



Rheological Laminar Flow Dynamics of a Viscous Fluid with the Scrutinization of a Novel Stagnation Point

Jehad Alzabut^{1,2,*}, Sohail Nadeem^{1,3,4}, Bushra Ishtiaq³, Salman Saleem⁵

¹ Department of Mathematics and Sciences, Prince Sultan University, Riyadh 11586, Saudi Arabia

² Department of Industrial Engineering, OSTİM Technical University, Ankara 06374, Türkiye

³ Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan

⁴ Department of Mathematics, Wenzhou University, Wenzhou 325035, China

⁵ Department of Mathematics, King Khalid University, Abha, 61413, Saudi Arabia

Abstract. In the flow field of viscous fluids, a stagnation point always occurs on the surface of the object at $x = 0$. In this novel study, we examine another significant stagnation point at $x \rightarrow \infty$ in the boundary layer region relative to the flow mechanism of a viscous fluid in two dimensions. On the surface of a static sheet, the boundary layer flow phenomenon of a viscous fluid is investigated in two dimensions regarding the determination of two significant stagnation points in the flow field. Based on physical assumptions, the constitutive equations are formulated and transformed into a dimensionless form using appropriate variables. Through the significant condition, we investigated two stagnation points corresponding to the two cases of free stream velocity i.e. $\pi - \alpha > 0$ and $\pi - \alpha < 0$. If we consider the case of $\pi - \alpha > 0$, then at $x = 0$, the stagnation point develops and in the case of $\pi - \alpha < 0$, the stagnation point emerges at $x \rightarrow \infty$. Through the bvp4c technique in the MATLAB package, the dimensionless equations are numerically scrutinized regarding two cases of $\pi - \alpha > 0$ and $\pi - \alpha < 0$. Graphical scrutinization of these two cases is exhibited for the velocity field. The graphical result exhibits that the velocity field becomes more significant for the case of $\pi - \alpha > 0$ as compared to the case of $\pi - \alpha < 0$. A comparative study between present and previous findings is exhibited through Tables which show a strong relationship between present and previous findings.

2020 Mathematics Subject Classifications: 76D10, 76D05, 76M20, 34B15

Key Words and Phrases: Stagnation point flow, viscous fluid, boundary layer flow, significant stagnation points.

*Corresponding Author.

DOI: <https://doi.org/10.29020/nybg.ejpam.v19i2.6002>

Email addresses: jalzabut@psu.edu.sa (J. Alzabut)

sohail@qau.edu.pk (S. Nadeem), bushraishtiaq@math.qau.edu.pk (B. Ishtiaq)

1. Introduction

In the field of fluid flow, a stagnation point is where fluid exhibits zero velocity. The movement of fluid close to a stagnation region at the surface of a solid is described as stagnation point flow. There are numerous fields where stagnation point flows have significant implementations such as aircraft, drag reduction, artificial fibers, adhesive materials manufacturing, car designs, cooling systems, etc. Due to adverse applications of such flows, many researchers investigated the fluid dynamics close to the stagnation point. Tian et al. [1] explored the mechanism of a nanofluid influenced by physical effects corresponding to a stagnation point. By considering a stretched plate, Seth et al. [2] analyzed the behavior of a micropolar fluid by focusing on the area around a stagnation point. Adjacent to a stagnant point region at the surface of a stretched sheet, Harinath Reddy et al. [3] scrutinized an incompressible fluid with its thermal and flow features. They used an efficient technique to numerically perceive the flow problem. With the consideration of a linear stretched plate, the aim of Pal and Mandal's [4] work is to demonstrate the behavior of a Sisko nanofluid near a stagnant point. An exploration of the laminar flow mechanism of a nanofluid was manifested by Mahmood et al. [5], by taking into account the region of stagnation point. Nadeem et al. [6] work on a second-grade fluid to investigate its three-dimensional flow features adjacent to the neighborhood of stagnation point. The radiative thermal mechanism and magnetized flow problem developed around a stagnation point in a non-Newtonian fluid was elucidated by Abbas et al. [7]. More recent studies related to fluid flows close to the stagnation point are revealed in Refs. [8–12].

