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Smart Irrigation Frameworks Using Computational Complex Neutrosophic Fuzzy Hesitant Distance Measures

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Abstract. In decision-making and intelligent systems, accurately measuring distances in uncertain and high-dimensional environments remains a challenge. Traditional distance measures often fail to capture the hesitancy, indeterminacy, and blurriness intrinsic in practical statistical scenarios. To address this limitation, we propose novel complex neutrosophic fuzzy hesitance distance measures, specifically the complex neutrosophic fuzzy Euclidean distance measure with hesitance degree. These measures extend existing fuzzy distance concepts by incorporating truth, indeterminacy, and falsity components, allowing for more comprehensive and precise analysis of uncertainty. The paper presents Euclidean distance measure's definitions and its operations. Furthermore, through computational modeling, we integrate complex neutrosophic fuzzy Euclidean hesitance distance measure (CNFEHDM) into a MCDM framework, applied to a smart irrigation system for small-scale farmers. The proposed approach demonstrates enhanced precision and adaptability in managing vague and inconsistent data. Application results show that water availability is the most influential factor, enabling the smart irrigation system to optimize water usage, prevent over-irrigation, and support sustainable farming practices. A comparative analysis highlights the advantages of normalized Euclidean distance measure over existing methods, showcasing its superior ability to navigate complex, vague, and inconsistent data sets.

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1. Introduction

Zadeh (1965) introduces the fuzzy set where every element has a membership value from 0 and 1, enabling flexible classification. Key operations like union, intersection, and complement are extended to this framework. Additionally, another theorem for convex fuzzy sets is established with no need of disjointness [1]. Fuzzy sets plays a key role in computational intelligence, solving problems in control, pattern recognition, and planning. Pedrycz & Gomide's book, bridges theory and practices explaining their use, design, and real-world applications with minimal mathematical prerequisites [2]. Safety and reliability are assessed using probabilistic risk assessment methods, but data scarcity can pose challenges. Fuzzy set theory addresses this uncertainty through fuzzy techniques. Kabir and Papadopoulos reviewed these methods, their applications, and their role in enhancing reliability analysis in their paper [3].

In soil science, they are applied to classification, mapping, land evaluation, modeling, geostatistics, and soil quality assessment. Fuzzy systems also aid in simulating complex soil processes and human reasoning [4]. A definition of intuitionistic fuzzy set was discovered by Atanassov, the hindmost idea extends the idea of a "fuzzy set," and an illustrative example is provided. Several properties related to set operations, relations, model and topological operators defined on the set of IFSs are also established.[5]. Handling uncertain information is tough, but several factors in decision making makes decision-making smarter, Khan and Raza introduces priority-based operators and new Aczel-Alsina t-norms in fuzzy set theory. To tackle data loss in rough fuzzy sets, IFRS is used, leading to powerful new aggregation operators. Applied to medical diagnosis, this approach outperforms existing methods, proving its superiority [6]. Intuitionistic fuzzy sets enhance uncertainty handling but lack a fully effective distance measure. Xiao proposes a new IFS distance measure using Jensen-Shannon divergence, ensuring compliance with axioms and improving discrepancy detection. Numerical examples demonstrate its superiority over existing methods. Additionally, a pattern classification algorithm is developed, offering a robust solution for inference problems [7].

Zeng introduces an recently developed distance measure for IFS, considering membership, non-membership, their difference, and an exponential distance measure to prevent data missing. It satisfies axiomatic requirements and outperforms existing methods. Applied to pattern recognition, results show improved accuracy and broader applicability [8]. Pythagorean fuzzy sets generalize intuitionistic fuzzy sets, and thus they are more useful in decision science because of their special treatment of indeterminacy. Ejegwa and Awolola presents new distance measures, considering most important parameters and a guarantee of reliability. A numerical illustration proves their efficacy, and comparisons show that they outperform others. Applications to pattern recognition exemplify their usability in decision-making under uncertainty [9]. Selecting the best database management system is a difficult MADM problem. Rehman introduces probability-based aggregation operators in complex fuzzy sets and presents a MADM method to deal with fuzzy data. A numerical example on database management system selection confirms its efficacy, with comparisons indicating its superiority [10]. Ramot et al. investigates complex fuzzy set, whose membership values go beyond [0,1]. It presents an intuitive interpretation of complex valued membership and its applications, like solar activity modeling and signal processing. Basic operations like intersection, union and complement are investigated, as well as concepts such as set rotation and reflection. Also studied are complex fuzzy relations, with emphasis placed on the mathematical properties of this system [11].

Liue et al. investigates information measures in complex fuzzy sets for decision and machine learning production. They investigates the interplay between cross-entropy measures and distance measures, introducing complex fuzzy discrimination measures and symmetry cross-entropy measures. The research confirms that satisfy distance measure requirements and justifies their efficiency on the basis of real-life instances [12]. Hu et al. explores distance measures for complex fuzzy sets, where membership values will be on the circle of complex plane. Several distance measures are introduced and applied to discrete faults in complex fuzzy operations [13].

Ma et al. introduces new operations and laws for complex fuzzy sets, where membership values extend. Basic results on intersection, union, and complement functions are presented with examples. Complex fuzzy sets are applied to signal processing, resembling Fourier transforms in certain cases. A new algorithm using inverse discrete Fourier transforms is developed to identify reference signals from detected signals in a digital receiver [14]. Dai introduces the CIFOWD measure for complex intuitionistic fuzzy sets, enhancing decision-making by handling uncertain data. It includes special cases like CIFOWGD and CIFOWED, reducing extreme deviations. The approach is demonstrated with a COVID-19 vaccine selection example and compared with existing methods [15]. Paulraj and Tamilarasi introduces recently developed accumulation operators in a neutrosophic environment for multi-attribute decision-making. The single-valued trapezoidal neutrosophic generalized ordered weighted harmonic averaging operator extends the operator. The proposed methods are tested with illustrative examples, demonstrating their effectiveness [16]. Smarandache extend the notion of neutrosophic sets and various functions [17].

Ye introduces single-valued neutrosophic linguistic sets, numbers as extensions of linguistic and intuitionistic linguistic concepts. They define their operational rules, propose a generalized distance measure, and evolve a elongated TOPSIS method for multi-attribute group decision-making. A case study on investment alternatives demonstrates the approach's effectiveness [18]. Chi and Liu extends the TOPSIS method to interval neutrosophic Sets for handling partial, uncertain, and conflicting information in MADM. Since traditional TOPSIS relies on crisp numbers, they propose an expanded approach that accommodates INS-valued attributes with unknown weights and define INS operations, introduce a distance measure, determine attribute weights using the Maximizing Deviation method. This elongated TOPSIS steps was then applied to another levels, with built model validating its effectiveness [19]. Nagarajan et al. introduced a process to simplify MGDM by reducing criteria overload. Using neutrosophic ensembles, our approach handles uncertainty and ranks alternatives efficiently. Applied to 3D printer selection, it integrates neutrosophic numbers with hybrid score and accuracy functions, demonstrating its effectiveness [20]. Bui et al. introduces entropy-based measures for neutrosophic fuzzy sets and a refined decision-making algorithm. Applied to Vietnam's high school exam subject selection, it outperforms previous methods, with results rigorously analyzed [21]. Ali et al. proposes novel Hausdorff distance measures for single-valued neutrosophic sets and applies them to MCDM. This method effectively ranks alternatives, demonstrating its practicality in practical issues [22]. Mondal et al. introduces three similarity measures-cosine, Jaccard, and Dice for complex neutrosophic fuzzy sets to handle uncertainty in periodic decision-making. They extend these to weighted measures, prove key properties, and propose a tangent basis to regulate attribute amount. The approach is applied to MADM, along a student stream selection example demonstrating its effectiveness [23]. Paul [24] proposed the drawbacks of existing methods, satisfying PFCC axioms and enhancing reliability and precision in multi-criteria decision-making. Bilal [25], Complex intuitionistic fuzzy sets manage uncertainty using membership and non-membership values, with a distance measure defining s-equalities for effective real-world decisions. Kousar [26], Agritourism supports rural diversification in Pakistan; a fuzzy rough set based MCDM study identified farm tours, stays and retail activities as key drivers of sustainable development. Vrioni [27], Complex cubic neutrosophic sub-bisemiring explores structural and homomorphic properties, showing that its images and pre-images preserve the same form, ensuring mathematical consistency. Kamran [28], A neutrosophic Z-number fuzzy solid transportation model was proposed, handling interval and fuzzy uncertainties through parametric and sensitivity analysis. Kamran [29], The Fermatean fuzzy soft linear space models uncertainty with defined structures and theorems, enhanced by machine learning for better decision performance. Kamran [30], Recent studies use Neutrosophic Z-rough numbers with sine-trigonometric functions to enhance reliability and handle uncertainty in MCDM for sustainable and Industry 4.0 decisions. Kaviyarasu [31], TpVFFNs efficiently handle uncertainty in decision-making. Dombi t-norm and t-conorm-based operators ensure symmetric aggregation, supporting MAGDM applications like transportation service evaluation.

The primary objective of the reseach is initiate novel decision-making models under complex neutrosophic fuzzy sets integrating with Euclidean distance measures referred to as CNFEDM. Several definition and theorem with example are explored within the framework of the proposed CNFEDMs with hesitance degree, leveraging the complement, max-min complex neutrosophic fuzzy operations. Additionally, we examine the fuctional use of CNFEHDMs in solving decision-making problems by integrating a newly developed algorithm that incorporates CNFEHDMs within the decision framework. This integration provides a robust approach to handling real-world uncertainties and complexities. Furthermore, a comparative observation has been done in between CNHEDMs and CNEDMs to highlight their advantages and productiveness.

1.1. Research Gap

The potential of CNFS is still underutilized, especially in distance based models, despite notable developments in fuzzy and neutrosophic set theories for multi attribute decision-making. The hesitation degree, a crucial element in complicated contexts with imprecise information, is not adequately taken into account by current Euclidean distance measures. Furthermore, there is a methodological vacuum in effectively representing real-world uncertainty because no previous work has systematically integrated hesitance based similarity and distance functions into the CNFS structure for applying in smart irrigation . This restriction limits the adaptability and sensitivity of MADM applications under CNFS, especially in dynamic settings like environmental evaluation, medical diagnostics, and intelligent irrigation. To enable more expressive, flexible, and reliable decision-making frameworks, a mathematically sound and hesitancy aware Euclidean distance measure specifically designed for CNFS is urgently needed.

1.2. Motivation for the Research:

- The primary motivation for employing CNFEHDM is their capability to handle complex and uncertain scenarios where ambiguity and imprecision exist in the relationships between elements.
- CNFS, integrating the Euclidean distance measure, provides a structured approach for assessing and quantifying varying levels of confidence, hesitation, and uncertainty in interconnected systems.
- This framework facilitates the fusion and differentiation of uncertain information, making it particularly effective for decision-making applications that involve multiple uncertain and imprecise inputs day-to-day contexts.
- This approach finds application in different like risk analysis, resolved support systems, and enhanced problems, for achieving a balance between practical utility and unresolved complexities is crucial.

