EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2026, Vol. 19, Issue 1, Article Number 6399
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Arterial Hemodynamics of Two Phase Blood Flow for Medical Diagnostics

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DOI: https://doi.org/10.29020/nybg.ejpam.v19i1.6399

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Abstract. Impact of thermal radiation on two-phase fluctuating blood flow and wall shear stress has applications in multiple directions. In medical science, areas such as hyperthermia, vascular diagnostics, and prosthetics benefit from advanced simulations of vascular behavior under thermal conditions. Wall shear stress (WSS), which arises due to the frictional force exerted by blood flow on arterial walls, plays a central role in regulating endothelial cell function and vascular homeostasis. Abnormalities in WSS are directly linked to the development and progression of atherosclerosis and other cardiovascular diseases. Specifically, regions with low or oscillatory WSS are more susceptible to plaque formation, while elevated WSS levels can lead to vascular remodeling or rupture of vulnerable plaques. In this study, electrically conducting dust particles that generate a high magnetic field are used to investigate how WSS affects two-phase fluctuating blood flow and heat transfer. An inclined channel with a certain inclination along the x-axis is used to model the flow. The flow in the channel is due to one of the walls fluctuating due to WSS. In order to investigate systematic solutions, the Poincaré-Lighthill perturbation technique (PLPT) is used. Mathcad-15 software is used to obtain the numerical results and related graphs. A graphic representation is presented to illustrate the effect of various embedded parameters on fluid and dust particle velocity profiles as well as temperature profiles. It is found that the increase in inclination angle controls both the flows. The boundary layer thins out with increased blood flow, decreasing both velocity profiles.

