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Assessment of Hospital Performance Using Complex Spherical Fuzzy Soft ELECTRE and TOPSIS

Muhammad Shazib Hameed[1], Shahzaib Ashraf[1], Vladimir Simic[2,*], Chiranjibe Jana[3], Dragan Pamucar[4,5,6,*], Nebojsa Bacanin[7,3]

[1] *Institute of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan 64200, Pakistan*
[2] *Széchenyi István University, Egyetem tér 1, 9026 Gyor, Hungary*
[3] *Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences (SIMATS), Chennai 602105, Tamil Nadu, India*
[4] *Department of Applied Mathematical Science, College of Science and Technology, Korea University, Sejong, 30019, Republic of Korea*
[5] *Faculty of Engineering, Dogus University, 34775 Umraniye, Istanbul, Türkiye*
[6] *Applied Artificial Intelligence Research Center, Azerbaijan State University of Economics (UNEC), Baku, Azerbaijan*
[7] *Singidunum University, Danijelova 32, Belgrade, 11000, Serbia*

Abstract. Delineating the tactical approach of the "ELiminating Et Choice Translating REality" (ELECTRE) technique and the "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS) for multi-attribute group decision-making in terms of complex spherical fuzzy soft sets is the focus of this study. The decision-making effectiveness and ranking quality of the proposed methods are greatly improved by the distinctive, pioneering, and significant structure of proposed set. This makes it an outstanding and proficient approach for addressing multi-attribute group decision-making. This is a result of the fact that suggested work is able to provide a comprehensive performance with a useful and more sophisticated competence method. In addition to the presented approach, a number of non-fundamental features of the complex spherical fuzzy soft weighted averaging operators are investigated. These qualities include shift invariance, homogeneity, linearity, and additive properties. The individual views are validated into an acceptable form by the use of the this operator, and the aggregated opinions are further analyzed by the suggested complex spherical fuzzy soft ELECTRE I technique and the CSFS-TOPSIS technique. Normalized Euclidean distances of complex spherical fuzzy soft numbers are also taken into account as part of the approach that has been developed. A decision graph is formed on the basis of an aggregated outranking matrix to get the solutions that are outranked and the optimal option. This article offers a supplemental strategy at the very end of the process, which aims to provide a linear ranking order for the many possibilities. An example case study taken from the medical field helps to illustrate the adaptability and practicability of the strategy that is introduced.

*Corresponding Author.
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Email addresses: shazib.hameed@kfueit.edu.pk (M. S. Hameed),
sssssssssssssssssssssssssssssssssssssssssssssssssss shahzaib.ashraf@kfueit.edu.pk (S. Ashraf)
simic.vladimir@sze.hu (V. Simic), jana.chiranjibe7@gmail.com (C. Jana)
dragan.pamucar@gmail.com (D. Pamucar), nbacanin@singidunum.ac.rs (N. Bacanin)

1. Introduction

The available fuzzy, intuitionistic and spherical fuzzy based extensions of TOPSIS and ELECTRE can only deal with uncertainty in the real space, without considering phase and interaction effects in the complicated assessments. They also do not obtain an organized method of handling multi-attribute data of numerous specialists. Complex Spherical Fuzzy Soft Set (CSFSS) model suggested is an integration of the parametric flexibility of soft sets and the multidimensional uncertainty representation of complex spherical fuzziness. The innovation is with the CSFS-ELECTRE I and CSFS-TOPSIS procedures which get the magnitude and the phase of the expert ratings to get more precise and understandable rankings. The decision-making (DM) process incorporates the usage of MAGDM. With their assistance, a variety of decisions relating to daily life can be made. Existing methodologies place an emphasis on DM evaluation data as a means of selecting the most effective strategy. However, the ever-increasing complexity of the DM environment and the subjective ambiguity of DMs make it difficult for them to accurately represent their genuine preference data. Zadeh [1] created fuzzy sets (FSs) to accommodate for evaluation imprecision and uncertainty. Many scientists have studied and implemented FSs [2–4]. Researchers are now trying to better represent DM evaluation preferences. FSs have been classified to suit distinct application circumstances. Intuitionistic FSs (IFSs), developed by Atanassov [5], combine supporting and non-supporting grades with a constraint that their aggregate cannot exceed a unit interval. In an attempt to address the limitations of existing FS theories, researchers have explored various alternative approaches. While IFSs restrict the sum of supporting and non-supporting grades within a unit interval, this condition is deemed too restrictive for certain decision-makers. To overcome this, Yager introduced Pythagorean FSs (PyFSs) [6], where the sum of the squares of both grades must not exceed a unit interval. Both IFSs and PyFSs have found numerous applications in various fields. However, another theory emerged, known as picture FSs (PFSs), introduced by Cuong and Kreinovich [7]. In PFSs, the total grade cannot go over a certain unit interval, which takes into account the degrees of falsehood, abstention, and truth. If the student gives an inaccurate response to any question, they will get a score of zero for that question. Despite their value, PFSs have been criticized for allegedly misrepresenting the preferences of decision-makers. This holds true even if PFSs turn out to be helpful. To address these limitations, Mahmood et al. [8] introduced the concept of spherical fuzzy sets (SFSs), providing a more flexible framework, which was later extended to interval-valued T-SFSs by Sarfraz [9]. SFS is preferable to earlier versions of the FS theory when applied to higher grades since it guarantees that the total squares for all grades are contained within a single interval. For FS theory, this is a tremendous achievement. Given the challenges posed by DMs inherent fuzziness and ambiguity, this is a strategic step. Since SFSs provide a flexible and trustworthy solution to the problem of handling complex and puzzling data, they are useful in many different settings. It has been discovered that SFSs may be helpful in a number of different contexts. The mathematical framework known as "soft set theory" has found use in a wide range of contexts because of its ability to accommodate ambiguous and imprecise data. Soft set theory has many exciting appli-

cations, including decision-making [10, 11], medical diagnosis [12], image processing [13], assessment methods [14], machine learning [15], commerce and economics [16].
The research that was done on the DM process revealed that the generally recognized theories, despite the fact that they perform well when it comes to addressing the role of informational risk, do not perform well when it comes to addressing the influence of time. Because data sources, such as clinical tests and biometric databases, are always under development, it is necessary to make use of methodologies that are more adaptable. The term complex FS (CFS) was used by Ramot et al. [17] in order to refer to the solution they came up with to these problems. With CFS, the range of the supporting grade is extended from a real subset to the unit circle of the complex plane, making it helpful for dealing with time-varying information. The adaptability of CFSs has led to widespread adoption in a wide variety of settings. Later, Alkouri and Salleh [18] created complex intuitionistic FS (CIFS) to provide decision-makers with more opportunity for debate. One limitation of CIFS is that the sum of the real and imaginary parts of both grades cannot exceed a unit interval, which may be problematic for some decision-makers. To get over this limitation, Ullah et al. [19] proposed complex PyFS (CPyFS), which makes sure the grade sum of the real and imaginary sections stays inside the interval of one. Both CIFS and CPyFS can be used for a variety of purposes [20–22]. In cases where decision-makers are presented with a wide variety of responses stated as complex numbers, classical DM theories like IFS, PyFS, CIFS, and CPyFS may not be sufficient. For instance, when voters can be classified into four categories: in favor, abstain, against, or refuse the democratic process, such a structure has limitations in analyzing such complex scenarios. To address these intricacies, the researchers delve into the theory of complex SFSs to assess proficiency and capabilities. The complex SFS [23] introduces a novel approach for effectively handling diverse and intricate situations, while the TOPSIS method [24] serves as a powerful tool for DM problems. This approach shows promise in solving the unique problems that arise in complex DM scenarios when several answer types are represented as complex numbers. The combination of complex SFSs with soft set theory has been very helpful to the advancement of fuzzy logic. Expanding soft set theory's usefulness is a top priority, and this study presents the unifying idea of complex SFS merging with soft set. The CSFSS [25] is a cross-disciplinary hybrid method for managing and organizing challenges. Building on this foundation, the essay has helped us make significant progress in our understanding and use of fuzzy logic. In several academic fields, including economics, sociology, health, computer science, physics, ecology, management science, and engineering, real-world examples illuminate study limitations. Research in these fields is essential because of the world's inherent fuzziness, imprecision, and ambiguity. To address the difficulties brought on by these features, researchers have turned to mathematical methods that can take them into consideration. In this research, we starting with a brief introduction to CSFSSs, this study goes on to provide an innovative approach to DM by combining CSFSSs with aggregation operators. In a complicated CSFSS context, this method utilizes the aforementioned aggregation procedures to determine a grade. To demonstrate the value of our modified proposed ELECTRE I method and emphasize its efficacy and dependability, we provide a comprehensive example. Furthermore, we do comprehensive testing of the DM process

to demonstrate its reliability. These assessments show its worth in tense DM scenarios and emphasize its potential. Through this research, our primary goal is to enhance the understanding and utilization of CSFSS by introducing novel aggregation operators and DM methodologies. By doing so, our aim is to promote the widespread application of CSFSS in various domains and contribute to the advancement of fuzzy logic in practical problem-solving contexts in the near future.

The application domain is the area of the hospital performance assessment because it is a critical area in the society and in management. Hospitals are the backbone of any healthcare system, and successful evaluation of a hospital directly determines patient safety, quality of service, and policy making. At the societal level, comparisons of hospital performance guarantee fair access to high-quality care, effective distribution of resources, and better health outcomes of people. In terms of management, it helps administrators and policymakers understand where they can cut costs in their operations, where to invest, and improve the general healthcare standards of the delivery.

CSFSS are especially suitable in this field, as the assessment of hospitals entails indeterminate and vague opinions, which are sometimes conflicting, of various professionals. Patient satisfaction, quality of care and technological efficiency are in their nature vague and subjective. CSFSS provides a mathematically enriched framework that is able to express the levels of true, indeterminate and false in complex situations of DM. CSFSS is able to model dynamic expert opinions and uncertainty using complex numbers better than the classical or real-valued fuzzy methods by incorporating the phase and amplitude components of complex numbers.

Current fuzzy, intuitionistic and Pythagorean DM models can capture uncertainty partially but are usually restricted in their ability to model complex and multidimensional expert opinions. The classical fuzzy sets are not able to differentiate between various sources of indeterminacy. Pythagorean and intuitionistic-based fuzzy sets have constraining membership requirements that limit flexibility. Complex fuzzy sets give the representations in terms of phase components and do not fully model neutrality and hesitation. These shortcomings hamper their capacity to represent the dynamic and uncertain character of the hospital evaluation setting, comprising people with multiple professions and criteria. CSFSS framework builds on the previous fuzzy models, including the amplitude and phase of the membership degrees in a soft set framework, which enables the multi-attribute multi-expert decision-making with a greater degree of accuracy. Key advantages include: Increased ability to show indeterminacy, impartiality, and inconsistency in the judgments of experts. Aggregation flexibility with CSFSWA operator properties such as shift invariance, linearity, and homogeneity. Better ranking and discrimination capability with the hybrid CSFS-ELECTRE and CSFS-TOPSIS procedures relative to the previous fuzzy MCDM approaches.

The paper leads to the literature in the following respects: Presents the CSFSS framework as the development of fuzzy soft set theory to complex DM problems. Presents two new hybrid techniques, CSFS-ELECTRE I and CSFS-TOPSIS, to evaluate the performance of hospitals. Demonstrates mathematical properties of the CSFSWA operator in order to provide accuracy in aggregation. Watches the suggested techniques on a practical case of a hospital performance and shows their effectiveness and readability. Presents a comparative analysis in relation to the existing methods, pointing to the strength and flexibility of CSFSS when used in real-life healthcare assessment.

Motivation of the Research: The process of measuring hospital performance is a complicated DM process, which assumes having several conflicting criteria and ambiguous human judgments. Conventional assessment schemes tend to overlook the subtle viewpoints of the decision-makers in instances where they are in form of ambiguous or unclear words. The rationale of this study is to develop a well-founded and versatile system that is capable of managing the vagueness and ambiguity of healthcare assessment. This motivation is further enhanced by the fact that the medical field requires credible, data-driven, and transparent methods of performance measurement.

As a review of published literature shows, the majority of the existing decision-making frameworks do not effectively deal with multi-dimensional uncertainty and contextual vagueness in healthcare assessment. There are insufficient phase relationships and interdependencies between attributes and expert opinions that are represented in previous fuzzy-based approaches. This void inspires the emergence of CSFSS that combine the flexibility of soft sets with complex spherical fuzziness and provide a more realistic, flexible, and mathematically consistent system in the evaluation of hospital performance in multiple attributes. The paper aims to investigate and explore various aspects of CSFSSs and their applications, as well as establish a modified ELECTRE I and TOPSIS method by using these aspects. The specific objectives and organization of the paper are as follows: In Section 2, we discussed some fundamental concepts. In Section 3, we investigate the basic definitions of the novel concept of CSFSSs and their fundamental laws, along with their score and accuracy functions, and also discuss the operational laws and properties associated with these concepts. In Section 4, the focus shifts to proposing new operators based on CSFSS, specifically some important properties (shift invariance, homogeneity, linearity, and additive property) of the CSFSWA operator. We introduce a novel approach to MAGDM by combining the concept of CSFSS with the ELECTRE I method in Section 5. This new approach is referred to as the CSFS-ELECTRE I. By integrating CSFSS into the ELECTRE I method, we aim to enhance the DM process in scenarios involving multiple attributes and decision-makers. The CSFS-ELECTRE I method provides a more comprehensive and effective way to handle complex DM situations where uncertainty and imprecision are prevalent. Section 6 of the article introduces a compelling and comprehensive example, showcasing the application of the CSFS-ELECTRE I method in real-life DM scenarios. This exemplar case study exemplifies the power of innovative ideas at work. The problem description offers a clear and engaging narrative, outlining the objectives, criteria, and alternatives involved. The attribute selection process skillfully identifies the most relevant factors critical to the DM process, setting the stage for an in-depth evaluation. At the end, we present the results of our discussion. We provided comparative analysis in Section 7. In this section, a single MAGDM problem is tackled and successfully resolved using the CSFS-TOPSIS technique. This section includes both numerical and theoretical comparisons of the proposed approach with the CSFS-ELECTRE I technique. In the last

Section 8, our findings and summary are presented. In this section, we have also addressed the limitations of the study and provided insights into future directions.
Objectives of the Research:

- To create a new decision-making model that involves CSFSS with uncertainty-based hospital performance evaluation.
- To plan and implement the CSFS-ELECTRE I and CSFS-TOPSIS techniques of ranking hospitals using several qualitative and quantitative characteristics.
- To improve the accuracy and consistency of the DM process by aggregated operators.
- To test the feasibility and versatility of the suggested solution with a real-life example in the medical field.