1.3. Novelty of Work:

- The incorporation of neutrosophic components in CNFS introduces a refined mechanism for handling hesitation and uncertainty, enabling a structured representation of ambiguous relationships.
- Utilizing the Euclidean distance measure enhances the evaluation of interactions, ensuring that the selection of elements and their associations is guided by a quantified confidence level that accounts for uncertainty and vagueness.
- This framework facilitates a systematic approach to managing ambiguity by distinctly differentiating between strong and uncertain relationships, allowing for more precise decision-making.
- A multi-dimensional analysis is achievable within CNFS, where different levels of uncertainty and hesitation are mapped using specific neutrosophic parameters. This approach provides a comprehensive evaluation of relationships, ensuring clarity in complex decision-making scenarios.

Structure of the Paper: Section 1 contains the brief introduction for my research work. Some basic definition of CNFS and theorems are covered in section 2. Then we proposed definition of CNEDM with their examples and theorem in Section 3. The application of smart irrigation for small scale farmers and its result calculation was worked out in Section 4 and the final portion of the paper, Section 5 contains the conclusion.

2. Preliminaries

Definition 1. *Let $\mathcal{S}$ be a nonempty set. A FS $\mathfrak{J}$ is defined as:* $\mathfrak{J} = \{\langle \mathrm{j}, \mathcal{F}_{\mathfrak{J}}(\mathrm{j})\rangle : \mathrm{j} \in \mathcal{S}\}$, *where* $\mathcal{F}_{\mathfrak{J}}(\mathrm{j}) : \mathcal{S} \to [0,1]$

Definition 2. *An NS $\mathfrak{J}$ is defined as:* $\mathfrak{J} = \{\langle \mathrm{j}, \nu_{\mathfrak{J}}(\mathrm{j}), \sigma_{\mathfrak{J}}(\mathrm{j}), \mu_{\mathfrak{J}}(\mathrm{j})\rangle : \mathrm{j} \in \mathcal{S}\}$, $\nu_{\mathfrak{J}}(\mathrm{j}) : \mathcal{S} \to [0,1]$ *denote the truth grade value,* $\sigma_{\mathfrak{J}}(\mathrm{j}) : \mathcal{S} \to [0,1]$ *denote the indeterminacy grade value,* $\mu_{\mathfrak{J}}(\mathrm{j}) : \mathcal{S} \to [0,1]$ *denote the falsity grade value, where satisfies the condition* $0 \leq (\nu_{\mathfrak{J}}(\mathrm{j})) + (\sigma_{\mathfrak{J}}(\mathrm{j})) + (\mu_{\mathfrak{J}}(\mathrm{j})) \leq 1$.

Definition 3. *A CNFS is characterised by a grade value* $\nu_{\mathfrak{J}}(\mathrm{j}) = T_{\mathfrak{J}}(\mathrm{j})e^{iu_{\mathfrak{J}}(\mathrm{j})}, \sigma_{\mathfrak{J}}(\mathrm{j}) = I_{\mathfrak{J}}(\mathrm{j})e^{iv_{\mathfrak{J}}(\mathrm{j})}, \mu_{\mathfrak{J}}(\mathrm{j}) = F_{\mathfrak{J}}(\mathrm{j})e^{iw_{\mathfrak{J}}(\mathrm{j})}$. *The Hesitance degree of CNFS is defined by* $h_{\mathfrak{J}}(\mathrm{j}) = \{|(1 - T_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - u_{\mathfrak{J}}(\mathrm{j})))|, |(1 - I_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - v_{\mathfrak{J}}(\mathrm{j})))|, |(1 - F_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - w_{\mathfrak{J}}(\mathrm{j})))|\}$ *written as*

$$\begin{aligned}\mathfrak{J} =&\{\langle \mathrm{j}, \nu_{\mathfrak{J}}(\mathrm{j}), \sigma_{\mathfrak{J}}(\mathrm{j}), \mu_{\mathfrak{J}}(\mathrm{j})\rangle : \mathrm{j} \in \mathcal{S}\},\\ \mathfrak{J} =&T_{\mathfrak{J}}(\mathrm{j})e^{iu_{\mathfrak{J}}(\mathrm{j})}, I_{\mathfrak{J}}(\mathrm{j})e^{iv_{\mathfrak{J}}(\mathrm{j})}, F_{\mathfrak{J}}(\mathrm{j})e^{iw_{\mathfrak{J}}(\mathrm{j})}\end{aligned}$$

Definition 4. *Let $\mathfrak{J}_n$ be n number of CNFS defined on $\mathcal{S}$ and* $\nu_{\mathfrak{J}}(\mathrm{j}) = T_{\mathfrak{J}}(\mathrm{j})e^{iu_{\mathfrak{J}}(\mathrm{j})}, \sigma_{\mathfrak{J}}(\mathrm{j}) = I_{\mathfrak{J}}(\mathrm{j})e^{iv_{\mathfrak{J}}(\mathrm{j})}, \mu_{\mathfrak{J}}(\mathrm{j}) = F_{\mathfrak{J}}(\mathrm{j})e^{iw_{\mathfrak{J}}(\mathrm{j})}$ *their grade value. The CNF cartesian product of $\mathfrak{J}_n$ represented by* $\times_{i=1}^{n}\mathfrak{J}_i = \mathfrak{J}_1 \times \mathfrak{J}_2 \times \cdots \times \mathfrak{J}_n$ *is defined by the function*

$$\begin{aligned}\nu_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j}) =&T_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})e^{iu_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})}\\ =&\{T_{\mathfrak{J}_1}(\mathrm{j}) \wedge T_{\mathfrak{J}_2}(\mathrm{j}) \wedge \cdots \wedge T_{\mathfrak{J}_n}(\mathrm{j})\}e^{i\{u_{\mathfrak{J}_1}(\mathrm{j}) \wedge u_{\mathfrak{J}_2}(\mathrm{j}) \wedge \cdots \wedge u_{\mathfrak{J}_n}(\mathrm{j})\}}\\ \sigma_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j}) =&I_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})e^{iv_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})}\\ =&\{I_{\mathfrak{J}_1}(\mathrm{j}) \wedge I_{\mathfrak{J}_2}(\mathrm{j}) \wedge \cdots \wedge I_{\mathfrak{J}_n}(\mathrm{j})\}e^{i\{v_{\mathfrak{J}_1}(\mathrm{j}) \wedge v_{\mathfrak{J}_2}(\mathrm{j}) \wedge \cdots \wedge v_{\mathfrak{J}_n}(\mathrm{j})\}}\\ \mu_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j}) =&F_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})e^{iw_{\times_{i=1}^{n}\mathfrak{J}_i}(\mathrm{j})}\\ =&\{F_{\mathfrak{J}_1}(\mathrm{j}) \vee F_{\mathfrak{J}_2}(\mathrm{j}) \vee \cdots \vee F_{\mathfrak{J}_n}(\mathrm{j})\}e^{i\{w_{\mathfrak{J}_1}(\mathrm{j}) \vee w_{\mathfrak{J}_2}(\mathrm{j}) \vee \cdots \vee w_{\mathfrak{J}_n}(\mathrm{j})\}}\end{aligned}$$

Definition 5. *Let $\mathfrak{J}_1$ and $\mathfrak{J}_2$ be any two CNFS on S then the intersection of two CNFS is written as*

$$\begin{aligned}\mathfrak{J}_1 \cap \mathfrak{J}_2 =&T_{\mathfrak{J}_1}(\mathrm{j})e^{iu_{\mathfrak{J}_1}(\mathrm{j})} \cap T_{\mathfrak{J}_2}(\mathrm{j})e^{iu_{\mathfrak{J}_2}(\mathrm{j})}\\ =&[T_{\mathfrak{J}_1}(\mathrm{j}) \wedge T_{\mathfrak{J}_2}(\mathrm{j})]e^{i[u_{\mathfrak{J}_1}(\mathrm{j}) \wedge u_{\mathfrak{J}_2}(\mathrm{j})]}\\ \mathfrak{J}_1 \cap \mathfrak{J}_2 =&I_{\mathfrak{J}_1}(\mathrm{j})e^{iv_{\mathfrak{J}_1}(\mathrm{j})} \cap I_{\mathfrak{J}_2}(\mathrm{j})e^{iv_{\mathfrak{J}_2}(\mathrm{j})}\end{aligned}$$

$$=[I_{\mathfrak{J}_1}(\mathrm{j}) \wedge I_{\mathfrak{J}_2}(\mathrm{j})]e^{i[v_{\mathfrak{J}_1}(\mathrm{j}) \wedge v_{\mathfrak{J}_2}(\mathrm{j})]}$$

$$\mathfrak{J}_1 \cap \mathfrak{J}_2 = F_{\mathfrak{J}_1}(\mathrm{j})e^{iw_{\mathfrak{J}_1}(\mathrm{j})} \cap F_{\mathfrak{J}_2}(\mathrm{j})e^{iw_{\mathfrak{J}_2}(\mathrm{j})}$$

$$=[F_{\mathfrak{J}_1}(\mathrm{j}) \vee F_{\mathfrak{J}_2}(\mathrm{j})]e^{i[w_{\mathfrak{J}_1}(\mathrm{j}) \vee w_{\mathfrak{J}_2}(\mathrm{j})]}$$

Definition 6. *Let $\mathfrak{J}_1$ and $\mathfrak{J}_2$ be any two CNFS on S then the union of two CNFS is written as*

$$\mathfrak{J}_1 \cup \mathfrak{J}_2 = T_{\mathfrak{J}_1}(\mathrm{j})e^{iu_{\mathfrak{J}_1}(\mathrm{j})} \cup T_{\mathfrak{J}_2}(\mathrm{j})e^{iu_{\mathfrak{J}_2}(\mathrm{j})}$$

$$=[T_{\mathfrak{J}_1}(\mathrm{j}) \vee T_{\mathfrak{J}_2}(\mathrm{j})]e^{i[u_{\mathfrak{J}_1}(\mathrm{j}) \vee u_{\mathfrak{J}_2}(\mathrm{j})]}$$

$$\mathfrak{J}_1 \cup \mathfrak{J}_2 = I_{\mathfrak{J}_1}(\mathrm{j})e^{iv_{\mathfrak{J}_1}(\mathrm{j})} \cup I_{\mathfrak{J}_2}(\mathrm{j})e^{iv_{\mathfrak{J}_2}(\mathrm{j})}$$

$$=[I_{\mathfrak{J}_1}(\mathrm{j}) \vee I_{\mathfrak{J}_2}(\mathrm{j})]e^{i[v_{\mathfrak{J}_1}(\mathrm{j}) \vee v_{\mathfrak{J}_2}(\mathrm{j})]}$$

$$\mathfrak{J}_1 \cup \mathfrak{J}_2 = F_{\mathfrak{J}_1}(\mathrm{j})e^{iw_{\mathfrak{J}_1}(\mathrm{j})} \cup F_{\mathfrak{J}_2}(\mathrm{j})e^{iw_{\mathfrak{J}_2}(\mathrm{j})}$$

$$=[F_{\mathfrak{J}_1}(\mathrm{j}) \wedge F_{\mathfrak{J}_2}(\mathrm{j})]e^{i[w_{\mathfrak{J}_1}(\mathrm{j}) \wedge w_{\mathfrak{J}_2}(\mathrm{j})]}$$