1. Introduction

In magnetohydrodynamics, electric and magnetic properties are studied in fluids in relationship to one another. Magnetofluids include liquid metals, saltwater, plasmas, and electrolytes. Flow metres, blood flow, heat transfer increase in gas cooling systems, MHD generators, accelerators, and free convection are just a few of the many fluid engineering issues that may benefit from this combination [1]. In view of the aforementioned uses, a plethora of research investigated the consequences of natural convective flow with MHD applied in different directions [2]. Recent studies such as Abbas et al. [3, 4] and Mehmood et al. [5] have highlighted the significant impact of Lorentz forces, chemical reactions, thermal-diffusion effects, and activation energy on heat and mass transfer in non-Newtonian and nanofluid flows within porous media. These works support the relevance of incorporating such complex interactions, as done in our study, to better capture real-world behaviors in magnetized, reactive, and thermally sensitive fluid systems. In their study, Khan et al. [6] examined the effects of heat transfer and changing wall shear stress on the flow of viscoelastic blood fluid in a mixed-heat-dissipation (MHD) system with a vertical submerged in a porous media. Using a viscoelastic blood fluid under a wall shear stress state, Anwar et al. [7] investigate the effects of radiation heat flux and heat injection/suction processes on MHD free convective fluid flow. Additionally, in relation to a nonlinear permeable stretched cylinder, Ullah et al. [8] investigate the combined effects of heat absorption, chemical reaction, and thermal transfer on the MHD slip flow of blood. In their investigation of blood-dusty fluid flow in the presence of fluid-particle suspension, Mahanthesh et al. [9] examines the effects of Joule heating and thermal radiation. Using heat radiation as an external stimulus, Mohyud-din et al. [10] investigated how magnetic field lines affected the viscous flow of micropolar fluid. Non-Newtonian fluids, on the other hand, have received much interest recently due to the terms of behaviour wide range of industries in which they can be used, including such solid matrix heat configuration, chemical catalytic reactors, ground water hydrology, waste of nuclear disposal, production of geothermal energy, and transpiration cooling down of petroleum reservoirs. Because of the non-linear relation between stress and strain rate, these fluids are more challenging than the fluids that obey Newtonian's law. The non-Newtonian fluids have been explored using a variety of models, but no one model has yet been developed that encompasses all of their features. The Blood fluid is among several non-Newtonian fluids that occur. In the theory of fluid mechanics, a fluid is described as having an infinite range of viscosity, strain and stress at which there is no flow occur. It is shear thinning fluid with an infinite rate of shear and an infinite yield stress that causes no flow to occur [11]. Blood established the Blood model, which is frequently referred to as the rheological model [12], for the viscous suspension of cylindrical particles [13]. Blood fluids contain soup, sauce of tomato, jelly, and fluid of concentrated fruit. Furthermore, Blood fluid has yield stress and is used to biomechanics and produce the polymers [14]. The effects of ramping surface concentration and wall temperature on the magnetohydrodynamic flow of viscoelastic blood fluid and an accelerated parallel plate embedded in a porous media with chemical reaction and heat absorption were studied by Kataria and Patel [15]. In addition to a moving vertical plate with free convection and MHD, Khan et al. [16] investigation of the Blood fluid flow also considered the effects of chemical processes, Newtonian heating, and a magnetic field. The effects of varying temperature on generalised magnetohydrodynamic Blood dusty fluid flow in two parallel plates were examined by Ali et al. [17]. As a result of Newtonian heating, Khan et al. [18] studied the MHD of a two-phase fluctuating Newtonian dusty fluid flow using a Lighthill perturbation technique. Ali et al. [19] used the Lighthill perturbation technique to explored the effect of wall shear stress and heat absorption on non-Newtonian viscoelastic fluid flow on an inclined parallel plate with MHD. Furthermore, Gireesha et al. [20] studied the impacts of heat transfer on the MHD dusty fluid flow across a stretched sheet. An interaction flow between two distinct phases in a channel, each of which represents a mass or volume of matter, is called "two-phase flow." Two-phase flow fountains can resemble the effects of rain, waves at sea, and bubbles. The applications of two-phase flow have been studied by a number of researchers [21–25]. The purpose of this review was only to examine the use of micromodels in the study of two-phase flow in porous media. There are several approaches for creating patterns for usage in micromodels. An example of each of these patterns is the perfect pattern, the partly regular pattern, or the fractal pattern. Micromodels are created using a variety of manufacturing processes and materials, each with its own set of advantages and limitations [26]. There is a review of recent research on ammonia, novel refrigerants, and CO_2, as well as characteristics of two-phase flow patterns and flow-pattern maps at adiabatic and diabatic temperatures. Following that, basic studies of gas-liquid flow patterns and flow-pattern maps are provided by Cheng et al. [27]. The JAX-Fluids code was shown to be a complete differentiable high-order solution for a compressible two-phase flow by Bezgin et al. [28]. Cancès and Matthes [29] investigated the Two-phase flow with unique energy constructed using gradient flow techniques. With the use of non-conducting parallel plates, Ali et al. [30] investigated how heat transfer affected MHD two-phase viscoelastic dusty fluid flow. For a more in-depth investigation, see the articles [31–34]. As the temperature differential between an object and its environment changes, the rate of heat loss from that object will also change, according to Newton's law of heating (or cooling) [35]. While convection-driven convection can be used to cool and heat forced air or pushed fluid, the general law only holds almost true for buoyancy-driven convection in which flow velocity increases with temperature difference. Finally, when heat is transported via thermal radiation, the Newton law of cooling (or heating) only applies to very small temperature differences. Some frequent assumptions used in MHD two-phase fluctuation flow modelling between parallel plates are constant surface temperature and ramping wall temperatures [36–40]. Furthermore, in many practical cases when the Newtonian heating condition is necessary and heat transmission is inversely proportional to surface temperature, the aforementioned assumptions are not valid. Merkin [41] was the first to investigate four different types of wall temperature distributions, including Newtonian heating. Conjugated heat transport around fins, solar radiation, heat exchangers, and the oil industry all need Newtonian heating conditions. Khan et al. [42] reported on the effects of chemical reactions, fluid viscoelasticity, magnetic fields, heat production, and Newtonian heating on natural convection flow between vertical plates. Regarding the flow of viscoelastic blood fluid between oscillating vertical plates, Hussanan et al. [43] demonstrated the impact of Newtonian heating. The viscoelastic flow of blood fluid over a cylinder, including the effects of heat absorption and Newtonian heating, was studied by Loganathan et al. [44]. Blood fluid flows in two dimensions across a stretched sheet; Hussanan et al. [45] investigated the effects of a magnetic field and Newtonian heating on this flow. According to Manjula and Sekhar [46], heat transmission and thermal expansion affect MHD. The effect of Newtonian heating on blood flow on a vertical surface. Dhairiyasamy et al.[47] examined nanofluid modifications on heat pipe thermal performance. Researchers in the aforementioned works looked for dusty fluids that were non-Newtonian, incompressible, electrically conducting, and transporting by free convection, Newtonian heating, and maximum heat transfer (MHD). So far as we are aware, no research has been published that integrates the momentum equations of fluids, dust particles, and a viscoelastic blood dusty fluid, using the Light-Hill method. Viscoelastic blood fluid flows between two inclined parallel plates and exhibits heat transfer and flow characteristics that have several technical and industrial uses. Within a two-phase fluctuating free convection flow between plates containing suspended electrically conducting particles, this research examines the wall shear stress and varied temperature impact of blood-dusty fluid flow.

