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Impact of Rectified Sine Pulses on the Motion of Fractionalized MHD Oldroyd-B Fluid with Second Order Slip

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Abstract. Precise prediction of viscoelastic fluid behavior under oscillatory forcing is serious in polymer dispensation, porous media transport, and biomedical systems. This study investigates the influence of first- and second-order slip conditions on the magnetohydrodynamic (MHD) flow of a fractionalized Oldroyd-B fluid over a porous plate impelled by rectified sine pulses. Using fractional calculus and Laplace transforms, analytical solutions for the velocity field and shear stress are derived in terms of generalized M-functions. The solutions recover several subclasses of fluids, including classical/non-fractional Oldroyd-B, Maxwell, second-grade, and Newtonian fluids. Graphical analyses expose that increasing slip coefficients decreases velocity up to 25-35 % while fractional parameters ψ and ϕ significantly modify relaxation and retardation effects associated with the classical case. The relaxation time $\mathcal{R}_1$ is found to have a dominant influence on both velocity and shear stress associated with other parameters. The results highlight that second-order slip produces stronger damping than first-order slip, mostly at longer oscillation periods. The novelty of this work lies in being the first analytical treatment of fractionalized Oldroyd-B MHD flow with second-order slip under rectified sine pulses, providing standard solutions that can guide numerical and experimental validation. Potential applications include polymer extrusion, porous media flow control, and microfluidic design.

2020 Mathematics Subject Classifications: AMS classification codes

Key Words and Phrases: MHD Oldroyd-B fluid, fractional derivative, smart grid, unsteady flow, velocity field, shear stress and Laplace transform.

1. Introduction

Navier-Stokes equations are used to describe Newtonian fluids, but multiple governing equations have been developed for non-Newtonian fluids, which can be categorized into rate, differential, and integral models. There are numerous applications for non-Newtonian fluids, specifically polymeric melts and dilute solutions, in numerous fields, including material processing, bio engineering, geophysics, and a wide range of industries, including chemical, nuclear, oil, and food processing. The treatment of memory effects is one of the key differences between integral-type models and their rate or differential counterparts. There is a marked memory effect observed in integral models, as seen in polymeric melts, whereas rate or differential models exhibit minimal memory effects, characteristic of dilute polymer solutions where stress is independent of past deformations. A prominent example of a rate-type fluid that adheres to the in-variance requirements of such models is the Oldroyd [1] fluid model, however it does not incorporate a thermodynamic framework. Maxwell [2] was the first to envisage the viscoelastic features of these fluids, while Rajagopal [3] and others made significant advancements that resulted in the Oldroyd-B fluid model. This model is a generalization of the upper-convicted Maxwell fluid and can be used to precisely characterize the rheological features of different polymeric fluids. Its capacity to forecast stress relaxation and retardation times which are critical for precisely characterizing their rheological characteristics has drawn attention. However, due to this model, shear thickening and thinning behaviors are not considered. The Oldroyd-B fluid model has various benefits, such as being easy to use and applicable to both theoretical and experimental research. Energy storage and plasticity are captured by two dissipative mechanisms that depict a physical interaction between two viscous fluids and a linearized elastic solid. Moreover, Newtonian fluids, Maxwell fluids, and second-grade fluids are subclasses included in the Oldroyd-B model [4, 5].

Fractional calculus is necessary to comprehend the intricate dynamics and viscoelastic characteristics of gels and other gum-like non-Newtonian fluids. Researchers frequently use fractional forms of flow regulating equations to improve their modeling of experimental data [6, 7]. Time derivatives of integer order are substituted for fractional order using a variety of fractional operators, mainly related to Riemann-Liouville and Caputo [8]. Germant [9] was among the first to use fractional derivatives to study viscoelastic dynamics, which opened the door for later research in this area [10, 11]. In the setting of unstable unidirectional flow, Qi [12] analyzed Oldroyd-B fluids using fractional derivatives and showed that fractional factors significantly impacted the flow behavior. A similar effect was also observed in helical flows of fractionalized Oldroyd-B fluids. A series of solutions was provided by Jamil [13] for the fractionalized Oldroyd-B model in unsteady helical flow. In recent decades, the study of MHD fluid flow through porous media has received considerable attention due to its wide-ranging applications in the medical, chemical, and engineering fields [14, 15]. Several critical processes are included in this, such as using electromagnetic fluids to cool strips and using magnetic fields to extract non-metallic elements from molten metal. MHD fluid flow is an essential component of many industrial processes, including the extraction of non-metallic elements from molten metals, the cooling of strips in series by passing them through electromagnetic fluid, and many others. In the presence of a magnetic field and porous media, many researchers have studied the flow of blood and other biological fluids through various cardiovascular channels. In the presence of a magnetic field, the flow of blood through porous geometry with double stenosis is modeled mathematically [16]. Oldroyd-B fluid models are appropriate for describing physiological fluids due to their diverse enclosure limitations. The respiratory, cardiac, circulatory, and other natural systems have pulsating flow, which consists of steady and time-dependent oscillation components. Hung [17] studies the properties of nonlinear pulsatile flow. Bestman [18] examines the flow of tooth pulses in a hydromagnetic channel. Ghosh [19] used the Laplace transform method to determine the exact solution to the pulse flow field. Khan [20] and Ghosh [21] studied MHD flow of Oldroyd-B fluid in porous media produced by sawtooth pulses, while Ghosh [22] also investigated the same for rectified sine pulses, concluding that pulses increase the fluid velocity in correlation with an increase in an elastic parameter. All the studies described above did not consider the slip effects of first order or second order. A significant characteristic of most polymer-based rheological flow is wall slip, which is the disturbance of velocity layers on the solid-fluid interface. According to Mooney [23], based on the hydrodynamic theory of Newtonian fluid within a capillary viscometer, velocity increases linearly with normal stress. Beavers [24] proposed the slip flow condition near the boundary. Vieru [25, 26] studied the wall slip condition for Stokes and Couette flows in Maxwell fluid. Zheng [27] studied first order slip effects for fractionalized MHD Oldroyd-B fluids. Several researchers have been studying the second-order slip in recent years. Liu [28] investigated the effect of second slip over a fractionalized Maxwell flow and concluded that the fluid has a lower velocity with second slip than with the first slip. The second slip of nanofluid flow has been corrected by Zhang and Vinutha [27][29]. The second-order slip flow of Casson fluid with consideration of heat and mass transfer was discussed by Rahman [30]. Fetecau [31] studies the motion problem of ECIOBFs in an infinite circular cylinder moving along its axis, taking into account magnetic and porous effects. The most recent papers that examined the effects of slip on first and second orders were published in [32, 33]. Several authors have discussed the flow problems over the pale or between two walls over the plate using Sawtooth pulses or rectified sine pulses. Currently, no one has examined the effect of second-order or even first-order slip in the MHD Oldroyd-B fluid flow resulting from rectified sine pulses on the bottom porous plate, which is the topic of this study.

Although numerous studies have examined the pulsatile flow of viscoelastic fluids under rectified sine pulses, the majority neglected slip boundary properties and fractional derivatives. This study works on Oldroyd-B, Maxwell, and second-grade fluids mainly focused on fractional-order models, thus overlooking the memory effects that fractional operators can capture more accurately. Moreover, while first-order slip has been investigated in some circumstances, the combined influence of fractional derivatives, magnetohydrodynamics (MHD), porous media, and second-order slip under rectified sine pulses remains unexplored [**?**]. To the best of our knowledge, no analytical study has yet addressed this configuration. The novelty of this research lies in providing, for the first time, exact analytical solutions for fractionalized Oldroyd-B MHD flow over a porous plate with both first- and second-order slip conditions driven by rectified sine pulses. The solutions are articulated in terms of generalized M-functions, which not only recover well-known fluid subclasses (Newtonian, Maxwell, second-grade, and non-fractional Oldroyd-B) as preventive cases but also expose the distinct role of fractional parameters in modulating flow and shear stress. This influence fills a clear gap in the literature and offers benchmark results that can serve as validation references for numerical [34, 35] and experimental studies in fluid mechanics, polymer processing, and biomedical flow applications. In biomedical contexts, first-order slip may approximate partial slip of blood along endothelial surfaces or in porous tissues. Recent advancements in CFD and heat transfer have significantly enhanced the understanding of complex thermal systems. Analytical and numerical investigations of hybrid and tetra-hybrid nanofluids have revealed notable improvements in

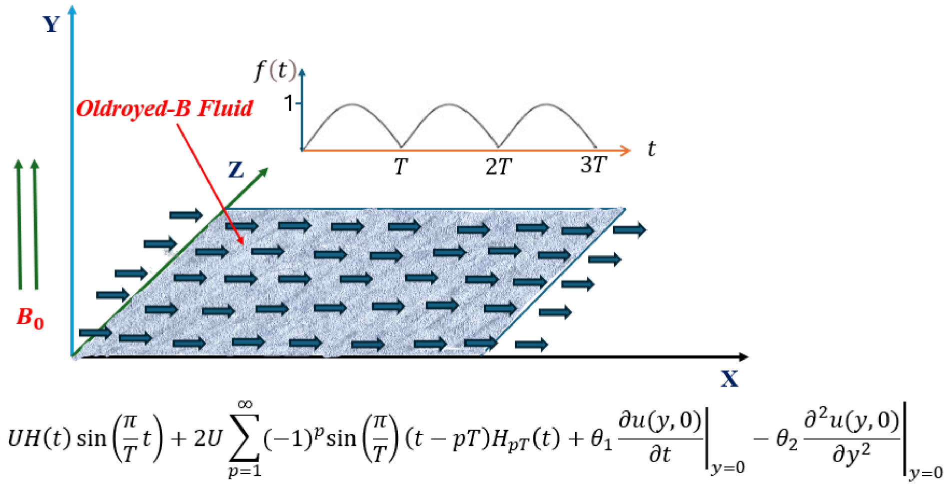

Figure 1: Geometry of the motion of fractionalized MHD Oldroyed-B fluid due to rectified sine pulses moving plate with second order slip effects

thermal performance and energy efficiency under the influence of magnetohydrodynamic, thermocapillary, and radiative effects [36, 37]. In particular, analytical formulations addressing non-Newtonian fluid behaviors such as Williamson and couple stress models have provided valuable insights into flow and heat transfer mechanisms within porous and nonporous media [38]. Likewise, optimization of heat exchanger networks through graph-theoretic and design-based analytical methodologies has played a pivotal role in achieving minimal energy utilization and system efficiency enhancement [39, 40]. Furthermore, advanced numerical and simulation-based approaches continue to contribute to understanding convective and radiative heat transfer in energy storage and combustion processes [41–43]. Collectively, these analytical and computational developments are instrumental in formulating efficient thermal management and energy optimization strategies. In industrial systems such as porous coatings, filtration processes, and MHD-based cooling devices, this property can be used to stabilize flow and regulate transport. Likewise, in biomedical contexts, porous tissues or engineered scaffolds moderate pulsatile viscoelastic flows, where higher porosity permits deeper oscillatory penetration while lower porosity dampens fluctuations more strongly. Thus, the porous medium serves as a practical control parameter in both engineering and biological applications.