2. Preliminaries and Basic Concepts

Definition 1. *[26] Let Z be the non-empty universal set of objects and $\mathfrak{Y}$ be the set of attributes, for any non-empty set $K \subseteq \mathfrak{Y}$. A pair (ϖ, K) is called soft set over Z if there is a mapping $\varpi : K \to P(Z)$ that $P(Z)$ denotes the set of all subsets of Z. Thus, the soft set is a parametric family of the subsets of a universal set. For each $k_j \in K$, we can interpret $\varpi(k_j)$ as a subset of universal set Z. We can also consider (k_j) as a mapping $\varpi(k_j) : Z \to \{0,1\}$ and then $\varpi(k_j)(u_i) = 1$ equivalent to $u_i \in \varpi(k_j)$, otherwise $\varpi(k_j)(u_i) = 0$. Molodtsov considered many examples in [26] to illustrate the soft set.*

Definition 2. *[27] Let Z be universal set. Let $\mathfrak{Y}$ be a set of parameters, and $P(Z)$ be the power set of FS of Z. For any non empty set $K \subseteq \mathfrak{Y}$, a pair (ϖ, K) is called a fuzzy soft set over Z where ϖ is a mapping given by $\varpi : K \to P(Z)$.*

Definition 3. *[28] Let Z be a universal set. A SFS ϖ on Z is interpreted as*

$$\begin{aligned} \varpi &= \langle (z, \chi_\varpi(z), \vartheta_\varpi(z), \xi_\varpi(z)) | z \in Z \rangle \\ &= \langle z, \mu_\varpi(z), \nu_\varpi(z), \omega_\varpi(z) | z \in Z \rangle, \end{aligned}$$

where $\mu_\varpi,\ \nu_\varpi,\ \omega_\varpi : \varpi \to [0,1]$ are said to be degree of positive membership, degree of neutral membership, and degree of negative membership of z in Z. Also $\forall\ z \in Z,\ 0 \leq \mu_\varpi(z)^2 + \nu_\varpi(z)^2 + \omega(z)^2 \leq 1$. For $z \in Z$, the degree of refusal of is described as: $\Psi_\varpi(z) = \sqrt{1 - (\mu_\varpi(z)^2 + \nu_\varpi(z)^2 + \omega_\varpi(z)^2)}$.

Definition 4. *[29] Let Z be universal set. Let $\mathfrak{Y}$ be a set of parameters, and $SFS(Z)$ be the power set of SFS of Z. For any non-empty set $K \subseteq \mathfrak{Y}$, a pair (ϖ, K) is called a SFSS over Z where ϖ is a mapping given by $\varpi : K \to SFS(Z)$.*
Note: *If the degree of neutral membership will become 0, $\forall\ z \in Z$, then SFS will become a PyFS and (ϖ, K) will become a Pythagorean fuzzy soft set if it is true for all $k_j \in K$. We denote the set of all SFSs over Z by SFSS(Z).*

Definition 5. *[17] A CFS ϖ defined on a universe of discourse Z is characterized by a membership function that allocates each element of Z to a unit circle C^* in complex plane and is written as $\varpi = \{\mu_\varpi(z)e^{i2\pi f_\varpi(z)} | z \in Z\}$, such that $i = \sqrt{-1}$ where $\mu_\varpi(z)$ denotes the real-valued function from Z to the closed unit interval and $e^{i2\pi f_\varpi(z)}$ is a periodic function whose periodic law and principal period are 2π and $0 < arg_\varpi(z) \leq 2\pi$, respectively. Note that $f_\varpi(z) = arg_\varpi(z) + 2\kappa\pi,\ \kappa \in Z$ and $arg_\varpi(z)$ is the principal argument.*

Definition 6. *[30] Let Z be the universal set. Let $\mathfrak{Y}$ be the set of parameters under consideration, and $CFS(Z)$ denote the set of all complex fuzzy subset of Z. For any non-empty set $K \subseteq \mathfrak{Y}$, a pair (ϖ, K) is called a complex fuzzy soft set (CFSS) over Z, where ϖ is a mapping given by $\varpi : K \to CFS(Z)$, $\varpi(k_j) = \phi$; that is, $\chi_{\varpi(k_j)}(z) = 0$, for all $z \in Z$, if $k_j \notin K$.*

Definition 7. *[19] A CIFS ϖ on universal set Z is listed below:*

$$\begin{aligned}\varpi &= \langle (z, \chi_\varpi(z), \vartheta_\varpi(z) | z \in Z \rangle \\ &= \langle z, \mu_\varpi(z)e^{i2\pi f_\varpi(z)}, \nu_\varpi(z)e^{i2\pi g_\varpi(z)} | z \in Z \rangle,\end{aligned}$$

where $i = \sqrt{-1},\ \mu_\varpi(z), \nu_\varpi(z)$, identifying the amplitude terms, and $f_\varpi(z), g_\varpi(z)$ identifying the periodic terms, are limited to the interval of closed units, with the clauses that for each $z \in Z,\ \mu_\varpi(z) + \nu_\varpi(z) \leq 1$ and $f_\varpi(z) + g_\varpi(z) \leq 1$.

Definition 8. *[31] Let Z be the universal set. Let $\mathfrak{Y}$ be the set of parameters under consideration, and $CIFS(Z)$ denote the set of all complex intuitionistic fuzzy subsets of Z. For any non-empty set $K \subseteq \mathfrak{Y}$, a pair (ϖ, K) is called a complex intuitionistic fuzzy soft set (CIFSS) over Z, where ϖ is a mapping given by $\varpi : K \to CIFS(Z)$ such that $\varpi(k_j) = \phi$; that is, $\chi_{\varpi(k_j)}(z) = 0$ and $\vartheta_{\varpi(k_j)}(z) = 1$, for all $z \in Z$, if $k_j \notin K$.*

Definition 9. *[18] A CPyFS ϖ on a universal set Z is listed below:*

$$\begin{aligned}\varpi &= \langle (z, \chi_\varpi(z), \vartheta_\varpi(z) | z \in Z \rangle \\ &= \langle z, \mu_\varpi(z)e^{i2\pi f_\varpi(z)}, \nu_\varpi(z)e^{i2\pi g_\varpi(z)} | z \in Z \rangle,\end{aligned}$$

where $i = \sqrt{-1},\ \mu_\varpi(z), \nu_\varpi(z)$, identifying the amplitude terms, and $f_\varpi(z), g_\varpi(z)$ identifying the periodic terms, are limited to the interval of closed units, with the clauses that for each $z \in Z,\ \mu_\varpi(z)^2 + \nu_\varpi(z)^2 \leq 1$ and $f_\varpi(z)^2 + g_\varpi(z)^2 \leq 1$.
The degree of refusal of $z \in Z$ is outlined as
$\Psi_\varpi(z) = \sqrt{1 - (\mu_\varpi(z)^2 + \nu_\varpi(z)^2)}e^{i2\pi\sqrt{1-(f_\varpi(z)^2 + g_\varpi(z)^2)}}$.
For ease, the pair $(\chi_\varpi, \vartheta_\varpi) = (\mu_\varpi e^{i2\pi h_\varpi}, \nu_\varpi e^{i2\pi g_\varpi})$ is defined as complex Pythagorean fuzzy number (CPyFN).

Definition 10. *[32] Let Z be the universal set. Let $\mathfrak{Y}$ be the set of parameters under consideration and $CPyFS(Z)$ denote the set of all complex Pythagorean fuzzy subset of Z. For any non empty set $K \subset \mathfrak{Y}$, a pair (ϖ, K) is called a complex Pythagorean fuzzy soft set (CPyFSS) over Z, where ϖ is a mapping given by $\varpi : K \to CPyFS(Z)$ such that $\varpi(k_j) = \phi$; that is, $\chi_{\varpi(k_j)}(z) = 0$ and $\vartheta_{\varpi(k_j)}(z) = 1$, for all $z \in Z$, if $k_j \notin K$.*

3. Construction of Complex Spherical Fuzzy Soft Set

Definition 11. *[33] A complex SFS ϖ on a universal set Z is listed below:*

$$\begin{aligned}\varpi &= \langle (z, \chi_{\varpi}(z), \vartheta_{\varpi}(z), \xi_{\varpi}(z)) | z \in Z\rangle \\ &= \langle z, \mu_{\varpi}(z)e^{i2\pi f_{\varpi}(z)}, \nu_{\varpi}(z)e^{i2\pi g_{\varpi}(z)}, \omega_{\varpi}(z)e^{i2\pi h_{\varpi}(z)} | z \in Z\rangle,\end{aligned}$$

where $i = \sqrt{-1}$, $\mu_{\varpi}(z), \nu_{\varpi}(z), \omega_{\varpi}(z)$, identifying the amplitude terms, and $f_{\varpi}(z), g_{\varpi}(z), h_{\varpi}(z)$, identifying the periodic terms, are limited to the interval of closed units, with the clauses that for each $z \in Z$, $\mu_{\varpi}(z)^2 + \nu_{\varpi}(z)^2 + \omega(z)^2 \leq 1$ and $f_{\varpi}(z)^2 + g_{\varpi}(z)^2 + h_{\varpi}(z)^2 \leq 1$. The degree of refusal $z \in Z$ is outlined as
$\Psi_{\varpi}(z) = \sqrt{1 - (\mu_{\varpi}(z)^2 + \nu_{\varpi}(z)^2 + \omega_{\varpi}(z)^2)}e^{i2\pi\sqrt{1-(f_{\varpi}(z)^2+g_{\varpi}(z)^2+h_{\varpi}(z)^2)}}$.

Definition 12. *Let Z be the universal set. Let $\mathfrak{Y}$ be the set of parameters under consideration and $CSFS(Z)$ denote the set of all complex spherical fuzzy subset of Z. For any non empty set $K \subset \mathfrak{Y}$, a pair (ϖ, K) is called a CSFSS over Z, where ϖ is a mapping given by $\varpi : K \to CSFS(Z)$, $\varpi(k_j) = \phi$; that is, $\chi_{\varpi(k_j)}(z) = 0$ and $\vartheta_{\varpi(k_j)}(z) = 0$, and $\xi_{\varpi(k_j)}(z) = 1$ for all $z \in Z$, if $k_j \notin K$.*
$(\varpi_j, K_j) = \varpi_j = (\chi_j, \vartheta_j, \xi_j) = (\mu_j e^{i2\pi f_j}, \nu_1 e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ is defined as a complex spherical fuzzy soft number (CSFSN).

Example 1. *Suppose we have a set of objects representing different fruits, and we want to define a CSFSS that describes the quality of each fruit. For this, we choose the family of a fixed set*
$Z = \{z_1(\text{apple}), z_2(\text{mango}), z_3(\text{apricot}), z_4(\text{grapes})\}$ and corresponding of parameters
$\varpi = \{\varpi_1(\text{sweetness}), \varpi_2(\text{ripeness}), \varpi_3(\text{skin color}), \varpi_4(\text{absence of defects})\}$. Then the rating values of each value of the fixed set concerning parameters are discussed in Table 1.