Definition 7. *Let $\mathfrak{J}$ be any two CNFS on S then the complement of CNFS $\mathfrak{J}$ is written as*

$$\mathfrak{J}^c = T_{\mathfrak{J}^c}(\mathrm{j})e^{iu_{\mathfrak{J}^c}(\mathrm{j})} = [1 - T_{\mathfrak{J}}(\mathrm{j})]e^{iu_{\mathfrak{J}}(\mathrm{j})}$$

$$\mathfrak{J}^c = I_{\mathfrak{J}^c}(\mathrm{j})e^{iv_{\mathfrak{J}^c}(\mathrm{j})} = [1 - I_{\mathfrak{J}}(\mathrm{j})]e^{iv_{\mathfrak{J}}(\mathrm{j})}$$

$$\mathfrak{J}^c = F_{\mathfrak{J}^c}(\mathrm{j})e^{iw_{\mathfrak{J}^c}(\mathrm{j})} = [1 - F_{\mathfrak{J}}(\mathrm{j})]e^{iw_{\mathfrak{J}}(\mathrm{j})}$$

Theorem 1. *For any of these CNFS $\mathfrak{J}_1, \mathfrak{J}_2, \mathfrak{J}_3, \mathfrak{J}_4$ on S. The standard intersection, union and complement satisfy the following*

i. $\mathfrak{J}_1 \cap \mathfrak{J}_2 = \mathfrak{J}_2 \cap \mathfrak{J}_1$

ii. $\mathfrak{J}_1 \cup \mathfrak{J}_2 = \mathfrak{J}_2 \cup \mathfrak{J}_1$

iii. $\mathfrak{J}_1 \cap (\mathfrak{J}_2 \cap \mathfrak{J}_3) = (\mathfrak{J}_1 \cap \mathfrak{J}_2) \cap \mathfrak{J}_3$

iv. $\mathfrak{J}_1 \cup (\mathfrak{J}_2 \cup \mathfrak{J}_3) = (\mathfrak{J}_1 \cup \mathfrak{J}_2) \cup \mathfrak{J}_3$

v. $\mathfrak{J}_1 \cap (\mathfrak{J}_2 \cup \mathfrak{J}_3) = (\mathfrak{J}_1 \cap \mathfrak{J}_2) \cup (\mathfrak{J}_1 \cap \mathfrak{J}_3)$

vi. $\mathfrak{J}_1 \cup (\mathfrak{J}_2 \cap \mathfrak{J}_3) = (\mathfrak{J}_1 \cup \mathfrak{J}_2) \cap (\mathfrak{J}_1 \cup \mathfrak{J}_3)$

vii. $(\mathfrak{J}_1 \cap \mathfrak{J}_2)^c = \mathfrak{J}_1^c \cup \mathfrak{J}_2^c$

viii. $(\mathfrak{J}_1 \cup \mathfrak{J}_2)^c = \mathfrak{J}_1^c \cap \mathfrak{J}_2^c$

Definition 8. *Let $\mathfrak{J}_1$ and $\mathfrak{J}_2$ be two CNFSs on S and Then CNF product of $\mathfrak{J}_1$ and $\mathfrak{J}_2$ is $\mathfrak{J}_1 \circ \mathfrak{J}_2$ defined by*

$$\mathfrak{J}_1 \circ \mathfrak{J}_2 = T_{\mathfrak{J}_1}(\mathrm{j})e^{iu_{\mathfrak{J}_1}(\mathrm{j})} \circ T_{\mathfrak{J}_2}(\mathrm{j})e^{iu_{\mathfrak{J}_2}(\mathrm{j})} = [T_{\mathfrak{J}_1}(\mathrm{j}).T_{\mathfrak{J}_2}(\mathrm{j})]e^{i2\pi[\frac{u_{\mathfrak{J}_1}(\mathrm{j})}{2\pi}.\frac{u_{\mathfrak{J}_2}(\mathrm{j})}{2\pi}]}$$

$$\mathfrak{J}_1 \circ \mathfrak{J}_2 = I_{\mathfrak{J}_1}(\mathrm{j})e^{iv_{\mathfrak{J}_1}(\mathrm{j})} \circ I_{\mathfrak{J}_2}(\mathrm{j})e^{iv_{\mathfrak{J}_2}(\mathrm{j})} = [I_{\mathfrak{J}_1}(\mathrm{j}).I_{\mathfrak{J}_2}(\mathrm{j})]e^{i2\pi[\frac{v_{\mathfrak{J}_1}(\mathrm{j})}{2\pi}.\frac{v_{\mathfrak{J}_2}(\mathrm{j})}{2\pi}]}$$

$$\mathfrak{J}_1 \circ \mathfrak{J}_2 = F_{\mathfrak{J}_1}(\mathrm{j})e^{iw_{\mathfrak{J}_1}(\mathrm{j})} \circ F_{\mathfrak{J}_2}(\mathrm{j})e^{iw_{\mathfrak{J}_2}(\mathrm{j})} = [F_{\mathfrak{J}_1}(\mathrm{j}).F_{\mathfrak{J}_2}(\mathrm{j})]e^{i2\pi[\frac{w_{\mathfrak{J}_1}(\mathrm{j})}{2\pi}.\frac{w_{\mathfrak{J}_2}(\mathrm{j})}{2\pi}]}$$

3. Complex Neutrosophic Fuzzy Euclidean Hesitance Distance Measures

The concept of hesitance degree plays a vital role in decision-making and fuzzy logic, as it quantifies the unreliability and inconclusive linked to a specific option and selection. When integrated with distance measures (DMs), hesitance values become especially significant in cases where preferences or similarities are not rigidly defined but instead exist on a spectrum. In MCDM, these values help accommodate uncertainty in the significance or weighting assigned to each criterion, ensuring a more flexible and nuanced evaluation process.

Definition 9. *A Euclidean hesitance distance measure of CNFSs is a function* $d_{EH} : \mathfrak{J}(S) \times \mathfrak{J}(S) \to [0,1]$ *with the condition for the collections of CNFSs* $\mathfrak{J}_1, \mathfrak{J}_2, \mathfrak{J}_3 \in \mathfrak{J}(S)$

i. $0 \leq d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) \leq 1$,

ii. $d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = 0 \iff \mathfrak{J}_1 = \mathfrak{J}_2$,

iii. $d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = d_{EH}(\mathfrak{J}_2, \mathfrak{J}_1)$,

iv. $d_{EH}(\mathfrak{J}_1, \mathfrak{J}_3) \leq d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) + d_{EH}(\mathfrak{J}_2, \mathfrak{J}_3)$.

The Euclidean hesitance distance measure is defined as

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (T_{\mathfrak{J}_1}(\mathrm{j}_i) - T_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i) - u_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}} \tag{3.1}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (I_{\mathfrak{J}_1}(\mathrm{j}_i) - I_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i) - v_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}} \tag{3.2}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (F_{\mathfrak{J}_1}(\mathrm{j}_i) - F_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i) - w_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}} \tag{3.3}$$

Example 1. *Let us take*

$$\mathfrak{J}_1 = \frac{0.6e^{i0.4\pi}, 0.8e^{i0.3\pi}, 0.2e^{i0.3\pi}}{\mathrm{j}_1} + \frac{0.3e^{i0.6\pi}, 0.6e^{i0.5\pi}, 0.3e^{i1.4\pi}}{\mathrm{j}_2} + \frac{0.7e^{i0.4\pi}, 0.8e^{i1.3\pi}, 0.5e^{i0.4\pi}}{\mathrm{j}_3}$$
$$\mathfrak{J}_2 = \frac{0.7e^{i1.3\pi}, 0.7e^{i0.4\pi}, 0.6e^{i1.5\pi}}{\mathrm{j}_1} + \frac{0.9e^{i1.2\pi}, 0.9e^{i0.4\pi}, 0.1e^{i1.0\pi}}{\mathrm{j}_2} + \frac{0.5e^{i0.2\pi}, 0.7e^{i1.6\pi}, 0.4e^{i0.3\pi}}{\mathrm{j}_3}$$

we know that,

$$h_{\mathfrak{J}}(\mathrm{j}) = \{|(1 - T_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - u_{\mathfrak{J}}(\mathrm{j})))|, |(1 - I_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - v_{\mathfrak{J}}(\mathrm{j})))|, |(1 - F_{\mathfrak{J}}(\mathrm{j}) - \frac{1}{2\pi}(2\pi - w_{\mathfrak{J}}(\mathrm{j})))|\}$$

then for truth,

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) = \sqrt{\frac{\begin{bmatrix} ((0.6-0.7)^2+\frac{1}{2\pi^2}(0.4-1.3)^2+(0.4-0.05)^2) \\ ((0.3-0.9)^2+\frac{1}{2\pi^2}(0.6-1.2)^2+(0-0.3)^2) \\ ((0.7-0.5)^2+\frac{1}{2\pi^2}(0.4-0.2)^2+(0.5-0.4)^2) \end{bmatrix}}{3(3)}}$$
$$= \sqrt{\frac{0.01+0.405+0.12+0.36+0.18+0.09+0.04+0.02+0.01}{9}}$$
$$= \sqrt{\frac{1.235}{9}} = 0.3701$$

for indeterminacy,

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) = \sqrt{\frac{\begin{bmatrix} ((0.8-0.7)^2+\frac{1}{2\pi^2}(0.3-0.4)^2+(0.65-0.5)^2) \\ ((0.6-0.9)^2+\frac{1}{2\pi^2}(0.5-0.4)^2+(0.35-0.7)^2) \\ ((0.8-0.7)^2+\frac{1}{2\pi^2}(1.3-1.6)^2+(0.15-0.1)^2) \end{bmatrix}}{3(3)}}$$
$$= \sqrt{\frac{0.01+0.005+0.02+0.09+0.005+0.12+0.01+0.04+0.0025}{9}}$$
$$= \sqrt{\frac{0.3075}{9}} = 0.1846$$

for falsity,

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) = \sqrt{\frac{\begin{bmatrix} ((0.2-0.6)^2+\frac{1}{2\pi^2}(0.3-1.5)^2+(0.05-0.15)^2) \\ ((0.3-0.1)^2+\frac{1}{2\pi^2}(1.4-1.0)^2+(0.4-0.4)^2) \\ ((0.5-0.4)^2+\frac{1}{2\pi^2}(0.4-0.3)^2+(0.3-0.25)^2) \end{bmatrix}}{3(3)}}$$
$$= \sqrt{\frac{0.16+0.405+0.01+0.04+0.08+0+0.01+0.005+0.0025}{9}}$$
$$= \sqrt{\frac{0.7125}{9}} = 0.2812$$

Theorem 2. *The function d_{EH} defined in (3.1),(3.2) and (3.3) is a distance function.*

Proof.

i. The condition $d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) \leq 0$ is obviously true, Then

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\begin{bmatrix} (T_{\mathfrak{J}_1}(\mathrm{j}_i)-T_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}[1+1+1]}{3n}}=\sqrt{\frac{1}{n}}$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_1}(\mathrm{j}_i)-I_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}[1+1+1]}{3n}}=\sqrt{\frac{1}{n}}$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_1}(\mathrm{j}_i)-F_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}[1+1+1]}{3n}}=\sqrt{\frac{1}{n}}$$

If $n=1$ then $d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)=1$ otherwise $d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)<1$. Therefore $0\leq d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)\leq 1$.

ii.