2. Mathematical Description

2.1. Problem Formulation

We consider the unsteady flow of an electrically conducting, incompressible blood fluid with suspended dust particles between two inclined parallel plates. The plates are inclined at an angle α with the x-axis. A uniform transverse magnetic field of strength B_0 is applied. The flow is driven by a free stream velocity $U(\tau)$ at the right plate and a wall shear stress at the left plate, as shown in Figure 1. The left plate is heated with a time-dependent temperature $T_d + (T_w - T_d)A\tau$, while the right plate is maintained at the constant temperature T_d. Radiative heat transfer is also taken into account.

The governing equations of motion, energy, and dust particle momentum are

$$\frac{\partial u}{\partial \tau} = \left(1 + \frac{1}{\beta}\right) \nu \frac{\partial^2 u}{\partial \eta^2} + \frac{K_0 N_0}{\rho}(v - u) - \frac{\sigma B_0^2}{\rho} u + g\beta_T \big[T(\eta, \tau) - T_d\big] \cos(\alpha), \tag{1}$$

$$\frac{\rho c_p}{k} \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial \eta^2} - \frac{1}{k} \frac{\partial q_r}{\partial \eta}, \tag{2}$$

$$m \frac{\partial v}{\partial \tau} = K_0(u - v). \tag{3}$$

Using the Rosseland approximation, the radiative heat flux is

$$-\frac{\partial q_r}{\partial \eta} = 4\alpha_0(T - T_d). \tag{4}$$

Boundary conditions:

$$[\mu \frac{\partial u(0, \tau)}{\partial \eta} = f(\tau), \quad \tau > 0, u(1, \tau) \quad = U(\tau), T(0, \tau) = T_d + (T_w - T_d)A\tau, T(1, \tau) \quad = T_d,], \tag{5}$$

where the free stream velocity is prescribed as

$$U(\tau) = u_0 \left[1 + \epsilon \cos(\omega \tau)\right].$$

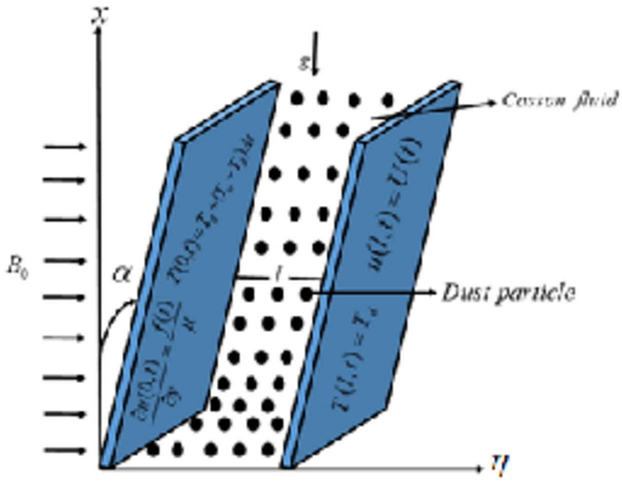

Figure 1: The flow geometry.

Dimensionless variables:

$$\eta^* = \frac{\eta}{d}, \quad u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{u_0}, \quad \theta^* = \frac{T - T_d}{T_w - T_d}, \quad \tau^* = \frac{u_0 \tau}{d}. \tag{6}$$

Dropping the asterisks for convenience, the governing equations become

$$\mathrm{Re}\,\frac{\partial u}{\partial \tau} = \beta_1 \frac{\partial^2 u}{\partial \eta^2} - Mu + K(v - u) + \mathrm{Gr}\,\theta\cos(\alpha), \tag{7}$$

$$\mathrm{Pe}\,\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + N^2\theta, \tag{8}$$

$$\frac{\partial v}{\partial \tau} = P_m(u - v). \tag{9}$$

The corresponding dimensionless boundary conditions are

$$[\frac{\partial u(0,\tau)}{\partial \eta} = f(\tau), \qquad u(1,\tau) = U(\tau), \theta(0,\tau) \qquad = \tau, \theta(1,\tau) \qquad = 0.]. \tag{10}$$