Table 1: Expressions of the CSFSNs

Z	ϖ_1	ϖ_2
z_1	$(0.20e^{i2\pi(0.56)}, 0.14e^{i2\pi(0.39)}, 0.25e^{i2\pi(0.37)})$	$(0.72e^{i2\pi(0.23)}, 0.41e^{i2\pi(0.65)}, 0.35e^{i2\pi(0.27)})$
z_2	$(0.37e^{i2\pi(0.57)}, 0.21e^{i2\pi(0.34)}, 0.61e^{i2\pi(0.32)})$	$(0.11e^{i2\pi(0.15)}, 0.18e^{i2\pi(0.68)}, 0.73e^{i2\pi(0.52)})$
z_3	$(0.26e^{i2\pi(0.35)}, 0.18e^{i2\pi(0.53)}, 0.31e^{i2\pi(0.46)})$	$(0.41e^{i2\pi(0.32)}, 0.29e^{i2\pi(0.48)}, 0.67e^{i2\pi(0.41)})$
z_4	$(0.46e^{i2\pi(0.35)}, 0.32e^{i2\pi(0.42)}, 0.45e^{i2\pi(0.29)})$	$(0.28e^{i2\pi(0.27)}, 0.33e^{i2\pi(0.44)}, 0.25e^{i2\pi(0.32)})$
Z	ϖ_3	ϖ_4
z_1	$(0.22e^{i2\pi(0.45)}, 0.53e^{i2\pi(0.32)}, 0.27e^{i2\pi(0.37)})$	$(0.42e^{i2\pi(0.40)}, 0.45e^{i2\pi(0.17)}, 0.48e^{i2\pi(0.60)})$
z_2	$(0.52e^{i2\pi(0.20)}, 0.17e^{i2\pi(0.45)}, 0.30e^{i2\pi(0.18)})$	$(0.31e^{i2\pi(0.44)}, 0.17e^{i2\pi(0.37)}, 0.38e^{i2\pi(0.31)})$
z_3	$(0.26e^{i2\pi(0.20)}, 0.42e^{i2\pi(0.56)}, 0.21e^{i2\pi(0.34)})$	$(0.23e^{i2\pi(0.50)}, 0.31e^{i2\pi(0.55)}, 0.33e^{i2\pi(0.44)})$
z_4	$(0.50e^{i2\pi(0.10)}, 0.22e^{i2\pi(0.19)}, 0.49e^{i2\pi(0.41)})$	$(0.39e^{i2\pi(0.18)}, 0.17e^{i2\pi(0.44)}, 0.39e^{i2\pi(0.36)})$

Definition 13. *Let $\varpi_1 = (\mu_1 e^{i2\pi f_1}, \nu_1 e^{i2\pi g_1}, \omega_1 e^{i2\pi h_1})$ and $\varpi_2 = (\mu_2 e^{i2\pi f_2}, \nu_2 e^{i2\pi g_2}, \omega_2 e^{i2\pi h_2})$ be two CSFSNs and $\upsilon > 0$, then the related operations associated with CSFSNs are*

(1) $\varpi_1 \oplus \varpi_2 = (\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}e^{i2\pi\sqrt{f_1^2+f_2^2-f_1^2f_2^2}}, \nu_1\nu_2 e^{i2\pi g_1 g_2}, \omega_1\omega_2 e^{i2\pi h_1 h_2})$,

(2) $\varpi_1 \otimes \varpi_2 = (\mu_1\mu_2 e^{i2\pi f_1 f_2}, \sqrt{\nu_1^2+\nu_2^2-\nu_1^2\nu_2^2}e^{i2\pi\sqrt{g_1^2+g_2^2-g_1^2g_2^2}}, \sqrt{\omega_1^2+\omega_2^2-\omega_1^2\omega_2^2}e^{i2\pi\sqrt{h_1^2+h_2^2-h_1^2h_2^2}}),$

(3) $\upsilon\varpi_1 = (\sqrt{1-(1-\mu_1^2)^\upsilon}e^{i2\pi\sqrt{1-(1-f_1^2)}^\upsilon}, \nu_1^\upsilon e^{i2\pi g_1^\upsilon}, \omega_1^\upsilon e^{i2\pi h_1^\upsilon}),$

(4) $\varpi_1^\upsilon = (\mu_1^\upsilon e^{i2\pi f_1^\upsilon}, \sqrt{1-(1-\nu_1^2)^\upsilon}e^{i2\pi\sqrt{1-(1-g_1^2)^\upsilon}}, \sqrt{1-(1-\omega_1^2)^\upsilon}e^{i2\pi\sqrt{1-(1-h_1^2)^\upsilon}}).$

Definition 14. *The normalized Euclidean distance between two CSFSNs* $\varpi_1 = (\mu_1 e^{i2\pi f_1}, \nu_1 e^{i2\pi g_1}, \omega_1 e^{i2\pi h_1})$ *and* $\varpi_2 = (\mu_2 e^{i2\pi f_2}, \nu_2 e^{i2\pi g_2}, \omega_2 e^{i2\pi h_2})$ *is calculated as follows:*

$$d(\varpi_1,\varpi_2) = \sqrt{\frac{(\mu_1^2-\mu_2^2)^2+(\nu_1^2-\nu_2^2)^2+(\omega_1^2-\omega_2^2)^2+(f_1^2-f_2^2)^2+(g_1^2-g_2^2)^2+(h_2^1-h_2^2)^2}{3}}.$$

Definition 15. *Let* $\varpi = (\mu_\varpi e^{i2\pi f_\varpi}, \nu_\varpi e^{i2\pi g_\varpi}, \omega_\varpi e^{i2\pi h_\varpi})$ *be CSFSN. The score function* $Sc(\varpi)$ *is*
$Sc(\varpi) = \mu_\varpi(z)^2 - \nu_\varpi(z)^2 - \omega_\varpi(z)^2 + f_\varpi(z)^2 - g_\varpi(z)^2 - h_\varpi(z)^2$*, where* $Sc(\varpi) \in [-2,2]$.
The accuracy function $Ac(\varpi)$ *is defined as* $Ac(\varpi) = \mu_\varpi(z)^2 + \nu_\varpi(z)^2 + \omega_\varpi(z)^2 + f_\varpi(z)^2 + g_\varpi(z)^2 + h_\varpi(z)^2$ *where* $Ac(\varpi) \in [0,2]$.

Definition 16. *Consider* $\varpi_1 = (\mu_1 e^{i2\pi f_1}, \nu_1 e^{i2\pi g_1}, \omega_1 e^{i2\pi h^1})$ *and* $\varpi_2 = (\mu_2 e^{i2\pi f_2}, \nu_2 e^{i2\pi g_2}, \omega_2 e^{i2\pi h_2})$ *be two CSFSNs.*

(1) If $Sc(\varpi_1) < Sc(\varpi_2)$*, then* $\varpi_1 \prec \varpi_2$*(*ϖ_1 *is inferior to* ϖ_2*).*

(2) If $Sc(\varpi_1) > Sc(\varpi_2)$*, then* $\varpi_1 \succ \varpi_2$*(*ϖ_1*is superior to* ϖ_2*).*

(3) If $Sc(\varpi_1) = Sc(\varpi_2)$*, then*

(i) $Ac(\varpi_1) < Ac(\varpi_2)$*, then* $\varpi_1 \prec \varpi_2$*(*ϖ_1 *is inferior to* ϖ_2*);*

(ii) $Ac(\varpi_1) > Ac(\varpi_2)$*, then* $\varpi_1 \succ \varpi_2$*(*ϖ_1 *is superior to* ϖ_2*);*

(iii) $Ac(\varpi_1) = Ac(\varpi_2)$*, then* $\varpi_1 \sim \varpi_2$*(*ϖ_1 *is equivalent to* ϖ_2*).*

Definition 17. *Let* $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})(j = 1,2,...,l)$ *be a collection of CSFSNs and* $\vartheta = (\vartheta_1, \vartheta_2, ...\vartheta_l)^T$ *be the weight vector (w.v) of* ϖ_j *with* $\vartheta > 0$ *and* $\sum_{j=1}^l \vartheta_l = 1$*. The CSFSWA operator is indicated by*

$$CSFSWA(\varpi_1,\varpi_2,...,\varpi_l) = \vartheta_1\varpi_1 \oplus \vartheta_2\varpi_2 \oplus ...... \oplus \vartheta_l\varpi_l$$
$$= \begin{pmatrix} \sqrt{1-\prod_{j=1}^l (1-\mu_j)^{\vartheta_j}}\, e^{i2\pi\sqrt{1-\prod_{j=1}^l (1-f_j)^{\vartheta_j}}}, (\prod_{j=1}^l (\nu_j)^{\vartheta_j}) e^{i2\pi(\prod_{j=1}^l (g_j)^{\vartheta_j})}, \\ (\prod_{j=1}^l (\omega_j)^{\vartheta_j}) e^{i2\pi(\prod_{j=1}^l (h_j)^{\vartheta_j})} \end{pmatrix} \tag{1}$$

4. Some Properties of Complex Spherical Fuzzy Soft Weighted Averaging Operator

Some of the mathematical characteristics of the CSFSWA operator have been outlined here.

Theorem 1. *(Shift invariant) Let* $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ $(j = 1, 2, ..., l)$ *be a collection of CSFSNs and* $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_l)^T$ *be the normalized w.v of* ϖ_j*, and then,* $\forall \ \ h \in (0, 1), \ \forall \ j.$
$CSFSWA(\varpi_1 + \hbar, \varpi_2 + \hbar, ..., \varpi_l + \hbar) = CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l) + \hbar.$

Proof. Let collection of CSFSNs be $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ $(j = 1, 2, 3, ..., l)$ and $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_l)^T$ be the normalized w.v of ϖ_j along with $h \in (0, 1)$, afterwards

$$\begin{aligned}
CSFSWA(\varpi_1 + \hbar, \varpi_2 + \hbar, ..., \varpi_l + \hbar) &= \vartheta_1(\varpi_1 + \hbar) \oplus \vartheta_2(\varpi_2 + \hbar) \oplus ... \oplus \vartheta_l(\varpi_l + \hbar) \\
&\quad \text{(using Equation 1)} \\
&= (\vartheta_1\varpi_1 \oplus ... \oplus \vartheta_l\varpi_l) + (\vartheta_1\hbar + ... + \vartheta_l\hbar) \\
&= (\vartheta_1\varpi_1 \oplus \vartheta_2\varpi_2 \oplus ... \oplus \vartheta_l\varpi_l) + \hbar \qquad (\because \sum_{j=1}^{l} \vartheta_1 = 1) \\
&= CSFSWA(\varpi_l, \varpi_2, ..., \varpi_l) + \hbar.
\end{aligned}$$

Theorem 2. *(Homogenous) Let collection of CSFSNs be* $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ $(j = 1, 2, 3, ..., l)$*, and* $\vartheta = (\vartheta_1, \vartheta_2, ...\vartheta_l)^T$ *be the normalized w.v of* ϖ_j*, and next,* $\forall \ \ \hbar \in (0, 1), \forall \ \ j$

$$CSFSWA(\hbar\varpi_1, \hbar\varpi_2, ..., \hbar\varpi_l) = \hbar(CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l)).$$

Proof. Let $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ $(j = 1, 2, ..., l)$ be a collection of CSFSNs and $\vartheta = (\vartheta\_1, \vartheta_2, ...\vartheta_l)^T$ be the normalized w.v of ϖ_j along with $\hbar \in (0, 1)$, and now

$$\begin{aligned}
CSFSWA(\hbar\varpi_1, \hbar\varpi_2, ..., \hbar\varpi_l) &= \vartheta_1(\hbar\varpi_1) \oplus \vartheta_1(\hbar\varpi_1) \oplus ... \oplus \vartheta_1(\hbar\varpi_1) \\
&\quad \text{(using Equation 1)} \\
&= \hbar(\vartheta_1\varpi_1 \oplus \vartheta_2\varpi_2 \oplus ... \oplus \vartheta_l\varpi_l) \\
&= \hbar(CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l)).
\end{aligned}$$

Theorem 3. *(Linearity) Let* $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})(j = 1, 2, 3, ..., l)$ *be a collection of CSFSNs and* $\vartheta = (\vartheta_1, \vartheta_2, ...\vartheta_l)^T$ *be the normalized w.v of* ϖ_j*, and then,*$\forall \ \ \hbar, b \in (0, 1), \forall \ \ j$

$$CSFSWA(\hbar\varpi_1 + b, \hbar\varpi_2 + b, ..., \hbar\varpi_l + b) = b(CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l)) + b.$$

Proof. Let collection of CSFSNs be $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})$ $(j = 1, 2, 3, ..., l)$ and $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_l)^T$ be the normalized w.v of ϖ_j along with $\hbar, b \in (0, 1)$, after that

$$\begin{aligned}
CSFSWA(\hbar\varpi_1 + b, \hbar\varpi_2 + b, ..., \hbar\varpi_l + b) &= \vartheta_1(\hbar\varpi_1 + b) \oplus \vartheta_2(\hbar\varpi_2 + b) \oplus ... \oplus \vartheta_l(\hbar\varpi_l + b) \\
&\quad \text{(using Equation 1)} \\
&= (\vartheta_1\hbar\varpi_1 \oplus ... \oplus \vartheta_l\hbar\varpi_l) + (\vartheta_1 b + ... + \vartheta_l b) \\
&= \hbar(\vartheta_1\varpi_1 \oplus ... \oplus \vartheta_l\varpi_l) + b \qquad (\because \sum_{j=1}^{l} \vartheta_l = 1) \\
&= \hbar(CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l)) + b.
\end{aligned}$$

Remark 1. *CSFSWA operator is linear if and only if CSFSWA operator is*

(i) shift invariant and

(ii) homogenous.

Theorem 4. *(Additive property) Let two families of CSFSNs be ϖ_j and $\aleph_j (j = 1, 2, ..., l)$, and $\vartheta = (\vartheta_1, \vartheta_2, ...\vartheta_l)^T$ be the normalized w.v of both ϖ_j and $\aleph_j$, and then $\forall$ j $CSFSWA(\varpi_1 \oplus \aleph_1, \varpi_2 \oplus \aleph_2, ..., \varpi_l \oplus \aleph_l) = CSFSWA(\varpi_1, \varpi_2, ..., \varpi_l) \oplus CSFSWA(\aleph_1, \aleph_2, ..., \aleph_l)$.*

Proof. Let collection of CSFSNs be $\varpi_j = (\mu_j e^{i2\pi f_j}, \nu_j e^{i2\pi g_j}, \omega_j e^{i2\pi h_j})(j = 1, 2, 3, ..., l)$, and $\vartheta = (\vartheta_1, \vartheta_2, ...\vartheta_l)^T$ be the normalized w.v of ϖ_j along with $\hbar \in (0, 1)$, and after that

$$\begin{aligned}
CSFSWA(\varpi_1 \oplus \aleph_1, \varpi_2 \oplus \aleph_2, ..., \varpi_l \oplus \aleph_l) &= \vartheta_1(\varpi_1 \oplus \aleph_1) \oplus \vartheta_2(\varpi_2 \oplus \aleph_2) \oplus ... \oplus \vartheta_l(\varpi_l \oplus \aleph_l) \\
&\quad \text{(using Equation 1)} \\
&= (\vartheta_1\varpi_1 \oplus ... \oplus \vartheta_l\varpi_l) \oplus (\vartheta_1\aleph_1 \oplus .... \oplus \vartheta_l\aleph_l) \\
&= CSFSWA(\varpi_1, ..., \varpi_l) \oplus CSFSWA(\aleph_1, ..., \aleph_l).
\end{aligned}$$

5. Complex Spherical Fuzzy Soft-ELECTRE I Method

MAGDM is a methodical strategy used to assess and choose the optimal choice from a set of possibilities, taking into account numerous criteria or characteristics. It entails the simultaneous analysis of several elements to make well-informed judgments. MAGDM strategies use mathematical models, ranking systems, or optimization algorithms to evaluate and compare alternatives based on their performance across several criteria. MAGDM helps in determining the best appropriate option that corresponds with desired goals or preferences by evaluating and ranking various aspects. It provides a systematic framework for making decisions in complicated settings.

