EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2026, Vol. 19, Issue 1, Article Number 6973
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Evaluation of Availability in Symmetric Bridge System with Weibull Lifetime Distribution

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Abstract. This paper evaluates the availability of a symmetric bridge system composed of five identical repairable components. The system is modeled as a coherent structure that cannot be reduced to a pure series or parallel configuration. The failure and repair times of each component are assumed to follow the Weibull distribution. Five improvement techniques are considered: failure rate reduction, repair rate increase, hot, warm, and cold duplication. The availability equivalence factor is computed to compare the effectiveness of these methods. Simulation results are presented to distinguish among the improvement, revealing that cold duplication provides the highest improvement in system availability.

2020 Mathematics Subject Classifications: 60K10, 90B25, 62N05
Key Words and Phrases: Availability analysis, bridge system, Weibull distribution, failure and repair rates, improving methods, availability equivalence factors

1. Introduction

Availability and reliability are two fundamental performance measures in system engineering and maintenance planning. Availability refers to the ability of a system to operate when required, while reliability measures the ability to perform its intended function without failure for a specified period. For repairable systems, availability is influenced by both failure and repair behavior over time. Traditional models of availability often rely on exponential failure distributions due to their mathematical convenience; however, these models assume a constant failure rate, which does not accurately capture real-world behaviors such as early-life failures (infant mortality) or aging- related degradation (wear-out). In contrast, the Weibull distribution offers greater flexibility by accommodating increasing, decreasing, or constant failure rates. To improve availability, two main techniques are widely adopted: reduction, which aims to decrease the failure rates of certain components, and

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DOI: https://doi.org/10.29020/nybg.ejpam.v19i1.6973

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redundancy, where additional backup components are added in forms such as hot, warm, and cold redundancy. in repairable systems, availability can also be enhanced by increasing the repair rates of specific components. Numerous studies have addressed the modeling and improvement of system reliability, availability, and maintainability. Early foundational contributions were made by Råde (1993)[1] investigated reliability theory using probabilistic frameworks and statistical methods. Abdelfattah et al. (2009, 2014)[2],[3] investigated the reliability equivalence of systems with components that have mixed Weibull failure rates, proposing mathematical methods for constructing equivalent systems that maintain the same reliability. Alghamdi et al. (2017)[4] studied series-parallel systems using Exponentiated Weibull lifetimes, while El-Damcese et al. (2015)[5] analyzed systems with two distinct types of failure rates in series-parallel configurations and calculated the corresponding reliability equivalence factors. El-Faheem et al. (2024)[6] developed a reliability enhancement approach for radar systems using the two parameter Weibull distribution. El-Faheem and Denguir (2023)[7] applied a mixture lifetime distribution with time delay to enhance the reliability modeling of radar system designs. Sarhan et al. (2000, 2005, 2008, 2009, 2013)[[8],[9],[10],[11],[12],[13]] conducted extensive work on the reliability equivalence factors of general series-parallel systems, offering analytical formulations for both reliability and availability equivalence, particularly for systems with dependent and non-identical component lifetimes. In the context of availability analysis, Alghazo et al. (2020)[14] applied availability equivalence analysis to repairable bridge network systems using simulation methods, while El-Ghamry et al. (2022)[15] provided analytical formulas for the availability and reliability of k-out-of-n warm standby systems with fuzzy failure rates. El-Faheem et al. (2022)[16] focused on enhancing radar system reliability by utilizing Rayleigh distribution-based models. Bahri et al. (2009)[17] modeled asymptotic availability where both failure and repair rates follow Gamma distributions. Nabwey et al. (2025)[18] evaluated and upgraded the performance of a bridge system with Rayleigh lifetime distribution. Pogány et al. (2013)[19] evaluated hot duplication strategies versus survivor equivalence in systems governed by Gamma-Weibull distributions. Sridharan (2007)[20] examined the availability of series systems with cold standby components under general repair-time assumptions. Gu et al. (2006)[21] studied the reliability metrics of an n-unit cold standby repairable system with two repair facilities. Liu, Y. et al. (2010)[22] analyzed the reliability of warm standby repairable systems with n identical units and k repair facilities. Hu et al. (2012, 2016)[23],[24] conducted in-depth analyses of availability equivalence in repairable multi-state parallel-series systems with variable performance rates, extending these models to systems with mixed performance states and multi-state components—such as bridge networks—that undergo gradual degradation. Kumar et al. (2007)[25] explored system reliability improvement through alternative design choices via a practical case study. Tian et al. (2009)[26] proposed a joint reliability–redundancy optimization approach for multi-state series-parallel systems. Xia et al. (2007)[27] applied reliability equivalence factor analysis in systems governed by Gamma distributions. In this paper, we aim to evaluate the availability function and determine the availability equivalence factors of symmetric bridge structure consisting of identical and independent repairable components. To improve the overall availability, five improvement techniques are considered: failure rate reduction, repair rate increase, hot, warm, and cold duplication. The availability equivalence factor is computed for each method to measure its impact. Additionally, simulation examples using the MATHEMATICA application are conducted to validate the analytical results.

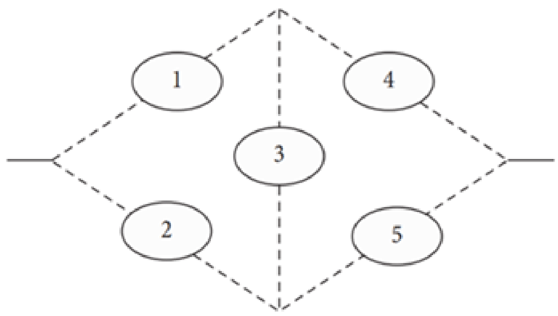

Figure 1: Bridge network system.