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_1}(\mathrm{j}_i)-T_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_2}(\mathrm{j}_i)-T_{\mathfrak{J}_1}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_2}(\mathrm{j}_i)-u_{\mathfrak{J}_1}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=d_{EH}(\mathfrak{J}_2,\mathfrak{J}_1)$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_1}(\mathrm{j}_i)-I_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_2}(\mathrm{j}_i)-I_{\mathfrak{J}_1}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_2}(\mathrm{j}_i)-v_{\mathfrak{J}_1}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=d_{EH}(\mathfrak{J}_2,\mathfrak{J}_1)$$

$$
\begin{aligned}
d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2) =& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_1}(\mathrm{j}_i)-F_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
=& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_2}(\mathrm{j}_i)-F_{\mathfrak{J}_1}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_2}(\mathrm{j}_i)-w_{\mathfrak{J}_1}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
=& d_{EH}(\mathfrak{J}_2,\mathfrak{J}_1)
\end{aligned}
$$

iii.

$$
\begin{aligned}
d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3) =& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_1}(\mathrm{j}_i)-T_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
=& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_1}(\mathrm{j}_i)-T_{\mathfrak{J}_2}(\mathrm{j}_i)+T_{\mathfrak{J}_2}(\mathrm{j}_i)-T_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_2}(\mathrm{j}_i)+u_{\mathfrak{J}_2}(\mathrm{j}_i)-u_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i)+h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}
\end{aligned}
$$

$$
\begin{aligned}
\leq& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_1}(\mathrm{j}_i)-T_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}+\\
& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(T_{\mathfrak{J}_2}(\mathrm{j}_i)-T_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(u_{\mathfrak{J}_2}(\mathrm{j}_i)-u_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3) \leq& d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)+d_{EH}(\mathfrak{J}_2,\mathfrak{J}_3)
\end{aligned}
$$

$$
\begin{aligned}
d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3) =& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_1}(\mathrm{j}_i)-I_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
=& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_1}(\mathrm{j}_i)-I_{\mathfrak{J}_2}(\mathrm{j}_i)+I_{\mathfrak{J}_2}(\mathrm{j}_i)-I_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_2}(\mathrm{j}_i)+v_{\mathfrak{J}_2}(\mathrm{j}_i)-v_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i)+h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}\\
\leq& \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_1}(\mathrm{j}_i)-I_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}+
\end{aligned}
$$

$$\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(I_{\mathfrak{J}_2}(\mathrm{j}_i)-I_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(v_{\mathfrak{J}_2}(\mathrm{j}_i)-v_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3)\leq d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)+d_{EH}(\mathfrak{J}_2,\mathfrak{J}_3)$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_1}(\mathrm{j}_i)-F_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_1}(\mathrm{j}_i)-F_{\mathfrak{J}_2}(\mathrm{j}_i)+F_{\mathfrak{J}_2}(\mathrm{j}_i)-F_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_2}(\mathrm{j}_i)+w_{\mathfrak{J}_2}(\mathrm{j}_i)-w_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i)+h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$\leq\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_1}(\mathrm{j}_i)-F_{\mathfrak{J}_2}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_1}(\mathrm{j}_i)-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}+$$

$$\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(F_{\mathfrak{J}_2}(\mathrm{j}_i)-F_{\mathfrak{J}_3}(\mathrm{j}_i))^2+\frac{1}{2\pi^2}(w_{\mathfrak{J}_2}(\mathrm{j}_i)-w_{\mathfrak{J}_3}(\mathrm{j}_i))^2\\+(h_{\mathfrak{J}_2}(\mathrm{j}_i)-h_{\mathfrak{J}_3}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1,\mathfrak{J}_3)\leq d_{EH}(\mathfrak{J}_1,\mathfrak{J}_2)+d_{EH}(\mathfrak{J}_2,\mathfrak{J}_3)$$

Definition 10. *Let d_{EH} be the Euclidean hesitance measure of CNFSs. Then, the complement Euclidean hesitance distance measure $d_{EH}((\mathfrak{J}_1)^c,(\mathfrak{J}_2)^c)$ of two CNFSs is defined as*

$$d_{EH}((\mathfrak{J}_1)^c,(\mathfrak{J}_2)^c)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(1-T_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-T_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+\frac{1}{2\pi}(u_{\mathfrak{J}_1}(\mathrm{j}_i)-u_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(1-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$d_{EH}((\mathfrak{J}_1)^c,(\mathfrak{J}_2)^c)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(1-I_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-I_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+\frac{1}{2\pi}(v_{\mathfrak{J}_1}(\mathrm{j}_i)-v_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(1-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

$$d_{EH}((\mathfrak{J}_1)^c,(\mathfrak{J}_2)^c)=\sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c}(1-F_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-F_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+\frac{1}{2\pi}(w_{\mathfrak{J}_1}(\mathrm{j}_i)-w_{\mathfrak{J}_2}(\mathrm{j}_i))^2\\+(1-h_{\mathfrak{J}_1}(\mathrm{j}_i))^2-(1-h_{\mathfrak{J}_2}(\mathrm{j}_i))^2\end{array}\right]}{3n}}$$

Definition 11. *Let d_{EH} be the Euclidean hesitance measure of CNFSs. Then, the max Euclidean hesitance distance measure of two CNFSs $\mathfrak{J}_1$ and $\mathfrak{J}_2$ is denoted by $d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_2)$ or $d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cup \mathfrak{J}_2)$ and fulfill the requirements*

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} ((T_{\mathfrak{J}_1}(\mathrm{j}_i) \vee T_{\mathfrak{J}_2}(\mathrm{j}_i)) - T_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(u_{\mathfrak{J}_1}(\mathrm{j}_i) \vee u_{\mathfrak{J}_2}(\mathrm{j}_i) - u_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} ((I_{\mathfrak{J}_1}(\mathrm{j}_i) \vee I_{\mathfrak{J}_2}(\mathrm{j}_i)) - I_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(v_{\mathfrak{J}_1}(\mathrm{j}_i) \vee v_{\mathfrak{J}_2}(\mathrm{j}_i) - v_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} ((F_{\mathfrak{J}_1}(\mathrm{j}_i) \vee F_{\mathfrak{J}_2}(\mathrm{j}_i)) - F_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(w_{\mathfrak{J}_1}(\mathrm{j}_i) \vee w_{\mathfrak{J}_2}(\mathrm{j}_i) - w_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{array}\right]}{3n}}$$

and

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} (T_{\mathfrak{J}_1}(\mathrm{j}_i) - (T_{\mathfrak{J}_2}(\mathrm{j}_i) \vee T_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(u_{\mathfrak{J}_1}(\mathrm{j}_i) - (u_{\mathfrak{J}_2}(\mathrm{j}_i) \vee u_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} (I_{\mathfrak{J}_1}(\mathrm{j}_i) - (I_{\mathfrak{J}_2}(\mathrm{j}_i) \vee I_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(v_{\mathfrak{J}_1}(\mathrm{j}_i) - (v_{\mathfrak{J}_2}(\mathrm{j}_i) \vee v_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} (F_{\mathfrak{J}_1}(\mathrm{j}_i) - (F_{\mathfrak{J}_2}(\mathrm{j}_i) \vee F_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(w_{\mathfrak{J}_1}(\mathrm{j}_i) - (w_{\mathfrak{J}_2}(\mathrm{j}_i) \vee w_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{array}\right]}{3n}}$$

Definition 12. *Let d_{EH} be the Euclidean hesitance measure of CNFSs. Then, the min Euclidean hesitance distance measure of two CNFSs $\mathfrak{J}_1$ and $\mathfrak{J}_2$ is denoted by $d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_2)$ or $d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cap \mathfrak{J}_2)$ and fulfill the requirements*

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\left[\begin{array}{c} ((T_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge T_{\mathfrak{J}_2}(\mathrm{j}_i)) - T_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(u_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge u_{\mathfrak{J}_2}(\mathrm{j}_i) - u_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{array}\right]}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((I_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge I_{\mathfrak{J}_2}(\mathrm{j}_i)) - I_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(v_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge v_{\mathfrak{J}_2}(\mathrm{j}_i) - v_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((F_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge F_{\mathfrak{J}_2}(\mathrm{j}_i)) - F_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi}(w_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge w_{\mathfrak{J}_2}(\mathrm{j}_i) - w_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

and

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (T_{\mathfrak{J}_1}(\mathrm{j}_i) - (T_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge T_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(u_{\mathfrak{J}_1}(\mathrm{j}_i) - (u_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge u_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (I_{\mathfrak{J}_1}(\mathrm{j}_i) - (I_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge I_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(v_{\mathfrak{J}_1}(\mathrm{j}_i) - (v_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge v_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} (F_{\mathfrak{J}_1}(\mathrm{j}_i) - (F_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge F_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}(w_{\mathfrak{J}_1}(\mathrm{j}_i) - (w_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge w_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - (h_{\mathfrak{J}_2}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

Definition 13. *Let d_{EH} be the Euclidean hesitance measure of CNFSs. Then, the max-min Euclidean hesitance distance measure of two CNFSs $\mathfrak{J}_1$ and $\mathfrak{J}_2$ is denoted by $d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_1 \cap \mathfrak{J}_2)$ or $d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_1 \cup \mathfrak{J}_2)$ and fulfill the requirements*

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_1 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((T_{\mathfrak{J}_1}(\mathrm{j}_i) \vee T_{\mathfrak{J}_2}(\mathrm{j}_i), T_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge T_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((u_{\mathfrak{J}_1}(\mathrm{j}_i) \vee u_{\mathfrak{J}_2}(\mathrm{j}_i), u_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge u_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_1 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((I_{\mathfrak{J}_1}(\mathrm{j}_i) \vee I_{\mathfrak{J}_2}(\mathrm{j}_i), I_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge I_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((v_{\mathfrak{J}_1}(\mathrm{j}_i) \vee v_{\mathfrak{J}_2}(\mathrm{j}_i), v_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge v_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cup \mathfrak{J}_2, \mathfrak{J}_1 \cap \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((F_{\mathfrak{J}_1}(\mathrm{j}_i) \vee F_{\mathfrak{J}_2}(\mathrm{j}_i), F_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge F_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((w_{\mathfrak{J}_1}(\mathrm{j}_i) \vee w_{\mathfrak{J}_2}(\mathrm{j}_i), w_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge w_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

and

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_1 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((T_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge T_{\mathfrak{J}_2}(\mathrm{j}_i), T_{\mathfrak{J}_1}(\mathrm{j}_i) \vee T_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((u_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge u_{\mathfrak{J}_2}(\mathrm{j}_i), u_{\mathfrak{J}_1}(\mathrm{j}_i) \vee u_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_1 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((I_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge I_{\mathfrak{J}_2}(\mathrm{j}_i), I_{\mathfrak{J}_1}(\mathrm{j}_i) \vee I_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((v_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge v_{\mathfrak{J}_2}(\mathrm{j}_i), v_{\mathfrak{J}_1}(\mathrm{j}_i) \vee v_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1 \cap \mathfrak{J}_2, \mathfrak{J}_1 \cup \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n} \begin{bmatrix} ((F_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge F_{\mathfrak{J}_2}(\mathrm{j}_i), F_{\mathfrak{J}_1}(\mathrm{j}_i) \vee F_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +\frac{1}{2\pi}((w_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge w_{\mathfrak{J}_2}(\mathrm{j}_i), w_{\mathfrak{J}_1}(\mathrm{j}_i) \vee w_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \\ +(((h_{\mathfrak{J}_1}(\mathrm{j}_i) \wedge h_{\mathfrak{J}_2}(\mathrm{j}_i), h_{\mathfrak{J}_1}(\mathrm{j}_i) \vee h_{\mathfrak{J}_2}(\mathrm{j}_i)))^2 \end{bmatrix}}{3n}}$$

4. Application of Complex Neutrosophic Fuzzy Set in Decision Making Process

Various decision-making algorithms are tailored to address specific problems and scenarios. This section introduces a novel decision-making approach that utilizes CNFSs in conjunction with CNFDMs. CNFS theory enhances conventional fuzzy and neutrosophic set methodologies by incorporating complex numbers, which allows for a more comprehensive representation of uncertainty, indeterminacy, and inconsistency in decision-making. This framework provides decision-makers with sophisticated tools to navigate complex and ambiguous situations, resulting in well-informed and rational decisions. By utilizing CNFSs, uncertain information, conflicting criteria, and varying degrees of truth, indeterminacy, and falsity can be modeled effectively. The useof complex numbers enlarge the scope of decision-making,offering deeperinsights into uncertainties. Additionally, the proposed algorithm,as described in this paper,applies CNFDM store fine the evaluation of alternatives,ensuring a more precise assessment of uncertainty, imprecision, and complexity. This methodological improvement enhances the decision-making process,making it more adaptable to real-world applications. The given flow chart makes the algorithm more easier to understand (Figure 1) .