This novel approach to pursuing MAGDM, called the CSFS-ELECTRE I technique, combines the CSFS and the ELECTRE I methods, with CSFSNs representing both the allocations of alternatives relative to criteria and the weights of the criteria. The CSFS-ELECTRE I technique is a novel strategy for pursuing MAGDM. The aforementioned is the MAGDM annotation for the CSFS-ELECTRE I method:

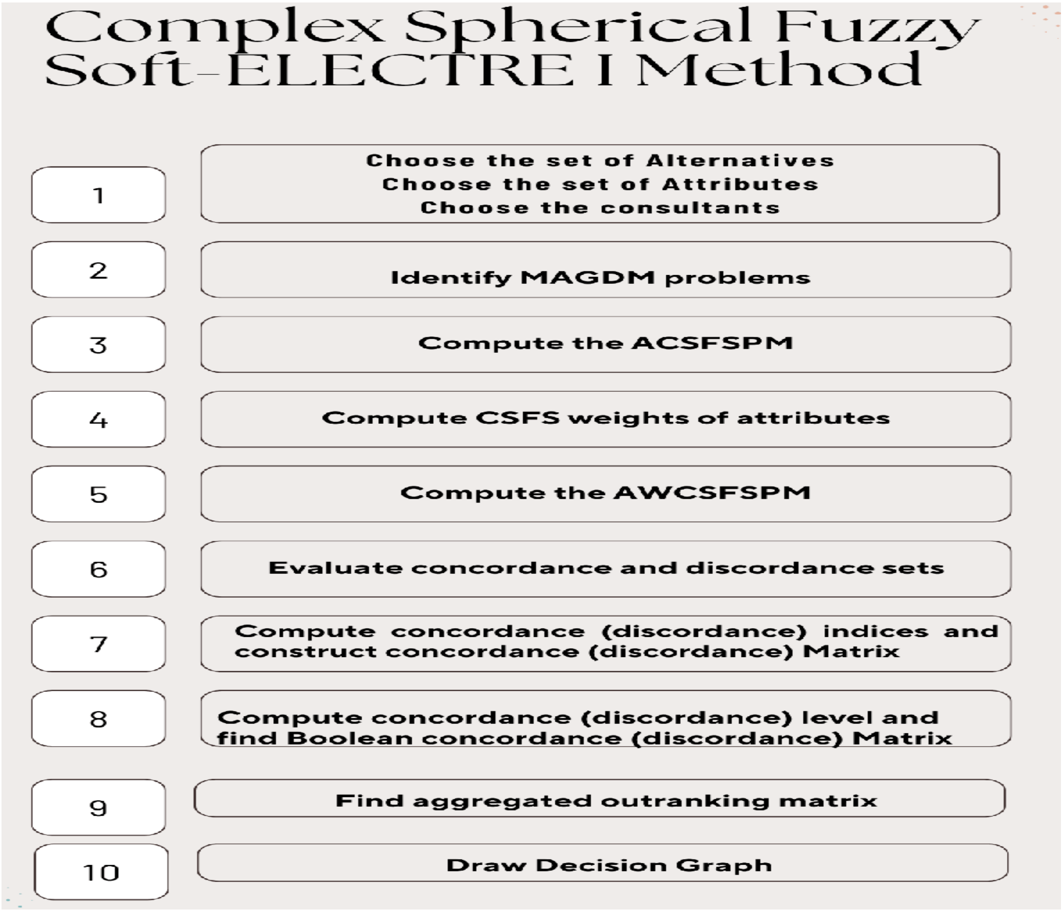

Figure 1: Flowchart illustrating the steps of the proposed CSFS-ELECTRE I method

Step 01: Construct the CSFS performance matrix (CSFSPM).

Let $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, ..., \mathfrak{R}_\mathfrak{r}$ resemble the alternatives and $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3, ..., \mathfrak{Y}_\mathfrak{y}$ represent the decision attributes based on which the consultants $\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, ..., \mathfrak{Q}_\mathfrak{q}$ allocate CSFSNs representing language phrases given by the MAGDM delegation. The CSFSPM $\mathfrak{S}^{(\mathfrak{s})}$ with the entries $\mathfrak{S}^{(s)}_{\mathfrak{d}\mathfrak{t}} = (\chi^{(s)}_{\mathfrak{d}\mathfrak{t}}, \vartheta^{(s)}_{\mathfrak{d}\mathfrak{t}}, \xi^{(s)}_{\mathfrak{d}\mathfrak{t}}) = (\mu^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi f^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}}}, \nu^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi g^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}}}, \omega^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi h^{\mathfrak{s}}_{\mathfrak{d}\mathfrak{t}}})$ are in accordance with expert $\mathfrak{Q}_\mathfrak{s}$. The MAGDM concerns can be delineated by the proceeding decision matrix

$$\mathfrak{S}^{(\mathfrak{s})} = \begin{pmatrix} (\chi^{(\mathfrak{s})}_{11}, \vartheta^{(\mathfrak{s})}_{11}, \xi^{(\mathfrak{s})}_{11}) & (\chi^{(\mathfrak{s})}_{12}, \vartheta^{(\mathfrak{s})}_{12}, \xi^{(\mathfrak{s})}_{12}) & . & . & . & (\chi^{(\mathfrak{s})}_{1\mathfrak{y}}, \vartheta^{(\mathfrak{s})}_{1\mathfrak{y}}, \xi^{(\mathfrak{s})}_{1\mathfrak{y}}) \\ (\chi^{(\mathfrak{s})}_{21}, \vartheta^{(\mathfrak{s})}_{21}, \xi^{(\mathfrak{s})}_{21}) & (\chi^{(\mathfrak{s})}_{22}, \vartheta^{(\mathfrak{s})}_{22}, \xi^{(\mathfrak{s})}_{22}) & . & . & . & (\chi^{(\mathfrak{s})}_{2\mathfrak{y}}, \vartheta^{(\mathfrak{s})}_{2\mathfrak{y}}, \xi^{(\mathfrak{s})}_{2\mathfrak{y}}) \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ (\chi^{(\mathfrak{s})}_{\mathfrak{r}1}, \vartheta^{(\mathfrak{s})}_{\mathfrak{r}1}, \xi^{(\mathfrak{s})}_{\mathfrak{r}1}) & (\chi^{(\mathfrak{s})}_{\mathfrak{r}2}, \vartheta^{(\mathfrak{s})}_{\mathfrak{r}2}, \xi^{(\mathfrak{s})}_{\mathfrak{r}2}) & . & . & . & (\chi^{(\mathfrak{s})}_{\mathfrak{r}\mathfrak{y}}, \vartheta^{(\mathfrak{s})}_{\mathfrak{r}\mathfrak{y}}, \xi^{(\mathfrak{s})}_{\mathfrak{r}\mathfrak{y}}) \end{pmatrix}$$

where $\mathfrak{d} \in \{1, 2, ..., \mathfrak{r}\}$, $\mathfrak{t} \in \{1, 2, ..., \mathfrak{y}\}$ and $\mathfrak{s} \in \{1, 2, ..., \mathfrak{q}\}$.

Step 02: Construct the aggregated CSFS performance matrix.
In the MAGDM context, each consultant $\mathfrak{Q}_{\mathfrak{s}}$ has self-endorsement, so the MAGDM deputation assigned linear and normalized w.v $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_l)^T$ to the panel of consultants. The opinions of the consultants are assembled using CSFSWA operator in the following way:

$$\begin{aligned}
\mathfrak{S}_{\mathfrak{d}\mathfrak{t}} &= CSFSWA_\tau(\mathfrak{S}^1_{\mathfrak{d}\mathfrak{t}}, \mathfrak{S}^2_{\mathfrak{d}\mathfrak{t}}, ..., \mathfrak{S}^{\mathfrak{q}}_{\mathfrak{d}\mathfrak{t}}) \\
&= \tau_1 \mathfrak{S}^1_{\mathfrak{d}\mathfrak{t}} \oplus \tau_2 \mathfrak{S}^2_{\mathfrak{d}\mathfrak{t}} \oplus ... \oplus \tau_{\mathfrak{q}} \mathfrak{S}^{\mathfrak{q}}_{\mathfrak{d}\mathfrak{t}} \\
&= \begin{pmatrix} \sqrt{1 - \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}(1-(\mu(^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}})^2)^{\tau_{\mathfrak{s}}}} e^{i2\pi\sqrt{1-\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}(1-(f(^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}})^2)^{\tau_{\mathfrak{s}}}}}, \\ \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[\nu^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_{\mathfrak{s}}} e^{i2\pi\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[g^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_{\mathfrak{s}}}}, \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[\omega^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_{\mathfrak{s}}} e^{i2\pi\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[h^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_{\mathfrak{s}}}} \end{pmatrix} \\
&= (\mu_{\mathfrak{d}\mathfrak{t}} e^{i2\pi f_{\mathfrak{d}\mathfrak{t}}}, \nu_{\mathfrak{d}\mathfrak{t}} e^{i2\pi g_{\mathfrak{d}\mathfrak{t}}}, \omega_{\mathfrak{d}\mathfrak{t}} e^{i2\pi h_{\mathfrak{d}\mathfrak{t}}})
\end{aligned}$$

The aggregated CSFSPM (ACSFSPM) is constructed as follows:

$$\mathfrak{S} = \begin{pmatrix}
(\chi_{11}, \vartheta_{11}, \xi_{11}) & (\chi_{12}, \vartheta_{12}, \xi_{12}) & . \; . \; . & (\chi_{1\mathfrak{y}}, \vartheta_{1\mathfrak{y}}, \xi_{1\mathfrak{y}}) \\
(\chi_{21}, \vartheta_{21}, \xi_{21}) & (\chi_{22}, \vartheta_{22}, \xi_{22}) & . \; . \; . & (\chi_{2\mathfrak{y}}, \vartheta_{2\mathfrak{y}}, \xi_{2\mathfrak{y}}) \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
(\chi_{\mathfrak{r}1}, \vartheta_{\mathfrak{r}1}, \xi_{\mathfrak{r}1}) & (\chi_{\mathfrak{r}2}, \vartheta_{\mathfrak{r}2}, \xi_{\mathfrak{r}2}) & . \; . \; . & (\chi_{\mathfrak{r}\mathfrak{y}}, \vartheta_{\mathfrak{r}\mathfrak{y}}, \xi_{\mathfrak{r}\mathfrak{y}})
\end{pmatrix}$$

Step 03: Assess the weights for the attributes. The relevance of the characteristics is determined by their respective linguistic weights, which are then assessed by the consultants and given CSFSNs. The judgements made by the consultants are compiled by the CSFSWA operator, which is then used in the process of formulating the CSFS-weights of the characteristics. $\mathfrak{p}^{(s)}_{\mathfrak{t}} = (\mu^{\mathfrak{s}}_{\mathfrak{t}} e^{i2\pi f^{\mathfrak{s}}_{\mathfrak{t}}}, \nu^{\mathfrak{s}}_{\mathfrak{t}} e^{i2\pi g^{\mathfrak{s}}_{\mathfrak{t}}}, \omega^{\mathfrak{s}}_{\mathfrak{t}} e^{i2\pi h^{\mathfrak{s}}_{\mathfrak{t}}})$ outlined below:

$$\begin{aligned}
\mathfrak{p}_{\mathfrak{t}} &= CSFSWA_\tau(\mathfrak{p}^1_{\mathfrak{t}}, \mathfrak{p}^2_{\mathfrak{t}}, ..., \mathfrak{p}^{\mathfrak{q}}_{\mathfrak{t}}) \\
&= \tau_1 \mathfrak{p}^1_{\mathfrak{t}} \oplus \tau_2 \mathfrak{p}^2_{\mathfrak{t}} \oplus ... \oplus \tau_{\mathfrak{q}} \mathfrak{p}^{\mathfrak{q}}_{\mathfrak{t}} \\
&= \begin{pmatrix} \sqrt{1 - \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}(1-(\mu(^{\mathfrak{s})}_{\mathfrak{t}})^2)^{\tau_{\mathfrak{s}}}} e^{i2\pi\sqrt{1-\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}(1-(f(^{\mathfrak{s})}_{\mathfrak{t}})^2)^{\tau_{\mathfrak{s}}}}}, \\ \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[\nu^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_{\mathfrak{s}}} e^{i2\pi\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[g^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_{\mathfrak{s}}}}, \Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[\omega^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_{\mathfrak{s}}} e^{i2\pi\Pi^{\mathfrak{q}}_{\mathfrak{s}=1}[h^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_{\mathfrak{s}}}} \end{pmatrix} \\
&= (\mu_{\mathfrak{t}} e^{i2\pi f_{\mathfrak{t}}}, \nu_{\mathfrak{t}} e^{i2\pi g_{\mathfrak{t}}}, \omega_{\mathfrak{t}} e^{i2\pi h_{\mathfrak{t}}})
\end{aligned}$$