2. Bridge network System

Bridge configuration consists of five components arranged as illustrated in Figure 1. The component i, i=1, 2, . . . , 5, has Weibull distribution with failure rates θ and κ. Also, the repair time is Weibull with repair rates α and β. For the Weibull distribution, the probability density function (pdf) can be expressed as:

$$f(t;\theta,\kappa)=\left(\frac{\kappa}{\theta}\right)\left(\frac{t}{\theta}\right)^{\kappa-1}e^{-\left(\frac{t}{\theta}\right)^{\kappa}},\ t\geq 0;\theta,\kappa>0 \tag{1}$$

The survival (reliability) and failure rate functions are respectively given by:

$$S(t;\theta,\kappa\ )=e^{-\left(\frac{t}{\theta}\right)^{\kappa}},\quad h(t;\theta,\kappa)=\left(\frac{\kappa}{\theta}\right)\left(\frac{t}{\theta}\right)^{\kappa-1} \tag{2}$$

The Weibull mean time to failure (MTTF) and mean time to repair (MTTR) can be calculated as:

$$MTTF=\int_0^{\infty}t f\left(t\right)dt=\theta\ \Gamma\left(1+\frac{1}{\kappa}\right),\quad MTTR=\alpha\ \Gamma\left(1+\frac{1}{\beta}\right) \tag{3}$$

where $\Gamma(.)$ is the gamma function. We can calculate the availability for any component as:

$$\mathcal{A}=\frac{MTTF}{MTTR+MTTF} \tag{4}$$

Each of the five system components is assumed to be independent and identical, with availability denoted as $\mathcal{A}_i$ for i = 1, 2, . . . , 5. and calculated using the following formula:

$$\mathcal{A}_i=\frac{\theta\ \Gamma\left(1+\frac{1}{\kappa}\right)}{\theta\ \Gamma\left(1+\frac{1}{\kappa}\right)+\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}=\frac{1}{1+\eta} \tag{5}$$

Where $\eta = \frac{\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$. To construct the structure function for this system, some methods such as Tie Sets, can be used to assess the bridge network systems. A Tie Set is the minimal path where, in our case the sets [1,4],[2,5],[1,3,5], and [2,3,4] are the minimal paths. Since it is sufficient for the system to operate, all elements of at least one of the four sets shown above must operate. So, the structural planning of this system can be redesigned as in Fig 2. The equivalent design consists of four parallel paths, where each path its elements are connected in series. Let $\mathcal{A}_{sys}$ represents the availability of the bridge

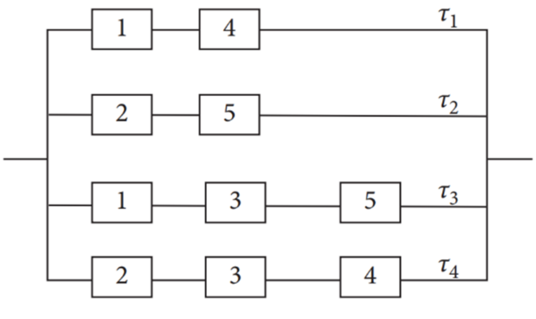

Figure 2: The equivalent design (Tie sets) for the bridge network system.

system, then $\mathcal{A}_{sys}$ can be determined as

$$\mathcal{A}_{sys} = \mathcal{A}_1\mathcal{A}_4 + \mathcal{A}_2\mathcal{A}_5 + \mathcal{A}_2\mathcal{A}_3\mathcal{A}_4 + \mathcal{A}_1\mathcal{A}_3\mathcal{A}_5 - \sum_{i=1}^{5}\prod_{j\in N_i}\mathcal{A}_i + 2\prod_{j\in N_i}\mathcal{A}_i \tag{6}$$

From equations (5) and (6), we get:

$$\mathcal{A}_{sys} = \frac{1+5\eta+8\eta^2+2\eta^3}{(1+\eta)^5} \tag{7}$$

3. Improving Methods

The following methods can be used to improve system availability:

(1) Reduction method

(2) Increase method

(3) Hot duplication method

(4) Warm duplication method

(5) Cold duplication method

3.1. Reduction Method

Suppose that the system will improve if the failure rates of the components in the set R are decreased by a factor ρ, $0<\rho<1$. Let $\mathcal{A}_{R,\rho}$ be the reduction method compliant availability function of the upgraded system. The availability $\mathcal{A}_{i,\rho}$ is provided for a component i∈R by

$$\mathcal{A}_{i,\ \rho} = \frac{\theta\ \Gamma\left(1+\frac{1}{\kappa}\right)}{\theta\ \Gamma\left(1+\frac{1}{\kappa}\right) +\ \rho\ .\ \alpha\ \Gamma\left(1+\frac{1}{\beta}\right)} = \frac{1}{1+\rho\ .\ \ \eta} \tag{8}$$

Where $\eta = \frac{\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$. The following values of R can be used to determine the availability $\mathcal{A}_{R,\rho}$

(1) R∈ $S_1 = \{\{3\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{1+2\eta^3\rho+4\eta^2(1+\rho)+\eta(4+\rho)}{(1+\eta)^4(1+\eta\rho)} \tag{9}$$