2. Governing equations and problem statement

Consider an incompressible fractionalized MHD Oldroyed-B fluid live in the space over an infinitely enlarged plate which is located in the (x, z) plane and at right angles to the y-axis. at the start, this system is at rest and at the instant $t = 0^+$ the plate is start to move to the rectified sine pulses velocity $U \sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT)$ [27, 28] in its own plane. Due to the shearing forces, the fluid above the plate is gradually moved. The velocity and extra stress tensor has the following mathematical set up

$$\mathcal{V} = (u(y,t), 0, 0), \quad \mathcal{S} = \mathcal{S}(y,t), \tag{1}$$

whereas appropriate governing equations [21] for $y, t > 0$ are given by

$$\left(1+\Re_1\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial t} = \nu\left(1+\Re_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u(y,t)}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(1+\Re_1\frac{\partial}{\partial t}\right)u(y,t) - \frac{\nu\phi}{\kappa}\left(1+\Re_2\frac{\partial}{\partial t}\right)u(y,t) \tag{2}$$

$$\left(1+\Re_1\frac{\partial}{\partial t}\right)\tau(y,t) = \mu\left(1+\Re_2\frac{\partial}{\partial t}\right)\frac{\partial u(y,t)}{\partial y}, \tag{3}$$

where $\tau(y,t) = S_{xy}(y,t)$ denotes the non-zero shear stress component. The fluid parameters are defined as follows: $\mu = \nu\rho$ is the dynamic viscosity, ν is the kinematic viscosity, and ρ is the fluid density. The parameters $\Re_1$ and $\Re_2$ represent, the relaxation and retardation times of the Oldroyd-B fluid. The magnetic and porosity effects are characterized by $K_m = \dfrac{\sigma B_0^2}{\rho}$ and $K_p = \dfrac{\nu\phi}{\kappa}$, where σ is the electrical conductivity, B_0 is the applied magnetic field strength, κ is the permeability of the porous medium, and ϕ is the porosity parameter. The fractional parameters ψ and ϕ govern the memory effects of the model, and the fractional time derivatives D_t^{ψ} and D_t^{ϕ} are defined in the sense of Caputo [33]. Accordingly, the governing equations for the incompressible fractionalized MHD Oldroyd-B fluid performing the prescribed oscillatory motion can be expressed as:

$$\left(1+\Re_1^{\psi}D_t^{\psi}\right)\frac{\partial u(y,t)}{\partial t} = \nu\left(1+\Re_2^{\phi}D_t^{\phi}\right)\frac{\partial^2 u(y,t)}{\partial y^2} - K_m\left(1+\Re^{\psi}D_t^{\psi}\right)u(y,t) - K_p\left(1+\Re_2^{\phi}D_t^{\phi}\right)u(y,t), \tag{4}$$

$$\left(1+\Re_1^{\psi}D_t^{\psi}\right)\tau(y,t) = \mu\left(1+\Re_2^{\phi}D_t^{\phi}\right)\frac{\partial u(y,t)}{\partial y}. \tag{5}$$

The appropriate initial and boundary conditions for this class of flows are given by

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0; \quad \tau(y,0) = 0, \quad y > 0, \tag{6}$$

$$u(0,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT) + \Omega_1 \frac{\partial u(y,0)}{\partial t}\bigg|_{y=0} - \Omega_2 \frac{\partial^2 u(y,0)}{\partial y^2}\bigg|_{y=0} \tag{7}$$

where $t \geq 0$ and $H(t)$ is the Heaviside function

$$H_{pT}(t) = \begin{cases} 0, & t \leq pT; \\ 1, & t > pT. \end{cases}$$

further, the natural conditions

$$u(y,t),\ \frac{\partial u(y,t)}{\partial y} \to 0 \text{ as } y \to \infty \text{ and } t > 0, \tag{8}$$

For convenience, and to avoid lengthy calculations involving contour integrals and residues, the discrete method of the inverse Laplace transform is employed in the following analysis.

Model Assumptions

The present problem is developed under the following assumptions:

- The fluid is incompressible, electrically conducting, and follows the fractionalized Oldroyd-B constitutive model.
- Flow occurs over an infinitely extended flat porous plate located in the (x, z)-plane, normal to the y-axis.
- The fluid and plate system is initially at rest, i.e., $u(y, 0) = 0$.
- At $t = 0^+$, the plate begins to oscillate with a rectified sine pulse velocity:

$$U\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT).$$

- The flow is unidirectional with velocity field $\mathcal{V} = (u(y,t), 0, 0)$ depending only on y and t.

- First- and second-order slip boundary conditions are imposed at the wall.
- The porous medium is homogeneous and isotropic.
- A uniform transverse magnetic field B_0 is applied perpendicular to the plate.
- The flow is considered isothermal, neglecting thermal, chemical, and mass transfer effects.

2.1. Calculation of the velocity field

Application of the Laplace transform to Eq. (2) and using the initial conditions $(6)_{1,2}$ and boundary condition (7), we obtain

$$\frac{\partial^2 \overline{u}(y,q)}{\partial y^2} - \frac{(q+K_m)(1+\Re_1^{\psi} q^{\psi}) + K_p(1+\Re_2^{\phi} q^{\phi})}{\nu(1+\Re_2^{\phi} q^{\phi})}\overline{u}(y,q) = 0, \tag{9}$$

subject to boundary conditions

$$\overline{u}(0,t) = U\frac{\frac{\pi}{T}}{q^2+(\frac{\pi}{T})^2}\left[1+2\sum_{p=1}^{\infty}(-1)^p e^{-pTq}\right] + \Omega_1 \frac{\partial \overline{u}(y,0)}{\partial y}\bigg|_{y=0} - \Omega_2 \frac{\partial^2 \overline{u}(y,0)}{\partial y^2}\bigg|_{y=0}, \tag{10}$$

and natural constraints

$$\overline{u}(y,q),\ \frac{\partial \overline{u}(y,q)}{\partial y} \to 0 \text{ as } y \to \infty, \tag{11}$$

where q is the transform parameter and $\overline{u}(y,q)$is the Laplace transformed function of $u(y,t)$. Solving Eqs.(9) utilizing conditions(10) and (11), we get

$$\overline{u}(y,q) = \frac{U(\frac{\pi}{T})}{(q^2+(\frac{\pi}{T})^2)\left\{1+\Omega_1\left[\frac{(q+K_m)(1+\Re_1^{\psi} q^{\psi})+K_p(1+\Re_2^{\phi} q^{\phi})}{\nu(1+\Re_2^{\phi} q^{\phi})}\right]^{\frac{1}{2}} + \Omega_2\left[\frac{(q+K_m)(1+\Re_1^{\psi} q^{\psi})+K_p(1+\Re_2^{\phi} q^{\phi})}{\nu(1+\Re_2^{\phi} q^{\phi})}\right]\right\}}$$

$$\times\left[1+2\sum_{p=1}^{\infty}(-1)^p e^{-pTq}\right]\exp\left\{-\left[\frac{(q+K_m)(1+\Re_1^{\psi} q^{\psi})+K_p(1+\Re_2^{\phi} q^{\phi})}{\nu(1+\Re_2^{\phi} q^{\phi})}\right]^{\frac{1}{2}} y\right\}. \tag{12}$$

To find inverse Laplace transform of above equation and avoid to calculate the contour integrals and related burdensome calculation. We apply here discrete inverse Laplace transform, for this purpose, first we write above equation in series form as

$$\overline{u}(y,q)=\frac{U(\frac{\pi}{T})}{(q^2+(\frac{\pi}{T})^2)}\left[1+2\sum_{p=1}^{\infty}(-1)^p e^{-pTq}\right]$$

$$+U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{i_8=0}^{\infty}\frac{\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2}{2})\Gamma(i_5-\frac{i_1+i_2}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2}{2}-i_4)(-\Re_2^{\phi})^{-i_8}}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2}{2})\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(\frac{i_1+i_2}{2}-i_4)}$$

$$\times\frac{1}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2}{2}-i_4)+i_5+\psi i_6+2i_7+2}}+U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{i_8=0}^{\infty}\frac{\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3}{2})\Gamma(i_5-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2+i_3}{2}-i_4)(-\Re_2^{\phi})^{-i_8}}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3}{2})\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3}{2}-i_4)}$$

$$\times\frac{1}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+i_5+\psi i_6+2i_7+2}}+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p$$

$$\times\sum_{i_8=0}^{\infty}\frac{\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2}{2})\Gamma(i_5-\frac{i_1+i_2}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2}{2}-i_4)(-\Re_2^{\phi})^{-i_8}}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2}{2})\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(\frac{i_1+i_2}{2}-i_4)}$$

$$\times\frac{e^{-pTq}}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2}{2}-i_4)+i_5+\psi i_6+2i_7+2}}+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p$$

$$\times\sum_{i_8=0}^{\infty}\frac{\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3}{2})\Gamma(i_5-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2+i_3}{2}-i_4)(-\lambda_r^{\phi})^{-i_8}}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3}{2})\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3}{2}-i_4)}$$

$$\times\frac{e^{-pTq}}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+i_5+\psi i_6+2i_7+2}}. \tag{13}$$

Now we are able to apply the inverse Laplace transform in discrete senses of above equation, we get