Step 04: Instigate the aggregated weighted CSFSPM (AWCSFSPM).
The entities of AWCSFSPM $\overline{\mathfrak{S}} = (\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}})_{\mathfrak{r}\times\mathfrak{y}} = (\overline{\mu}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi\overline{f}_{\mathfrak{d}\mathfrak{t}}}, \overline{\nu}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi\overline{g}_{\mathfrak{d}\mathfrak{t}}}, \overline{\omega}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi\overline{h}_{\mathfrak{d}\mathfrak{t}}})$ are assessed by employing the ACSFSPM and weightage of attributes $\mathfrak{p}_{\mathfrak{t}}$ are outlined as:

$$\begin{aligned}
\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}} &= \mathfrak{S}_{\mathfrak{d}\mathfrak{t}} \otimes \mathfrak{p}_{\mathfrak{t}} \\
&= (\mu_{\mathfrak{d}\mathfrak{t}}\mu_{\mathfrak{t}} e^{i2\pi f_{\mathfrak{d}\mathfrak{t}} f_{\mathfrak{t}}}, \sqrt{\nu^2_{\mathfrak{d}\mathfrak{t}} + \nu^2_{\mathfrak{t}} - \nu^2_{\mathfrak{d}\mathfrak{t}}\nu^2_{\mathfrak{t}}} e^{i2\pi\sqrt{g^2_{\mathfrak{d}\mathfrak{t}} + g^2_{\mathfrak{t}} - g^2_{\mathfrak{d}\mathfrak{t}} g^2_{\mathfrak{t}}}}, \sqrt{\omega^2_{\mathfrak{d}\mathfrak{t}} + \omega^2_{\mathfrak{t}} - \omega^2_{\mathfrak{d}\mathfrak{t}}\omega^2_{\mathfrak{t}}} e^{i2\pi\sqrt{h^2_{\mathfrak{d}\mathfrak{t}} + h^2_{\mathfrak{t}} - h^2_{\mathfrak{d}\mathfrak{t}} h^2_{\mathfrak{t}}}})
\end{aligned} \tag{2}$$

$$= (\overline{\mu}_{\mathfrak{dt}} e^{i2\pi \overline{f}_{\mathfrak{dt}}}, \overline{\nu}_{\mathfrak{dt}} e^{i2\pi \overline{g}_{\mathfrak{dt}}}, \overline{\omega}_{\mathfrak{dt}} e^{i2\pi \overline{h}_{\mathfrak{dt}}})$$
$$= (\overline{\chi}_{\mathfrak{dt}}, \overline{\vartheta}_{\mathfrak{dt}}, \overline{\xi}_{\mathfrak{dt}})$$

Then the matrix can be formed as following:

$$\overline{\mathfrak{S}} = \begin{pmatrix} (\overline{\chi}_{11}, \overline{\vartheta}_{11}, \overline{\xi}_{11}) & (\overline{\chi}_{12}, \overline{\vartheta}_{12}, \overline{\xi}_{12}) & . \quad . \quad . & (\overline{\chi}_{1\mathfrak{y}}, \overline{\vartheta}_{1\mathfrak{y}}, \overline{\xi}_{1\mathfrak{y}}) \\ (\overline{\chi}_{21}, \overline{\vartheta}_{21}, \overline{\xi}_{21}) & (\overline{\chi}_{22}, \overline{\vartheta}_{22}, \overline{\xi}_{22}) & . \quad . \quad . & (\overline{\chi}_{2\mathfrak{y}}, \overline{\vartheta}_{2\mathfrak{y}}, \overline{\xi}_{2\mathfrak{y}}) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ (\overline{\chi}_{\mathfrak{r}1}, \overline{\vartheta}_{\mathfrak{r}1}, \overline{\xi}_{\mathfrak{r}1}) & (\overline{\chi}_{\mathfrak{r}2}, \overline{\vartheta}_{\mathfrak{r}2}, \overline{\xi}_{\mathfrak{r}2}) & . \quad . \quad . & (\overline{\chi}_{\mathfrak{r}\mathfrak{y}}, \overline{\vartheta}_{\mathfrak{r}\mathfrak{y}}, \overline{\xi}_{\mathfrak{r}\mathfrak{y}}) \end{pmatrix}$$

Step 05: The CSFS concordance sets (CNSs) and discordance sets (DCNSs) will now be launched. The fundamental idea behind the operation of CSFS-ELECTRE I method includes the pairwise comparison of alternatives related to some adequate attributes. To compare two CSFSNs, the score values, accuracy values, and degree of refusal of CSFSNs are used in CSFS-ELECTRE I method. An alternative with a higher score value, a higher accuracy value but least refusal value is a preferable option as compared to other feasible choices. The CSFS concordance (CN) and DCNSs are described as strong, mid-range, and weak CN and DCNSs, respectively, that rely on the score degree, accuracy degree, and refusal degree of CSFSN. The CSFS CNS contains indices of all the attributes whereby the alternative $\mathfrak{R}_\kappa$ is better than the $\mathfrak{R}_\tau$.

(i) The CSFS strong CNS $\mathfrak{C}_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{C}_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}),\ Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}),\ \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{3}$$

(ii) The CSFS mid-range CNS $\mathfrak{C}'_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{C}'_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{4}$$

In the mid-range CNS, the concordance is lower than in the strong CNS, primarily due to the higher refusal value of the κ-th alternative concerning the τ-th alternative with respect to the $\mathfrak{t}$-th criteria.

(iii) The CSFS weak CNS $\mathfrak{C}''_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{C}''_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{5}$$

In a similar vein, $\mathfrak{C}'_{\kappa\tau}$ demonstrates higher CN than $\mathfrak{C}''_{\kappa\tau}$ due to the lower accuracy value of alternative $\mathfrak{R}_\kappa$ in the weak CNS compared to the mid-range CNS. The CSFS DCNSs serve as complementary subsets of the CNSs, containing the indices of all attributes where alternative $\mathfrak{R}_\kappa$ is not superior to $\mathfrak{R}_\tau$.

(i) The CSFS strong DCNS $\mathfrak{D}_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{D}_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{6}$$

(ii) The CSFS mid-range DCNS $\mathfrak{D}'_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{D}'_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{7}$$

(iii) The CSFS weak DCNS $\mathfrak{D}''_{\kappa\tau}$ can be worked out in this way:

$$\mathfrak{D}''_{\kappa\tau} = \{\mathfrak{t} : Sc(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < Sc(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), Ac(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) \geq Ac(\overline{\mathfrak{S}}_{\tau\mathfrak{t}}), \Psi(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}) < \Psi(\overline{\mathfrak{S}}_{\tau\mathfrak{t}})\} \tag{8}$$

$\mathfrak{D}_{\kappa\tau}$ exhibits the highest level of DCN compared to $\mathfrak{D}'_{\kappa\tau}$ and $\mathfrak{D}''_{\kappa\tau}$, where $(\kappa, \tau = 1, 2, 3, ..., \mathfrak{r}, \kappa \neq \tau)$ and $\mathfrak{t} = \mathfrak{1}, \mathfrak{2}, \mathfrak{3}, ...\mathfrak{y}$.

Step 06: Determine the CSFS CN index and matrix.
The probable CSFS CN indices $\Im_{\kappa\tau}$ $(\kappa, \tau = 1, 2, 3, ..., \mathfrak{r}, \kappa \neq \tau)$ are evaluated by equipping the weights $\lambda_{\mathfrak{C}_{\kappa\tau}}$, $\lambda_{\mathfrak{C}'_{\kappa\tau}}$, and $\lambda_{\mathfrak{C}''_{\kappa\tau}}$ to the CSFS strong, mid-range, and weak CNSs, respectively. In CSFS-ELECTRE I method, the CN index

$$\begin{aligned} \Im_{\kappa\tau} &= (m^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \chi^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \vartheta^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \xi^{(\mathfrak{h})}_{\Im_{\kappa\tau}}) = (m^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \chi^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \vartheta^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \xi^{(\mathfrak{h})}_{\Im_{\kappa\tau}}) \\ &= (m^{(\mathfrak{h})}_{\Im_{\kappa\tau}}, \mu^{(\mathfrak{h})}_{\Im_{\kappa\tau}} e^{i2\pi f^{(\mathfrak{h})}_{\Im_{\kappa\tau}}}, \nu^{(\mathfrak{h})}_{\Im_{\kappa\tau}} e^{i2\pi g^{(\mathfrak{h})}_{\Im_{\kappa\tau}}}, \omega^{(\mathfrak{h})}_{\Im_{\kappa\tau}} e^{i2\pi h^{(\mathfrak{h})}_{\Im_{\kappa\tau}}}), \end{aligned}$$

the indicator of the superiority of alternative $\mathfrak{R}_\kappa$ over alternative $\mathfrak{R}_\tau$ is illustrated as given:

$$\Im_{\kappa\tau} = (\lambda_{\mathfrak{C}_{\kappa\tau}} \otimes \sum_{\mathfrak{t}\in\mathfrak{C}_{\kappa\tau}} \mathfrak{p}_\mathfrak{t}) \oplus (\lambda_{\mathfrak{C}'_{\kappa\tau}} \otimes \sum_{\mathfrak{t}\in\mathfrak{C}'_{\kappa\tau}} \mathfrak{p}_\mathfrak{t}) \oplus (\lambda_{\mathfrak{C}''_{\kappa\tau}} \otimes \sum_{\mathfrak{t}\in\mathfrak{C}''_{\kappa\tau}} \mathfrak{p}_\mathfrak{t}) \tag{9}$$

where, $\mathfrak{p}_\mathfrak{t} = (\mu_\mathfrak{t} e^{i2\pi f_\mathfrak{t}}, \nu_\mathfrak{t} e^{i2\pi g_\mathfrak{t}}, \omega_\mathfrak{t} e^{i2\pi h_\mathfrak{t}})$ reveals the CSFS-weightage of the vth attribute. In addition, the $\Gamma_{\Im_{\kappa\tau}}$ CSFS CN matrix is built in this way:

$$\Gamma_{\Im_{\kappa\tau}} = \begin{pmatrix} - & (\chi_{\Im_{12}}, \vartheta_{\Im_{12}}, \xi_{\Im_{12}}) & . & . & . & (\chi_{\Im_{1\mathfrak{y}}}, \vartheta_{\Im_{1\mathfrak{y}}}, \xi_{\Im_{1\mathfrak{y}}}) \\ (\chi_{\Im_{21}}, \vartheta_{\Im_{21}}, \xi_{\Im_{21}}) & - & . & . & . & (\chi_{\Im_{2\mathfrak{y}}}, \vartheta_{\Im_{2\mathfrak{y}}}, \xi_{\Im_{2\mathfrak{y}}}) \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ (\chi_{\Im_{\mathfrak{r}2}}, \vartheta_{\Im_{\mathfrak{r}2}}, \xi_{\Im_{\mathfrak{r}2}}) & (\chi_{\Im_{\mathfrak{r}2}}, \vartheta_{\Im_{\mathfrak{r}2}}, \xi_{\Im_{\mathfrak{r}2}}) & . & . & . & - \end{pmatrix}$$

Step 08: The feasible CSFS DCN indices $\Xi_{\kappa\tau}(\kappa, \tau = 1, 2, 3, ..., \mathfrak{r},\ \kappa \neq \tau)$ are evaluated by employing the weights $\lambda_{\mathfrak{D}_{\kappa\tau}}$, $\lambda_{\mathfrak{D}'_{\kappa\tau}}$, and $\lambda_{\mathfrak{D}''_{\kappa\tau}}$ of the CSFS strong, mid-range, and weak DCNSs, respectively. In CSFS-ELECTRE I method, the DCN index
$\Upsilon_{\kappa\tau} = (m^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}, \chi^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}, \vartheta^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}, \xi^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}) = (m^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}, \mu^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}} e^{i2\pi f^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}}, \nu^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}} e^{i2\pi g^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}}, \omega^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}} e^{i2\pi h^{(\mathfrak{h})}_{\Upsilon_{\kappa\tau}}}),$

indicating the extent to which alternative $\mathfrak{R}_\kappa$ is less desirable than alternative $\mathfrak{R}_\tau$ is determined as subsequent:

$$\Upsilon_{\kappa\tau} = \frac{\max_{\mathfrak{t}\in(\mathfrak{D}_{\kappa\tau}\cup\mathfrak{D}'_{\kappa\tau}\cup\mathfrak{D}'_{\kappa\tau})}\left\{\lambda_{\mathfrak{D}_{\kappa\tau}} \otimes d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}}), \lambda_{\mathfrak{D}'_{\kappa\tau}} \otimes d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}}), \lambda_{\mathfrak{D}''_{\kappa\tau}} \otimes d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}})\right\}}{\max_{\mathfrak{t}} d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}})} \tag{10}$$

where the normalized Euclidean distance $d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}})$ between two entities $\overline{S}_{\kappa\mathfrak{t}}$ and $\overline{S}_{\tau\mathfrak{t}}$ of AWCSFSPM can be calculated as follows:

$$d(\overline{S}_{\kappa\mathfrak{t}}, \overline{S}_{\tau\mathfrak{t}}) = \sqrt{\frac{(\overline{\mu}^2_{\kappa\mathfrak{t}} - \overline{\mu}^2_{\tau\mathfrak{t}})^2 + (\overline{\nu}^2_{\kappa\mathfrak{t}} - \overline{\nu}^2_{\tau\mathfrak{t}})^2 + (\overline{\omega}^2_{\kappa\mathfrak{t}} - \overline{\omega}^2_{\tau\mathfrak{t}})^2 + (\overline{f}^2_{\kappa\mathfrak{t}} - \overline{f}^2_{\tau\mathfrak{t}})^2 + (\overline{g}^2_{\kappa\mathfrak{t}} - \overline{g}^2_{\tau\mathfrak{t}})^2 + (\overline{h}^2_{\kappa\mathfrak{t}} - \overline{h}^2_{\tau\mathfrak{t}})^2}{3}}$$

The CSFS DCN matrix $\Gamma_{\Upsilon_{\kappa\tau}}$ is assembled in a straightforward manner:

$$\Gamma_{\Upsilon_{\kappa\tau}} = \begin{pmatrix} - & \Upsilon_{12} & . & . & . & \Upsilon_{1\mathfrak{y}} \\ \Upsilon_{21} & - & . & . & . & \Upsilon_{2\mathfrak{y}} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ \Upsilon_{\mathfrak{r}1} & \Upsilon_{\mathfrak{r}2} & & & \Upsilon_{\mathfrak{r}(\mathfrak{y}-1)} & - \end{pmatrix}$$