The primary boundary layer flow has several natural implementations in different disciplines including plastic film drawing, wire coating, meteorology, crystal growth, electronic equipment cooling, aerodynamics, etc. Following the emergence of the boundary layer flow concept, various researchers focused on such flow problems by taking different physical effects. Gangadhar et al. [13] investigated how viscous dissipation influences the performance of a nanofluid within the flow region of a boundary layer. On the surface of a flat sheet, Belhocine et al. [14] demonstrated the boundary layer mechanism in a laminar flow phenomenon of a viscous fluid. By considering variable physical properties, the mechanism of thermal transport and flow of a viscous fluid within the boundary layer region was inspected by Jha and Samaila [15]. Eldabe et al. [16] used the concept of the boundary layer to demonstrate the magnetized flow features of a micropolar fluid. More investigations into the importance of boundary layer flows are depicted in Refs. [17–20].

Viscous fluids incorporate those fluids that exhibit viscosity characteristics and have internal opposition to their movement. Viscous fluids have implementations in different disciplines including biological systems, damping systems, food industry, engine oils, and cleaning agents. The flow mechanism of viscous fluids has been extensively studied by numerous researchers. By taking a curved medium, Sanni et al. [21] discussed the dynamics of a viscous fluid in two dimensions. Based on the modified mechanism of heat transport, the transient nature of a viscous fluid was manifested by Shen et al. [22]. The purpose of Nadeem et al. [23] work was to scrutinize the radiative thermal phenomenon of a viscous fluid regarding a vertical plane. In the context of a non-inertial frame of reference, Yasir

and Khan [24] discussed the magnetized flow in three dimensions relative to a viscous fluid.

After analyzing the above-mentioned studies, it is observed that in the stagnation point flow problems, the stagnation point only arises at $x = 0$. In this novel study, we examine the stagnation point flow of a viscous fluid in which the stagnation point not only exists at $x = 0$ but also exist at $x \rightarrow \infty$. No one has determined the existence of a stagnation point at $x \rightarrow \infty$ until now. In this study, two stagnation points in the flow field of a viscous flow are investigated. Graphical illustration of these two stagnant points also examined corresponding to the fluid motion.

2. Problem's formulation

We examine the boundary layer flow behavior at a stagnation point for a viscous incompressible fluid by considering a static sheet. In the Cartesian coordinates system, the stationary sheet is aligned along the direction of the x -axis. The y -axis is oriented normally to the x -axis (See Fig. 1). The free stream velocity is considered to be $u_e = U_0 \frac{\pi}{\alpha} (x)^{\frac{\pi-\alpha}{\alpha}}$, while the plate is at rest position. Two stagnation points occur in this flow problem, at $x = 0$ and $x \rightarrow \infty$.

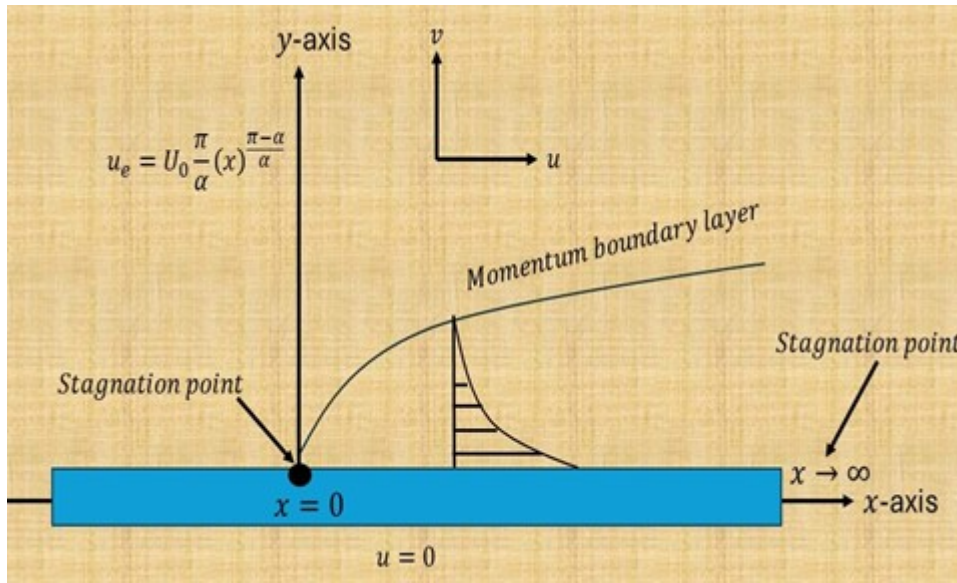


Figure 1: Problem's geometry.