Dimensionless parameters:

$$\mathrm{Re} = \frac{u_0 d}{\nu}, \qquad M = \frac{\sigma B_0^2 d^2}{\rho \nu}, \quad P_m = \frac{K_0 d}{m u_0},$$
$$K = \frac{K_0 N_0 d^2}{\rho \nu}, \quad N^2 = \frac{4\alpha_0^2 d^2}{k}, \quad \beta_1 = 1 + \frac{1}{\beta}, \mathrm{Pe} \quad = \frac{\rho c_p u_0 d}{k}, \mathrm{Gr} \quad = \frac{g \beta_T d^2 (T_w - T_d)}{u_0 \nu}.$$

Dust particle velocity: Assuming a solution of the form

$$v(\eta,\tau) = w_0(\eta) e^{i\omega\tau} + O(\epsilon), \tag{11}$$

substituting into Eq. (9) gives

$$v(\eta,\tau) = \left(\frac{P_m}{i\omega + P_m} \right) u(\eta,\tau). \tag{12}$$

Temperature distribution: Assuming

$$\theta(\eta,\tau) = \Theta_0(\eta) + \epsilon \Theta_1(\eta) e^{i\omega\tau} + O(\epsilon^2), \tag{13}$$

and substituting into Eq. (8), we obtain

$$\theta(\eta,\tau) = \frac{\tau \sinh(N - N\eta)}{\sinh(N)}. \tag{14}$$

Reduced momentum equation: Using Eqs. (12) and (13) in Eq. (7), the governing momentum equation reduces to

$$\mathrm{Re} \frac{\partial u}{\partial \tau} = \beta_1 \frac{\partial^2 u}{\partial \eta^2} + a_0 u + \mathrm{Gr} \cos(\alpha) \frac{\tau \sinh(N - N\eta)}{\sinh(N)}, \tag{15}$$

where

$$a_0 = \frac{K(M+1)(i\omega + P_m) - P_m}{i\omega + P_m}.$$

2.2. Solution of the Problem

The solution of Eq. (15) is obtained using the perturbation technique [19]. We assume

$$u(\eta,\tau) = \xi_0(\eta) + \frac{\epsilon}{2}\Big[\xi_1(\eta)e^{i\omega\tau} + \xi_2(\eta)e^{-i\omega\tau}\Big] + O(\epsilon^2). \tag{16}$$

Substituting Eq. (16) into Eq. (15), the zeroth- and first-order solutions are obtained as

$$\xi_0(\eta) = \left(1 - \frac{\sinh(\sqrt{a_1})}{\sqrt{a_1}}\right)\big(f(\tau) - G_1\big)\frac{\cosh(\sqrt{a_1}\eta)}{\cosh(\sqrt{a_1})} + \big(f(\tau) - G_1\big)\frac{\sinh(\sqrt{a_1}\eta)}{\sqrt{a_1}} + G\frac{\sinh(N - N\eta)}{\sinh(N)}, \tag{17}$$

where

$$G = \frac{\tau\,\mathrm{Gr}\cos(\alpha)}{a_0}, \quad a_1 = \frac{a_0}{\beta_1}, \quad G_1 = \frac{NG\cosh(N)}{\sinh(N)}.$$

$$\xi_1(\eta) = \frac{\cosh\big(\eta\sqrt{a_2}\big)}{\cosh\big(\sqrt{a_2}\big)}, \quad a_2 = \frac{a_0 - \mathrm{Re}\, i\omega}{\beta_1}, \tag{18}$$

$$\xi_2(\eta) = \frac{\cosh\big(\eta\sqrt{a_3}\big)}{\cosh\big(\sqrt{a_3}\big)}, \quad a_3 = \frac{a_0 + \mathrm{Re}\, i\omega}{\beta_1}. \tag{19}$$

Finally, substituting Eqs. (17)–(19) into Eq. (16), the complete solution is obtained as

$$\begin{aligned} u(\eta,\tau) = {} & \left(1 - \frac{\sinh(\sqrt{a_1})}{\sqrt{a_1}}\right)\big(f(\tau) - G_1\big)\frac{\cosh(\sqrt{a_1}\eta)}{\cosh(\sqrt{a_1})} + G\frac{\sinh(N - N\eta)}{\sinh(N)} \\ & + \frac{\epsilon}{2}\left(\frac{\cosh(\eta\sqrt{a_2})}{\cosh(\sqrt{a_2})}\right)e^{i\omega\tau} + \frac{\epsilon}{2}\left(\frac{\cosh(\eta\sqrt{a_3})}{\cosh(\sqrt{a_3})}\right)e^{-i\omega\tau}. \end{aligned} \tag{20}$$

Special Cases:

Case (1): For blood parameter $\beta \to \infty$, $\beta_1 = 1$ and Eq. (14) reduces to

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \eta^2} + K(v - u) - Mu + \mathrm{Gr}\cos(\alpha)\,\theta.$$

The corresponding solution follows analogously with modified coefficients.