4.1. Algorithm

Step 1: Consider a set of i alternatives, denoted as $(\alpha_i, \beta_i, \gamma_i) = \{(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2), \cdots, (\alpha_i, \beta_i, \gamma_i)\}$, and a set of j attributes, represented as $\mathfrak{J}_j = (\mathfrak{J}_1, \mathfrak{J}_2, \cdots, \mathfrak{J}_j)$. Additionally, let $Z = \{Z_1, Z_2, \cdots, Z_n\}$ be a group of n experts, where each Z_n corresponds to an expert participating in the decision-making process.

Step 2: The attribute value assigned to an alternative $(\alpha_i, \beta_i, \gamma_i)$ for a specific criterion $\mathfrak{J}_j$ is expressed as $T_{ij} = \mathfrak{j}T_{ij}(T_j)e^{iuT_{ij}(T_j)}; I_{ij} = \mathfrak{j}I_{ij}(I_j)e^{ivI_{ij}(I_j)} and F_{ij} = \mathfrak{j}F_{ij}(F_j)e^{iwF_{ij}(F_j)}$. This value represents the assessment given by expert Z_m regarding alternative $(\alpha_i, \beta_i, \gamma_i)$ in relation to criterion $\mathfrak{J}_j$.

Step 3: Structure the table derived from these evaluated values serves as a fundamental representation of the decision-making data.

Z_i	$\alpha_i, \beta_i, \gamma_i$	T_1, I_1, F_1	T_2, I_2, F_2	$\cdots$	T_j, I_j, F_j
Z_1	$\alpha_1, \beta_1, \gamma_1$	$(\mathfrak{j}_{T_1T_1}(T_1)e^{iuT_1(T_1)}, \mathfrak{j}_{I_1I_1}(I_1)e^{ivI_1(I_1)}, \mathfrak{j}_{F_1F_1}(F_1)e^{iwF_1(F_1)})$	$(\mathfrak{j}_{T_1T_2}(T_2)e^{iuT_1(T_2)}, \mathfrak{j}_{I_1I_2}(I_2)e^{ivI_1(I_2)}, \mathfrak{j}_{F_1F_2}(F_2)e^{iwF_1(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_1T_j}(T_j)e^{iuT_1(T_j)}, \mathfrak{j}_{I_1I_j}(I_j)e^{ivI_1(I_j)}, \mathfrak{j}_{F_1F_j}(F_j)e^{iwF_1(F_j)})$
	$\alpha_2, \beta_2, \gamma_2$	$(\mathfrak{j}_{T_2T_1}(T_1)e^{iuT_2(T_1)}, \mathfrak{j}_{I_2I_1}(I_1)e^{ivI_2(I_1)}, \mathfrak{j}_{F_2F_1}(F_1)e^{iwF_2(F_1)})$	$(\mathfrak{j}_{T_2T_2}(T_2)e^{iuT_2(T_2)}, \mathfrak{j}_{I_2I_2}(I_2)e^{ivI_2(I_2)}, \mathfrak{j}_{F_2F_2}(F_2)e^{iwF_2(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_2T_j}(T_j)e^{iuT_2(T_j)}, \mathfrak{j}_{I_2I_j}(I_j)e^{ivI_2(I_j)}, \mathfrak{j}_{F_2F_j}(F_j)e^{iwF_2(F_j)})$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(\mathfrak{j}_{T_iT_1}(T_1)e^{iuT_i(T_1)}, \mathfrak{j}_{I_iI_1}(I_1)e^{ivI_i(I_1)}, \mathfrak{j}_{F_iF_1}(F_1)e^{iwF_i(F_1)})$	$(\mathfrak{j}_{T_iT_2}(T_2)e^{iuT_i(T_2)}, \mathfrak{j}_{I_iI_2}(I_2)e^{ivI_i(I_2)}, \mathfrak{j}_{F_iF_2}(F_2)e^{iwF_i(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_iT_j}(T_j)e^{iuT_i(T_j)}, \mathfrak{j}_{I_iI_j}(I_j)e^{ivI_i(I_j)}, \mathfrak{j}_{F_iF_j}(F_j)e^{iwF_i(F_j)})$
Z_2	$\alpha_1, \beta_1, \gamma_1$	$(\mathfrak{j}_{T_1T_1}(T_1)e^{iuT_1(T_1)}, \mathfrak{j}_{I_1I_1}(I_1)e^{ivI_1(I_1)}, \mathfrak{j}_{F_1F_1}(F_1)e^{iwF_1(F_1)})$	$(\mathfrak{j}_{T_1T_2}(T_2)e^{iuT_1(T_2)}, \mathfrak{j}_{I_1I_2}(I_2)e^{ivI_1(I_2)}, \mathfrak{j}_{F_1F_2}(F_2)e^{iwF_1(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_1T_j}(T_j)e^{iuT_1(T_j)}, \mathfrak{j}_{I_1I_j}(I_j)e^{ivI_1(I_j)}, \mathfrak{j}_{F_1F_j}(F_j)e^{iwF_1(F_j)})$
	$\alpha_2, \beta_2, \gamma_2$	$(\mathfrak{j}_{T_2T_1}(T_1)e^{iuT_2(T_1)}, \mathfrak{j}_{I_2I_1}(I_1)e^{ivI_2(I_1)}, \mathfrak{j}_{F_2F_1}(F_1)e^{iwF_2(F_1)})$	$(\mathfrak{j}_{T_2T_2}(T_2)e^{iuT_2(T_2)}, \mathfrak{j}_{I_2I_2}(I_2)e^{ivI_2(I_2)}, \mathfrak{j}_{F_2F_2}(F_2)e^{iwF_2(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_2T_j}(T_j)e^{iuT_2(T_j)}, \mathfrak{j}_{I_2I_j}(I_j)e^{ivI_2(I_j)}, \mathfrak{j}_{F_2F_j}(F_j)e^{iwF_2(F_j)})$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(\mathfrak{j}_{T_iT_1}(T_1)e^{iuT_i(T_1)}, \mathfrak{j}_{I_iI_1}(I_1)e^{ivI_i(I_1)}, \mathfrak{j}_{F_iF_1}(F_1)e^{iwF_i(F_1)})$	$(\mathfrak{j}_{T_iT_2}(T_2)e^{iuT_i(T_2)}, \mathfrak{j}_{I_iI_2}(I_2)e^{ivI_i(I_2)}, \mathfrak{j}_{F_iF_2}(F_2)e^{iwF_i(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_iT_j}(T_j)e^{iuT_i(T_j)}, \mathfrak{j}_{I_iI_j}(I_j)e^{ivI_i(I_j)}, \mathfrak{j}_{F_iF_j}(F_j)e^{iwF_i(F_j)})$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
Z_i	$\alpha_1, \beta_1, \gamma_1$	$(\mathfrak{j}_{T_1T_1}(T_1)e^{iuT_1(T_1)}, \mathfrak{j}_{I_1I_1}(I_1)e^{ivI_1(I_1)}, \mathfrak{j}_{F_1F_1}(F_1)e^{iwF_1(F_1)})$	$(\mathfrak{j}_{T_1T_2}(T_2)e^{iuT_1(T_2)}, \mathfrak{j}_{I_1I_2}(I_2)e^{ivI_1(I_2)}, \mathfrak{j}_{F_1F_2}(F_2)e^{iwF_1(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_1T_j}(T_j)e^{iuT_1(T_j)}, \mathfrak{j}_{I_1I_j}(I_j)e^{ivI_1(I_j)}, \mathfrak{j}_{F_1F_j}(F_j)e^{iwF_1(F_j)})$
	$\alpha_2, \beta_2, \gamma_2$	$(\mathfrak{j}_{T_2T_1}(T_1)e^{iuT_2(T_1)}, \mathfrak{j}_{I_2I_1}(I_1)e^{ivI_2(I_1)}, \mathfrak{j}_{F_2F_1}(F_1)e^{iwF_2(F_1)})$	$(\mathfrak{j}_{T_2T_2}(T_2)e^{iuT_2(T_2)}, \mathfrak{j}_{I_2I_2}(I_2)e^{ivI_2(I_2)}, \mathfrak{j}_{F_2F_2}(F_2)e^{iwF_2(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_2T_j}(T_j)e^{iuT_2(T_j)}, \mathfrak{j}_{I_2I_j}(I_j)e^{ivI_2(I_j)}, \mathfrak{j}_{F_2F_j}(F_j)e^{iwF_2(F_j)})$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	$\alpha_i, \beta_i, \gamma_i$	$(\mathfrak{j}_{T_iT_1}(T_1)e^{iuT_i(T_1)}, \mathfrak{j}_{I_iI_1}(I_1)e^{ivI_i(I_1)}, \mathfrak{j}_{F_iF_1}(F_1)e^{iwF_i(F_1)})$	$(\mathfrak{j}_{T_iT_2}(T_2)e^{iuT_i(T_2)}, \mathfrak{j}_{I_iI_2}(I_2)e^{ivI_i(I_2)}, \mathfrak{j}_{F_iF_2}(F_2)e^{iwF_i(F_2)})$	$\cdots$	$(\mathfrak{j}_{T_iT_j}(T_j)e^{iuT_i(T_j)}, \mathfrak{j}_{I_iI_j}(I_j)e^{ivI_i(I_j)}, \mathfrak{j}_{F_iF_j}(F_j)e^{iwF_i(F_j)})$

Table 1: Collection of attributes

Step 4: Construct distance matrix using the definition of Euclidean distance measure,