Step 08: Create the efficacy CSFSCN matrix.

$$\varsigma = \frac{1}{\mathfrak{r}(\mathfrak{r}-1)} \max[Sc(\Upsilon_{\kappa\tau})], \ (\kappa, \tau = 1, 2, 3, ..., \mathfrak{r}, \kappa \neq \tau). \tag{11}$$

The CSFS CN indices, arranged in the CSFS CN matrix $\Gamma_{\Upsilon_{\kappa\tau}}$, are compared using the score function. This leads to two possibilities: either the score value of the CN index exceeds or falls below the threshold value ς. If the CSFS CN index $\Upsilon_{\kappa\tau}$ is greater than ς, it indicates a higher likelihood of preferring alternative $\mathfrak{R}_\kappa$ over $\mathfrak{R}_\tau$. Conversely, if the CSFS CN index $\Upsilon_{\kappa\tau}$ has a value lower than ς, the chances of preferring the alternative $\mathfrak{R}_\kappa$ over $\mathfrak{R}_\tau$ diminish. The CN Boolean matrix $\sigma = (\sigma_{\kappa\tau})_{\mathfrak{r}\times\mathfrak{r}}$ can be compiled as follows:

$$\sigma = \begin{pmatrix} - & \sigma_{12} & . & . & . & \sigma_{1\mathfrak{y}} \\ \sigma_{21} & - & . & . & . & \sigma_{2\mathfrak{y}} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ \sigma_{\mathfrak{r}1} & \sigma_{\mathfrak{r}2} & & & \sigma_{\mathfrak{r}(\mathfrak{y}-1)} & - \end{pmatrix}$$

where

$$\sigma_{\kappa\tau} = \begin{cases} 1 & , \text{ if } Sc(\Upsilon_{\kappa\tau}) \geq \varsigma, \\ 0 & , \text{ if } Sc(\Upsilon_{\kappa\tau}) < \varsigma. \end{cases}$$

Step 09: Create the efficacy CSFS DCN matrix.

For the purpose of evaluating the dominant alternative from all possible choices, the discordant level $\mathfrak{P}$ assisted and compared with the items of the CSFS concordant matrix containing the concordant indices $\Upsilon_{\kappa\tau}$ that represents the degree of inferiority of the alternative $\mathfrak{R}_{\kappa}$ over the $\mathfrak{R}_{\tau}$. The discordant level $\mathfrak{P}$ is formalized as follows:

$$\mathfrak{P} = \frac{1}{\mathfrak{r}(\mathfrak{r}-1)} \sum_{\kappa,\kappa\neq\tau} \sum_{\tau,\tau\neq\kappa} (\Upsilon)_{\kappa\tau}, \ (\kappa, \tau = 1, 2, 3, ..., \mathfrak{r}, \kappa \neq \tau). \tag{12}$$

When the value of $\daleth_{\kappa\tau}$ exceeds the DCN level $\mathfrak{P}$, it strengthens the notion that alternative $\mathfrak{R}_{\tau}$ is more dispensable than alternative $\mathfrak{R}_{\kappa}$. The efficacy DCN matrix $\gimel = (\gimel_{\kappa\tau})_{\mathfrak{r}c\times\mathfrak{r}}$ can be compiled as follows:

$$\gimel = \begin{pmatrix} - & \gimel_{12} & . & . & . & \gimel_{1\mathfrak{y}} \\ \gimel_{21} & - & . & . & . & \gimel_{2\mathfrak{y}} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ \gimel_{\mathfrak{r}1} & \gimel_{\mathfrak{r}2} & & & \gimel_{\mathfrak{r}(\mathfrak{y}-1)} & - \end{pmatrix}$$

where

$$\gimel_{\kappa\tau} = \begin{cases} 1 & , \ \text{if } \Upsilon_{\kappa\tau} \leq \mathfrak{P}, \\ 0 & , \ \text{if } \Upsilon_{\kappa\tau} > \mathfrak{P}. \end{cases}$$

Step 10: Develop a Boolean matrix of aggregate rankings.

The aggregated outranking Boolean matrix contributes to strengthen the facts about the ordinate relations of the feasible alternatives subject to the contentious attributes that legitimize the dominant alternative. The CSFS effective CN and DCN matrices are integrated to accomplish the aggregated outranking Boolean matrix as follows:

$$\mho = \begin{pmatrix} - & \mho_{12} & . & . & . & \mho_{1\mathfrak{y}} \\ \mho_{21} & - & . & . & . & \mho_{2\mathfrak{y}} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ \mho_{\mathfrak{r}1} & \mho_{\mathfrak{r}2} & & & \mho_{\mathfrak{r}(\mathfrak{y}-1)} & - \end{pmatrix}$$

where $\mho_{\kappa\tau} = \sigma_{\kappa\tau} \times \gimel_{\kappa\tau}$.

If $\mho_{\kappa\tau} = 1$, it implies that $\mathfrak{R}_{\kappa}$ is a better choice than $\mathfrak{R}_{\tau}$, indicating that $\mathfrak{R}_{\kappa}$ is preferred over $\mathfrak{R}_{\tau}$.

Step 11: Show the order of precedence using a graphical representation.

A graph is a powerful visual aid because it provides an overview of data, facilitates comparisons, and highlights connections within the data set. Therefore, a directed graph is used in the final step of the CSFS-ELECTRE I technique to choose the optimal solution. Let's say $G = (V, E)$ is a directed graph, where V (the vertex set)

represents the set of options and E (the edge set) represents the connections among the options. Following these use cases, we can design the directed graph:
1. If $\mho_{\kappa\tau} = 1$, an arc is drawn from vertex $\mathfrak{R}_\kappa$ directed towards $\mathfrak{R}_\tau$, showing out that $\mathfrak{R}\kappa$ outperforms $\mathfrak{R}_\tau$, i.e., $(\mathfrak{R}_\kappa > \mathfrak{R}_\tau)$.
2. If $\mho_{\kappa\tau} = 1$, and $\mho_{\tau\kappa} = 1$, to depict the analogous connection that exists between the vertices, a circular path in both directions is drawn $\mathfrak{R}_\kappa$ and $\mathfrak{R}_\tau$, i.e., $(\mathfrak{R}_\kappa \approx \mathfrak{R}_\tau)$.
3. If $\mho_{\kappa\tau} = 0$, no arc will be drawn between the vertices, indicating that $\mathfrak{R}_\kappa$ and $\mathfrak{R}_\tau$ are inferiority, i.e., $(\mathfrak{R}\kappa ? \mathfrak{R}\tau)$. The three states of affection, lack of interest, and inferiority are shown in detail in the Figure 2, which does an excellent task of illustrating the aforementioned issues.

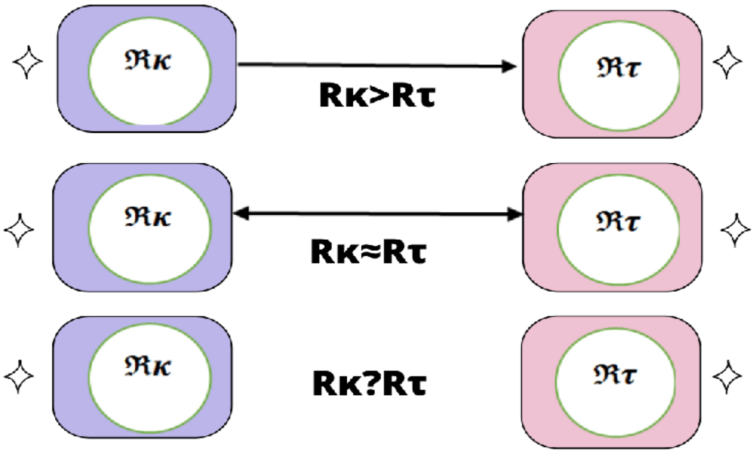

Figure 2: Graphical illustration of the outranking decision graph

Step 12: Execute an analysis of the linear ranking order.

Decision graph provides partial-precedence ordering of the alternatives and helps to eradicate the less beneficial alternatives but impotent to discriminate the preference orders of feasible alternatives through the proposed CSFS-ELECTRE I methodology. The linear ranking order of the alternatives eliminates the partial-precedence dilemma that can be evaluated using the net outranking indices to strengthen the proposed ELECTRE I method. The CN outranking relation $\overbrace{\Lambda_{\mathfrak{d}}}$ is determined by utilizing the CN matrix $\Gamma_{\Im_{\kappa\tau}}$ and Eq. 13. It can be stated as such:

$$\overbrace{\Lambda_{\mathfrak{d}}} = \sum_{\kappa=1,\ \kappa\neq\mathfrak{d}}^{\mathfrak{t}} Sc(\Gamma_{\Im_{\mathfrak{d}\kappa}}) - \sum_{\tau=1,\ \tau\neq\mathfrak{d}}^{\mathfrak{t}} Sc(\Gamma_{\Im_{\tau\mathfrak{d}}}) \tag{13}$$

Let $\underbrace{\Lambda_{\mathfrak{d}}}$ be DCN outranking relation, computed by employing the DCN matrix $\Gamma_{\Im_{\kappa\tau}}$ and Eq. 14, is defined as follows:

$$\underbrace{\Lambda_{\mathfrak{d}}} = \sum_{\kappa=1,\ \kappa\neq\mathfrak{d}}^{\mathfrak{t}} Sc(\Gamma_{\Upsilon_{\mathfrak{d}\kappa}}) - \sum_{\tau=1,\ \tau\neq\mathfrak{d}}^{\mathfrak{t}} Sc(\Gamma_{\Upsilon_{\tau\mathfrak{d}}}) \tag{14}$$

It can be argued that the best option is the one with the smallest value of $\underbrace{\Lambda_{\mathfrak{d}}}$ and the largest value of $\overbrace{\Lambda_{\mathfrak{d}}}$. It is not a truism that the option with the smallest $\underbrace{\Lambda_{\mathfrak{d}}}$ also has the largest $\overbrace{\Lambda_{\mathfrak{d}}}$. Therefore, the following definition of the net outranking index is used to get the ultimate linear ranking order:

$$\maltese_{\mathfrak{d}} = \overbrace{\Lambda_{\mathfrak{d}}} - \underbrace{\Lambda_{\mathfrak{d}}} \tag{15}$$

The alternatives are shown in decreasing order from the highest to the lowest value of their respective net outranking indexes. The solution that offers the highest possible net outranking index is the one that has been determined to be the best option for the MAGDM challenge. A graphical illustration of the CSFS-ELECTRE I approach that is suggested to be used in the context of MAGDM difficulties can be seen in the Figure 1.

6. Application

The need for transparent measurement and reporting of hospital performance is a significant concern in many countries, including Pakistan. With the aim of providing quality healthcare, various organizations and initiatives have emerged to rate hospitals and healthcare systems. However, the abundance of rating systems can be overwhelming for healthcare consumers, who often lack information about how these systems compare or which ones align with their specific information needs. This part analyzes the conceptual evaluation of choosing the city's finest hospital to illustrate the technique's utility. A city's medical care depends on its hospital. We provided a case study to demonstrate the CSFS-ELECTRE I method.

6.1. Case Study: Selection of a best hospital in Pakistan

6.1.1. Problem description

Carefully choose the top hospital in Pakistan, as the complexity of the country's healthcare system may be intimidating for patients and their loved ones. Therefore, when choosing a hospital, it is important to consider several key factors. These include the facility's background, accreditation, medical technology, treatment quality, healthcare professionals expertise, and patient outcomes. Think about things like proximity, affordability, and the need for specialist treatment. Before deciding on a hospital, patients should do their research, consider their options, and read reviews from other patients. By choosing the best hospital, patients in Pakistan have access to the highest standard of medical and psychological treatment available. Fifteen healthcare professionals administered linguistic tests transformed to CSFS numbers were used to get the criterion weights. They were evaluated using the CSFS Weighted Averaging operator that is the most appropriate to combine quantitative and qualitative evaluations of hospital performance because it is able to capture diversity and uncertainty. The Department of Health and Human Services assembled a panel of 15 experts, including $\mathfrak{Q}_1$: six high-ranking hospital administrators, to aid in selecting the best facility, $\mathfrak{Q}_2$: four local medical organizations, $\mathfrak{Q}_3$: The top five delegates from government health agencies. Three sets of consultants, $\mathfrak{Q}_1$, $\mathfrak{Q}_2$, and $\mathfrak{Q}_3$, have reached a consensus on four hospitals, $\mathfrak{R}_1$, $\mathfrak{R}_2$, $\mathfrak{R}_3$, and $\mathfrak{R}_4$.

6.1.2. Attribute Selection

In the quest for finding the best hospital for treatment, it is essential to take into account the following eight attributes:

$\mathfrak{Y}_1$: Ambulatory surgical centers:

Ambulatory surgical centers, also known as outpatient surgical facilities, offer patients the opportunity to undergo certain surgical procedures outside of a hospital setting. This allows patients to receive care in a different environment. Opting for these centers provides the advantage of lower surgical expenses compared to hospitals and reduces the risk of exposure to infectious diseases. Unlike hospitals, ambulatory surgical centers do not provide diagnostic procedures or function as clinics. Instead, their main focus is on delivering specialized surgical treatment. Their main objective is to offer effective and specialized surgical services, and they primarily serve patients who have received a hospital's or doctor's recommendation for surgery.

$\mathfrak{Y}_2$: Birth Centers:

As stated by the American Association of Birth Centers, birth centers are specialized healthcare facilities specifically designed to cater to the needs of child birth, adhering closely to the principles and practices of the midwifery profession. These centers have a primary objective of establishing a nurturing and supportive environment for women throughout the child birth process, with the aim of providing an experience that is not only comfortable but also affordable and inclusive for the entire family.