(2) R∈ $S_2 = \{\{1\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{1+\eta^3(1+\rho)+\eta(4+\rho)+\eta^2(5+3\rho)}{(1+\eta)^4(1+\eta\rho)} \tag{10}$$

(3) R∈ $S_3 = \{\{1,3\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{1+\eta^3\rho(1+\rho)+\eta(3+2\rho)+\eta^2(2+5\rho+\rho^2)}{(1+\eta)^3(1+\eta\rho)^2} \tag{11}$$

(4) R∈ $S_4 = \{\{1,2\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{1+2\eta^3\rho+\eta(3+2\rho)+\eta^2(2+6\rho)}{(1+\eta)^3(1+\eta\rho)^2} \tag{12}$$

(5) R∈ $S_5 = \{\{1,4\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{1+\eta(3+2\rho)+\eta^3(1+\rho^2)+\eta^2(3+4\rho+\rho^2)}{(1+\eta)^3(1+\eta\rho)^2} \tag{13}$$

3.2. Increase Method

Suppose that $\mathcal{A}_{I,\sigma}$ represents the availability function of the improved system after increasing the repair rates of some of the set I system elements by a factor σ, $\sigma>1$. The availability for element i∈I following an increase in repair rate can be expressed as follows:

$$\mathcal{A}_{i,\ \sigma} = \frac{\sigma\ .\ \theta\ \Gamma\left(1+\frac{1}{\kappa}\right)}{\sigma\ .\ \ \theta\ \Gamma\left(1+\frac{1}{\kappa}\right) +\ \alpha\ \Gamma\left(1+\frac{1}{\beta}\right)} = \frac{\sigma}{\sigma+\ \eta} \tag{14}$$

Where $\eta = \frac{\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$. The following values of I can be used to determine the availability $\mathcal{A}_{I,\sigma}$:

(1) I$\in S_1 = \{\{3\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\eta + 2\eta^3 + \sigma + 4\eta\sigma + 4\eta^2(1+\sigma)}{(1+\eta)^4(\eta+\sigma)} \tag{15}$$

(2) I$\in S_2 = \{\{1\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\eta + \sigma + 4\eta\sigma + \eta^3(1+\sigma) + \eta^2(3+5\sigma)}{(1+\eta)^4(\eta+\sigma)} \tag{16}$$

(3) I$\in S_3 = \{\{1,3\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\sigma^2 + \eta^3(1+\sigma) + \eta\sigma(2+3\sigma) + \eta^2(1+5\sigma+2\sigma^2)}{(1+\eta)^3(\eta+\sigma)^2} \tag{17}$$

(4) I$\in S_4 = \{\{1,2\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\sigma(2\eta^3 + \sigma + 2\eta^2(3+\sigma) + \eta(2+3\sigma))}{(1+\eta)^3(\eta+\sigma)^2} \tag{18}$$

(5) I$\in S_5 = \{\{1,4\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\sigma^2 + \eta\sigma(2+3\sigma) + \eta^3(1+\sigma^2) + \eta^2(1+4\sigma+3\sigma^2)}{(1+\eta)^3(\eta+\sigma)^2} \tag{19}$$

3.3. Hot Duplication Method

Let $\mathcal{A}_B^H$ represent the availability of the enhanced system when hot duplication is applied to elements within the set B. in this context, each element i$\in$B exhibits an availability described by the following expression:

$$\mathcal{A}_i^H = 1 - (1-\mathcal{A}_i)^2 = 1 - \left(\frac{\eta}{1+\eta}\right)^2 \tag{20}$$

So, it is possible to determine the system availability $\mathcal{A}_B^H$:

(1) B$\in S_1 = \{\{3\}\}$:

$$\mathcal{A}_B^H = \frac{1 + 6\eta + 13\eta^2 + 12\eta^3 + 2\eta^4}{(1+\eta)^6} \tag{21}$$

(2) B$\in S_2 = \{\{1\}\}$:

$$\mathcal{A}_B^H = \frac{1 + 6\eta + 14\eta^2 + 14\eta^3 + 3\eta^4}{(1+\eta)^6} \tag{22}$$

(3) B$\in S_3 = \{\{1,3\}\}$:

$$\mathcal{A}_B^H = \frac{1 + 7\eta + 20\eta^2 + 29\eta^3 + 20\eta^4 + 3\eta^5}{(1+\eta)^7} \tag{23}$$

(4) B$\in S_4 = \{\{1,2\}\}$:

$$\mathcal{A}_B^H = \frac{(1+2\eta)(1+5\eta+10\eta^2+10\eta^3+2\eta^4)}{(1+\eta)^7} \tag{24}$$

(5) B$\in S_5 = \{\{1,4\}\}$:

$$\mathcal{A}_B^H = \frac{1+7\eta+21\eta^2+33\eta^3+25\eta^4+5\eta^5}{(1+\eta)^7} \tag{25}$$

3.4. Warm Duplication Method

Assume that each component in set B is linked to an identical warm standby component (with a failure rate λ and ν) using a perfect switch. Under this configuration, the availability of the element i$\in$B is denoted by $\mathcal{A}_i^W$. Using a Markov process, $\mathcal{A}_i^W$ can be derived as shown in Liu and Zheng (2010)[22].