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}$$

$$\times\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times t^{-(\psi-\phi+1)(\frac{i_1+i_2}{2}-i_4)+i_5+\psi i_6+2i_7+1}\sum_{i_8=0}^{\infty}\frac{\left(\frac{t^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2}{2})\Gamma(i_5-\frac{i_1+i_2}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2}{2}+i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2}{2})\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(-\frac{i_1+i_2}{2}+i_4)\Gamma(\frac{i_1+i_2}{2}-i_4)}$$

$$\times \frac{\Gamma(i_8 + \frac{i_1+i_2}{2} - i_4)}{\Gamma(\phi i_8 - (\psi - \phi + 1)(\frac{i_1+i_2}{2} - i_4) + i_5 + \psi i_6 + 2i_7 + 2)} + UH(t)(\frac{\pi}{T}) \sum_{i_1=0}^{\infty} (-1)^{i_1} \sum_{i_2=0}^{\infty} \frac{1}{i_2!} \left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2} \left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\times \sum_{i_3=1}^{\infty} \frac{1}{i_3!} \left(-\frac{y}{\sqrt{\nu}}\right)^{i_3} \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4 - \frac{i_1+i_2+i_3}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_7=0}^{\infty} (-1)^{i_7} \left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \sum_{i_6=0}^{\infty} \frac{(-1)^{i_6}}{i_6!} \Re_1^{\psi(\frac{i_1+i_2+i_3}{2} - i_4 - i_6)} t^{-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2} - i_4) + i_5 + \psi i_6 + 2i_7 + 1}$$

$$\times \sum_{i_8=0}^{\infty} \frac{\left(\frac{t^{\phi}}{\Re_2^{\phi}}\right)^{i_8} \Gamma(i_2 - i_1)\Gamma(i_4 - \frac{i_1+i_2+i_3}{2})\Gamma(i_5 - \frac{i_1+i_2+i_3}{2} + i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3}{2})\Gamma(-\frac{i_1+i_2+i_3}{2} + i_4)\Gamma(-\frac{i_1+i_2+i_3}{2} + i_4)\Gamma(\frac{i_1+i_2+i_3}{2} - i_4)}$$

$$\times \frac{\Gamma(i_6 - \frac{i_1+i_2+i_3}{2} + i_4)\Gamma(i_8 + \frac{i_1+i_2+i_3}{2} - i_4)}{\Gamma(\phi i_8 - (\psi - \phi + 1)(\frac{i_1+i_2+i_3}{2} - i_4) + i_5 + \psi i_6 + 2i_7 + 2)} + 2U \left(\frac{\pi}{T}\right) \sum_{i_1=1}^{\infty} (-1)^{i_1} \sum_{i_2=0}^{\infty} \frac{1}{i_2!} \left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2} \left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\times \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4 - \frac{i_1+i_2}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_6=0}^{\infty} \frac{(-1)^{i_6}}{i_6!} \Re_1^{\psi(\frac{i_1+i_2}{2} - i_4 - i_6)} \sum_{i_7=0}^{\infty} (-1)^{i_7} \left(\frac{\pi}{T}\right)^{2i_7} \sum_{p=1}^{\infty} (-1)^p H_{pT}(t)$$

$$\times t^{-(\psi-\phi+1)(\frac{i_1+i_2}{2} - i_4) + i_5 + \psi i_6 + 2i_7 + 1} \sum_{i_8=0}^{\infty} \frac{\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8} \Gamma(i_2 - i_1)\Gamma(i_4 - \frac{i_1+i_2}{2})\Gamma(i_5 - \frac{i_1+i_2}{2} + i_4)\Gamma(i_6 - \frac{i_1+i_2}{2} + i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2}{2})\Gamma(-\frac{i_1+i_2}{2} + i_4)\Gamma(-\frac{i_1+i_2}{2} + i_4)\Gamma(\frac{i_1+i_2}{2} - i_4)}$$

$$\times \frac{\Gamma(i_8 + \frac{i_1+i_2}{2} - i_4)}{\Gamma(\phi i_8 - (\psi - \phi + 1)(\frac{i_1+i_2}{2} - i_4) + i_5 + \psi i_6 + 2i_7 + 2)} + 2U(\frac{\pi}{T}) \sum_{i_1=0}^{\infty} (-1)^{i_1} \sum_{i_2=0}^{\infty} \frac{1}{i_2!} \left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2} \left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\times \sum_{i_3=1}^{\infty} \frac{1}{i_3!} \left(-\frac{y}{\sqrt{\nu}}\right)^{i_3} \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4 - \frac{i_1+i_2+i_3}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_6=0}^{\infty} \frac{(-1)^{i_6}}{i_6!} \Re_1^{\psi(\frac{i_1+i_2+i_3}{2} - i_4 - i_6)} \sum_{i_7=0}^{\infty} (-1)^{i_7} \left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \sum_{p=1}^{\infty}(-1)^p H_{pT}(t) t^{-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+i_5+\psi i_6+2i_7+1} \sum_{i_8=0}^{\infty} \frac{\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8} \Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3}{2})}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3}{2})\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)}$$

$$\times \frac{\Gamma(i_5-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2+i_3}{2}-i_4)}{\Gamma(-\frac{i_1+i_2+i_3}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3}{2}-i_4)\Gamma(\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+i_5+\psi i_6+2i_7+2)} \tag{14}$$

In order to write Eq. (14) in more compact form, we introduce here the generalized **M** function, we have

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT) + UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}$$

$$\times\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\Bigg|_{\substack{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\\ \left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)}}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{5,7}^{1,5}\Big[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Bigg|_{\substack{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\\ \left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)}}^{\substack{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\\ \left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)}}\Big]. \tag{15}$$

Using generalized H function [33], we can define new generalized $\mathbf{M}$ function as

$$\mathbf{M}_{k,n+1}^{1,k}\left[z\Bigg|_{(0,1),(1-d_1,D_1),...(1-d_n,D_n)}^{(1-c_1,C_1),...(1-c_k,C_k)}\right]=t^{d_n-1}\sum_{\ell=0}^{\infty}\frac{(-z)^{\ell}\prod_{j=1}^{k}\Gamma(c_j+C_j\ell)}{\ell!\prod_{j=1}^{n}\Gamma(d_j+D_j\ell)}$$

$$=t^{d_n-1}H_{k,n+1}^{1,k}\left[z\Bigg|_{(0,1),(1-d_1,D_1),...(1-d_n,D_n)}^{(1-c_1,C_1),...(1-c_k,C_k)}\right]. \tag{16}$$

.

2.2. Calculation of the shear stress

Using the initial conditions $(6)_3$, we find the Laplace transform to Eq. (3) as

$$\overline{\tau}(y,q)=\mu\frac{(1+\Re_2^{\phi}q^{\phi})}{(1+\Re_1^{\psi}q^{\psi})}\frac{\partial\overline{u}(y,q)}{\partial y}, \tag{17}$$

where $\overline{\tau}(y,q)$ is the Laplace transform of $\tau(y,t)$. Utilizing Eq. (12) in the above equation, we have

$$\overline{\tau}(y,q)=-\frac{U\mu(\frac{\pi}{T})(1+\Re_2^{\phi}q^{\phi})\left[\frac{(q+K_{i_5})(1+\Re_1^{\psi}q^{\psi})+K_p(1+\Re_2^{\phi}q^{\phi})}{\nu(1+\Re_2^{\phi}q^{\phi})}\right]^{\frac{1}{2}}}{(q^2+(\frac{\pi}{T})^2)(1+\Re_1^{\psi}q^{\psi})\left\{1+\Omega_1\left[\frac{(q+K_{i_5})(1+\Re_1^{\psi}q^{\psi})+K_p(1+\Re_2^{\phi}q^{\phi})}{\nu(1+\Re_2^{\phi}q^{\phi})}\right]^{\frac{1}{2}}+\Omega_2\left[\frac{(q+K_{i_5})(1+\Re_1^{\psi}q^{\psi})+K_p(1+\Re_2^{\phi}q^{\phi})}{\nu(1+\Re_2^{\phi}q^{\phi})}\right]\right\}}$$

$$\times\left[1+2\sum_{p=1}^{\infty}(-1)^pe^{-pTq}\right]\exp\left\{-\left[\frac{(q+K_{i_5})(1+\Re_1^{\psi}q^{\psi})+K_p(1+\Re_2^{\phi}q^{\phi})}{\nu(1+\Re_2^{\phi}q^{\phi})}\right]^{\frac{1}{2}}y\right\}. \tag{18}$$

Series representation of above expression yields the following result

$$\overline{\tau}(y,q)=-\frac{U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{i_8=0}^{\infty}\frac{\left(\Re_2^{\phi}\right)^{i_8}\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3+1}{2})\Gamma(i_5-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3-1}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2+i_3-1}{2}-i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3+1}{2})\Gamma(-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3-1}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3-1}{2}-i_4)}$$

$$\times\frac{1}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+2}}-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}$$

$$\times\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{i_8=0}^{\infty}\frac{\left(\Re_2^{\phi}\right)^{i_8}\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3+1}{2})\Gamma(i_5-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3-1}{2}+i_4)\Gamma(i_8+\frac{i_1+i_2+i_3-1}{2}-i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3+1}{2})\Gamma(-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3-1}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3-1}{2}-i_4)}$$

$$\times\sum_{p=1}^{\infty}(-1)^p\frac{e^{-pTq}}{q^{\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+2}}. \tag{19}$$

Inverse Laplace transform in discreet sense provide the Eq. (20)

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}$$

$$\times\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}t^{-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+1}$$

$$\times\sum_{i_8=0}^{\infty}\frac{\left(\frac{t^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3+1}{2})\Gamma(i_5-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3-1}{2}+i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3+1}{2})\Gamma(-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3-1}{2}+i_4)\Gamma(\frac{i_1+i_2+i_3-1}{2}-i_4)}$$

$$\times\frac{\Gamma(i_8+\frac{i_1+i_2+i_3-1}{2}-i_4)}{\Gamma(\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+2)}$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\sum_{i_8=0}^{\infty}\frac{\left(\frac{(t-pT)^\phi}{\Re_2^\phi}\right)^{i_8}\Gamma(i_2-i_1)\Gamma(i_4-\frac{i_1+i_2+i_3+1}{2})\Gamma(i_5-\frac{i_1+i_3+1}{2}+i_4)\Gamma(i_6-\frac{i_1+i_2+i_3-1}{2}+i_4)}{i_8!\Gamma(-i_1)\Gamma(-\frac{i_1+i_2+i_3+1}{2})\Gamma(-\frac{i_1+i_2+i_3+1}{2}+i_4)\Gamma(-\frac{i_1+i_2+i_3-1}{2}+i_4)}$$

$$\times\frac{\Gamma(i_8+\frac{i_1+i_2+i_3-1}{2}-i_4)t^{-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+1}}{\Gamma(\frac{i_1+i_2+i_3-1}{2}-i_4)\Gamma(\phi i_8-(\psi-\phi+1)(\frac{i_1+i_2+i_3}{2}-i_4)+\frac{1}{2}(\psi-\phi-1)+i_5+\psi i_6+2i_7+2)}. \tag{20}$$

Expressing in the form of generalized **M**-function, we get

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{t^\phi}{\Re_2^\phi}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]. \tag{21}$$

3. Some interesting particular cases

3.1. Ordinary MHD Oldroyd-B fluid in porus medium

For $\psi\longrightarrow 1$ and $\phi\longrightarrow 1$, the Eqs. (15) and (21) reduces to

