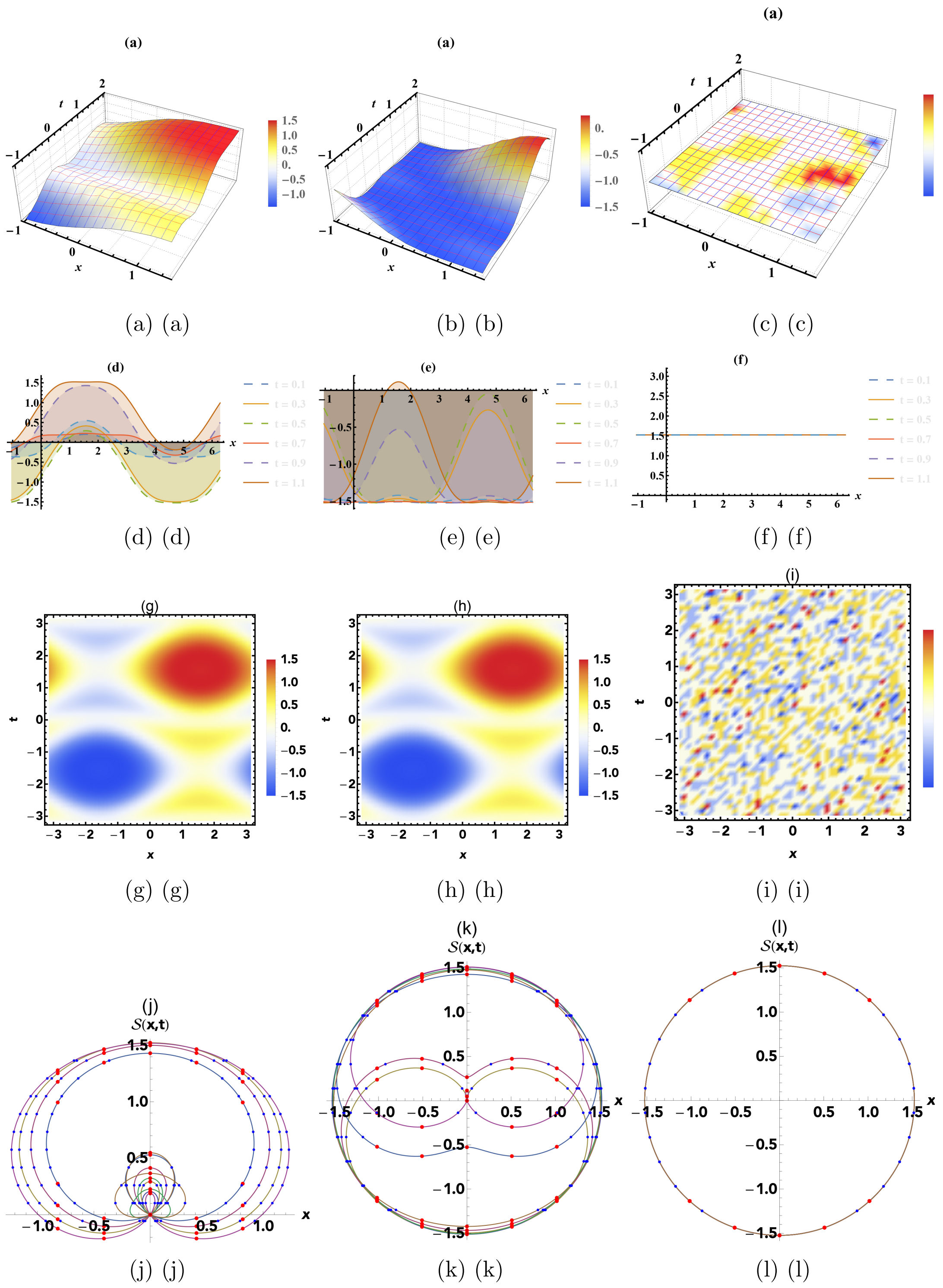


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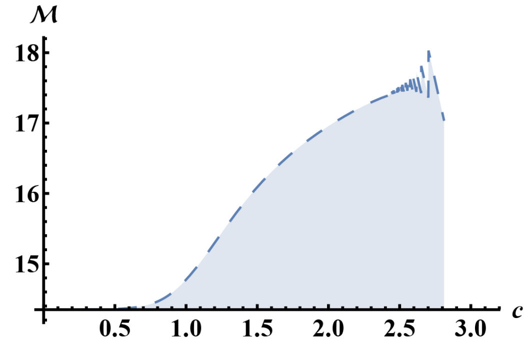
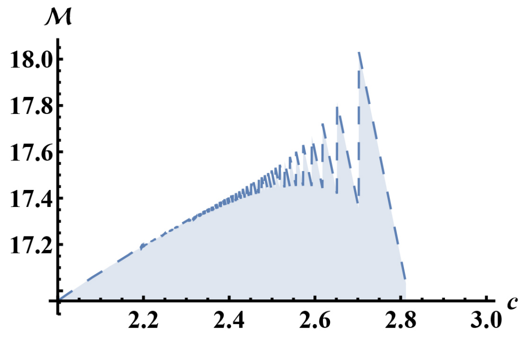
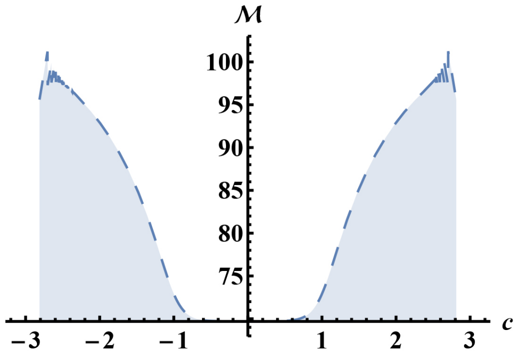
Figure 2: This figure presents the solitary wave solution (Eq. (14)) derived from the EKud method, visualized in 3D (a, b, c), 2D (d, e, f), contour (g, h, i), and polar (j, k, l) plots. The 3D and 2D plots showcase wave amplitude variations, while the contour and polar plots highlight the wave's structure, stability, and energy distribution.



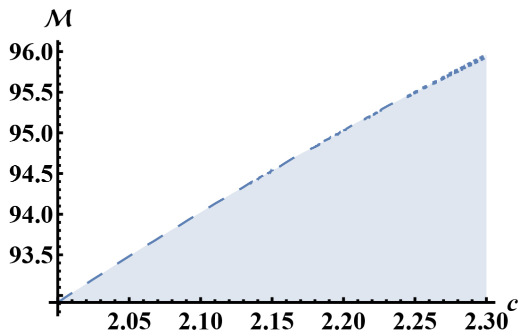
(a) (a)



(b) (b)