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\begin{bmatrix} (T_{\mathfrak{J}_1}(\mathrm{j}_i) - T_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(u_{\mathfrak{J}_1}(\mathrm{j}_i) - u_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\begin{bmatrix} (I_{\mathfrak{J}_1}(\mathrm{j}_i) - I_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(v_{\mathfrak{J}_1}(\mathrm{j}_i) - v_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(\mathfrak{J}_1, \mathfrak{J}_2) = \sqrt{\frac{\sum_{i=1}^{n}\begin{bmatrix} (F_{\mathfrak{J}_1}(\mathrm{j}_i) - F_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +\frac{1}{2\pi^2}(w_{\mathfrak{J}_1}(\mathrm{j}_i) - w_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \\ +(h_{\mathfrak{J}_1}(\mathrm{j}_i) - h_{\mathfrak{J}_2}(\mathrm{j}_i))^2 \end{bmatrix}}{3n}}$$

The distance matrix is demonstrted as

$$[D_M]_{3\times 3} = \begin{pmatrix} (d(T_1^{(Z_1)}, T_1^{(Z_2)}), d(I_1^{(Z_1)}, I_1^{(Z_2)}), d(F_1^{(Z_1)}, F_1^{(Z_2)})) & (d(T_1^{(Z_1)}, T_1^{(Z_3)}), d(I_1^{(Z_1)}, I_1^{(Z_3)}), d(F_1^{(Z_1)}, F_1^{(Z_3)})) & (d(T_1^{(Z_2)}, T_1^{(Z_3)}), d(I_1^{(Z_2)}, I_1^{(Z_3)}), d(F_1^{(Z_2)}, F_1^{(Z_3)})) \\ (d(T_2^{(Z_1)}, T_2^{(Z_2)}), d(I_2^{(Z_1)}, I_2^{(Z_2)}), d(F_2^{(Z_1)}, F_2^{(Z_2)})) & (d(T_2^{(Z_1)}, T_2^{(Z_3)}), d(I_2^{(Z_1)}, I_2^{(Z_3)}), d(F_2^{(Z_1)}, F_2^{(Z_3)})) & (d(T_2^{(Z_2)}, T_2^{(Z_3)}), d(I_2^{(Z_2)}, I_2^{(Z_3)}), d(F_2^{(Z_2)}, F_2^{(Z_3)})) \\ (d(T_3^{(Z_1)}, T_3^{(Z_2)}), d(I_3^{(Z_1)}, I_3^{(Z_2)}), d(F_3^{(Z_1)}, F_3^{(Z_2)})) & (d(T_3^{(Z_1)}, T_3^{(Z_3)}), d(I_3^{(Z_1)}, I_3^{(Z_3)}), d(F_3^{(Z_1)}, F_3^{(Z_3)})) & (d(T_3^{(Z_2)}, T_3^{(Z_3)}), d(I_3^{(Z_2)}, I_3^{(Z_3)}), d(F_3^{(Z_2)}, F_3^{(Z_3)})) \end{pmatrix}$$

Step 5: Find score function $S(\mathfrak{J}_i)$ of alternative $\mathfrak{J}_i$ where $i = 1, 2, \cdots, n$ is defined as

$$S(\mathfrak{J}_i) = \sum_{i=1}^{3}\left[\frac{d(T_i^{(Z_j)}, T_i^{(Z_k)})}{3}, \frac{d(I_i^{(Z_j)}, I_i^{(Z_k)})}{3}, \frac{d(F_i^{(Z_j)}, F_i^{(Z_k)})}{3}\right]$$

4.2. Problem Description

4.2.1. Euclidean Distance Measure in Smart Irrigation for Small-Scale Farmers

Chad is a landlocked country in Central Africa, heavily relies on agriculture, with around 80% of its population engaged in farming and livestock. However, poor irrigation infrastructure severely limits agricultural productivity. Most farming depends on unpredictable rainfall, and prolonged droughts worsen food insecurity. The country faces water scarcity, desertification, and inadequate irrigation systems, making it difficult to grow crops consistently. The lack of modern technology, such as drip irrigation and water pumps, forces farmers to rely on inefficient traditional methods. Chadian farmers face low yields of their crops, which results in mass food shortages and economic misery. The traditional approaches to decision-making do not embrace the uncertainty that comes

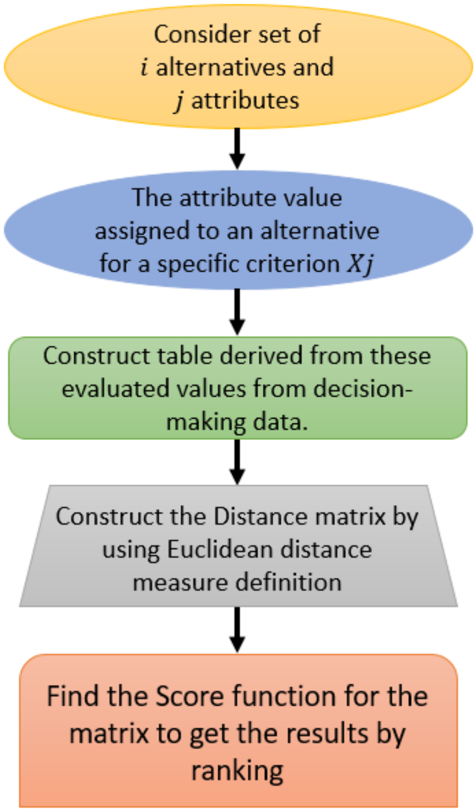

Figure 1: Graphical Algorithm

with climate change and the management of water resources. Owing to the poor irrigation facilities and extreme climatic conditions, the majority of Chadians are plagued by long-term food insecurity and malnutrition. Clean water is also scarce, an issue that impacts agricultural production as well as daily life. Consequently, most communities continue to rely on humanitarian assistance to survive.

Enhancing irrigation via measures like rainwater harvesting, solar pumps, and effective water management could greatly boost food yields and living conditions nationwide. In response to these problems, this study suggests a CNFS model to irrigate optimally. With a three-parameter Euclidean distance measure, the model offers a more effective and powerful decision-making platform under uncertainty, indeterminacy, and inconsistent information.

Let $\mathfrak{J} = \{\mathfrak{J}_1 =$ climate adaptability , $\mathfrak{J}_2 =$ Soil moisture retention , $\mathfrak{J}_3 =$ Water availability $\}$ be a set of three most important decision-making criteria. In order to analyze irrigation methods in Chad, we establish a decision-making scheme based on CNFS parameters:

Let $Y = \{Y_1, Y_2, Y_3\}$ be a set of three irrigation strategies, where Y_1 represents Traditional rain-fed irrigation, Y_2 represents Solar-powered drip irrigation, Y_3 represents

Rainwater harvesting with storage tanks. The CNFS-based Euclidean distance measure to examine the irrigation issues unique to Chad:

- Dataset: Farmer's feedback on the effectiveness of various irrigation methods, grouped into CNFS values according to their perceptions and experiences.
- Evaluation: CNFS distance measures classify regions based on irrigation challenges.
- Decision-Making: Prioritization of areas needing urgent irrigation improvements.

Let $Z = \{Z_1, Z_2, Z_3\}$ be a set of three farmers involved in the diagnosis process. Assume that a land presents with uncertain and overlapping symptoms, makes harvest challenging. The experts evaluate each irrigation alternative $\mathfrak{J}_i$ based on the three strategies. A CNFS is used to express expert evaluations, where truth membership represent how true it is that irrigation is needed. Indeterminacy membership meant that the uncertainty in decision-making (e.g., unpredictable rain, sensor errors). Falsity membership is represent how false it is that irrigation is needed. Each expert provides complex neutrosophic fuzzy numbers (CNFNs), incorporating both magnitude and phase components to reflect their confidence and hesitation in their assessments.

Euclidean Distance Measure Computation Using CNFS:

To analyze the consistency between the expert evaluations, a CNFDM is applied. The distance measure calculates the similarity or divergence between the farmers assessments to the yield crops.

1 The current environmental condition is compared with an ideal irrigation scenario using Euclidean distance measure in the CNFS.

2 The closer the current conditions are to the ideal conditions, the less irrigation is needed.

3 If the distance is large, more irrigation is applied. The Euclidean distance helps weigh the factors to optimize irrigation decisions dynamically. This approach ensures water efficiency, prevents over-irrigation, and supports sustainable farming for small-scale farmers.

Z_i	T_i, I_i, F_i	Y_1	Y_2	Y_3
Z_1	T_1, I_1, F_1	$(0.5e^{i0.2\pi}, 0.2e^{i0.3\pi}, 0.4e^{i0.2\pi})$	$(0.2e^{i0.4\pi}, 0.4e^{i0.6\pi}, 0.6e^{i0.5\pi})$	$(0.1e^{i0.7\pi}, 0.7e^{i0.2\pi}, 0.8e^{i0.3\pi})$
	T_2, I_2, F_2	$(0.6e^{i0.3\pi}, 0.5e^{i0.1\pi}, 0.7e^{i0.2\pi})$	$(0.8e^{i0.1\pi}, 0.9e^{i0.2\pi}, 1.0e^{i1.1\pi})$	$(0.7e^{i0.3\pi}, 0.8e^{i0.7\pi}, 0.9e^{i0.2\pi})$
	T_3, I_3, F_3	$(0.9e^{i0.8\pi}, 0.7e^{i0.3\pi}, 0.6e^{i0.5\pi}$	$(0.5e^{i0.4\pi}, 0.4e^{i0.2\pi}, 0.1e^{i0.3\pi})$	$(0.7e^{i0.4\pi}, 0.9e^{i0.1\pi}, 0.5e^{i0.5\pi})$
Z_2	T_1, I_1, F_1	$(0.5e^{i0.2\pi}, 0.2e^{i0.1\pi}, 0.3e^{i0.2\pi})$	$(0.3e^{i0.1\pi}, 0.4e^{i0.2\pi}, 0.3e^{i0.5\pi})$	$(0.4e^{i0.1\pi}, 0.6e^{i0.4\pi}, 0.5e^{i0.1\pi})$
	T_2, I_2, F_2	$(0.6e^{i0.3\pi}, 0.8e^{i0.4\pi}, 0.9e^{i0.5\pi})$	$(0.8e^{i0.1\pi}, 0.7e^{i0.2\pi}, 0.9e^{i0.3\pi})$	$(0.5e^{i0.2\pi}, 0.6e^{i0.3\pi}, 0.7e^{i0.4\pi})$
	T_3, I_3, F_3	$(0.4e^{i0.2\pi}, 0.9e^{i1.2\pi}, 0.8e^{i1.1\pi})$	$(0.7e^{i1.4\pi}, 0.3e^{i0.2\pi}, 0.5e^{i0.7\pi})$	$(0.7e^{i0.3\pi}, 0.8e^{i0.3\pi}, 0.6e^{i1.1\pi})$
Z_3	T_1, I_1, F_1	$(0.2e^{i0.1\pi}, 0.3e^{i0.4\pi}, 0.5e^{i0.2\pi})$	$(0.7e^{i0.4\pi}, 0.8e^{i0.1\pi}, 0.5e^{i0.1\pi})$	$(0.4e^{i0.2\pi}, 0.2e^{i0.3\pi}, 0.5e^{i0.4\pi})$
	T_2, I_2, F_2	$(0.8e^{i0.9\pi}, 0.7e^{i0.8\pi}, 0.6e^{i0.7\pi})$	$(0.9e^{i1.1\pi}, 0.7e^{i0.8\pi}, 1.0e^{i0.9\pi})$	$(0.7e^{i0.3\pi}, 0.6e^{i0.4\pi}, 0.5e^{i0.2\pi})$
	T_3, I_3, F_3	$(1.0e^{i0.9\pi}, 0.8e^{i0.2\pi}, 0.9e^{i1.1\pi})$	$(1.0e^{i1.1\pi}, 0.8e^{i0.7\pi}, 0.2e^{i0.4\pi})$	$(0.7e^{i0.3\pi}, 0.5e^{i0.3\pi}, 0.7e^{i0.4\pi})$