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)\sin\frac{\pi}{T}(t-pT)+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}$$

$$\times\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{i_4-\frac{i_1+i_2}{2}}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{t}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left(\frac{i_1+i_2}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{i_4-\frac{i_1+i_2+i_3}{2}}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2+i_3}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{5,7}^{1,5}\left[\frac{t}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\\\left(\frac{i_1+i_2+i_3}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{i_4-\frac{i_1+i_2}{2}}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times \mathbf{M}_{5,7}^{1,5}\left[\frac{(t-pT)}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left(\frac{i_1+i_2}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{i_4-\frac{i_1+i_2+i_3}{2}}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2+i_3}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times \mathbf{M}_{5,7}^{1,5}\left[\frac{(t-pT)}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\\\left(\frac{i_1+i_2+i_3}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right] \quad (22)$$

and the corresponding shear stress is

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{i_4-\frac{i_1+i_2+i_3-1}{2}}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2+i_3-1}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{t}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\\left(\frac{i_1+i_2+i_3+1}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{i_4-\frac{i_1+i_2+i_3-1}{2}}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\frac{i_1+i_2+i_3-1}{2}-i_4-i_6}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times\mathbf{M}_{5,7}^{1,5}\left[\frac{(t-pT)}{\Re_2}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\\left(\frac{i_1+i_2+i_3+1}{2}-i_4-i_5-i_6-2i_7-1,1\right)\end{matrix}\right].\tag{23}$$

3.2. Fractionalized Oldroyed-B fluid in porus medium

When $K_m\to 0$, the Eqs. (15) and (21) reduces to

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}$$

$$\times\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\Bigg|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}$$

$$\times\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\Bigg|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Bigg|_{\substack{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\\ \left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\psi i_6-2i_7-1,\phi\right)}}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)}\right] \tag{24}$$

and

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\Bigg|_{\substack{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\ \left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-\psi i_6-2i_7-1,\phi\right)}}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,1\right)}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]. \tag{25}$$

3.3. Fractionalized MHD Oldroyd-B fluid

When $K_p \to 0$, the Eqs. (15) and (21) simplified into

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT)+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}$$

$$\times\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\Re_2^{\phi(-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_6,0\right),\left(1-\frac{i_1+i_2}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2},0\right),\left(1-\frac{i_1+i_2}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\times\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\Re_2^{\phi(-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1-\frac{i_1+i_2+i_3}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-i_5-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2},1\right)}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\Re_2^{\phi(-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}$$

$$\times\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2},0\right),\left(1-\frac{i_1+i_2}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}\right)-i_5-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_6,0\right),\left(1-\frac{i_1+i_2}{2},1\right)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\Re_2^{\phi(-\frac{i_1+i_2+i_3}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1-\frac{i_1+i_2+i_3}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-i_5-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2},1\right)}\right]$$

and

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\Re_2^{\phi(-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_5,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3-1}{2},0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\Re_2^{\phi(-\frac{i_1+i_2+i_3-1}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_5,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3-1}{2},0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]. \quad (27)$$

3.4. Fractionalized Oldroyd-B fluid

Making $K_m, K_p \to 0$, Eqs. (15) and (21) yield the following expressions

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\Re_2^{\phi(-\frac{i_1+i_2}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_6)}$$

$$\times\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_6,0\right),\left(1-\frac{i_1+i_2}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1-\frac{i_1+i_2}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}\right)-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\Re_2^{\phi(-\frac{i_1+i_2+i_3}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1-\frac{i_1+i_2+i_3}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2},1\right)}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\Re_2^{\phi(-\frac{i_1+i_2}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\Re_1^{\psi(\frac{i_1+i_2}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1-\frac{i_1+i_2}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2}{2}\right)-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_6,0\right),\left(1-\frac{i_1+i_2}{2},1\right)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\Re_2^{\phi(-\frac{i_1+i_2+i_3}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\middle|_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1-\frac{i_1+i_2+i_3}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\psi i_6-2i_7-1,\phi\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3}{2},1\right)}\right],\quad(28)$$

and

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\Re_2^{\phi(-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3-1}{2},0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\frac{1}{2}(\psi-\phi-1)-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\Re_2^{\phi(-\frac{i_1+i_2+i_3-1}{2})}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_6,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3-1}{2},0\right),\left(1-\frac{i_1+i_2+i_3-1}{2},0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_2+i_3}{2}\right)-\frac{1}{2}(\psi-\phi-1)-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right].\quad(29)$$

3.5. Fractionalized MHD Oldroyd-B fluid in porus medium without seocond order slip

Making $\Omega_2\to 0$ in Eqs. (15) and (21), we get the velocity filed

$$u(y,t)=UH(t)\sin\left(\frac{\pi}{T}t\right)+2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}\left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}\left(1+\frac{i_1}{2}-i_4,0\right),\left(1+\frac{i_1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_1}{2},0\right),\left(1+\frac{i_1}{2}-i_4,0\right),\left(1+\frac{i_1}{2}-i_4,0\right),\left(1-\frac{i_1}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}\left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}$$

$$\times\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_1+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\Bigg|\begin{matrix}\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_3}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_1+i_3}{2},0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_3}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1+i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}\left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\Bigg|\begin{matrix}\left(1+\frac{i_1}{2}-i_4,0\right),\left(1+\frac{i_1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_1}{2},0\right),\left(1+\frac{i_1}{2}-i_4,0\right),\left(1+\frac{i_1}{2}-i_4,0\right),\left(1-\frac{i_1}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_1}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}\left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1}sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_1+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\ \Re_1^{\psi(\frac{i_1+i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Bigg|\begin{matrix}\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_3}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_1+i_3}{2},0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_3}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_1+i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right],\qquad(30)$$

and the respective shear stress is

$$\tau(y,q) = -\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}} \sum_{i_1=0}^{\infty} \left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1} \sum_{i_3=0}^{\infty} \frac{1}{i_3!} \left(-\frac{y}{\sqrt{\nu}}\right)^{i_3} \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!}$$

$$\times \Re_2^{\phi(i_4-\frac{i_1+i_3-1}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_6=0}^{\infty} \frac{(-1)^{i_6}}{i_6!} \Re_1^{\psi(\frac{i_1+i_3-1}{2}-i_4-i_6)} \sum_{i_7=0}^{\infty} (-1)^{i_7} \left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} \left(1+\frac{i_1+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_3+1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_3-1}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_3-1}{2}+i_4,1\right) \\ (0,1),\left(1+\frac{i_1+i_3+1}{2},0\right),\left(1+\frac{i_1+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_3-1}{2}-i_4,0\right),\left(1-\frac{i_1+i_3-1}{2}+i_4,0\right), \\ \left((\psi-\phi+1)\left(\frac{i_1+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right) \end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}} \sum_{i_1=0}^{\infty} \left(-\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1} \sum_{i_3=0}^{\infty} \frac{1}{i_3!} \left(-\frac{y}{\sqrt{\nu}}\right)^{i_3} \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4-\frac{i_1+i_3-1}{2})}$$

$$\times \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_6=0}^{\infty} \frac{(-1)^{i_6}}{i_6!} \Re_1^{\psi(\frac{i_1+i_3-1}{2}-i_4-i_6)} \sum_{i_7=0}^{\infty} (-1)^{i_7} \left(\frac{\pi}{T}\right)^{2i_7} \sum_{p=1}^{\infty} (-1)^p H_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} \left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_1+i_3}{2}+i_4,1\right) \\ (1,0),\left(1+\frac{i_1+i_3}{2},0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_3-1}{2}+i_4,0\right),\left(1+\frac{i_1+i_3}{2}-i_4,0\right), \\ \left((\psi-\phi+1)\left(\frac{i_1+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right) \end{matrix}\right]. \quad (31)$$

3.6. Fractionalized MHD Oldroyd-B fluid in porus medium without slip effects

The general solutions without slip effect can be obtained by taking $\Omega_1, \Omega_2 \to 0$ into Eqs. (15) and (21) yields

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}$$

$$\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle|\begin{matrix}\left(1+\frac{i_3}{2}-i_4,0\right),\left(1+\frac{i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_3}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_3}{2},0\right),\left(1+\frac{i_3}{2}-i_4,0\right),\left(1+\frac{i_3}{2}-i_4,0\right),\left(1-\frac{i_3}{2}+i_4,0\right),\left((\psi-\phi+1)\left(\frac{i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_3=1}^{\infty}\frac{1}{i_3!}+2U(\frac{\pi}{T})\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}$$

$$\times\Re_1^{\psi(\frac{i_3}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\middle|\begin{matrix}\left(1+\frac{i_3}{2}-i_4,0\right),\left(1+\frac{i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_3}{2}+i_4,1\right)\\(0,1),\left(1+\frac{i_3}{2},0\right),\left(1+\frac{i_3}{2}-i_4,0\right),\left(1+\frac{i_3}{2}-i_4,0\right),\left(1-\frac{i_3}{2}+i_4,0\right),\\\left((\psi-\phi+1)\left(\frac{i_3}{2}-i_4\right)-i_5-\psi i_6-2i_7-1,\phi\right)\end{matrix}\right], \quad (32)$$

and the attached shear stress is

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_3-1}{2})}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} \left(1+\frac{i_3+1}{2}-i_4,0\right),\left(1+\frac{i_3+1}{2}-i_4-i_5,0\right),\left(1+\frac{i_3-1}{2}-i_4-i_6,0\right),\left(1-\frac{i_3-1}{2}+i_4,1\right) \\ (0,1),\left(1+\frac{i_3+1}{2},0\right),\left(1+\frac{i_3+1}{2}-i_4,0\right),\left(1+\frac{i_3-1}{2}-i_4,0\right),\left(1-\frac{i_3-1}{2}+i_4,0\right), \\ \left((\psi-\phi+1)\left(\frac{i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right) \end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\Re_2^{\phi(i_4-\frac{i_3-1}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}$$

$$\times\sum_{i_6=0}^{\infty}\frac{(-1)^{i_6}}{i_6!}\Re_1^{\psi(\frac{i_3-1}{2}-i_4-i_6)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} \left(1+\frac{i_3}{2}-i_4,0\right),\left(1+\frac{i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_3}{2}-i_4-i_6,0\right),\left(1-\frac{i_3}{2}+i_4,1\right) \\ (1,0),\left(1+\frac{i_3}{2},0\right),\left(1+\frac{i_3}{2}-i_4,0\right),\left(1-\frac{i_3-1}{2}+i_4,0\right),\left(1+\frac{i_3}{2}-i_4,0\right), \\ \left((\psi-\phi+1)\left(\frac{i_3}{2}-i_4\right)-\frac{1}{2}(\psi-\phi-1)-i_5-\psi i_6-2i_7-1,\phi\right) \end{matrix}\right]. \quad (33)$$