In view of the aforementioned assumptions, the equations that govern the flow problem are manifested as follows [25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + u_e \frac{du_e}{dx}. \quad (2)$$

In Eqs. (1) and (2), density of fluid is ρ , velocity elements are u and v , μ is taken as dynamic viscosity, and free stream velocity is denoted as u_e .

The boundary conditions are delineated as follows:

$$v = 0, \quad u = 0, \quad \text{at} \quad y = 0, \quad (3)$$

$$u = u_e = U_0 \frac{\pi}{\alpha} (x)^{\frac{\pi-\alpha}{\alpha}}, \quad \text{as} \quad y \rightarrow \infty. \quad (4)$$

In Eq. (3), a specific form of free stream velocity is taken to determine the position of two stagnation points in the present flow field of a viscous fluid. We consider two cases of $\pi - \alpha > 0$ and $\pi - \alpha < 0$ in Eq. (3). If we consider $\pi - \alpha > 0$ then stagnation point occurs at $x = 0$ while in the case of $\pi - \alpha < 0$, stagnation point occurs at $x \rightarrow \infty$. This kind of free stream velocity provides two possibilities of stagnation points.

To obtain the equations in dimensionless form, we utilize the following similarity ansatz.

$$u = U_0 \frac{\pi}{\alpha} (x)^{\frac{\pi-\alpha}{\alpha}} F'(\eta), \quad \eta = y \sqrt{U_0 \frac{\pi}{\alpha} \left(\frac{\pi}{2\alpha} \right) \frac{x^{\frac{\pi}{\alpha}-2}}{\vartheta}}, \quad (5)$$

$$v = -\sqrt{\left(\frac{\pi}{2\alpha} \right) \vartheta U_0 \left(\frac{\pi}{\alpha} \right) x^{\frac{\pi}{\alpha}-2}} \left[\eta \left(\frac{\frac{\pi}{\alpha}-2}{\frac{\pi}{\alpha}} \right) F'(\eta) + F(\eta) \right]. \quad (6)$$

After using Eq. (4) in Eq. (2), we get the following dimensionless form of Eq. (2), while Eq. (1) is consistently satisfied.

$$F'''(\eta) - 2 \left(\frac{\pi-\alpha}{\pi} \right) (F'(\eta))^2 + F(\eta) F''(\eta) + 2 \left(\frac{\pi-\alpha}{\pi} \right) = 0. \quad (7)$$

Eqs. (3) and (4) yields the following equation.

$$F(\eta) = 0, \quad F'(\eta) = 0, \quad \text{at} \quad \eta = 0, \quad (8)$$

$$F'(\eta) \rightarrow 1, \quad \text{as} \quad \eta \rightarrow \infty. \quad (9)$$

The expression of the physical quantity of skin friction coefficient is deliberated as follows

$$C_f = \frac{\tau_w}{\rho u_e^2}, \quad \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (10)$$

Eqs. (4) and (7) yield the following result

$$C_f (Re x)^{0.5} = \sqrt{\frac{\pi}{2\alpha}} F''(0), \quad Re x = \frac{x u_e}{\vartheta}. \quad (11)$$