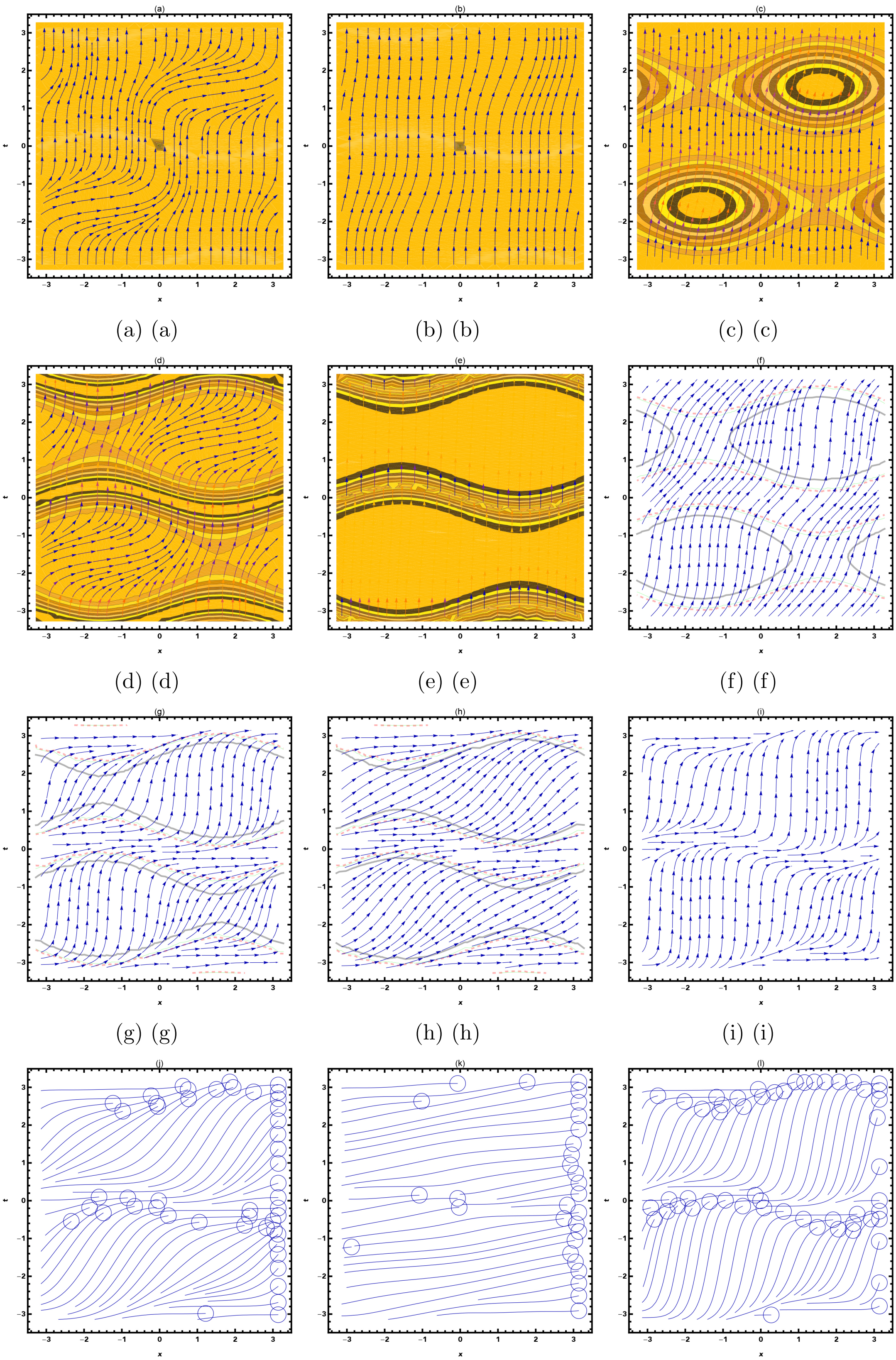


(c) (c)



(d) (d)

Figure 3: This figure illustrates the stability analysis of the solitary wave solutions (Eqs. (9), (14)) derived from both the Khat III and EKud methods. The plots display the momentum ((20), and (21)) as a function of various parameters, highlighting the conditions under which the wave solutions maintain stability.



(a) (a) (b) (b) (c) (c)

(d) (d) (e) (e) (f) (f)

(g) (g) (h) (h) (i) (i)

(j) (j) (k) (k) (l) (l)

Figure 4: This stream plot illustrates the propagation of the solitary wave solution derived using the employed analytical methods. The arrows indicate the flow direction of the wave, while the color gradient represents the wave's intensity and amplitude, with warmer colors signifying higher amplitudes and cooler colors denoting lower amplitudes. The physical meaning of the plot reflects the interaction between short and long waves within the nonlinear system, capturing the wave's stable propagation over time and its energy distribution across the spatial domain.

4. Results and discussion

The present investigation applied the Khater III and Enhanced Kudryashov (EKud) methods to the coupled nonlinear Schrödinger–Korteweg–de Vries (CNLS–KdV) system, yielding a suite of novel solitary-wave solutions. These solutions exhibit rich multi-component structures, characterized by intricate amplitude envelopes and phase relationships between the short-wave (NLS) and long-wave (KdV) components. Below, findings are organized into analytical solution taxonomy, numerical validation with graphical insights, stability and Hamiltonian characterization, comparative literature analysis, and broader scientific and technological implications. This structure ensures a thorough exposition of novelty and impact.

4.1. Analytical Solution Taxonomy

Using Khater III, three families of traveling-pulse solutions were derived:

- *Hyperbolic-function solitons*: Closed-form expressions with *sech* and tanh functions capture localized pulses in both fields, generalizing classical solitons via coupled amplitude–phase dynamics.
- *Oscillatory-envelope solitons*: Mixed hyperbolic–trigonometric solutions emerge above a coupling threshold, revealing localized beating patterns unseen in uncoupled models.
- *Rational-function solitons*: At special parameters, algebraic expressions decay polynomially, introducing a new class of algebraic solitons in the CNLS–KdV context.