In comparison to traditional hospitals, birth centers often operate with distinct levels of resources. Due to their focus on natural child birth, they may not possess the extensive personnel or advanced medical equipment necessary to address unforeseen complications that may arise during the child birth process. This can include the availability of surgeons specializing in caesarean sections or specialized neonatal critical care units. Consequently, birth centers typically have specific criteria in place when accepting patients, specifically targeting pregnant women who are in good health and have no known risk factors or complications that may require the comprehensive resources available at a hospital.

The operational philosophy underlying birth centers revolves around key principles such as prevention, sensitivity, safety, cost-effectiveness, and the provision of appropriate medical care when the need arises. By embracing these principles, birth centers strive to create an environment that is supportive, empowering, and conducive to natural child birth. The ultimate goal is to ensure the well-being and safety of both the mother and baby, while promoting a positive and fulfilling child birth experience for women and their families.

$\mathfrak{Y}_3$: Blood Banks:

Blood banks hold immense significance as they not only collect, store, and organize donated blood efficiently for patients in need but also play a vital role in promoting voluntary blood and platelet donations within the community.

The American Society of Hematology highlights that plasma contains specific proteins crucial for regulating coagulation and aiding in healing processes. Red blood cells are responsible for the vital task of oxygen transportation, while platelets play a key role in blood clotting. Some patients need massive transfusions, while others have specific component needs. The requirement for 100 pints of blood from a single accident victim is not unheard of.

In the case of a medical emergency, blood banks are crucial because they can preserve blood for later use. This is because it is hard to get human blood and typically depends on the kindness of donors. Blood is a precious commodity because of its use in life support and emergency medical care.

$\mathfrak{Y}_4$: Clinics and medical offices:

A clinic's major role is to provide diagnosis and treatment for patients who choose not to be hospitalized. Various sorts of health care facilities fall under this general category.

The majority of patient visits to clinics are for checkups. Clinics run by firms that are a part of a bigger healthcare network or hospital may also fall into this category.

Clinics provide comprehensive healthcare services. A paediatric speech therapy clinic may assist your child with articulation difficulties, a physical therapy clinic can treat your sports injury, and a dental clinic can alleviate your toothache.

If you require specialized expertise in a specific health area, chances are there is a clinic available to cater to your needs. The primary goal of these clinics is to provide individuals with preventive care and crucial diagnoses while prioritizing convenience.

This objective has led to the establishment of "walk-in" clinics in grocery stores, convenience stores, shopping malls, and even airports. These clinics allow patients to receive services like flu shots or obtain prescriptions without scheduling an appointment at their regular physician's office. While many healthcare providers believe that maintaining an ongoing relationship with a primary care provider is preferable for long-term health, walk-in clinics offer speed, convenience, and sometimes lower costs for immediate needs.

$\mathfrak{Y}_5$: Diabetes and dialysis education centers:

In the United States, diabetes is a serious health concern. Despite the fact that diabetes affects over 30 million people, many cases go undetected. According to the CDC's report, this is apparently the case. As a corollary, more than a third of the population has prediabetes, which substantially raises the risk of acquiring type 2 diabetes. Patients with diabetes must learn to control their condition to reduce their risk of developing life-threatening complications. Due to the widespread effects of diabetes, diabetes education institutes have sprung up to aid sufferers and provide preventive measures to those at risk. Diabetes education centers include courses, instructional materials, support groups, and equipment. These institutions may help diabetics manage and avoid problems. These programs educate high-risk individuals about diabetes and how to lower their risk. Dialysis treats renal failure. Dialysis filters and cleans blood like healthy kidneys. An estimated 14% of Americans have chronic renal illness, which may impair the kidneys' ability to appropriately filter the blood. To prevent potentially deadly complications, patients in this circumstance may need dialysis as often as three times each week. As the need for dialysis services has grown, specialized dialysis centers have emerged to better serve patients and relieve hospital resources. In order to meet the specific requirements of dialysis patients, these clinics provide specialized services and employ trained professionals. The healthcare system needs enough dialysis facilities to meet the demand for regular dialysis treatments without overwhelming hospitals and other medical facilities, ensuring that patients with kidney disease have access to the lifesaving care they need.

$\mathfrak{Y}_6$: Imaging and radiology centers:

These clinics function like mini-hospitals, offering diagnostic imaging to patients on a walk-in basis. X-rays, magnetic resonance imaging, computerized tomography scans, ultrasound, and many more treatments fall under the umbrella of diagnostic imaging. Outpatient facilities are often preferred by patients over in-house imaging services at hospitals and clinics because of their lower costs and better accessibility.

Hospitals often provide emergency imaging services, including MRI in the case of a severe head injury. However, outpatient imaging clinics provide an alternative for patients who need to schedule ahead of time for imaging treatments like an ultrasound to monitor a pregnancy.

Patients may get the diagnostic imaging services they need quickly and easily thanks to the work of outpatient imaging centers. These clinics provide the same level of care as hospitals for a fraction of the cost and with more convenient hours. As a result, patients' healthcare requirements may be met, patients' convenience is increased, and the hospital's resources may be put under less pressure.

$\mathfrak{Y}_7$: Orthopedic and other rehabilitation centers:

Orthopaedics is the medical specialty devoted to the diagnosis and treatment of disorders of the skeleton. Physical therapists treat patients with a broad variety of physical issues. To acquire an official diagnosis and treatment plan, someone with persistent lower back pain, for instance, might see a physical therapist at an orthopedic clinic or hospital.

Orthopedic hospitals provide more than just treatment for athletes; they also offer rehabilitation for those with physical impairments. These facilities typically provide comprehensive services, including evaluation, diagnosis, prevention, treatment, and rehabilitation for various bone, tendon, ligament, muscle, and joint conditions.

Depending on their specific focus, these healthcare facilities may go by different names. Some may be referred to as outpatient physical therapy centers, while others may be specialized as pediatric physical therapy clinics, sports medicine centers, or geriatric physical therapy clinics.

Rehabilitation centers are another type of facility where patients can receive different therapies aimed at restoring their abilities following an illness or injury. Physical therapy, occupational therapy, and speech therapy are all integral processes that help individuals regain the skills necessary for daily activities, such as movement, work, and communication.

Practitioners at rehabilitation centers collaborate with patients to maximize their mobility and independence during the recovery process. Outpatient rehabilitation centers play a crucial role in alleviating the burden on hospital rehabilitation floors, offering specialized care in a setting that is more focused on specific rehabilitation needs.

$\mathfrak{Y}_8$**:** Urgent care:

Urgent care (UR) facilities play a vital role in providing prompt healthcare services for non-emergency situations that require immediate attention, but do not necessitate a visit to the emergency room. These facilities serve as a convenient option when individuals face healthcare needs that are too urgent or concerning to wait for a scheduled appointment at a doctor's office, yet not severe enough to warrant emergency medical care.

Staff in urgent care clinics are acute care trained and have access to a broad variety of diagnostic and treatment options. Splinting broken limbs, diagnosing and treating infectious diseases, drawing blood, testing urine for infections, and setting fractures are just some of the emergency medical abilities they possess. They can evaluate a patient's state and provide an accurate diagnosis quickly, so they can go to work right once on relieving the patient's distress. A referral to a specialist will be made if the severity of the patient's condition exceeds what can be treated in an urgent care setting. They are able to swiftly refer patients to competent experts for further assessment and treatment, arrange for patients to be moved to a hospital, or call an ambulance if necessary.

Urgent care facilities are especially popular choices for parents when their children fall ill and require an immediate diagnosis or relief from symptoms. These facilities provide accessible and efficient healthcare services for acute conditions, filling the gap between primary care providers and emergency departments, ensuring that individuals receive timely and appropriate care for their urgent healthcare needs.

The linguistic expressions that the specialists use in order to assess the relative importance of the options and the qualities that correspond to them are included in the table that is referenced by the Table 2. These phrases were stated by the decision-makers using three numbers: the first number represents the degree to which they agreed, the second number indicates the degree to which they were neutral, and the third number indicates the degree to which they disagreed depending on the criteria that were important. The directors of each group were the ones responsible for collecting the data.

Table 2: Linguistic terms to the rate the hospital facilities

Linguistic Term	CSFSNs
Extremely Good (Ex Gd)	$\left(0.92e^{i2\pi(0.90)}, 0.14e^{i2\pi(0.16)}, 0.25e^{i2\pi(0.27)}\right)$
Good (Gd)	$\left(0.80e^{i2\pi(0.78)}, 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi(0.37)}\right)$
Medium (Md)	$\left(0.68e^{i2\pi(0.66)}, 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi0.47)}\right)$
Bad (Bd)	$\left(0.56e^{i2\pi(0.54)}, 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.57)}\right)$
Extremely Bad (Ex Bd)	$\left(0.44e^{i2\pi(0.42)}, 0.54e^{i2\pi(0.56)}, 0.65e^{i2\pi(0.67)}\right)$

6.1.3. Evaluation Process

The CSFS-ELECTRE I method entails a sequential sequence of procedures for determining which hospital is most suited for a certain patient's needs. They are as follows:

Step 01: The linguistic terms and their corresponding CSFSNs used by the experts to explain the relative significance of credible alternatives and relevant attributes are enclosed in Table 2. The experts thoroughly examine the four hospitals with respect to the attributes under consideration and reach a consensus on their decision, as presented in Table 3. The CSFSPMs related to the input from the three groups of consultants are comprised in Tables 4, 5, and 6, respectively.

Table 3: Linguistic evaluations of alternatives by decision-makers

Decision Maker	Criteria	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\mathfrak{Q}_1$	$\mathfrak{Y}_1$	Ex Gd	Md	Gd	Md
	$\mathfrak{Y}_2$	Ex Gd	Gd	Ex Gd	Md
	$\mathfrak{Y}_3$	Gd	Md	Md	Gd
	$\mathfrak{Y}_4$	Md	Md	Bd	Md
	$\mathfrak{Y}_5$	Gd	Gd	Md	Gd
	$\mathfrak{Y}_6$	Md	Bd	Bd	Bd
	$\mathfrak{Y}_7$	Bd	Ex Bd	Bd	Ex Bd
	$\mathfrak{Y}_8$	Md	Bd	Ex Bd	Bd
$\mathfrak{Q}_2$	$\mathfrak{Y}_1$	Ex Gd	Md	Gd	Gd
	$\mathfrak{Y}_2$	Ex Gd	Ex Gd	Gd	Gd
	$\mathfrak{Y}_3$	Gd	Md	Md	Md
	$\mathfrak{Y}_4$	Md	Md	Bd	Bd
	$\mathfrak{Y}_5$	Gd	Gd	Md	Md
	$\mathfrak{Y}_6$	Md	Bd	Bd	Bd
	$\mathfrak{Y}_7$	Bd	Ex Bd	Bd	Bd
	$\mathfrak{Y}_8$	Md	Bd	Ex Bd	Ex Bd
$\mathfrak{Q}_3$	$\mathfrak{Y}_1$	Ex Gd	Md	Gd	Gd
	$\mathfrak{Y}_2$	Ex Gd	Ex Gd	Gd	Gd
	$\mathfrak{Y}_3$	Gd	Md	Md	Md
	$\mathfrak{Y}_4$	Md	Md	Bd	Bd
	$\mathfrak{Y}_5$	Gd	Gd	Md	Md
	$\mathfrak{Y}_6$	Md	Bd	Bd	Bd
	$\mathfrak{Y}_7$	Bd	Ex Bd	Bd	Bd
	$\mathfrak{Y}_8$	Md	Bd	Ex Bd	Ex Bd

Step 02: The CSFSAW operator and consultant w.v $\vartheta = [0.314, 0.355, 0.331]^T$ are put to use for the formation of ACSFSPM $\mathfrak{S} = (\mathfrak{S}_{\mathfrak{d}\mathfrak{t}})_{\mathfrak{r}\times\mathfrak{y}}$, given in Table 7.

Step 03: The w.v of set of attributes are given in Table 8. The CSFS-w.v of attributes is calculated using the stance of the decision-makers, arranged in 9, accumulated by CSFSAW operator.

Step 04: Table 10 displays the AWCSFSPM $\mathfrak{S}$, calculated as the cumulative result of $\overline{\mathfrak{S}} = (\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}})_{\mathfrak{r}\times\mathfrak{y}}$ using Eq. 2.