Table 2: Evaluated values of alternatives

Complex Neutrosophic Fuzzy Euclidean Hesitance Distance Measures

$$d(T_1^{(Z_1)}, T_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2 + (0.4-0.4)^2)+ \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2 + (0-0.25)^2)+ \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2 + (0.25-0.35)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.0225 + 0.0625 + 0.09 + 0.09 + 0.01}{9}} = 0.177$$

$$d(I_1^{(Z_1)}, I_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2 + (0.4-0.4)^2)+ \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2 + (0-0.25)^2)+ \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2 + (0.25-0.35)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.01 + 0.04 + 0.04 + 0.04 + 0.01 + 0.01}{9}} = 0.133$$

$$d(F_1^{(Z_1)}, F_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2 + (0.4-0.4)^2)+ \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2 + (0-0.25)^2)+ \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2 + (0.25-0.35)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.01 + 0.0225 + 0.09 + 0.0225 + 0.09 + 0.04}{9}} = 0.176$$

$$d(T_1^{(Z_1)}, T_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2 + (0.4-0.15)^2)+ \\ ((0.2-0.7)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.4\pi)^2 + (0-0.5)^2)+ \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.2\pi)^2 + (0.25-0.3)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.09 + 0.0025 + 0.0625 + 0.025 + 0.025 + 0.09 + 0.0625 + 0.0025}{9}} = 0.300$$

$$d(I_1^{(Z_1)}, I_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.3\pi - 0.4\pi)^2 + (0.05-0.1)^2)+ \\ ((0.4-0.8)^2 + \frac{1}{2\pi^2}(0.6\pi - 0.1\pi)^2 + (0.1-0.75)^2)+ \\ ((0.7-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.3\pi)^2 + (0.6-0.05)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.0025 + 0.0025 + 0.16 + 0.0625 + 0.04225 + 0.25 + 0.0025}{9}} = 0.399$$

$$d(F_1^{(Z_1)}, F_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.4-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2 + (0.3-0.4)^2)+ \\ ((0.6-0.5)^2 + \frac{1}{2\pi^2}(0.8\pi - 0.1\pi)^2 + (0.2-0.45)^2)+ \\ ((0.8-0.5)^2 + \frac{1}{2\pi^2}(0.3\pi - 0.4\pi)^2 + (0.65-0.3)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.01 + 0.01 + 0.1225 + 0.0625 + 0.09 + 0.0025 + 0.1225}{9}} = 0.218$$

$$d(T_1^{(Z_2)}, T_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2 + (0.4-0.15)^2) + \\ ((0.3-0.7)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2 + (0.25-0.5)^2) + \\ ((0.4-0.4)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.2\pi)^2 + (0.35-0.3)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.09 + 0.0025 + 0.0625 + 0.16 + 0.25 + 0.0625 + 0.0025 + 0.0025}{9}} = 0.264$$

$$d(I_1^{(Z_2)}, I_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2 + (0.15-0.1)^2) + \\ ((0.4-0.8)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2 + (0.3-0.75)^2) + \\ ((0.6-0.2)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.3\pi)^2 + (0.4-0.05)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.01 + 0.0225 + 0.0025 + 0.16 + 0.0025 + 0.2025 + 0.16 + 0.0025 + 0.1225}{9}} = 0.275$$

$$d(F_1^{(Z_2)}, F_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.3-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2 + (0.2-0.4)^2) + \\ ((0.3-0.5)^2 + \frac{1}{2\pi^2}(0.5\pi - 0.1\pi)^2 + (0.05-0.45)^2) + \\ ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2 + (0.45-0.3)^2) \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{0.04 + 0.04 + 0.04 + 0.04 + 0.16 + 0.01 + 0.01 + 0.0225}{9}} = 0.197$$

Similarly,

$$\begin{array}{ccc} d(T_2^{(Z_1)}, T_2^{(Z_2)}) = 0.084 & d(T_2^{(Z_2)}, T_2^{(Z_3)}) = 0.249 & d(T_3^{(Z_1)}, T_3^{(Z_3)}) = 0.263 \\ d(I_2^{(Z_1)}, I_2^{(Z_2)}) = 0.180 & d(I_2^{(Z_2)}, I_2^{(Z_3)}) = 0.264 & d(I_3^{(Z_1)}, I_3^{(Z_3)}) = 0.190 \\ d(F_2^{(Z_1)}, F_2^{(Z_2)}) = 0.302 & d(F_2^{(Z_2)}, F_2^{(Z_3)}) = 0.243 & d(F_3^{(Z_1)}, F_3^{(Z_3)}) = 0.226 \\ d(T_2^{(Z_1)}, T_2^{(Z_3)}) = 0.292 & d(T_3^{(Z_1)}, T_3^{(Z_2)}) = 0.222 & d(T_3^{(Z_2)}, T_3^{(Z_3)}) = 0.364 \\ d(I_2^{(Z_1)}, I_2^{(Z_3)}) = 0.206 & d(I_3^{(Z_1)}, I_3^{(Z_2)}) = 0.280 & d(I_3^{(Z_2)}, I_3^{(Z_3)}) = 0.329 \\ d(F_2^{(Z_1)}, F_2^{(Z_3)}) = 0.233 & d(F_3^{(Z_1)}, F_3^{(Z_2)}) = 0.182 & d(F_3^{(Z_2)}, F_3^{(Z_3)}) = 0.233 \end{array}$$

$$[D_M]_{3\times 3} = \begin{pmatrix} (d(T_1^{(Z_1)}, T_1^{(Z_2)}), d(I_1^{(Z_1)}, I_1^{(Z_2)}), d(F_1^{(Z_1)}, F_1^{(Z_2)})) & (d(T_1^{(Z_1)}, T_1^{(Z_3)}), d(I_1^{(Z_1)}, I_1^{(Z_3)}), d(F_1^{(Z_1)}, F_1^{(Z_3)})) & (d(T_1^{(Z_2)}, T_1^{(Z_3)}), d(I_1^{(Z_2)}, I_1^{(Z_3)}), d(F_1^{(Z_2)}, F_1^{(Z_3)})) \\ (d(T_2^{(Z_1)}, T_2^{(Z_2)}), d(I_2^{(Z_1)}, I_2^{(Z_2)}), d(F_2^{(Z_1)}, F_2^{(Z_2)})) & (d(T_2^{(Z_1)}, T_2^{(Z_3)}), d(I_2^{(Z_1)}, I_2^{(Z_3)}), d(F_2^{(Z_1)}, F_2^{(Z_3)})) & (d(T_2^{(Z_2)}, T_2^{(Z_3)}), d(I_2^{(Z_2)}, I_2^{(Z_3)}), d(F_2^{(Z_2)}, F_2^{(Z_3)})) \\ (d(T_3^{(Z_1)}, T_3^{(Z_2)}), d(I_3^{(Z_1)}, I_3^{(Z_2)}), d(F_3^{(Z_1)}, F_3^{(Z_2)})) & (d(T_3^{(Z_1)}, T_3^{(Z_3)}), d(I_3^{(Z_1)}, I_3^{(Z_3)}), d(F_3^{(Z_1)}, F_3^{(Z_3)})) & (d(T_3^{(Z_2)}, T_3^{(Z_3)}), d(I_3^{(Z_2)}, I_3^{(Z_3)}), d(F_3^{(Z_2)}, F_3^{(Z_3)})) \end{pmatrix}$$

$$= \begin{bmatrix} (0.177, 0.133, 0.176) & (0.300, 0.399, 0.218) & (0.264, 0.275, 0.197) \\ (0.084, 0.180, 0.302) & (0.249, 0.264, 0.243) & (0.263, 0.190, 0.226) \\ (0.292, 0.206, 0.233) & (0.222, 0.280, 0.182) & (0.364, 0.329, 0.233) \end{bmatrix}$$

$$\mathcal{S}(\mathfrak{J}_1) = \left[\frac{0.177 + 0.300 + 0.264}{3}, \frac{0.133 + 0.399 + 0.275}{3}, \frac{0.176 + 0.218 + 0.197}{3}\right]$$

$$\mathcal{S}(\mathfrak{J}_1) = (0.247, 0.269, 0.197)$$

$$\mathcal{S}(\mathfrak{J}_2) = \left[\frac{0.084 + 0.249 + 0.263}{3}, \frac{0.180 + 0.264 + 0.190}{3}, \frac{0.302 + 0.243 + 0.226}{3}\right]$$

$$\mathcal{S}(\mathfrak{J}_2) = (0.198, 0.211, 0.257)$$
$$\mathcal{S}(\mathfrak{J}_3) = \left[\frac{0.292+0.222+0.364}{3}, \frac{0.206+0.280+0.329}{3}, \frac{0.233+0.182+0.233}{3}\right]$$
$$\mathcal{S}(\mathfrak{J}_3) = (0.292, 0.271, 0.216)$$
$$\mathcal{S}(\mathfrak{J}_1) < \mathcal{S}(\mathfrak{J}_2) < \mathcal{S}(\mathfrak{J}_3)$$

since $\mathfrak{J}_3$ (water availability) has the highest result, the smart irrigation system can prioritize minimal water usage, ensuring efficient irrigation while preventing overuse, conserving resources, and maintaining optimal soil moisture levels for sustainable farming (Figure 3).