3.7. Fractionalized MHD Second grade fluid in porous medium with second order slip

Putting $\Re_1 \to 0$, in Eqs. (15) and (21), we find velocity field and associated shear stress of the mentioned case

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\Bigg|^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)}_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left((1-\phi)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-2i_7-1,\phi\right)}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\Bigg|^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)}_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\left((1-\phi)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-2i_7-1,\phi\right)}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\Bigg|^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,1\right)}_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1-\frac{i_1+i_2}{2}+i_4,0\right),\left((1-\phi)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-2i_7-1,\phi\right)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\Re_2^{\phi(i_4-\frac{i_1+i_2+i_3}{2})}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\left(\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\right)^{i_8}\Bigg|^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right)}_{(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,0\right),\left((1-\phi)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-2i_7-1,\phi\right)}\right], \tag{34}$$

and

$$\tau(y,q) = -\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}} \sum_{i_1=0}^{\infty}(-1)^{i_1} \sum_{i_2=0}^{\infty} \frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2} \left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\sum_{i_3=0}^{\infty} \frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3} \sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} (1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,1\right) \\ (0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right), \\ \left((1-\phi)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)+\frac{1}{2}(1+\phi)-i_5-2i_7-1,\phi\right) \end{matrix}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}} \sum_{i_1=0}^{\infty}(-1)^{i_1} \sum_{i_2=0}^{\infty} \frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2} \left(-\frac{\Omega_2}{\nu}\right)^{i_2} \sum_{i_3=0}^{\infty} \frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\sum_{i_4=0}^{\infty} \frac{(-K_p)^{i_4}}{i_4!} \Re_2^{\phi(i_4-\frac{i_1+i_2+i_3-1}{2})} \sum_{i_5=0}^{\infty} \frac{(-K_{i_5})^{i_5}}{i_5!} \sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7} \sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times \mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\phi}}{\Re_2^{\phi}}\middle| \begin{matrix} (1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1-\frac{i_1+i_2+i_3}{2}+i_4,1\right) \\ (1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right), \\ \left((1-\phi)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)+\frac{1}{2}(1+\phi)-i_5-2i_7-1,\phi\right) \end{matrix}\right]. \tag{35}$$

3.8. Fractionalized MHD Maxwell fluid in porous medium with second order slip

Putting $\Re_2 \to 0$, in Eqs. (15) and (21), we find

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT) + UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}$$

$$\times\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\psi}}{\Re_1^{\psi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left((\psi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-2i_7-1,\psi\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\psi}}{\Re_1^{\psi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\\\left((\psi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-2i_7-1,\psi\right)\end{matrix}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\psi}}{\Re_1^{\psi}}\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left((\psi+1)\left(\frac{i_1+i_2}{2}-i_4\right)-i_5-2i_7-1,\psi\right)\end{matrix}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2+i_3}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\psi}}{\Re_1^{\psi}}\left|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\\\left((\psi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-i_5-2i_7-1,\psi\right)\end{matrix}\right.\right], \tag{36}$$

the velocity and the retrieved shear stress is

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{t^{\psi}}{\Re_1^{\psi}}\left|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,1\right)\\(0,1),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\\\left((\psi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-1)-i_5-2i_7-1,\psi\right)\end{matrix}\right.\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_5=0}^{\infty}\frac{(-K_{i_5})^{i_5}}{i_5!}\Re_1^{\psi(\frac{i_1+i_2+i_3-1}{2}-i_4)}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{4,6}^{1,4}\left[\frac{(t-pT)^{\psi}}{\Re_1^{\psi}}\left|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4-i_5,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1-\frac{i_1+i_2+i_3-1}{2}+i_4,0\right),\\\left((\psi+1)\left(\frac{i_1+i_2+i_3}{2}-i_4\right)-\frac{1}{2}(\psi-1)-i_5-2i_7-1,\psi\right)\end{matrix}\right.\right]. \tag{37}$$

3.9. MHD Newtonian fluid in porus medium with second order slip

Utilizing $\Re_1, \Re_2 \to 0$ into Eqs. (12) and (18), the revaluated velocity is

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\mathbf{M}_{3,5}^{1,3}\left[K_{i_5}t\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(\frac{i_1+i_2}{2}-i_4-2i_7-1,1\right)\end{matrix}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[K_{i_5}t\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(\frac{i_1+i_2+i_3}{2}-i_4-2i_7-1,1\right)\end{matrix}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times\mathbf{M}_{3,5}^{1,3}\left[K_{i_5}(t-pT)\middle|\begin{matrix}(1+i_1-i_2,0),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(1+\frac{i_1+i_2}{2}-i_4,1\right)\\(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2}{2},0\right),\left(1+\frac{i_1+i_2}{2}-i_4,0\right),\left(\frac{i_1+i_2}{2}-i_4-2i_7-1,1\right)\end{matrix}\right]+2U(\frac{\pi}{T})$$

$$\times\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}$$

$$\times\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[K_{i_5}(t-pT)\Bigg|_{(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3}{2},0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(\frac{i_1+i_2+i_3}{2}-i_4-2i_7-1,1\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3}{2}-i_4,1\right)}\right], \tag{38}$$

and the related shear stress is for MHD Newtonian fluid in porous medium is

$$\tau(y,q) = -\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[K_{i_5}t\Bigg|_{(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\left(\frac{i_1+i_2+i_3+1}{2}-i_4-2i_7-1,1\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,1\right)}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}$$

$$\times\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)$$

$$\times \mathbf{M}_{3,5}^{1,3}\left[K_{i_5}(t-pT)\Bigg|_{(1,0),(1+i_1,0),\left(1+\frac{i_1+i_2+i_3+1}{2},0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,0\right),\left(\frac{i_1+i_2+i_3+1}{2}-i_4-2i_7-1,1\right)}^{(1+i_1-i_2,0),\left(1+\frac{i_1+i_2+i_3+1}{2}-i_4,0\right),\left(1+\frac{i_1+i_2+i_3-1}{2}-i_4,1\right)}\right]. \tag{39}$$

3.10. Newtonian fluid under second order slip effect

Putting $K_m, K_p \to 0$ in the Eqs. (38) and(39) yields the results

$$u(y,t) = UH(t)\sin\left(\frac{\pi}{T}t\right) + 2U\sum_{p=1}^{\infty}(-1)^pH_{pT}(t)\sin\frac{\pi}{T}(t-pT)$$

$$+UH(t)\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu t}}\Bigg|_{(1,0),(1+i_1,0),\left(\frac{i_1}{2}-2i_7-1,-\frac{1}{2}\right)}^{(1+i_1,0)}\right]$$

$$+UH(t)(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\times\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu t}}\Bigg|_{(1,0),(1+i_1,0),\left(\frac{i_1+i_3}{2}-2i_7-1,-\frac{1}{2}\right)}^{(1+i_1,1)}\right]$$

$$+2U\left(\frac{\pi}{T}\right)\sum_{i_1=1}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)$$

$$\times\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu(t-pT)}}\Bigg|_{(1,0),(1+i_1,0)\left(\frac{i_1}{2}-2i_7-1,-\frac{1}{2}\right)}^{(1+i_1,0)}\right]$$

$$+2U(\frac{\pi}{T})\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=1}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{p=1}^{\infty}(-1)^{p}H_{pT}(t)\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu(t-pT)}}\Bigg|_{(1,0),(1+i_1,0),\left(\frac{i_1+i_3}{2}-2i_7-1,-\frac{1}{2}\right)}^{(1+i_1,1)}\right] \tag{40}$$

and the shear stress has the form

$$\tau(y,q)=-\frac{UH(t)\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}$$

$$\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu t}}\Bigg|_{(1,0),(1+i_1,0)\left(\frac{i_1+i_3+1}{2}-2i_6-1,-\frac{1}{2}\right)}^{(1+i_1,1)}\right]$$

$$-\frac{2U\mu(\frac{\pi}{T})}{\sqrt{\nu}}\sum_{i_1=0}^{\infty}(-1)^{i_1}\sum_{i_2=0}^{\infty}\frac{1}{i_2!}\left(\frac{\Omega_1}{\sqrt{\nu}}\right)^{i_1-i_2}\left(-\frac{\Omega_2}{\nu}\right)^{i_2}\sum_{i_3=0}^{\infty}\frac{1}{i_3!}\left(-\frac{y}{\sqrt{\nu}}\right)^{i_3}\sum_{i_4=0}^{\infty}\frac{(-K_p)^{i_4}}{i_4!}\sum_{i_7=0}^{\infty}(-1)^{i_7}\left(\frac{\pi}{T}\right)^{2i_7}$$

$$\times\sum_{p=1}^{\infty}(-1)^p H_{pT}(t)\mathbf{M}_{1,3}^{1,1}\left[\frac{\Omega_2}{\Omega_1\sqrt{\nu(t-pT)}}\Bigg|_{(1,0),(1+i_1,0)\left(\frac{i_1+i_3+1}{2}-2i_6-1,-\frac{1}{2}\right)}^{(1+i_1,1)}\right]. \qquad (41)$$

4. Numerical results and discussions

The flow behavior of MHD Oldroyd-B fluid over the boundlessly extended porous plate to which rectified sine pulses are applied is investigated for the impacts of both the first and the second-order slips. Fractional calculus and Laplace transforms are suitably invoked to acquire the exact solutions of the fractionalized Oldroyd-B fluid within the presence of MHD, porosity and slip components. Specialized results representing the similar motion of fractionalized second grade, Maxwell and the traditional Newtonian fluids are produced from general results. Developing and retarding the unidirectional motion of fluid within the interval of each is pictorially depicted and analyzed for the mentioned fluid types with numerically variational parameters of interest. The fluctuating flow is a natural phenomenon observed in blood within the arteries, suddenly effected heartbeats, earthquake geophysical motion and other natural pulsatile flow reveals interesting facts which are the main finding of this discussion. The analytical solutions obtained for velocity and shear stress are interpreted through graphical analyses (Figures 2–17). The influence of fractional parameters (ψ , ϕ), relaxation and retardation times ($\mathcal{R}_1$, $\mathcal{R}_2$), slip coefficients (Ω_1, Ω_2), magnetic and porosity parameters (K_m, K_p), and oscillation period (T) on the flow field are systematically examined. Variation of velocity profiles for different values of fractional parameter ψ. Unless otherwise specified, parameter values are chosen as $\nu = 2.526$, $\mu = 12.2425$, and related constants in accordance with [27, 28, 33].

Where appropriate, comparisons are drawn with limiting cases of classical (non-fractional) Oldroyd-B, Maxwell, second-grade, and Newtonian fluids.