3. Numerical method

For the current problem, the dimensionless equation (5) corresponding to Eq. (6) is numerically solved through an efficient `bvp4c` technique in the MATLAB package. The `bvp4c` technique is used to work out ordinary differential equations regarding boundary value problems. This technique is based on a finite difference method with a collocation approach. It uses a fourth-order collocation method which is based on the three-stage Lobatto IIIa formula. In this technique, the differential equations of higher order are transformed into the system of first-order ordinary differential equations by introducing new variables. To acquire the desirable solutions of these equations, the `bvp4c` technique requires a guess for an initial mesh. This initial mesh guess divulges the behavior of the desired solution. The code subsequently adjusts the mesh to achieve a precise numerical solution while maintaining a reasonable number of mesh points. Generating a sufficiently accurate initial guess is often the most challenging aspect of solving a boundary value problem. The `bvp4c` technique employs a unique error control strategy with residual-based error estimation that allows it to handle less precise initial guesses effectively. The `bvp4c` technique provides solutions to problems up to fourth-order accuracy. For the current problem, the numerical solution of Eq. (5) is acquired through the `bvp4c` technique. After successive iterations, this numerical solution is convergent and stable since the adequate mesh points are selected and the solution satisfies the given conditions.

To implement the `bvp4c` technique on the present problem corresponding to Eqs. (5) and (6), the following new variables are introduced as follow

$$F(\eta) = \zeta_1, \quad (12)$$

$$F'(\eta) = \zeta_2, \quad (13)$$

$$F''(\eta) = \zeta_3, \quad (14)$$

$$F'''(\eta) = \zeta'_3. \quad (15)$$

The following system of equations of first order is obtained after using Eq. (9).

$$\zeta'_1 = \zeta_2, \quad (16)$$

$$\zeta'_2 = \zeta_3, \quad (17)$$

$$\zeta'_3 = 2 \left(\frac{\pi - \alpha}{\pi} \right) (\zeta_2)^2 - \zeta_1 \zeta_3 - 2 \left(\frac{\pi - \alpha}{\pi} \right). \quad (18)$$

Eqs. (6) and (9) yields the following equation

$$\zeta_1(0) = 0, \quad \zeta_2(0) = 0, \quad \zeta_2(\infty) \rightarrow 1. \quad (19)$$

The complete procedure of the `bvp4c` technique is manifested through a flowchart in the following Fig. 2.

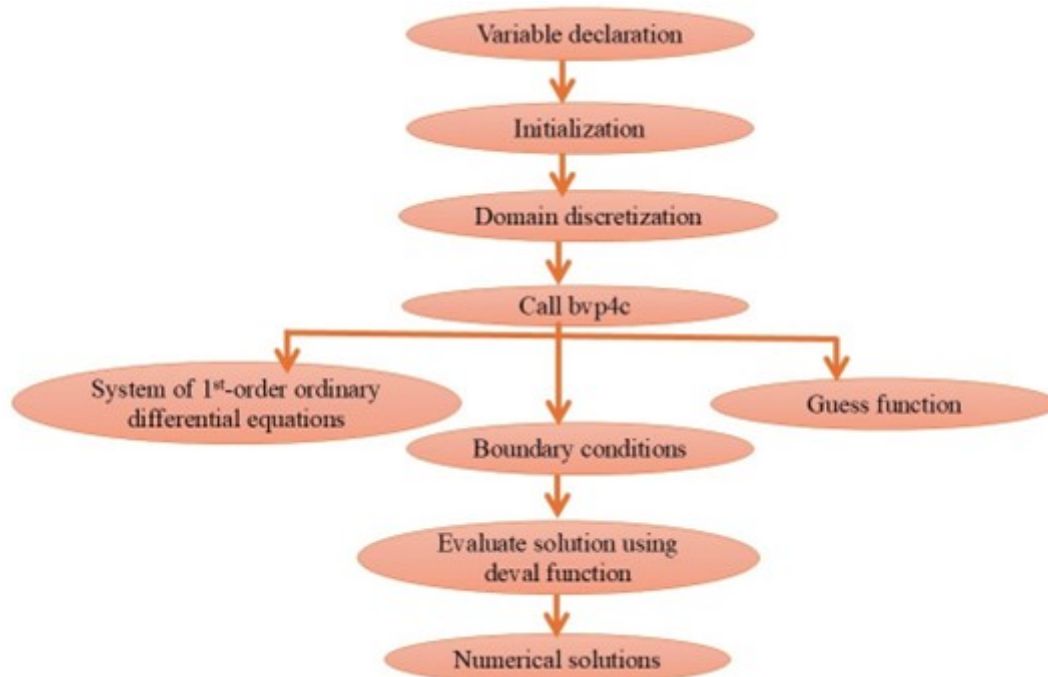


Figure 2: Flowchart of the bvp4c algorithm.