Case (2): (i) If $f(\tau) = \cos(\omega\tau)$, Eq. (19) reduces accordingly. (ii) If $f(\tau) = \sin(\omega\tau)$, Eq. (19) simplifies with sine terms instead of cosine.

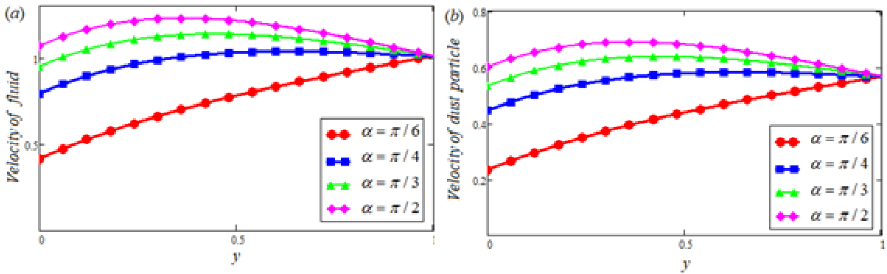

Figure 2: Effect of α on fluid velocity and dusty fluid velocity.

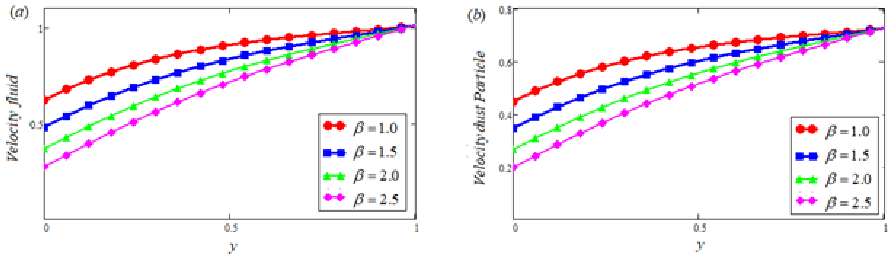

Figure 3: Effect of β on fluid velocity and dusty fluid velocity.

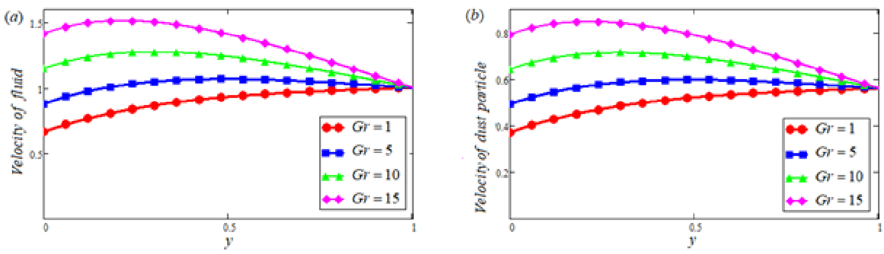

Figure 4: Effect of Gr on fluid velocity and dusty fluid velocity.

3. Graphical Results and Physical Interpretations

In this section, we'll talk about how different dimensionless factors affect the velocity distribution and particle velocity of blood dust. Theoretical analysis of the effects of temperature variation and wall shear stress on the two-phase fluctuating MHD flow of blood-dusty fluid is examined utilising PLPT methodology. The inclined parallel plates with the x-axis are slanted at an angle. Figures 2 and 9 show how various physical factors affect the temperature profile, for distribution of dust particle and blood fluid velocity.

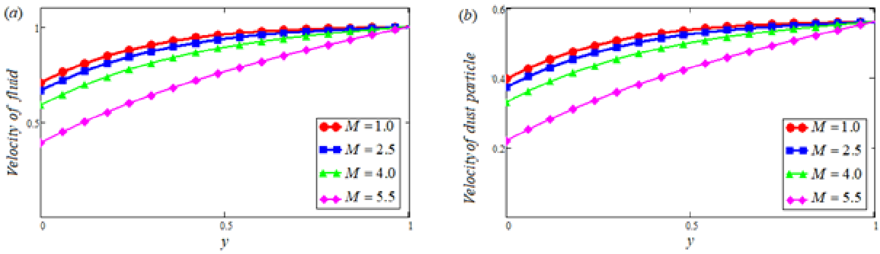

Figure 5: Effect of M on fluid velocity and dusty fluid velocity.