Figs: 2 and 3 are provided to view the increasing velocity of fluid particles that are in no distance with the pulsating plate, while the fluid motion is almost similar at a significant distance from the plate. Peak flow is observed near 6 seconds after the pulsation started with period $\pi/2$ radians, while flow velocity decreases at a distance from the plate and similar behavior is pursued by concerned shear stress. Figs: 4 and 5 reflect the expected opposite effects over the field; that is, the flow velocity and shear stress get low magnitude in fluctuations by enhancing the relaxing time $\Re_1$, whereas the flow magnitude goes high with increasing retardation time $\Re_2$. The magnetic and porous effects can be analyzed in Figs: 6 and 7, where velocity and shear stress decay with MHD and porous effects, whereas increasing the elastic factor ν grows the flow of fluid as shown in Fig: 8. The importance of the fractional parameters ψ and ϕ is highlighted in Figs: 9 and 10. Both the parameters react oppositely: the uplifting of ψ values accelerates the flow velocity and reduces the shear stress, but for a numerically increasing value of ϕ, a diminishing outcome in the velocity field is observed while shear stress moves high.

First and second-order slip effects are displayed in Figs: 11 and 12, flow is affected by slips. The second slip has even deeper numerical impacts than the first-order slip, whereas the two slips play a key role in reducing the flow field. Variations corresponding to change in the pulsating period of the plate in the flow field are given in figs: 13, which shows that the larger the pulsating period, the higher the velocity and shear stress. One can note that for low values of time, the dissimilarities among the velocities for various pulsation periods are small but these differences are at the peak after a few seconds of pulsation and decrease rapidly thereafter. A comparative analysis of flow field for fractionalized Oldroyd-B, second grade, Maxwell and traditional Newtonian fluid is graphically performed in Fig: 14, that predicts that fractionalized Maxwell fluid is fastest with rectified sine pulses while the second grade is slowest for smaller values of time but for large time values these differences diminish gradually with time as demonstrated in Fig: 16. Similar behavior is pursued by the ordinary type of fluids of the same kinds that are visualized in Figs: 15 and 17. Finally, we can conclude by the above discussion and analysis that velocity and shear stress of the fluid gets affected in decaying direction by slips and Maxwell fluid either fractional or ordinary, moves faster than Oldroyd-B in the flow field induced by rectified sine pulses.

5. Conclusion

This study presented an analytical investigation of the unsteady MHD flow of a fractionalized Oldroyd-B fluid over a porous plate subjected to rectified sine pulses with first- and second-order slip conditions. Using fractional calculus and Laplace transforms, exact solutions were obtained for velocity and shear stress in terms of generalized M-functions, encompassing several classical and fractional fluid subclasses.

The results demonstrate that:

- Slip effects (both first- and second-order) significantly reduce velocity and shear stress, with second-order slip exerting stronger damping.
- Fractional parameters (ψ , ϕ) provide a tunable mechanism to capture memory effects absent in classical models.
- Relaxation time ($\mathcal{R}_1$) strongly influences the flow field, dominating over retardation and porosity parameters.
- Magnetic field and porous medium effects further suppress motion, relevant for MHD control in engineering and biomedical contexts.
- Strongly modulates flow oscillations (T), with slip effects amplifying damping at longer periods.
- Practical implications suggest that slip coefficients and fractional parameters may be exploited to regulate flow in polymer extrusion, porous filters, and microfluidic channels.
- Assumptions and limitations of the present model include incompressibility, infinite plate geometry, absence of temperature and chemical effects, and purely analytical treatment without experimental validation.
- Future research directions include: (i) extension to three-dimensional and thermal flows, (ii) coupling with heat and mass transfer, (iii) experimental or CFD validation of the analytical results, and (iv) sensitivity-based optimization of parameters for industrial and biomedical applications.

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Conflicts of Interest

The authors do not have any conflict or competing interests.

Authors' contributions

1. **Ilyas Khan and Afaque Ahmed Bhutto:** Conceptualization, Methodology, results and discussion**J. Alzahrani:** Writing- Original draft, Visualization, Investigation, **Iftikhar Ahmed and Abeer H. Alzahrani:** Software, results and discussion; **lker Ozsahin:** Conceptualization, results and discussion; **Wei Sin Koh:** Revision, Reviewing and Editing; **and Israr Ahmed:** Software, Investigation. All the authors reviewed the manuscript.

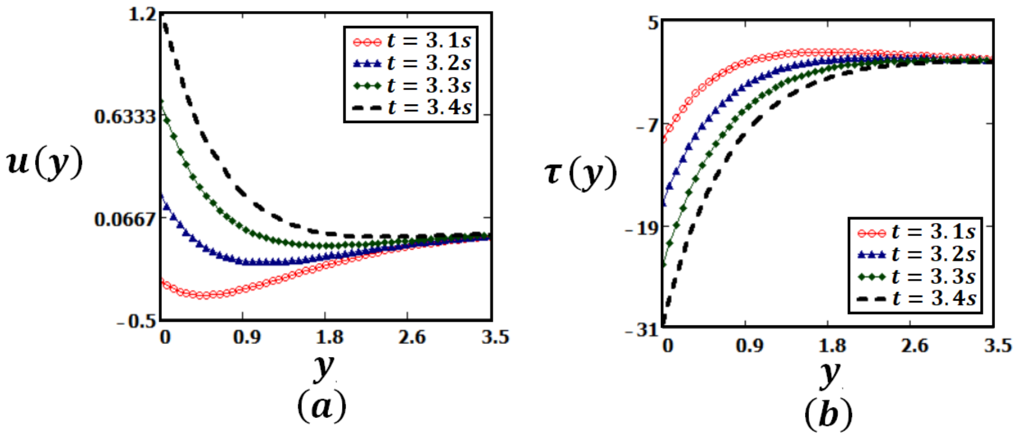

Figure 2: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles for the fractionalized Maxwell fluid based Eqs.(15) and (21). The result shown for $U = 1$, $\nu = 2.526$, $\mu = 12.2425$, $\Re_1 = 3$, $\Re_2 = 2.5$, $K_m = 0.5$, $K_p = 2$, $\psi = 0.5$, $\phi = 0.6$, $\Omega_1 = 0.2$, $\Omega_2 = 0.3$, $T = \pi/2$ at different time values.

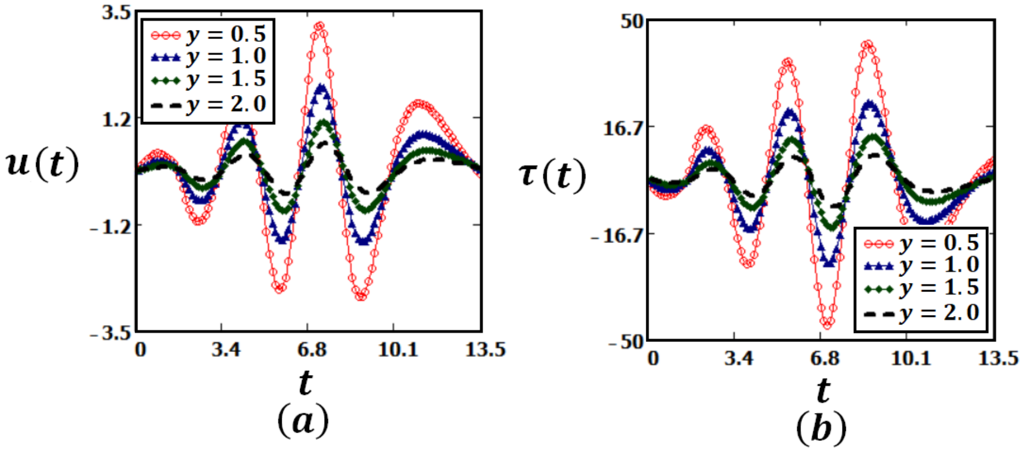

Figure 3: Velocity $u(y, t)$ and shear stress $\tau(y, t)$ profiles for the fractionalized Maxwell fluid based Eqs.(15) and (21). The results shown for $U = 1$, $\nu = 2.526$, $\mu = 12.2425$, $\Re_1 = 3$, $\Re_2 = 2.5$, $K_m = 0.5$, $K_p = 2$, $\psi = 0.5$, $\phi = 0.6$, $\Omega_1 = 0.2$, $\Omega_2 = 0.3$, $T = \pi/2$ at different values of y.

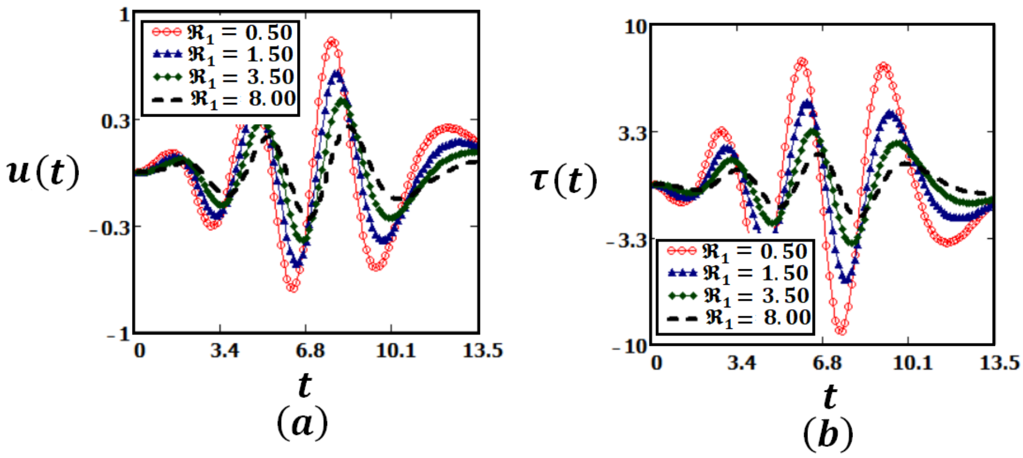

Figure 4: Velocity $u(y, t)$ and shear stress $\tau(y, t)$ profiles for the fractionalized Maxwell fluid based Eqs.(15) and (21). The results shown at different values of $\Re_1$.