$$\mathcal{A}_i^W = \frac{\theta^2\Gamma^2\left(1+\frac{1}{\kappa}\right)+\theta\alpha\,\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{1}{\beta}\right)+\theta\lambda\,\Gamma\left(1+\frac{1}{\nu}\right)\Gamma\left(1+\frac{1}{\kappa}\right)}{\theta^2\Gamma^2\left(1+\frac{1}{\kappa}\right)+\theta\alpha\,\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{1}{\beta}\right)+\ \theta\lambda\,\Gamma\left(1+\frac{1}{\nu}\right)\Gamma\left(1+\frac{1}{\kappa}\right)+0.5\ \alpha^2\ \Gamma^2\left(1+\frac{1}{\beta}\right)+0.5\alpha\lambda\,\Gamma\left(1+\right.}$$

$$= \frac{1+\eta+\xi}{1+\eta+\xi+0.5\eta^2+0.5\eta\xi} \tag{26}$$

Where $\eta = \frac{\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$ and $\xi = \frac{\lambda\ \Gamma\left(1+\frac{1}{\nu}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$. So, it is possible to determine the system availability $\mathcal{A}_B^W$:

(1) B$\in S_1 = \{\{3\}\}$:

$$\mathcal{A}_B^W = \frac{2+2\eta^4+2\xi+\eta^3(12+2\xi)+\eta(10+9\xi)+\eta^2(17+12\xi)}{(1+\eta)^4(2+2\eta+\eta^2+2\xi+\eta\xi)} \tag{27}$$

(2) B$\in S_2 = \{\{1\}\}$:

$$\mathcal{A}_B^W = \frac{2+3\eta^4+2\xi+\eta^3(15+3\xi)+\eta(10+9\xi)+\eta^2(19+13\xi)}{(1+\eta)^4(2+2\eta+\eta^2+2\xi+\eta\xi)} \tag{28}$$

(3) B$\in S_3 = \{\{1,3\}\}$:

$$\mathcal{A}_B^W = \frac{3\eta^5+4(1+\xi)^2+\eta^4(21+6\xi)+\eta^3(42+40\xi+3\xi^2)+\eta(20+36\xi+16\xi^2)+\eta^2(40+58\xi+19\xi^2)}{(1+\eta)^3(2+\eta^2+2\xi+\eta(2+\xi))^2} \tag{29}$$

(4) B$\in S_4 = \{\{1,2\}\}$:

$$\mathcal{A}_B^W = \frac{4\eta^5 + 4(1+\xi)^2 + \eta^4(24+8\xi) + \eta^3(44+44\xi+4\xi^2) + \eta(20+36\xi+16\xi^2) + \eta^2(40+60\xi+20\xi^2)}{(1+\eta)^3(2+\eta^2+2\xi+\eta(2+\xi))^2} \tag{30}$$

(5) B$\in S_5 = \{\{1,4\}\}$:

$$\mathcal{A}_B^W = \frac{5\eta^5 + 4(1+\xi)^2 + \eta^4(29+10\xi) + \eta^3(52+50\xi+5\xi^2) + \eta(20+36\xi+16\xi^2) + \eta^2(44+64\xi+21\xi^2)}{(1+\eta)^3(2+\eta^2+2\xi+\eta(2+\xi))^2} \tag{31}$$

3.5. Cold Duplication Method

Assume that each component in set B is linked to an identical cold standby component via a perfect switch. The cold duplication availability of the improved system is denoted by $\mathcal{A}_B^C$. the availability of the element i$\in$B under this method is denoted by $\mathcal{A}_i^C$. Using Markov process theory, $\mathcal{A}_i^C$ can be derived as presented in Gu and Wei (2006)[21].

$$\mathcal{A}_i^C = \frac{\theta^2\ \Gamma^2\left(1+\frac{1}{\kappa}\right) +\ \theta\alpha\ \Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{1}{\beta}\right)}{\theta^2\ \Gamma^2\left(1+\frac{1}{\kappa}\right) +\ \theta\alpha\ \Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{1}{\beta}\right) + 0.5\ \alpha^2\ \Gamma^2\left(1+\frac{1}{\beta}\right)}$$
$$= \frac{1+\eta}{1+\eta+0.5\ \eta^2} \tag{32}$$

where $\eta = \frac{\alpha\ \Gamma\left(1+\frac{1}{\beta}\right)}{\theta\ \Gamma\left(1+\frac{1}{k}\right)}$. So, it is possible to determine the system availability $\mathcal{A}_B^C$:

(1) B$\in S_1 = \{\{3\}\}$:

$$\mathcal{A}_B^C = \frac{2+10\eta+17\eta^2+12\eta^3+2\eta^4}{(1+\eta)^4(2+2\eta+\eta^2)} \tag{33}$$

(2) B$\in S_2 = \{\{1\}\}$:

$$\mathcal{A}_B^C = \frac{2+10\eta+19\eta^2+15\eta^3+3\eta^4}{(1+\eta)^4(2+2\eta+\eta^2)} \tag{34}$$

(3) B$\in S_3 = \{\{1,3\}\}$:

$$\mathcal{A}_B^C = \frac{4+16\eta+24\eta^2+18\eta^3+3\eta^4}{(1+\eta)^2(2+2\eta+\eta^2)^2} \tag{35}$$

(4) B$\in S_4 = \{\{1,2\}\}$:

$$\mathcal{A}_B^C = \frac{4(1+4\eta+6\eta^2+5\eta^3+\eta^4)}{(1+\eta)^2(2+2\eta+\eta^2)^2} \tag{36}$$

(5) B$\in S_5 = \{\{1,4\}\}$:

$$\mathcal{A}_B^C = \frac{4+16\eta+28\eta^2+24\eta^3+5\eta^4}{(1+\eta)^2(2+2\eta+\eta^2)^2} \tag{37}$$

4. Availability Equivalence Factors

In this section, we focus on two specific types of availability equivalence factor (AEF): the availability equivalence reducing factor (RAEF) and the Increasing Availability Equivalence Factor (IAEF). These factors serve as measures to evaluate system improvements. Definition1. AEF is the factor that needs to be reduced (increased) to equality the availability of a better system in terms of component failure (repair) rates.