4. Comparative analysis

To validate and confirm the existence of the current flow problem, the current problem is compared with the previous studies regarding the numerical values of the physical quantity of skin friction. In the present study, the boundary layer flow phenomenon of a viscous fluid is scrutinized by taking a static sheet but to compare the results of the current analysis with the previous studies of Wang [26] and Bachok et al. [27], now the present study is examined by taking a shrinking/stretching sheet. The velocity of the shrinking/stretching sheet for ongoing problem is assumed to be $u = u_w(x) = U_1 \frac{\pi}{\alpha}(x)^{\frac{\pi-\alpha}{\alpha}}$. Here $U_1 < 0$ denotes the shrinking sheet and $U_1 > 0$ presents the stretching sheet. With the involvement of the shrinking/stretching sheet in the current problem, the following constitutive equations are achieved [25, 27].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (20)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + u_e \frac{du_e}{dx}. \quad (21)$$

According to the flow assumptions, the boundary conditions are specified as follows.

$$u = u_w(x) = U_1 \frac{\pi}{\alpha}(x)^{\frac{\pi-\alpha}{\alpha}}, \quad v = 0, \quad \text{at} \quad y = 0, \quad (22)$$

$$u = u_e(x) = U_0 \frac{\pi}{\alpha}(x)^{\frac{\pi-\alpha}{\alpha}}, \quad \text{as} \quad y \rightarrow \infty. \quad (23)$$

The aforementioned equations in dimensionless form are acquired through the execution of the following transformations.

$$u = U_0 \frac{\pi}{\alpha} (x)^{\frac{\pi-\alpha}{\alpha}} F'(\eta), \quad \eta = y \sqrt{U_0 \frac{\pi}{\alpha} \left(\frac{\pi}{2\alpha} \right) \frac{x^{\frac{\pi}{\alpha}-2}}{\vartheta}}, \quad (24)$$

$$v = -\sqrt{\left(\frac{\pi}{2\alpha} \right) \vartheta U_0 \left(\frac{\pi}{\alpha} \right) x^{\frac{\pi}{\alpha}-2}} \left[\eta \left(\frac{\pi}{\alpha} - 2 \right) F'(\eta) + F(\eta) \right]. \quad (25)$$

After utilizing Eq. (15) into Eq. (13), the following form of Eq. (16) is acquired while Eq. (12) is consistently satisfied. Some studies dealing various builtin numerical schemes are presented [28–30]

$$F'''(\eta) - 2 \left(\frac{\pi - \alpha}{\pi} \right) (F'(\eta))^2 + F(\eta) F''(\eta) + 2 \left(\frac{\pi - \alpha}{\pi} \right) = 0. \quad (26)$$

Eqs. (14) and (15) yields the following equation.

$$F(\eta) = 0, \quad F'(\eta) = \varepsilon, \quad \text{at} \quad \eta = 0, \quad (27)$$

$$F'(\eta) = 1, \quad \text{as} \quad \eta \rightarrow \infty. \quad (28)$$

In Eq. (17), $\varepsilon = \frac{U_1}{U_0}$ denotes the shrinking parameter for $\varepsilon < 0$ and stretching parameter for $\varepsilon > 0$.

Eq. (16) takes exactly the same form as defined in previous studies of Wang [26] and Bachok et al. [27] by taking the value of the constant π is 2 and α is 1. So, the current problem is compared with the considered previous studies by taking $\pi = 2$ and $\alpha = 1$ regarding distinct values of parameter ε . The numerical values of the skin friction coefficient $F''(0)$ regarding different values of parameter ε are computed in Table 1 and compared with the previous findings of Wang [26] and Bachok et al. [27]. Table 1 yields an interesting fact that the current values of $F''(0)$ have an excellent bonding with the previous studies which confirm the existence and validation of the current problem.