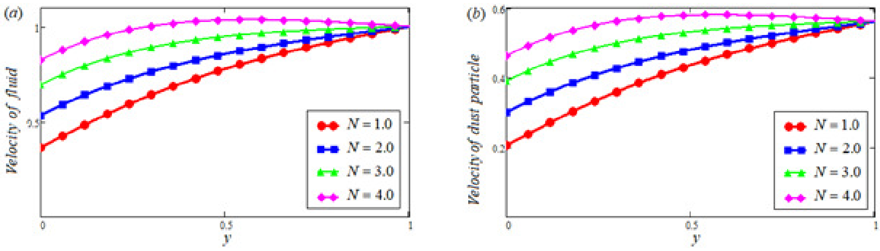

Figure 6: Effect of N on fluid velocity and dusty fluid velocity.

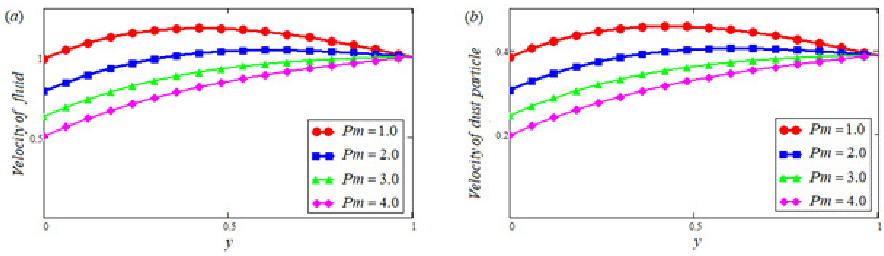

Figure 7: Effect of Pm on fluid velocity and dusty fluid velocity.

Graphical representations in Fig. 2(a) and 2(b), several values of the inclination angle are displayed. It is exposed that the dust particle and fluid's velocity is larger at $\alpha = 0^\circ$ as compared to $\alpha = 90^\circ$. Increased angle of inclination α causes friction, which reduces the velocities profiles of fluid as well as dust particles. The inclination angle represents how much the channel is tilted relative to the horizontal. When the inclination angle is small, the gravitational force acting against the flow is minimal, so the fluid and dust particles can move faster. As the inclination angle increases, the component of gravity opposing the flow direction becomes stronger, which effectively adds more frictional resistance along the walls. This additional resistance slows down the flow of both the blood fluid and the suspended particles, resulting in lower velocity profiles. Practically, this means that tilting or positioning of blood vessels, medical devices, or channels in biomedical applications

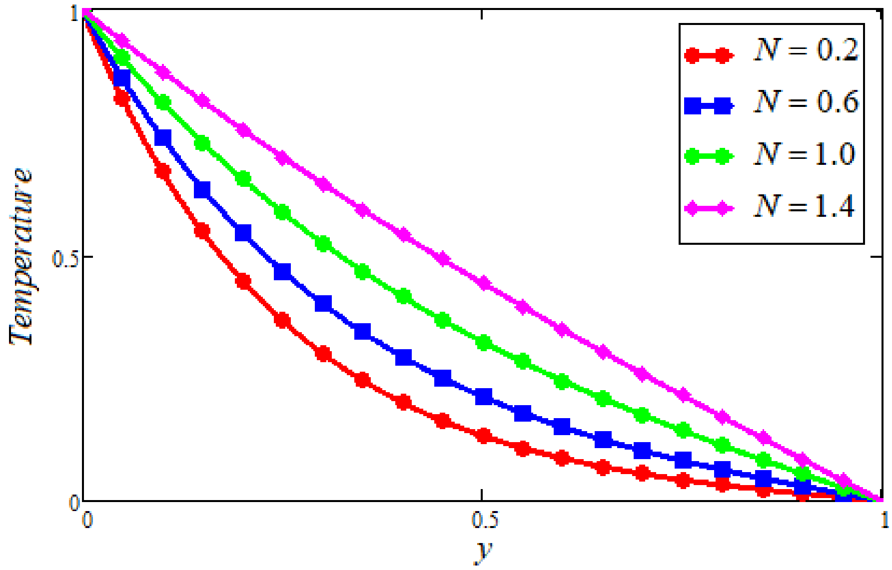

Figure 8: Effect of N on temperature.