4.1. The RAEF

Availability equivalence reducing factor represents the factor by which the failure rate of a specific component set R must be decreased so that the availability of the original system matches that of an enhanced system configurations, these improved configurations are obtained through the application of hot, warm, and cold duplication to a component subset B. formally, the factor is denoted as $\rho = \rho^{\mathcal{D}}_{R,B},\ \ \mathcal{D} = H,\ W,\ C$ corresponds to hot, warm, and cold duplication, respectively. This factor is determined by solving the relevant availability equation with respect to ρ.

$$\mathcal{A}_{R,\rho} = \mathcal{A}^{\mathcal{D}}_{B}, \mathcal{D} = H,\ W,\ C \tag{38}$$

We provide the many RAEF forms of the bridge system, which can be obtained from equation (38) by selecting specific subsets R of the system components.

(1) When R$\in S_1$:

$$\rho^{\mathcal{D}}_{R,B} = \frac{(1+\eta)^4\mathcal{A}^{\mathcal{D}}_{B} - (2\eta+1)^2}{\eta\left[2\eta^2+4\eta+1-(1+\eta)^4\mathcal{A}^{\mathcal{D}}_{B}\right]} \tag{39}$$

(2) When R$\in S_2$:

$$\rho^{\mathcal{D}}_{R,B} = \frac{(1+\eta)^4\mathcal{A}^{\mathcal{D}}_{B} - \left[\eta^3+5\eta^2+4\eta+1\right]}{\eta\left[\eta^2+3\eta+1-(1+\eta)^4\mathcal{A}^{\mathcal{D}}_{B}\right]} \tag{40}$$

(3) When R$\in S_3$:

$$\rho^{\mathcal{D}}_{R,B} = \frac{-b_1 \pm \sqrt{{b_1}^2 - 4\ a_1\ c_1}}{2\ a_1} \tag{41}$$

Where $a_1 = \eta^2(\eta+1)\left[1-(1+\eta)^2\mathcal{A}^{\mathcal{D}}_{B}\right], b_1 = \eta\left[\eta^2+5\eta+2-2(1+\eta)^3\mathcal{A}^{\mathcal{D}}_{B}\right],$ *and* $c_1 = (\eta+1)\left[2\eta+1-(1+\eta)\mathcal{A}^{\mathcal{D}}_{B}\right].$

(4) When R$\in S_4$:

$$\rho^{\mathcal{D}}_{R,B} = \frac{-b_2 \pm \sqrt{b_2^2 - 4\ a_2\ c_2}}{2\ a_2} \tag{42}$$

Where $a_2 = \eta^2(1+\eta)^3\mathcal{A}^{\mathcal{D}}_{B}\ , b_2 = 2\eta\left[(1+\eta)^3\mathcal{A}^{\mathcal{D}}_{B} - \left(\eta^2+3\eta+1\right)\right],$ *and* $c_2 = (1+\eta)\left[(1+\eta)^2\mathcal{A}^{\mathcal{D}}_{B} - \ (2\eta+1)\right].$

(5) When R$\in S_5$:

$$\rho^{\mathcal{D}}_{R,B} = \frac{-b_3 \pm \sqrt{b_3^2 - 4\ a_3\ c_3}}{2\ a_3} \tag{43}$$

Where $a_3 = \eta^2 (1+\eta) \left[1 - (1+\eta)^2 \mathcal{A}^{\mathcal{D}}_B\right]$, $b_3 = 2\eta \left[\left(2\eta^2+1\right) - (1+\eta)^3 \mathcal{A}^{\mathcal{D}}_B\right]$, *and* $c_3 = (1+\eta)^3 \left(1 - \mathcal{A}^{\mathcal{D}}_B\right)$.

The values for RAEFs, $\rho^{\mathcal{D}}_{R,B}$, may be determined from (39)- (43) for the values of η, ξ and $\mathcal{A}^{\mathcal{D}}_B$ for various B and $\mathcal{D} = H\,(W,\ C)$.

4.2. The IAEF

The increasing availability equivalence factor (IAEF) refers to the multiplier by which the repair rates of a selected subset I of components must be raised, so that the resulting system achieves the same availability level as an enhanced version of the original system—where improvements are introduced through hot, warm, or cold duplication of another component set B. This factor, denoted by $\sigma = \sigma^{\mathcal{D}}_{I,B}$, is determined by solving the relevant availability equation accordingly.

$$\mathcal{A}_{I,\sigma} = \mathcal{A}^{\mathcal{D}}_B, \mathcal{D} = H,\ W,\ C \tag{44}$$

For set I, the following versions of IAEF may be computed from equation (44):

(1) When I$\in S_1$:

$$\sigma^{\mathcal{D}}_{I,B} = -\frac{\eta\left[(1+\eta)^4 \mathcal{A}^{\mathcal{D}}_B - \left(2\eta^2+4\eta+1\right)\right]}{(1+2\eta)^2 - (1+\eta)^4 \mathcal{A}^{\mathcal{D}}_B} \tag{45}$$