Table 1: Comparative values of $F''(0)$ regarding distinct values of ε .

ε	Present findings	Wang [26]	Bachok et al. [27]
0	1.2325874	1.232588	1.2325877
0.1	1.1465608	1.14656	1.1465610
0.2	1.0511298	1.05113	1.0511300
0.5	0.71329486	0.71330	0.7132949
1	0	0	0
2	-1.8873068	-1.88731	-1.8873066

5. Results and discussion

In this study, we examine the flow behavior of a viscous fluid in two dimensions near a stagnation point on the surface of a static plate. In the boundary layer region, we investigate two stagnation points. One stagnation point occurs at $x = 0$ as in the usual situation while the other stagnation point occurs at $x \rightarrow \infty$ which has not been examined in any flow field until now. These two significant stagnation points develop with specific conditions. At $x = 0$, the formation of the stagnation point is observed when $\pi - \alpha > 0$, based on the free stream velocity of the fluid under consideration. Conversely, the possibility of a stagnation point at $x \rightarrow \infty$ is developed by taking $\pi - \alpha < 0$. By considering both cases of stagnation points, the free stream velocity of the considered viscous fluid becomes zero. For the ongoing problem, the nonlinear equation with conditions given in Eqs. (5) and (6) are graphically analyzed by taking two cases of $\pi - \alpha < 0$ and $\pi - \alpha > 0$. The curve of the velocity field corresponding to both cases of $\pi - \alpha > 0$ and $\pi - \alpha < 0$ is disclosed in Fig. 3. This graphic significantly exhibits the mechanism of the flow field subject to the cases of $\pi - \alpha < 0$ and $\pi - \alpha > 0$. The physical quantity with its numerical values corresponding to both considered cases is manifested in Table 2.

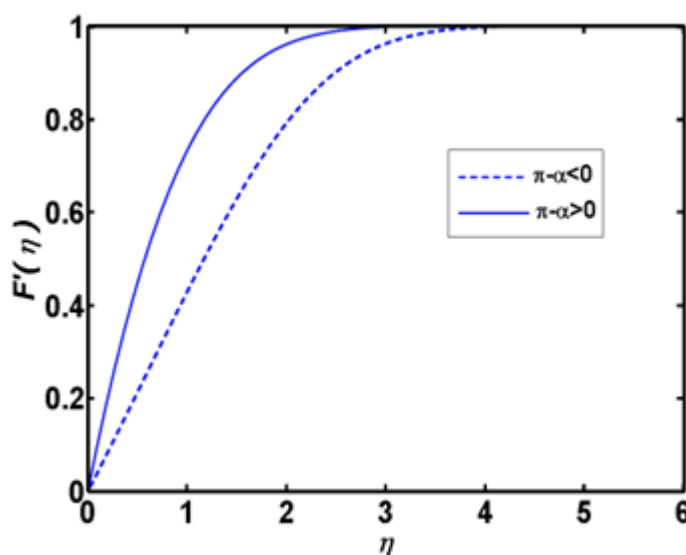


Figure 3: $F'(\eta)$ curve regarding $\pi - \alpha > 0$ and $\pi - \alpha < 0$.

Table 2: Numerical values of $C_f (Re_x)^{0.5}$ corresponding to $\pi - \alpha > 0$ and $\pi - \alpha < 0$.

	$\pi - \alpha > 0$ $\pi = 2, \alpha = 0.5$	$\pi - \alpha < 0$ $\pi = 1, \alpha = 2$
$C_f (Re_x)^{0.5}$	1.9077054	0.41215441

6. Conclusion

This study is organized to evaluate the location of the stagnation point other than $x = 0$ in the flow behavior of a viscous fluid near the region of a boundary layer in two dimensions. A significant stagnation point at $x \rightarrow \infty$ is investigated in this novel study as this position of stagnant point has not been studied until now. Two cases of $\pi - \alpha < 0$ and $\pi - \alpha > 0$ are discussed for stagnation points. For $\pi - \alpha > 0$, the location of stagnation point exists at $x = 0$ and for $\pi - \alpha < 0$, stagnation point develops at $x \rightarrow \infty$. The current problem is limited to the boundary layer flow phenomenon of a viscous fluid with the determination of two significant stagnation points subject to a static sheet. This study can be further investigated for the flow problem of different Newtonian and non-Newtonian fluid models by taking stretching/shrinking sheets. Moreover, the thermal mechanism can also be scrutinized with the current problem. Different physical effects can also be taken into account for the present study.