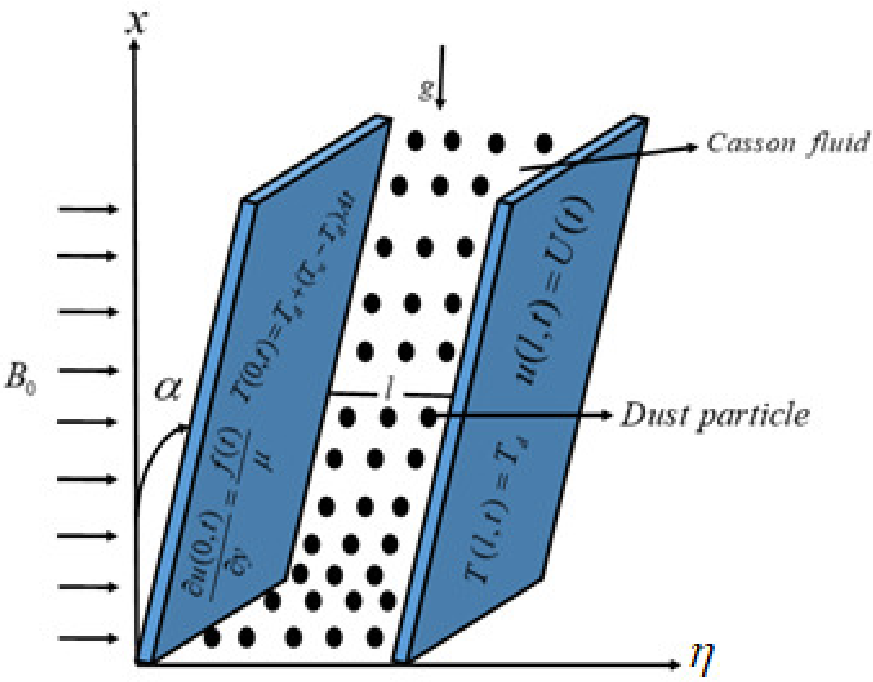

Figure 9: Flow geometry of the problem.

can significantly influence flow behavior due to gravity-induced friction, which is crucial for controlling blood circulation, targeted drug delivery, or designing efficient vascular implants.

Figures 3(a) and 3(b) illustrate that an increase in the blood fluid parameter β — which represents the non-Newtonian (viscoelastic) nature of blood — causes a noticeable reduction in both the fluid and dust particle velocities. Physically, this means that as blood becomes more viscoelastic, its internal resistance to flow increases, leading to a thinner momentum boundary layer. The thinning of this layer indicates that the fluid near the wall experiences more frictional drag and loses energy faster, resulting in slower overall flow. This behavior is relevant for real blood flow in narrow vessels or under pathological conditions where blood viscosity can change (such as in clotting disorders). Understanding this effect helps in predicting flow resistance in biomedical devices and in designing treatments for conditions where altered blood rheology plays a role.

Figures 4(a) and 4(b) depict the behaviour of Gr on dust particle and fluid velocities. Physically, this happens because higher buoyancy forces — due to stronger thermal gradients — push the fluid upward more vigorously, expanding the momentum boundary layer and accelerating the flow. This means that when thermal effects are significant, the heated fluid becomes lighter, rising and dragging particles with it. This behavior is practically important for biomedical heat treatments (like hyperthermia) and for understanding how localized heating can enhance blood flow and particle transport in medical therapies.

Figs. 5(a) and 5(b) demonstrate how magnetic parameters M affect fluid velocity profiles and dust particle velocity profiles. Physically, this is due to the Lorentz force: when an electrically conducting fluid like blood flows through a magnetic field, an opposing electromagnetic force is generated, resisting motion. This magnetic braking slows the fluid and suspended particles, thickening the boundary layer. This principle is relevant for medical applications involving magnetic fields — for example, using external magnets to control blood flow or guide magnetic drug carriers. It also has engineering implications for flow control in magnetohydrodynamic biomedical devices.

Figures 6(a) and 6(b) show that increasing the thermal radiation parameter N boosts the velocities of both the fluid and the dust particles. Physically, higher radiation means more heat is added to the fluid, raising its internal energy and temperature, which reduces viscosity and enhances flow speed. This direct link between temperature and flow behavior is crucial in medical contexts where external heating (e.g., laser therapy or hyperthermia) is used to modify blood flow for treatment, as well as in designing thermal control systems in biomedical devices.