(2) When I$\in S_2$:

$$\sigma^{\mathcal{D}}_{I,B} = \frac{\eta\left[(1+\eta)^4 \mathcal{A}^{\mathcal{D}}_B - \left(\eta^2+3\eta+1\right)\right]}{\eta^3+5\eta^2+4\eta+1 - (1+\eta)^4 \mathcal{A}^{\mathcal{D}}_B} \tag{46}$$

(3) When I$\in S_3$:

$$\sigma^{\mathcal{D}}_{I,B} = \frac{-e_1 \pm \sqrt{{e_1}^2 - 4d_1\ f_1}}{2d_1} \tag{47}$$

Where $d_1 = (1+\eta)\left[(2\eta+1) - (1+\eta)^2 \mathcal{A}^{\mathcal{D}}_B\right]$, $e_1 = \eta\left[(\eta 2+5\eta+2) - 2\,(1+\eta)^3 \mathcal{A}^{\mathcal{D}}_B\right]$, *and* $f_1 = \eta^2 (\eta+1)\ \left[1 - (1+\eta)^2 \mathcal{A}^{\mathcal{D}}_B\right]$.

(4) When I$\in S_4$:

$$\sigma^{\mathcal{D}}_{I,B} = \frac{-e_2 \pm \sqrt{{e_2}^2 - 4d_2\ f_2}}{2d_2} \tag{48}$$

Where $d_2 = (\eta+1)\left[(2\eta+1) - (1+\eta)^2 \mathcal{A}^{\mathcal{D}}_B\right]$, $e_2 = 2\eta\left[\eta^2+3\eta+1 - (1+\eta)^3 \mathcal{A}^{\mathcal{D}}_B\right]$, *and* $f_2 = -\eta^2 (\eta+1)^3\ \mathcal{A}^{\mathcal{D}}_B$.

(5) When I$\in S_5$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{-e_3 \pm \sqrt{{e_3}^2 - 4d_3\ f_3}}{2d_3} \tag{49}$$

Where $d_3 = (\eta+1)^3\left(1 - \ \mathcal{A}_B^{\mathcal{D}}\right),\ e_3 = 2\eta\left[2\eta + 1 - (1+\eta)^3\mathcal{A}_B^{\mathcal{D}}\right],$
$and\ \ f_3 = \eta^2\,(\eta+1)\left[1 - (1+\eta)^2\mathcal{A}_B^{\mathcal{D}}\right].$

The values for the IAEFs, $\sigma_{I,B}^{\mathcal{D}}$, between (44) and (49), may be determined using the stated values of η,ξ and $\mathcal{A}_B^{\mathcal{D}}$ for various B and $\mathcal{D} = H\,(W,\ C)$.

5. Numerical Results

This section presents a numerical example to illustrate the theoretical results obtained in the previous sections. We assume α=0.1, β=2.4, $\theta = 0.4$, $\kappa = 1$, $\lambda = 0.08$, $\nu = 2.6$. In this case, we have

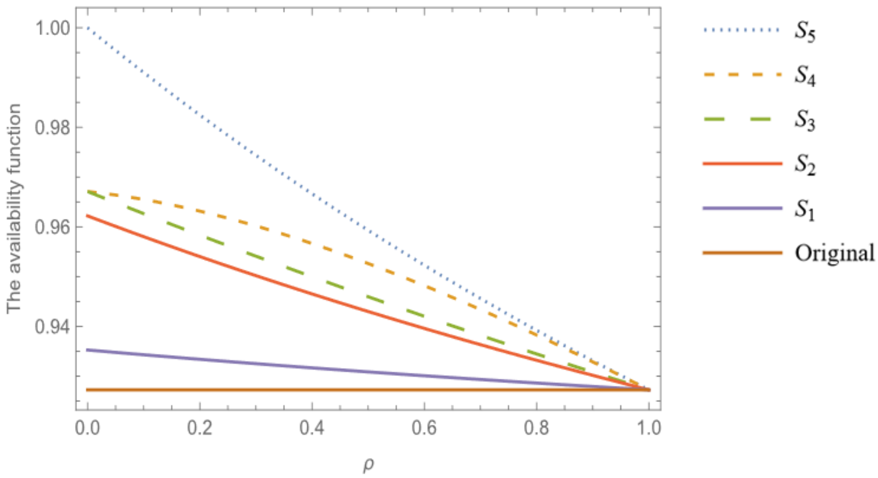

Figure 3: Availability $\mathcal{A}_{R,\rho}$ using the reduction method for $S_1$to S_5.

$$\eta = \frac{\alpha\,\Gamma\left(1+\frac{1}{\beta}\right)}{\theta\,\Gamma\left(1+\frac{1}{k}\right)} = 0.2216, and\ \xi = \frac{\lambda\,\Gamma\left(1+\frac{1}{\nu}\right)}{\theta\,\Gamma\left(1+\frac{1}{k}\right)} = 0.1776.$$

Figures 3 and 4 illustrate how different sets of components can be improved using the reduction (increase) method by the factors ρ and σ, respectively, to create a better system. Figures 3 and 4 allow us to draw the following conclusions:

(1) For all possible sets R, $\mathcal{A}_{R,\rho}$ decreases as increases ρ.

(2) For all possible sets I, $\mathcal{A}_{I,\sigma}$ increases as increases σ.