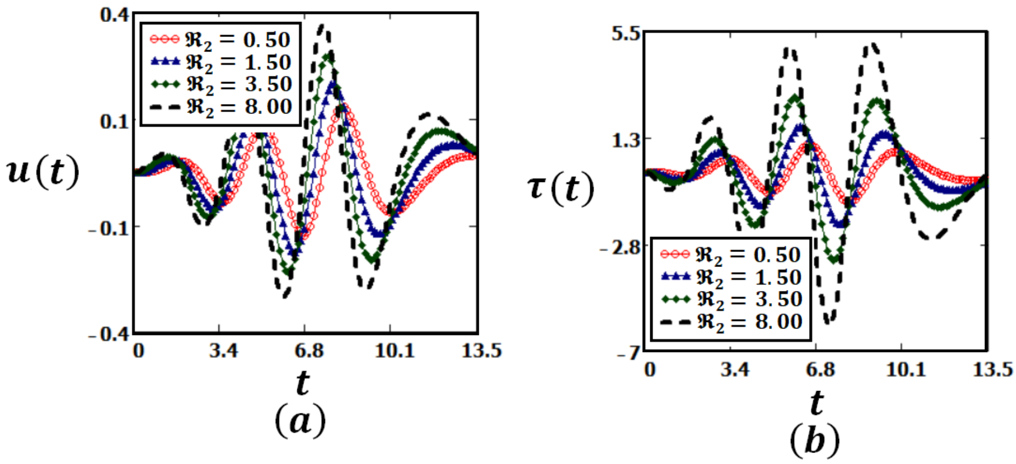

Figure 5: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles for the fractionalized Maxwell fluid based Eqs.(15) and (21) at different values of $\Re_1$.

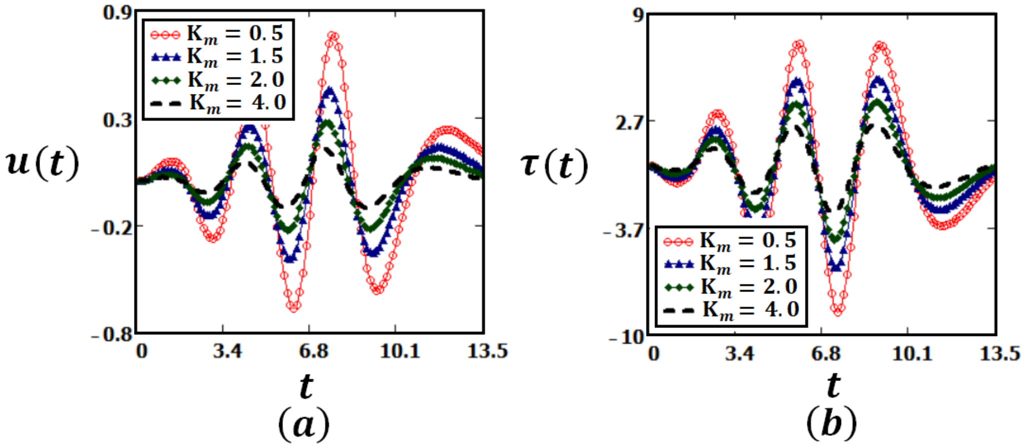

Figure 6: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of $\Re_2$.

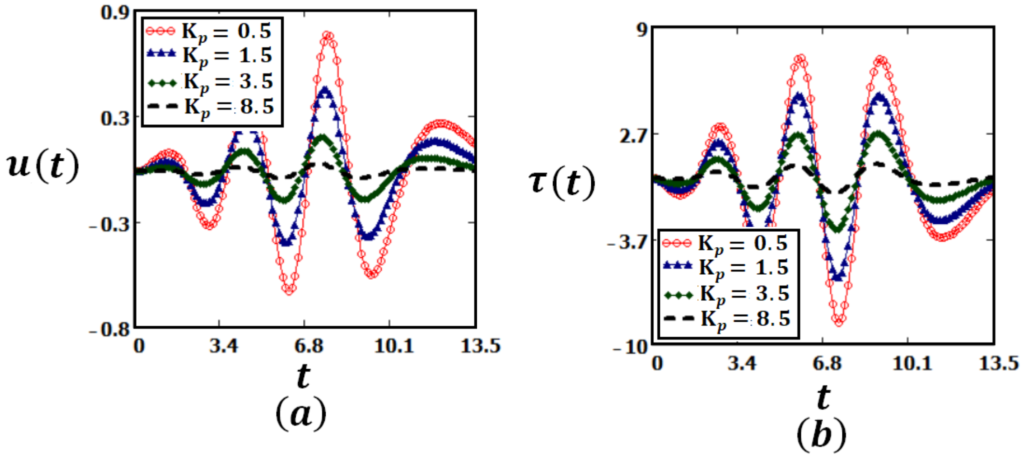

Figure 7: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of K_m.

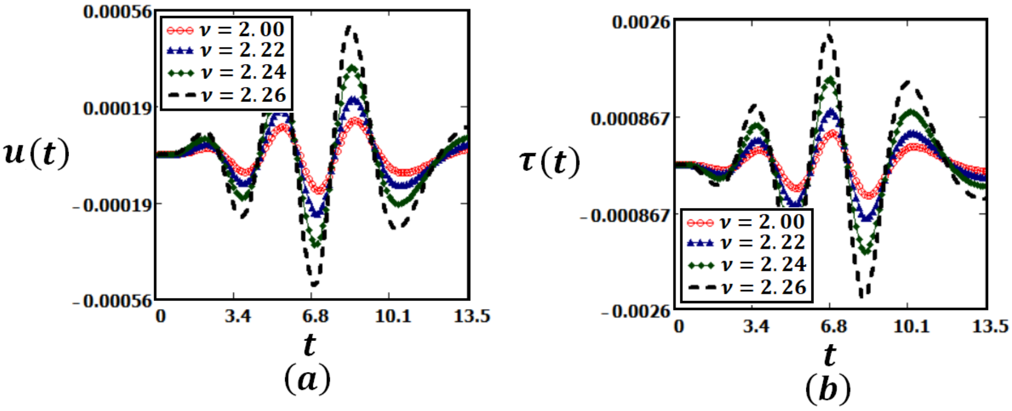

Figure 8: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of K_p.

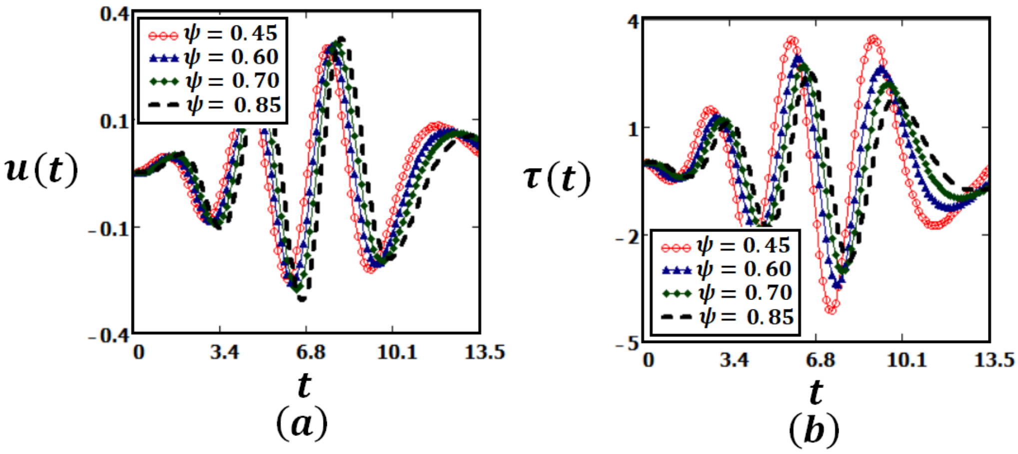

Figure 9: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of ν.

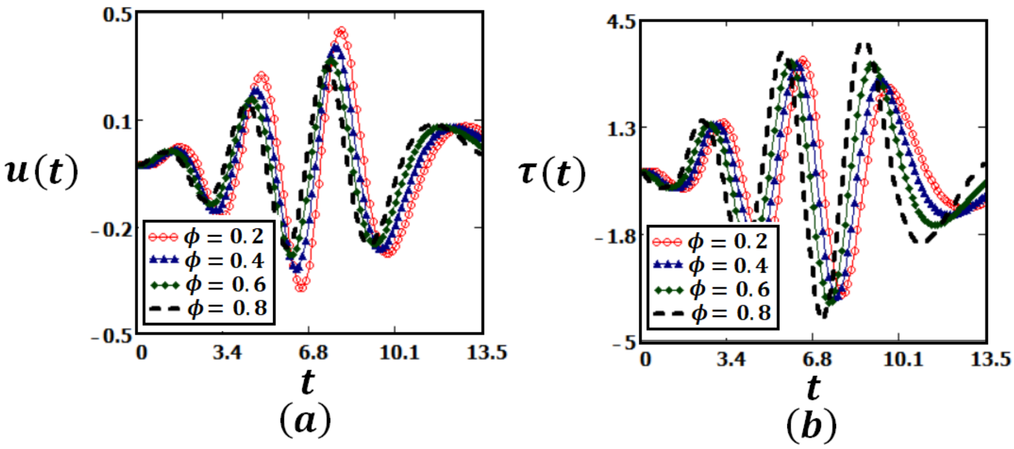

Figure 10: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of ψ.

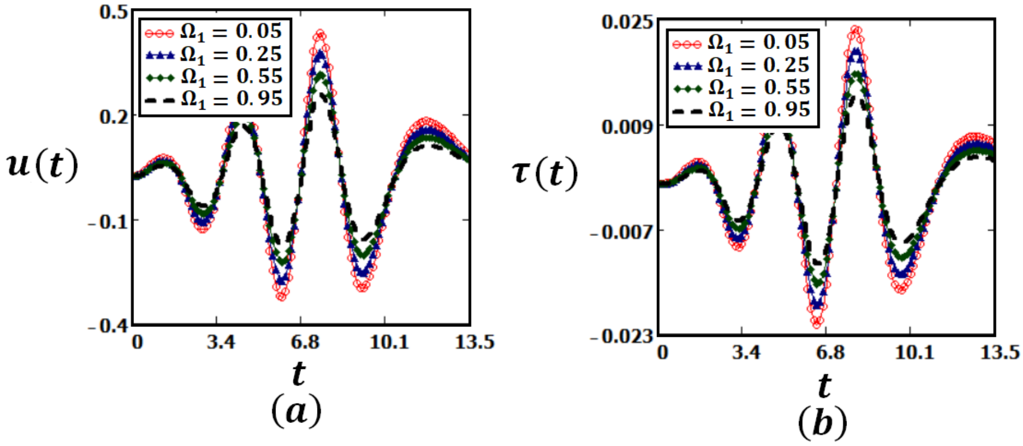

Figure 11: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of ϕ.