Acknowledgements

The authors J. Alzabut and S. Nadeem are thankful to PSU for its help and support.

References

- [1] X. Y. Tian, B. W. Li, and Z. M. Hu. Convective stagnation point flow of a mhd non-newtonian nanofluid towards a stretching plate. *International Journal of Heat and Mass Transfer*, 127:768–780, 2018.
- [2] G. S. Seth, B. Kumar, and R. Nandkeolyar. Mhd mixed convection stagnation point flow of a micropolar nanofluid adjacent to stretching sheet: a revised model with successive linearization method. *Journal of Nanofluids*, 8(3):620–630, 2019.
- [3] R. S. Harinath, N. K. Kumaraswamy, B. D. Harish, N. P. V. Satya, and M. C. Raju. Significance of chemical reaction on mhd near stagnation point flow towards a stretching sheet with radiation. *SN Applied Sciences*, 2:1–9, 2020.
- [4] D. Pal and G. Mandal. Magnetohydrodynamic stagnation-point flow of sisko nanofluid over a stretching sheet with suction. *Propulsion and Power Research*, 9(4):408–422, 2020.
- [5] Z. Mahmood, S. E. Alhazmi, A. Alhowaity, R. Marzouki, N. Al-Ansari, and U. Khan. Mhd mixed convective stagnation point flow of nanofluid past a permeable stretching sheet with nanoparticles aggregation and thermal stratification. *Scientific Reports*, 12(1):16020, 2022.
- [6] S Nadeem, B Ishtiaq, S Almutairi, and H A Ghazwani. Impact of cattaneo–christov double diffusion on 3d stagnation point axisymmetric flow of second-grade nanofluid towards a riga plate. *International Journal of Modern Physics B*, 36(29):2250205, 2022.
- [7] N Abbas, N Ul Huda, W Shatanawi, and Z Mustafa. Melting heat transfer of maxwell–

- sutterby fluid over a stretching sheet with stagnation region and induced magnetic field. *Modern Physics Letters B*, 38(15):2450085, 2024.
- [8] K Zheng, S I Shah, K K Naveed, E Tag-eldin, M Yasir, and M A Galal. Numerical simulation of melting heat transfer towards the stagnation point region over a permeable shrinking surface. *Scientia Iranica*, 30(3):1039–1048, 2023.
 - [9] B Ishtiaq and S Nadeem. Flow and thermal characteristics of jeffrey nanofluid in the fixed and moving frames of reference with newly developed reynolds model. *Case Studies in Thermal Engineering*, 56:104253, 2024.
 - [10] G Kenea and W Ibrahim. Nonlinear convection stagnation point flow of oldroyd-b nanofluid with non-fourier heat and non-fick’s mass flux over a spinning sphere. *Scientific Reports*, 14(1):841, 2024.
 - [11] G Karthik and P V Kumar. Emhd stagnation point flow of dusty hybrid nanofluid over a permeable stretching sheet: Radiative solar applications. *Case Studies in Thermal Engineering*, 54:104044, 2024.
 - [12] D Yang, M Yasir, and A Hamid. Thermal transport analysis in stagnation-point flow of casson nanofluid over a shrinking surface with viscous dissipation. *Waves in Random and Complex Media*, 34(4):3177–3191, 2024.
 - [13] K Gangadhar, T Kannan, G Sakthivel, and K DasaradhaRamaiah. Unsteady free convective boundary layer flow of a nanofluid past a stretching surface using a spectral relaxation method. *International Journal of Ambient Energy*, 41(6):609–616, 2020.
 - [14] A Belhocine, N Stojanovic, and O I Abdullah. Numerical simulation of laminar boundary layer flow over a horizontal flat plate in external incompressible viscous fluid. *European Journal of Computational Mechanics*, 30:337–386, 2021.
 - [15] B K Jha and G Samaila. Variable viscosity effect on boundary layer flow along continuously moving plate with the thermal boundary condition of the third kind. *International Journal of Applied and Computational Mathematics*, 7(3):78, 2021.
 - [16] N T Eldabe, M E Gabr, A Z Zaher, and S A Zaher. The effect of joule heating and viscous dissipation on the boundary layer flow of a magnetohydrodynamics micropolar–nanofluid over a stretching vertical riga plate. *Heat Transfer*, 50(5):4788–4805, 2021.
 - [17] S Nadeem, W Fuzhang, F M Alharbi, F Sajid, N Abbas, A S El-Shafay, and F S Al-Mubaddel. Numerical computations for buongiorno nano fluid model on the boundary layer flow of viscoelastic fluid towards a nonlinear stretching sheet. *Alexandria Engineering Journal*, 61(2):1769–1778, 2022.
 - [18] V R Komaravolu, R R Srinivasa, R T Govinda, and K S Balamurugan. Influence of viscous dissipation and spanwise cosinusoidally fluctuating temperature on mhd free convective boundary layer flow with radiation absorption and chemical reaction. *Heat Transfer*, 51(5):4714–4740, 2022.
 - [19] B Jalili, M Emad, P Jalili, D D Ganji, S Saleem, and E M Tag-eldin. Thermal analysis of boundary layer nanofluid flow over the movable plate with internal heat generation, radiation, and viscous dissipation. *Case Studies in Thermal Engineering*, 49:103203, 2023.
 - [20] B Ishtiaq and S Nadeem. Contraction/dilation flow analysis of non-newtonian fluid in