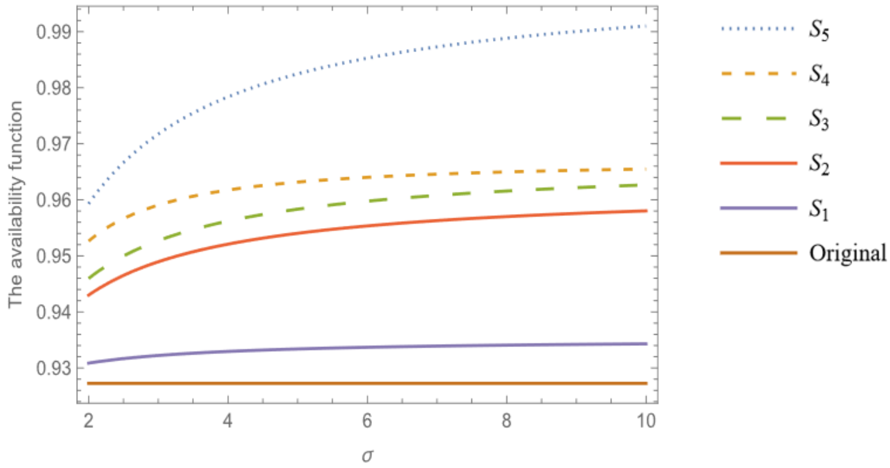

Figure 4: Availability $\mathcal{A}_{I,\sigma}$ using the increase method for $S_1$to S_5.

(3) $\mathcal{A}_{sys}<\mathcal{A}_{S_1,\rho}<\mathcal{A}_{S_2,\rho}<\mathcal{A}_{S_3,\rho}<\mathcal{A}_{S_4,\rho}<\mathcal{A}_{S_5,\rho}$, for all $0<\rho<1$.

(4) $\mathcal{A}_{sys}<\mathcal{A}_{S_1,\sigma}<\mathcal{A}_{S_2,\sigma}<\mathcal{A}_{S_3,\sigma}<\mathcal{A}_{S_4,\sigma}<\mathcal{A}_{S_5,\sigma}$, for all $\sigma>1$.

The availability of the original system is $\mathcal{A}_{sys} = 0.92726$, and $\mathcal{A}_B^{\mathcal{D}}$ for the improved system for all possible sets B are shown in Table 1. The following may be concluded from the results in Table 1:

(1) $\mathcal{A}_{sys}<\mathcal{A}_B^H<\mathcal{A}_B^W<\mathcal{A}_B^C$ for all B $\in S_i$.

(2) $\mathcal{A}_{sys}<\mathcal{A}_{S_1}^{\mathcal{D}}<\mathcal{A}_{S_2}^{\mathcal{D}}<\mathcal{A}_{S_3}^{\mathcal{D}}<\mathcal{A}_{S_4}^{\mathcal{D}}<\mathcal{A}_{S_5}^{\mathcal{D}}$, $\mathcal{D} = H\,(W,\ C)$.

Table 1: $\mathcal{A}_B^{\mathcal{D}}$, $\mathcal{D} = H\,(W,\ C)$ and B$\in S_i$,i=1,2,...,5.

$B \in S_i$	$\mathbf{A}_B^H$	$\mathbf{A}_B^W$	$\mathbf{A}_B^C$
S_1	0.93381	0.93391	0.93440
S_2	0.95586	0.95630	0.95841
S_3	0.96032	0.96080	0.96306
S_4	0.96432	0.96457	0.96567
S_5	0.98638	0.98731	0.99182

The AEFs, $\rho_{R,B}^{\mathcal{D}}$, and $\sigma_{R,B}^{\mathcal{D}}$ for different values of $\mathcal{D},\ R, I,\ and\ B$ are then computed using the previous theoretical formulas. AEFs values may be found in Tables 2–6.

6. Discussion

Based on the statistical outcomes presented in Tables 1–6, the following observations can be made:

Table 2: AEFs for various sets B and R $\in S_1$.

$B \in S_i$	$\rho^D_{R,B}$			$\sigma^D_{I,B}$		
	Hot	*Warm*	*Cold*	*Hot*	*Warm*	*Cold*
S_1	0.15356	0.14267	0.09071	6.51222	7.00923	11.02441
S_2	NA	NA	NA	NA	NA	NA
S_3	NA	NA	NA	NA	NA	NA
S_4	NA	NA	NA	NA	NA	NA
S_5	NA	NA	NA	NA	NA	NA

Table 3: AEFs for various sets B and R $\in S_2$.

$B \in S_i$	$\rho^D_{R,B}$			$\sigma^D_{I,B}$		
	Hot	*Warm*	*Cold*	*Hot*	*Warm*	*Cold*
S_1	0.78013	0.77693	0.76143	1.28182	1.28712	1.31331
S_2	0.15356	0.14267	0.09071	6.51222	7.00923	11.0244
S_3	0.04437	0.03314	NA	22.5346	30.1737	NA
S_4	NA	NA	NA	NA	NA	NA
S_5	NA	NA	NA	NA	NA	NA

(1) The system availability improves from 0.92726 to 0.93381 when the components within the subset B $\in S_1$ are improved using the hot duplication method (See Table 1). To achieve a system availability of $\mathcal{A}^H_B = 0.93381$, this result can alternatively be obtained by applying one the following methods:

(a) Reducing the failure rate of the components in: (i) R $\in S_1$ by the factor $\rho^H = 0.153558$, (ii) R $\in S_2$ by the factor $\rho^H = 0.78013$, (iii) R $\in S_3$ by the factor $\rho^H = 0.81779$, (iv) R $\in S_4$ by the factor $\rho^H = 0.88301$, (v) R $\in S_5$ by the factor $\rho^H = 0.88825$ (see Tables 2–6).