Figures 7(a) and 7(b) reveal that increasing the particle mass ratio parameter K increases both dust particle and fluid velocities. Physically, a higher particle mass ratio reduces the drag effect of particles on the fluid, allowing both phases to accelerate more freely. This behavior shows how the presence and mass of suspended particles — like drug carriers, clots, or cellular debris — can influence blood flow speed. This is important in biomedical engineering for designing drug delivery systems and understanding how micro-particles interact with blood under varying flow conditions.

Temperature is affected by the radiation parameter in figure 8. Figure 8 shows that the fluid temperature increases with higher values of the radiation parameter. Physically, this is straightforward: greater thermal radiation injects more heat into the fluid, elevating its temperature. In real-world applications, this demonstrates how radiation-based treatments can raise blood temperature locally, which can enhance perfusion or facilitate thermal therapies. It also highlights the importance of accurately managing thermal inputs in medical devices to avoid overheating and ensure patient safety.

Figure 9(a) compares velocity of Newtonian fluid to velocity of viscoelastic blood fluid. As can be seen, viscoelastic blood fluid has a much lower velocity than Newtonian fluid. With a greater blood parameter β, the boundary layer becomes thinner, slowing down blood fluid velocity. The value of $\beta \to \infty$ then $1/\beta = 0$ has been used for Newtonian fluid. To give you an idea of scale, the velocity of Newtonian fluid is much more magnified than the velocity of blood fluid in this particular situation.

Lastly, the connection between the current solutions and the solutions presented by Khan et al. [34] was shown in Fig. 9(b). Our present solution is brought down to Khan et al. [34] solution by taking into account $\beta \to \infty$, $M = 0$, $K = 0$, and $\alpha = 0°$. Both solutions clearly overlap, demonstrating the correctness and validity of our present solutions.

4. Conclusions

In this study, we employed the Poincaré-Lighthill perturbation technique (PLPT) to analytically explore the influence of wall shear stress and temperature variation on the magnetohydrodynamic (MHD) two-phase fluctuating flow of a viscoelastic blood-dusty fluid within an inclined channel, incorporating the effects of thermal radiation and key physical parameters. The results demonstrate that the inclination angle of the channel plays a significant role in regulating the movement of both the fluid and the suspended dust particles. As the inclination increases, gravitational resistance intensifies, resulting in reduced flow velocities due to increased friction along the walls. Moreover, the study highlights the critical role of the blood parameter, which reflects the viscoelastic behavior of the fluid. An increase in this parameter leads to the formation of a thinner boundary layer and a noticeable decrease in the velocities of both the fluid and dust phases. This reduction indicates enhanced internal resistance, a typical characteristic of non-Newtonian biological fluids like blood, particularly under pathological or high-shear conditions. Thermal radiation was also found to significantly impact the flow structure. Higher values of the radiation parameter increased the energy content of the system, thereby raising the temperature and enhancing the velocity profiles of both phases. On the other hand, the

application of a transverse magnetic field generated a Lorentz force that opposed the flow direction. This magnetic resistance effectively dampened the motion of the conducting fluid and the entrained particles, demonstrating the potential for magnetic fields to control or modulate blood flow in biomedical and engineering contexts. The outcomes of this work provide deeper insights into the dynamic behavior of dusty blood fluids under complex physical conditions. These findings are relevant for the design of biomedical devices, targeted drug delivery systems, and other clinical applications where thermal and electromagnetic effects play a pivotal role in fluid transport mechanisms.
However, this study has some limitations. The model assumes idealized boundary conditions, simplified geometry (parallel inclined plates), and neglects potential biological complexities such as vessel elasticity, pulsatile nature of real blood flow, and complex particulate interactions. Additionally, only steady-state temperature gradients are considered, and experimental validation is absent.

5. Future research

Future research could extend this work by incorporating more realistic vessel geometries, elastic wall effects, non-linear boundary conditions, and time-dependent flow behaviors. Moreover, adding experimental or computational validation (e.g., CFD simulations) would strengthen the practical relevance of the theoretical predictions. Exploring the effects of non-uniform particle distributions or chemical reactions may also provide further insight into biomedical and industrial applications.

Acknowledgements

The author Tareq Alqahtani extends the appreciation to the Deanship of Postgraduate Studies and Scientific Research at Majmaah University for funding this research work through the project number (R-2025-2227).

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