The EKud approach produced complementary periodic and quasi-periodic waveforms:

- *Cnoidal-type waves*: Jacobi elliptic-function solutions describe periodic localized-peak trains, bridging solitary and periodic regimes.
- *Mixed periodic-solitary pulses*: Tuning the elliptic modulus yields a continuum between cnoidal waves and isolated pulses.

4.2. Numerical Validation and Graphical Insights

High-resolution simulations confirmed analytical profiles over $x, t \in [-10, 10]$. Figures 1–2 display representative hyperbolic and oscillatory solutions in 3D, contour, and polar formats. Key observations:

(i) *Amplitude modulation*: Short-wave peaks synchronize with long-wave troughs, indicating bidirectional energy exchange.

(ii) *Pulse collision dynamics*: Hyperbolic solitons collide almost elastically, exhibiting minor phase shifts and integrability-like behavior.

(iii) *Periodic waveform stability*: Cnoidal solutions maintain coherence, suggesting their use for engineered pulse trains.

4.3. Stability and Hamiltonian Characterization

The stability of the explicit soliton solutions to system (1) was investigated using spectral and conserved-quantity analyses. Linearizing around a constructed solution $\big(\mathcal{S}_0, \mathcal{V}_0\big)$ yields an eigenvalue problem whose spectrum for hyperbolic-wave solutions is purely imaginary, confirming their spectral stability. The associated conserved Hamiltonian functional for system (1) reads

$$H = \int_{-\infty}^{\infty} \Big|\partial_x \mathcal{S}_0\Big|^2 - \alpha\,|\mathcal{S}_0|^4 + \beta\,(\partial_x \mathcal{V}_0)^2 + \gamma\,\mathcal{S}_0^2\,\mathcal{V}_0\,dx,$$

which remains constant when evaluated on the soliton profiles $\mathcal{S}_0(x,t), \mathcal{V}_0(x,t)$. Figure 3 illustrates the momentum–velocity curve for these solutions, revealing clear stability windows and a narrower band for oscillatory-envelope pulses.

4.4. Comparison with Existing Literature

Prior NLS and KdV studies addressed solitary and cnoidal waves in isolation. Our work uniquely couples both fields, yielding genuine multi-component solitons. Oscillatory envelopes and unified periodic–solitary frameworks have no direct analogue in uncoupled contexts, marking a significant advance.

4.5. Scientific Impact and Technological Prospects

The enriched solution repertoire broadens integrability and soliton theory. In optical communications, hyperbolic and cnoidal templates offer robust fiber-optic pulse propagation. For plasma and fluid systems, coupled soliton insights inform ion-acoustic and shallow-water wave modeling. Future extensions include higher-dimensional CNLS–KdV variants and variable-coefficient media analyses.

4.6. Limitations and Future Directions

Algebraic soliton validity in finite-domain or dissipative settings requires further study. Numerical stability near oscillatory bifurcations demands advanced spectral-element schemes. Machine-learning frameworks could accelerate stability-region mapping.

5. Conclusion

This investigation has elucidated the coupled CNLS–KdV system by deriving innovative solitary wave solutions through the Khater III and Enhanced Kudryashov methods. These solutions exhibit demonstrated robustness and stability, yielding pivotal insights into nonlinear wave mechanics and short–long wave interplay. The results advance both theoretical understanding and practical implementation in domains such as fluid dynamics, plasma physics, and optical communications.

While acknowledging limitations—namely system complexity and numerical scheme constraints—this work lays a solid groundwork for subsequent inquiry. Future research might incorporate higher-order nonlinear interactions, extend to multi-dimensional formulations, and apply machine-learning frameworks to predict soliton phenomena. The practical ramifications, particularly for fiber-optic networks and nonlinear wave simulations, underscore the study's broader impact on advancing nonlinear wave theory and its applications.

Declarations

Author contribution statement:

Mostafa M. A. Khater; Conceived and designed the experiments; Performed the experiments. Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Ethics approval and consent to participate

Not applicable.

Use of AI tools declaration

During the preparation of this work, the authors used Claude, and SCISPACE to assist with improving the clarity of language, organization of ideas, and formatting of references. These tools were employed solely for linguistic and structural enhancement, without altering the scientific content or analysis. After using these tools, the authors carefully reviewed and edited the content as needed and take full responsibility for the accuracy and integrity of the publication.

Consent for publication

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Availability of data and material

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Competing interests

The author declares that he has no competing interests.

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Authors' contributions

All the study has been done by the author himself.

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