(b) Increasing the repair rate of the components in: (i) I$\in S_1$ by the factor $\sigma^H = 6.51222$, (ii) I $\in S_2$ by the factor $\sigma^H = 1.28182$, (iii) I$\in S_3$ by the factor $\sigma^H = 1.22282$, (iv) I I$\in S_4$ by the factor $\sigma^H = 1.13249$, (v) I I$\in S_5$ by the factor $\sigma^H = 1.12581$ (see Tables 2–6).

(2) The system availability improves from 0.92726 to 0.93391 when the system compo-

Table 4: AEFs for various sets B and R $\in S_3$.

$B \in S_i$	$\rho^D_{R,B}$			$\sigma^D_{I,B}$		
	Hot	*Warm*	*Cold*	*Hot*	*Warm*	*Cold*
S_1	0.81779	0.81508	0.80198	1.22282	1.22687	1.24691
S_2	0.25808	0.24776	0.19819	3.87467	4.03609	5.04549
S_3	0.15355	0.14267	0.09071	6.51222	7.00923	11.0244
S_4	0.06204	0.05632	0.03169	16.1178	17.7561	31.5470
S_5	NA	NA	NA	NA	NA	NA

Table 5: AEFs for various sets B and R $\in S_4$.

$B \in S_i$	$\rho^D_{R,B}$			$\sigma^D_{I,B}$		
	Hot	$Warm$	$Cold$	Hot	$Warm$	$Cold$
S_1	0.88301	0.88118	0.87233	1.13249	1.13484	1.14635
S_2	0.42042	0.40913	0.35205	2.37856	2.44420	2.84050
S_3	0.29555	0.28083	0.20347	3.38352	3.56091	4.91466
S_4	0.15356	0.14267	0.09071	6.51222	7.00923	11.0244
S_5	NA	NA	NA	NA	NA	NA

Table 6: AEFs for various sets B and R $\in S_5$.

$B \in S_i$	$\rho^D_{R,B}$			$\sigma^D_{I,B}$		
	Hot	$Warm$	$Cold$	Hot	$Warm$	$Cold$
S_1	0.88825	0.88659	0.87857	1.12581	1.12792	1.13821
S_2	0.54839	0.54219	0.51242	1.82350	1.84437	1.95150
S_3	0.48567	0.47916	0.44810	2.05899	2.08700	2.23162
S_4	0.43101	0.42759	0.41293	2.32014	2.33866	2.42169
S_5	0.15356	0.14267	0.09071	6.51222	7.00923	11.0244

nents from the set B$\in S_1$ are warm duplicated (see Table 1). One of the following methods can be used to produce the design with $\mathcal{A}^W_B = 0.93391$:

(a) Reducing the failure rate of the components in: (i) R$\in S_1$, $\rho^W = 0.14267$, (ii) R0 $\in S_2$, $\rho^W = 0.77693$, (iii) R$\in S_3$, $\rho^W = 0.81508$, (iv) R$\in S_4$, $\rho^W = 0.88118$, (v) R$\in S_5$, $\rho^W = 0.88659$ (see Tables 2–6).

(b) Increasing the repair rate of the components in: (i) I$\in S_1$, $\sigma^W = 7.00923$, (ii) I$\in S_2$, $\sigma^W = 1.28712$, (iii) I$\in S_3$, $\sigma^W = 1.22687$, (iv) I$\in S_4$, $\sigma^W = 1.13484$, (v) I$\in S_5$, $\sigma^W = 1.12792$ (see Tables 2–6).

(3) The availability improves from 0.92726 to 0.93440 when the system components from the set B$\in S_1$ are cold duplicated (see Table 1). One of the following methods can be used to produce the design with $\mathcal{A}^C_B = 0.93440$:

(a) Reducing the failure rate of the components in: (i) R$\in S_1$, $\rho^C = 0.09071$, (ii) R$\in S_2$, $\rho^C = 0.76143$, (iii) R$\in S_3$, $\rho^C = 0.80198$, (iv) R$\in S_4$, $\rho^C = 0.87233$, (v) R$\in S_5$, $\rho^C = 0.87857$ (see Tables 2–6).

(b) Increasing the repair rate of the components in: (i) I$\in S_1$, $\sigma^C = 11.0244$, (ii) I$\in S_2$, $\sigma^C = 1.31331$, (iii) I$\in S_3$, $\sigma^C = 1.24691$, (iv) I$\in S_4$, $\sigma^C = 1.14635$, (v) I$\in S_5$, $\sigma^C = 1.13821$ (see Tables 2–6).

(4) The notation "NA" indicates that the availability equivalence factor is not available for the corresponding case.

7. Conclusion

In this study, a detailed availability analysis was conducted for a symmetric bridge system consisting of five identical, repairable components, with failure and repair rates modeled using the Weibull distribution to capture realistic system behavior. Five enhancement techniques: failure rate reduction, repair rate increase, hot, warm, and cold duplication. The availability equivalence factors were derived to quantify and compare the impact of each method. Both analytical and simulation-based evaluations were used to validate the effectiveness of these methods. Among all methods, cold duplication consistently demonstrated the highest potential in enhancing availability.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflict of interest.

Acknowledgements

The authors extend their appreciation to Prince Sattam bin Abdulaziz University for funding this research work through the project number (PSAU/2025/01/36908).

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