- a deformable channel with exponential permeable walls. *Chinese Journal of Physics*, 83:231–241, 2023.
- [21] K M Sanni, S Asghar, M Jalil, and N F Okechi. Flow of viscous fluid along a nonlinearly stretching curved surface. *Results in Physics*, 7:1–4, 2017.
- [22] M Shen, S Chen, and F Liu. Unsteady mhd flow and heat transfer of fractional maxwell viscoelastic nanofluid with cattaneo heat flux and different particle shapes. *Chinese Journal of Physics*, 56(3):1199–1211, 2018.
- [23] S Nadeem, S Akhtar, and N Abbas. Heat transfer of maxwell base fluid flow of nano-material with mhd over a vertical moving surface. *Alexandria Engineering Journal*, 59(3):1847–1856, 2020.
- [24] M Yasir and M Khan. Heat source/sink effect in radiative flow of hybrid nanofluid subject to noninertial frame. *Numerical Heat Transfer, Part A: Applications*, 2023.
- [25] K Mahmood, M Sajid, N Ali, and T Javed. Mhd mixed convection stagnation point flow of a viscous fluid over a lubricated vertical surface. *Industrial Lubrication and Tribology*, 69(4):527–535, 2017.
- [26] C. Y. Wang. Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics*, 43(5):377–382, 2008.
- [27] N. Bachok, A. Ishak, and I. Pop. Melting heat transfer in boundary layer stagnation-point flow towards a stretching/shrinking sheet. *Physics Letters A*, 374(40):4075–4079, 2010.
- [28] M. Sohail, F. Waseem, K. Abodayeh, and E. Rafiqu. Analysis of entropy generation on darcy-forchheimer squeezed hybrid nanofluid flow between two parallel rotating disks by considering thermal radiation and viscous dissipation. *Taylor and Francis Ltd.*, 2024.
- [29] K. Muhammad, N. Fatima, A. M. Shaima, and A. A. Haifa. Mixed convective magnetohydrodynamic flow of hybrid fluid with viscous dissipation: A numerical approach. *Journal of Magnetism and Magnetic Materials*, 573:170667, 2023.
- [30] N. Abbas, W. Shatanawi, and Z. Mustafa. Thermal analysis of non-newtonian fluid with radiation and mhd effects over permeable exponential stretching sheet. *Case Studies in Thermal Engineering*, 68:105895, 2025.