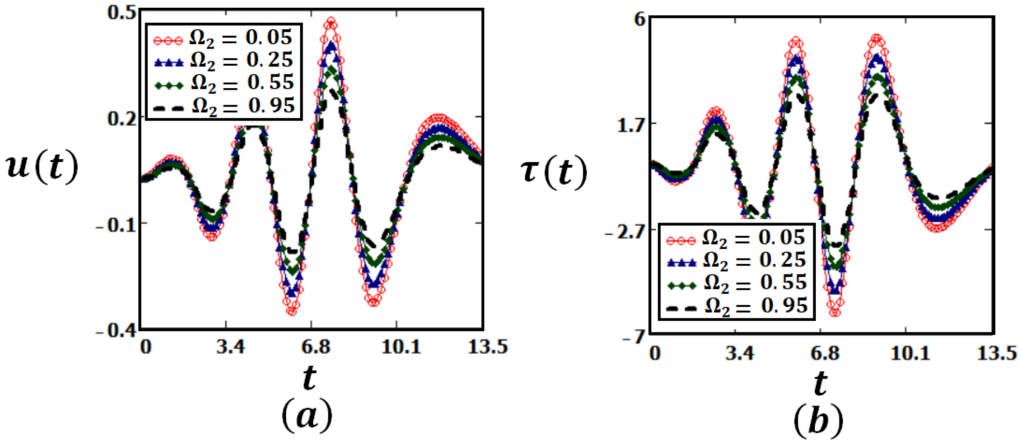

Figure 12: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of Ω_1.

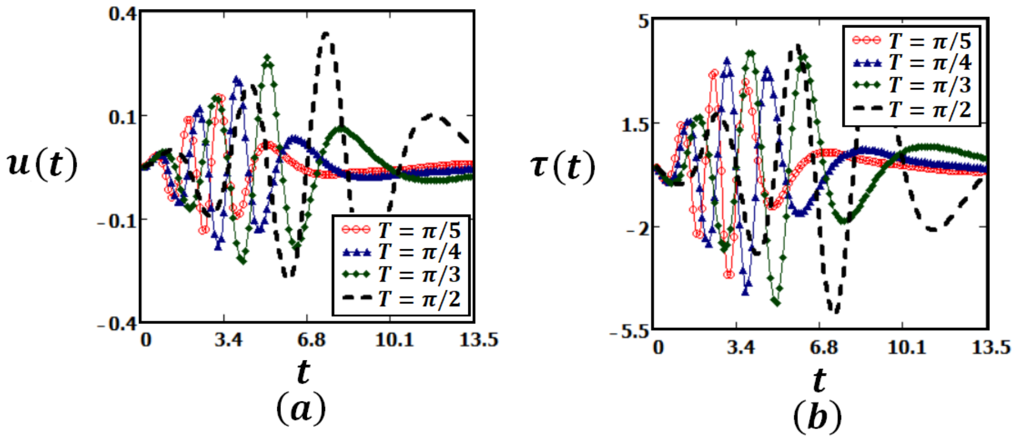

Figure 13: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of Ω_2.

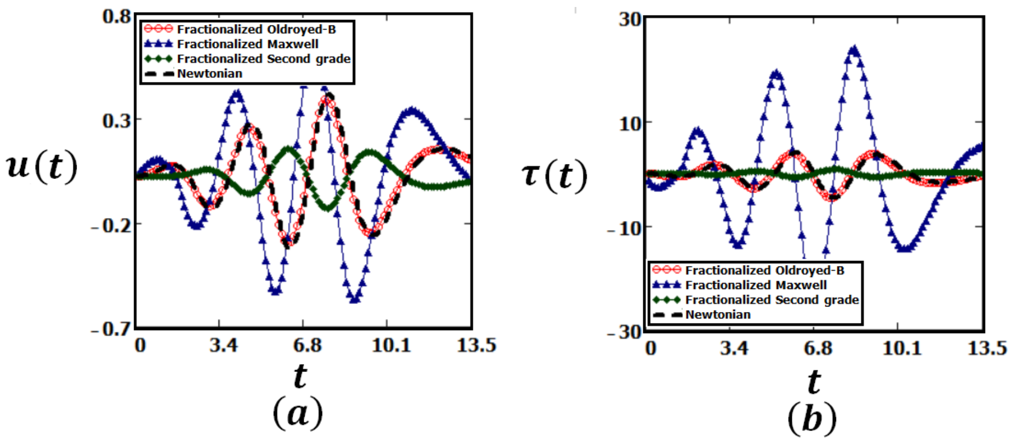

Figure 14: Velocity $u(y,t)$ and shear stress $\tau(y,t)$ profiles at different values of T.

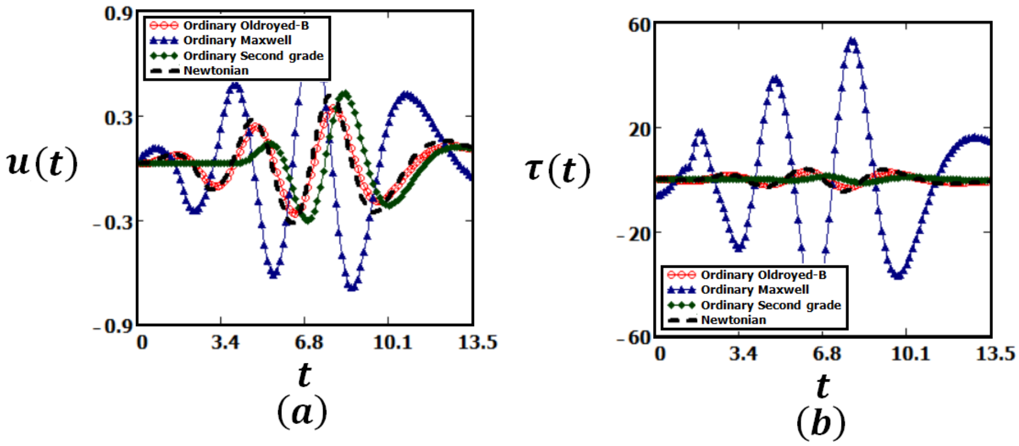

Figure 15: Velocity and shear stress profiles for fractionalized Oldroyd-B, Maxwell, Second grade, and Newtonian fluids.

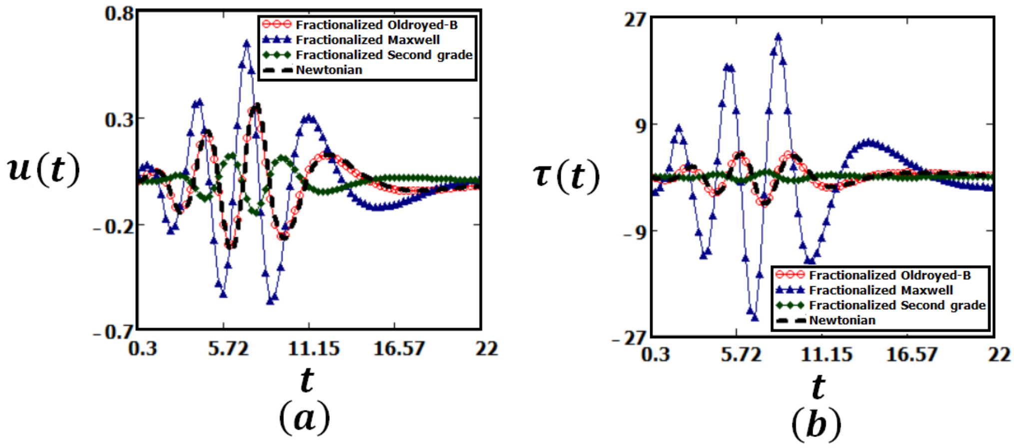

Figure 16: Velocity and shear stress profiles for ordinary Oldroyd-B, Maxwell, Second grade, and Newtonian fluids.

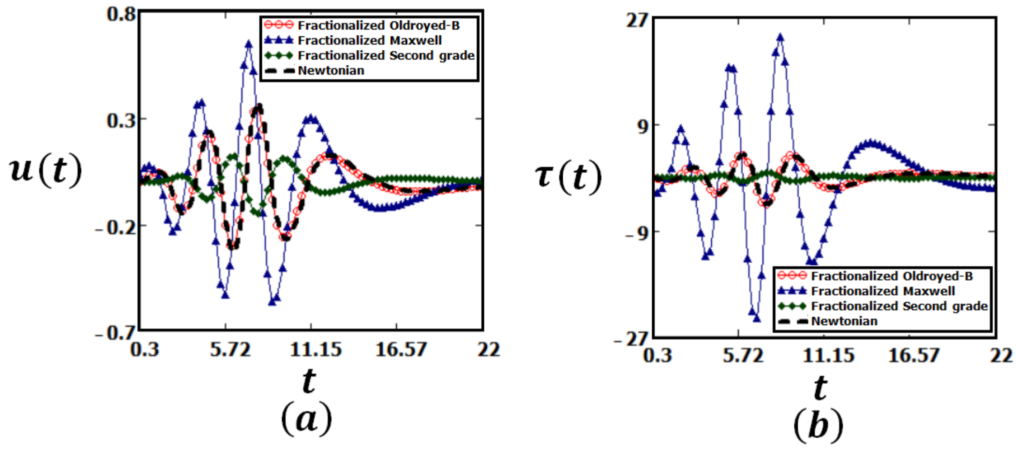

Figure 17: Velocity and shear stress profiles for fractionalized fluid models comparison.

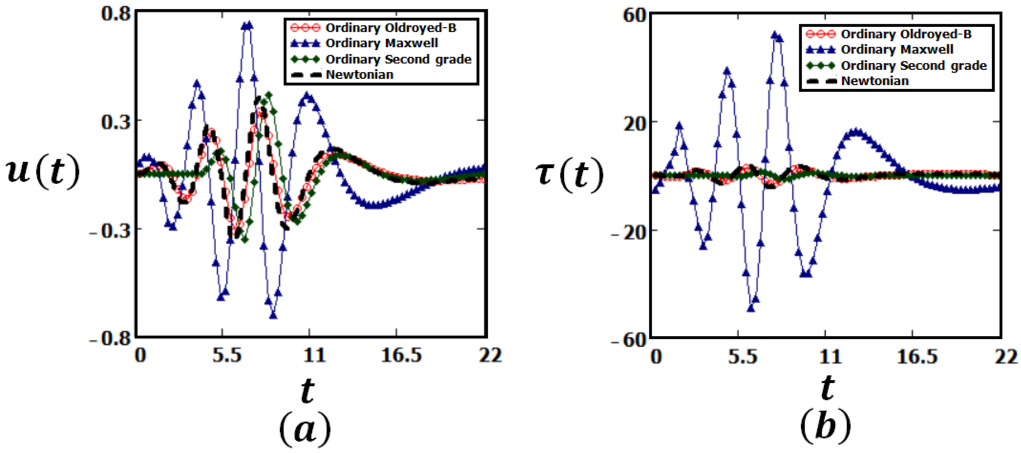

Figure 18: Velocity and shear stress profiles comparison for different fluid models.

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