4.3. Comparison Analysis

In this section, we analyze the comparison of CNFEHDM and CNFEDM. Since we already know the results of CNFEHDM, now we have to define the CNFEDM for the same problem. Then we can compare both results and get the accuracy level of this problem. And also we analyze the advantages and limitation of CNFEHDM.

$$d(T_1^{(Z_1)}, T_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2) + \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2) + \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2) \end{bmatrix}}{2(3)}}$$
$$= \sqrt{\frac{0.01+0.0225+0.09+0.09}{6}} = 0.188$$
$$d(I_1^{(Z_1)}, I_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2) + \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2) + \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.01+0.04+0.04+0.01}{6}} = 0.128$$
$$d(F_1^{(Z_1)}, F_1^{(Z_2)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2) + \\ ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.1\pi)^2) + \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.1\pi)^2) \end{bmatrix}}{2(3)}}$$
$$= \sqrt{\frac{0.01+0.0225+0.09+0.09+0.01}{6}} = 0.192$$
$$d(T_1^{(Z_1)}, T_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2) + \\ ((0.2-0.7)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.4\pi)^2) + \\ ((0.1-0.4)^2 + \frac{1}{2\pi^2}(0.7\pi - 0.2\pi)^2) \end{bmatrix}}{2(3)}}$$
$$= \sqrt{\frac{0.09+0.0025+0.025+0.09+0.0625}{6}} = 0.287$$

$$d(I_1^{(Z_1)}, I_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.3\pi - 0.4\pi)^2)+ \\ ((0.4-0.8)^2 + \frac{1}{2\pi^2}(0.6\pi - 0.1\pi)^2)+ \\ ((0.7-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.3\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.01 + 0.0025 + 0.16 + 0.0625 + 0.25 + 0.0025}{6}} = 0.285$$

$$d(F_1^{(Z_1)}, F_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.4-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2)+ \\ ((0.6-0.5)^2 + \frac{1}{2\pi^2}(0.8\pi - 0.1\pi)^2)+ \\ ((0.8-0.5)^2 + \frac{1}{2\pi^2}(0.3\pi - 0.4\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.01 + 0.01 + 0.1225 + 0.09 + 0.0025}{6}} = 0.197$$

$$d(T_1^{(Z_2)}, T_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.5-0.2)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2)+ \\ ((0.3-0.7)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2)+ \\ ((0.4-0.4)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.2\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.09 + 0.0025 + 0.16 + 0.25 + 0.0025}{6}} = 0.290$$

$$d(I_1^{(Z_2)}, I_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.2-0.3)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2)+ \\ ((0.4-0.8)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.1\pi)^2)+ \\ ((0.6-0.2)^2 + \frac{1}{2\pi^2}(0.4\pi - 0.3\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.01 + 0.0225 + 0.16 + 0.0025 + 0.16 + 0.0025}{6}} = 0.243$$

$$d(F_1^{(Z_2)}, F_1^{(Z_3)}) = \sqrt{\frac{\begin{bmatrix} ((0.3-0.5)^2 + \frac{1}{2\pi^2}(0.2\pi - 0.2\pi)^2)+ \\ ((0.3-0.5)^2 + \frac{1}{2\pi^2}(0.5\pi - 0.1\pi)^2)+ \\ ((0.5-0.5)^2 + \frac{1}{2\pi^2}(0.1\pi - 0.4\pi)^2) \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{0.04 + 0.04 + 0.04 + 0.01}{9}} = 0.146$$

Similarly,

$$\begin{array}{lll}
d(T_2^{(Z_1)}, T_2^{(Z_2)}) = 0.083 & d(T_2^{(Z_2)}, T_2^{(Z_3)}) = 0.268 & d(T_3^{(Z_1)}, T_3^{(Z_3)}) = 0.256 \\
d(I_2^{(Z_1)}, I_2^{(Z_2)}) = 0.196 & d(I_2^{(Z_2)}, I_2^{(Z_3)}) = 0.154 & d(I_3^{(Z_1)}, I_3^{(Z_3)}) = 0.262 \\
d(F_2^{(Z_1)}, F_2^{(Z_2)}) = 0.235 & d(F_2^{(Z_2)}, F_2^{(Z_3)}) = 0.204 & d(F_3^{(Z_1)}, F_3^{(Z_3)}) = 0.197 \\
d(T_2^{(Z_1)}, T_2^{(Z_3)}) = 0.305 & d(T_3^{(Z_1)}, T_3^{(Z_2)}) = 0.326 & d(T_3^{(Z_2)}, T_3^{(Z_3)}) = 0.400 \\
d(I_2^{(Z_1)}, I_2^{(Z_3)}) = 0.292 & d(I_3^{(Z_1)}, I_3^{(Z_2)}) = 0.213 & d(I_3^{(Z_2)}, I_3^{(Z_3)}) = 0.332 \\
d(F_2^{(Z_1)}, F_2^{(Z_3)}) = 0.189 & d(F_3^{(Z_1)}, F_3^{(Z_2)}) = 0.267 & d(F_3^{(Z_2)}, F_3^{(Z_3)}) = 0.206
\end{array}$$

$$[D_M]_{3\times 3} = \begin{pmatrix} (d(T_1^{(Z_1)}, T_1^{(Z_2)}), d(I_1^{(Z_1)}, I_1^{(Z_2)}), d(F_1^{(Z_1)}, F_1^{(Z_2)})) & (d(T_1^{(Z_1)}, T_1^{(Z_3)}), d(I_1^{(Z_1)}, I_1^{(Z_3)}), d(F_1^{(Z_1)}, F_1^{(Z_3)})) & (d(T_1^{(Z_2)}, T_1^{(Z_3)}), d(I_1^{(Z_2)}, I_1^{(Z_3)}), d(F_1^{(Z_2)}, F_1^{(Z_3)})) \\ (d(T_2^{(Z_1)}, T_2^{(Z_2)}), d(I_2^{(Z_1)}, I_2^{(Z_2)}), d(F_2^{(Z_1)}, F_2^{(Z_2)})) & (d(T_2^{(Z_1)}, T_2^{(Z_3)}), d(I_2^{(Z_1)}, I_2^{(Z_3)}), d(F_2^{(Z_1)}, F_2^{(Z_3)})) & (d(T_2^{(Z_2)}, T_2^{(Z_3)}), d(I_2^{(Z_2)}, I_2^{(Z_3)}), d(F_2^{(Z_2)}, F_2^{(Z_3)})) \\ (d(T_3^{(Z_1)}, T_3^{(Z_2)}), d(I_3^{(Z_1)}, I_3^{(Z_2)}), d(F_3^{(Z_1)}, F_3^{(Z_2)})) & (d(T_3^{(Z_1)}, T_3^{(Z_3)}), d(I_3^{(Z_1)}, I_3^{(Z_3)}), d(F_3^{(Z_1)}, F_3^{(Z_3)})) & (d(T_3^{(Z_2)}, T_3^{(Z_3)}), d(I_3^{(Z_2)}, I_3^{(Z_3)}), d(F_3^{(Z_2)}, F_3^{(Z_3)})) \end{pmatrix}$$

$$= \begin{bmatrix} (0.188, 0.128, 0.192) & (0.287, 0.285, 0.197) & (0.290, 0.243, 0.146) \\ (0.083, 0.196, 0.235) & (0.305, 0.292, 0.189) & (0.268, 0.154, 0.204) \\ (0.326, 0.213, 0.267) & (0.256, 0.262, 0.197) & (0.400, 0.332, 0.206) \end{bmatrix}$$

$$\begin{aligned}
\mathcal{S}(\mathfrak{J}_1) &= \left[\frac{0.188 + 0.287 + 0.290}{3}, \frac{0.128 + 0.285 + 0.243}{3}, \frac{0.192 + 0.197 + 0.146}{3}\right] \\
\mathcal{S}(\mathfrak{J}_1) &= (0.255, 0.218, 0.535) \\
\mathcal{S}(\mathfrak{J}_2) &= \left[\frac{0.083 + 0.305 + 0.268}{3}, \frac{0.196 + 0.292 + 0.154}{3}, \frac{0.235 + 0.189 + 0.204}{3}\right] \\
\mathcal{S}(\mathfrak{J}_2) &= (0.218, 0.214, 0.209) \\
\mathcal{S}(\mathfrak{J}_3) &= \left[\frac{0.326 + 0.256 + 0.400}{3}, \frac{0.213 + 0.262 + 0.332}{3}, \frac{0.267 + 0.197 + 0.206}{3}\right] \\
\mathcal{S}(\mathfrak{J}_3) &= (0.327, 0.269, 0.223) \\
&\mathcal{S}(\mathfrak{J}_1) < \mathcal{S}(\mathfrak{J}_2) < \mathcal{S}(\mathfrak{J}_3).
\end{aligned}$$

Since $\mathfrak{J}_3$ Farmer's feedback on the effectiveness of various irrigation methods, grouped into CNFS values according to their perceptions and experiences. By comparing CNFEHDM and CNFEDM we conclude that CNFEHDM has more accuracy level of ranking than CNFEDM.

Figure 2: Graphical Representation of CNFEHDM

The advantages of Euclidean hesitance distance measures in complex neutrosophic fuzzy systems is its versatility in uncertainty modeling. Decision-makers are able to express different

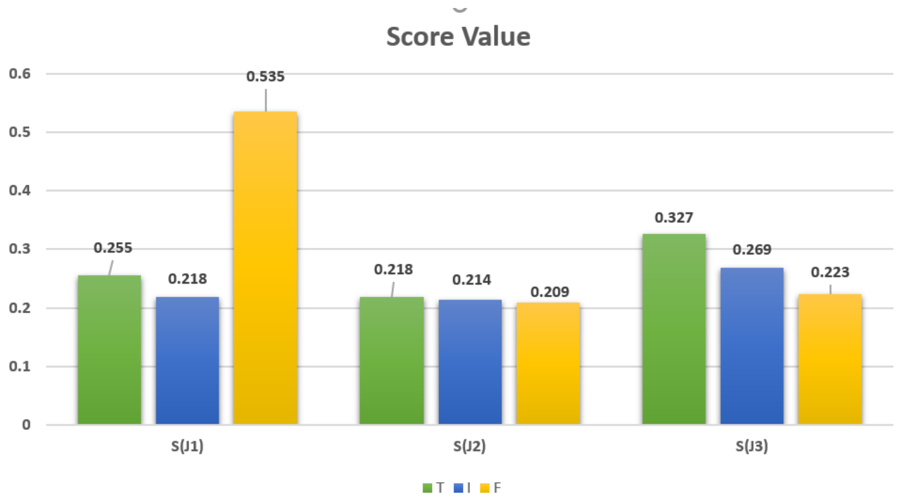

Figure 3: Graphical Representation of CNFEDM

levels of hesitancy towards diverse criteria, thereby grasping the complex and dynamic nature of decision problems in reality. This flexibility renders the approach particularly useful in situations where preferences are vague, contradictory, or affected by various interrelated factors.

While the CNFEHDM offers a richer and more accurate representation of uncertainty, it also has certain limitations. The method introduces higher computational complexity due to the handling of complex values and explicit hesitancy components. It also requires structured expert input, making implementation more time-consuming and demanding. Additionally, the approach lacks standardized software tools and is still in the early stages of acceptance within the broader research community, limiting its immediate applicability in large scale or real-time decision systems.

5. Conclusion

This study presents an innovative CNFEDM that incorporates hesitance degree, proficiently handling issues related to the modeling of uncertainty, indeterminacy, and expert reluctance in decision-making frameworks. The suggested model offers a more sophisticated tool for intelligent choice systems by expanding the conventional Euclidean distance to account for truth, indeterminacy, falsity, and hesitancy. The model effectively determined that water availability was the most important factor when applied to smart irrigation, allowing for the most efficient use of water resources and encouraging sustainable farming methods.

The novelty of this work lies in the integration of complex-valued neutrosophic information with hesitance sensitive distance measures, offering a multidimensional understanding of uncertainty and decision space. This structure supports more transparent and context aware decisions, particularly in environments where traditional models fall short due to data ambiguity and expert hesitation. Scalability and ease of use in large or real-time systems may be impacted by this model's drawbacks, which include greater computing complexity, limited tool support, and the requirement for structured expert inputs.

The Future approach can be expanded for this proposed concept by:

- Creating software libraries or automated tools for CNFEHDM.
- Investigating hybrid decision models that use CNF based architecture and machine learn-

ing.

- Extending the approach to other ambiguous fields like disaster response planning, climate risk assessment, and medical diagnostics.
- Including real-time data adaption and dynamic weighting to increase its usefulness in precision farming.

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