Step 05: The CSFS concordance sets are built upon the CSFS concordance matrices, which are in turn created from pairwise comparisons of alternatives according to each criteria. To get the CSFS effective concordance matrices, we use the CSFS concordance indices to evaluate the levels of concordance. An aggregated outranking matrix is the culmination of this procedure. The CSFS CNSs $\mathfrak{C}$, $\mathfrak{C}'$, and $\mathfrak{C}''$ are determined by employing Eqs. 3-5, which take into account score values, accuracy values, and refusal value the entities $\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}}$ of AWCSFSPM composed in Table 11. The CSFS strong CNS $\mathfrak{C}$, CSFS-mid-range CNS $\mathfrak{C}'$, and CSFS-weak CNS $\mathfrak{C}''$ are described below:

$$\mathfrak{C}_{\kappa\tau} = \begin{Bmatrix} - & \{1,8\} & \{1,7,8\} & \{4\} \\ \{5,6\} & - & \{2,4\} & \{3\} \\ \{5\} & \{\} & - & \{\} \\ \{2\} & \{\} & \{\} & - \end{Bmatrix}$$

$$\mathfrak{C}'_{\kappa\tau} = \begin{Bmatrix} - & \{\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} \\ \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & - \end{Bmatrix}$$

$$\mathfrak{C}''_{\kappa\tau} = \begin{Bmatrix} - & \{2,7\} & \{2\} & \{\} \\ \{3,4\} & - & \{3,5,8\} & \{\} \\ \{3,4,6\} & \{1,6,7\} & - & \{4,6\} \\ \{1,3,5,6,7,8\} & \{1,2,5,6,7,8\} & \{1,2,3,5,7,8\} & - \end{Bmatrix}$$

The CSFS DCNSs $\mathfrak{D}$, $\mathfrak{D}'$, and $\mathfrak{D}''$ are determined by employing Eqs. 6-8. The CSFS strong DCNS $\mathfrak{D}$, CSFS-mid-range CNS $\mathfrak{D}'$, and CSFS-weak DCNS $\mathfrak{D}''$ are accorded as follows:

$$\mathfrak{D}_{\kappa\tau} = \begin{Bmatrix} - & \{5\} & \{5\} & \{2\} \\ \{1,8\} & - & \{\} & \{\} \\ \{1,7,8\} & \{2,4\} & - & \{\} \\ \{4\} & \{3,4\} & \{\} & - \end{Bmatrix}$$

$$\mathfrak{D}'_{\kappa\tau} = \begin{Bmatrix} - & \{\} & \{\} & \{\} \\ \{\} & - & \{\} & \{\} \\ \{\} & \{\} & - & \{\} \\ \{\} & \{\} & \{\} & - \end{Bmatrix}$$

$$\mathfrak{D}''_{\kappa\tau} = \begin{Bmatrix} - & \{3,4,6,8\} & \{\} & \{1,3,5,6,7,8\} \\ \{2,7\} & - & \{1,6,7\} & \{1,2,5,6,7,8\} \\ \{2\} & \{3,5,8\} & - & \{1,2,3,5,7,8\} \\ \{\} & \{\} & \{4,6\} & - \end{Bmatrix}$$

Step 06: The CSFS strong, mid-range, and weak CNSs are examined, and their allocated weights, assigned by the consultants, are presented in Eq. 16.

$$(\lambda_{\mathfrak{C}_{\kappa\tau}}, \lambda'_{\mathfrak{C}_{\kappa\tau}}, \lambda''_{\mathfrak{C}_{\kappa\tau}}) = (0.3455, 0.3365, 0.318). \tag{16}$$

The CSFS CN indices $\Im_{\kappa\tau}$ are evaluated with the help of Eq. 9, which are further used to assemble CN matrix $\Gamma_{\Im_{\kappa\tau}}$. The CSFS CN matrix $\Gamma_{\Im_{\kappa\tau}}$ is organized in Table

12 and their score values are given by

$$Sc(\Gamma_{\Im_{\kappa\tau}}) = \begin{pmatrix} - & 1.1151 & 1.2140 & -0.7329 \\ 1.1980 & - & 0.8275 & -0.2349 \\ 1.1713 & -0.4260 & - & -1.3906 \\ 0.7792 & 0.5620 & 0.5792 & - \end{pmatrix} \tag{17}$$

Table 4: CSFSPM of decision-matrix $\mathfrak{Q}_1$

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$(0.86e^{i2\pi(0.76)}, 0.34e^{i2\pi(0.28)}, 0.25e^{i2\pi(0.23)})$	$(0.56e^{i2\pi(0.59)}, 0.47e^{i2\pi(0.44)}, 0.15e^{i2\pi(0.10)})$
$\mathfrak{R}_2$	$(0.83e^{i2\pi(0.62)}, 0.44e^{i2\pi(0.41)}, 0.27e^{i2\pi(0.25)})$	$(0.76e^{i2\pi(0.63)}, 0.50e^{i2\pi(0.66)}, 0.20e^{i2\pi(0.18)})$
$\mathfrak{R}_3$	$(0.69e^{i2\pi(0.65)}, 0.33e^{i2\pi(0.33)}, 0.18e^{i2\pi(0.13)})$	$(0.57e^{i2\pi(0.61)}, 0.53e^{i2\pi(0.57)}, 0.55e^{i2\pi(0.47)})$
$\mathfrak{R}_4$	$(0.52e^{i2\pi(0.54)}, 0.55e^{i2\pi(0.55)}, 0.21e^{i2\pi(0.24)})$	$(0.63e^{i2\pi(0.66)}, 0.33e^{i2\pi(0.48)}, 0.64e^{i2\pi(0.36)})$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$(0.79e^{i2\pi(0.72)}, 0.34e^{i2\pi(0.28)}, 0.36e^{i2\pi(0.32)})$	$(0.81e^{i2\pi(0.65)}, 0.45e^{i2\pi(0.39)}, 0.35e^{i2\pi(0.37)})$
$\mathfrak{R}_2$	$(0.84e^{i2\pi(0.80)}, 0.37e^{i2\pi(0.33)}, 0.11e^{i2\pi(0.10)})$	$(0.87e^{i2\pi(0.85)}, 0.35e^{i2\pi(0.38)}, 0.10e^{i2\pi(0.13)})$
$\mathfrak{R}_3$	$(0.73e^{i2\pi(0.66)}, 0.47e^{i2\pi(0.44)}, 0.24e^{i2\pi(0.25)})$	$(0.58e^{i2\pi(0.53)}, 0.17e^{i2\pi(0.19)}, 0.24e^{i2\pi(0.27)})$
$\mathfrak{R}_4$	$(0.65e^{i2\pi(0.61)}, 0.39e^{i2\pi(0.36)}, 0.21e^{i2\pi(0.24)})$	$(0.67e^{i2\pi(0.71)}, 0.52e^{i2\pi(0.55)}, 0.11e^{i2\pi(0.29)})$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$(0.56e^{i2\pi(0.76)}, 0.33e^{i2\pi(0.27)}, 0.22e^{i2\pi(0.18)})$	$(0.66e^{i2\pi(0.65)}, 0.55e^{i2\pi(0.44)}, 0.35e^{i2\pi(0.41)})$
$\mathfrak{R}_2$	$(0.69e^{i2\pi(0.89)}, 0.47e^{i2\pi(0.38)}, 0.32e^{i2\pi(0.23)})$	$(0.82e^{i2\pi(0.78)}, 0.44e^{i2\pi(0.38)}, 0.17e^{i2\pi(0.13)})$
$\mathfrak{R}_3$	$(0.78e^{i2\pi(0.72)}, 0.45e^{i2\pi(0.49)}, 0.41e^{i2\pi(0.33)})$	$(0.81e^{i2\pi 0.71)}, 0.37e^{i2\pi(0.23)}, 0.43e^{i2\pi(0.37)})$
$\mathfrak{R}_4$	$(0.74e^{i2\pi(0.57)}, 0.54e^{i2\pi(0.47)}, 0.29e^{i2\pi(0.27)})$	$(0.56e^{i2\pi(0.76)}, 0.61e^{i2\pi(0.57)}, 0.20e^{i2\pi(0.16)})$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$(0.78e^{i2\pi(0.73)}, 0.34e^{i2\pi(0.28)}, 0.10e^{i2\pi(0.14)})$	$(0.75e^{i2\pi(0.79)}, 0.37e^{i2\pi(0.34)}, 0.19e^{i2\pi(0.15)})$
$\mathfrak{R}_2$	$(0.67e^{i2\pi(0.74)}, 0.55e^{i2\pi(0.57)}, 0.29e^{i2\pi(0.26)})$	$(0.81e^{i2\pi(0.67)}, 0.45e^{i2\pi(0.53)}, 0.33e^{i2\pi(0.36)})$
$\mathfrak{R}_3$	$(0.79e^{i2\pi(0.71)}, 0.30e^{i2\pi(0.27)}, 0.16e^{i2\pi(0.19)})$	$(0.61e^{i2\pi(0.68)}, 0.55e^{i2\pi(0.55)}, 0.44e^{i2\pi(0.46)})$
$\mathfrak{R}_4$	$(0.73e^{i2\pi(0.75)}, 0.34e^{i2\pi(0.28)}, 0.18e^{i2\pi(0.25)})$	$(0.72e^{i2\pi(0.67)}, 0.34e^{i2\pi(0.28)}, 0.13e^{i2\pi(0.17)})$

Step 07: The CSFS strong, mid-range, and weak DCNSs are assigned weights based on expert viewpoints, as described in Equation 18. The CSFS DCN indices, denoted by $\Upsilon_{\kappa\tau}$, are then calculated using Equation 10 to construct the DCN matrix $\Gamma_{\Upsilon_{\kappa\tau}}$, wherein the normalized Euclidean distances $d(\overline{S}_{\kappa t}, \overline{S}_{\tau t})$ are set up in Table 13.

$$(\lambda_{\mathfrak{D}_{\kappa\tau}}, \lambda'_{\mathfrak{D}_{\kappa\tau}}, \lambda''_{\mathfrak{D}_{\kappa\tau}}) = (0.346, 0.441, 0.213). \tag{18}$$

The CSFS DCN matrix $\Gamma_{\Upsilon_{\kappa\tau}}$ is represented as follows:

$$\Gamma_{\Upsilon_{\kappa\tau}} = \begin{pmatrix} - & 0.5576 & 1 & 1 \\ 0.9192 & - & 0.3333 & 0.3333 \\ 0.0743 & 1 & - & 0.3333 \\ 1 & 1 & 0.3333 & - \end{pmatrix}$$

Table 5: CSFSPM of decision-matrix $\mathfrak{Q}_2$

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$\left(0.66e^{i2\pi(0.67)}, 0.41e^{i2\pi(0.27)}, 0.26e^{i2\pi(0.20)}\right)$	$\left(0.35e^{i2\pi(0.57)}, 0.67e^{i2\pi(0.63)}, 0.53e^{i2\pi(0.50)}\right)$
$\mathfrak{R}_2$	$\left(0.63e^{i2\pi(0.58)}, 0.34e^{i2\pi(0.43)}, 0.36e^{i2\pi(0.31)}\right)$	$\left(0.45e^{i2\pi(0.65)}, 0.73e^{i2\pi(0.54)}, 0.44e^{i2\pi(0.43)}\right)$
$\mathfrak{R}_3$	$\left(0.77e^{i2\pi(0.71)}, 0.44e^{i2\pi(0.51)}, 0.14e^{i2\pi(0.17)}\right)$	$\left(0.44e^{i2\pi(0.60)}, 0.79e^{i2\pi(0.68)}, 0.35e^{i2\pi(0.29)}\right)$
$\mathfrak{R}_4$	$\left(0.62e^{i2\pi(0.74)}, 0.18e^{i2\pi(0.21)}, 0.40e^{i2\pi(0.36)}\right)$	$\left(0.67e^{i2\pi(0.77)}, 0.44e^{i2\pi(0.37)}, 0.22e^{i2\pi(0.27)}\right)$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$\left(0.56e^{i2\pi(0.54)}, 0.45e^{i2\pi(0.54)}, 0.61e^{i2\pi(0.47)}\right)$	$\left(0.61e^{i2\pi(0.69)}, 0.54e^{i2\pi(0.47)}, 0.51e^{i2\pi(0.47)}\right)$
$\mathfrak{R}_2$	$\left(0.78e^{i2\pi(0.55)}), 0.37e^{i2\pi(0.27)}, 0.33e^{i2\pi(0.30)}\right)$	$\left(0.67e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.35)}, 0.26e^{i2\pi(0.29)}\right)$
$\mathfrak{R}_3$	$\left(0.65e^{i2\pi(0.69)}, 0.53e^{i2\pi(0.47)}, 0.27e^{i2\pi(0.24)}\right)$	$\left(0.56e^{i2\pi(0.72)}, 0.31e^{i2\pi(0.37)}, 0.36e^{i2\pi(0.26)}\right)$
$\mathfrak{R}_4$	$\left(0.69e^{i2\pi(0.63)}, 0.43e^{i2\pi(0.39)}, 0.27e^{i2\pi(0.21)}\right)$	$\left(0.57e^{i2\pi(0.71)}, 0.39e^{i2\pi(0.28)}, 0.45e^{i2\pi(0.21)}\right)$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$\left(0.74e^{i2\pi(0.61)}, 0.54e^{i2\pi(0.57)}, 0.33e^{i2\pi(0.37)}\right)$	$\left(0.64e^{i2\pi(0.73)}, 0.57e^{i2\pi(0.55)}, 0.41e^{i2\pi(0.37)}\right)$
$\mathfrak{R}_2$	$\left(0.63e^{i2\pi(0.72)}, 0.61e^{i2\pi(0.52)}, 0.27e^{i2\pi(0.21)}\right)$	$\left(0.65e^{i2\pi(0.67)}, 0.55e^{i2\pi(0.49)}, 0.40e^{i2\pi(0.46)}\right)$
$\mathfrak{R}_3$	$\left(0.71e^{i2\pi(0.79)}, 0.49e^{i2\pi(0.43)}, 0.41e^{i2\pi(0.35)}\right)$	$\left(0.67e^{i2\pi(0.70)}, 0.38e^{i2\pi(0.33)}, 0.37e^{i2\pi(0.36)}\right)$
$\mathfrak{R}_4$	$\left(0.68e^{i2\pi(0.63)}, 0.38e^{i2\pi(0.46)}, 0.11e^{i2\pi(0.22)}\right)$	$\left(0.71e^{i2\pi(0.68)}, 0.43e^{i2\pi(0.40)}, 0.28e^{i2\pi(0.31)}\right)$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$\left(0.61e^{i2\pi(0.85)}, 0.53e^{i2\pi(0.44)}, 0.17e^{i2\pi(0.16)}\right)$	$\left(0.71e^{i2\pi(0.62)}, 0.38e^{i2\pi(0.48)}, 0.38e^{i2\pi(0.21)}\right)$
$\mathfrak{R}_2$	$\left(0.69e^{i2\pi(0.68)}, 0.37e^{i2\pi(0.41)}, 0.24e^{i2\pi(0.27)}\right)$	$\left(0.60e^{i2\pi(0.55)}, 0.39e^{i2\pi(0.30)}, 0.39e^{i2\pi(0.29)}\right)$
$\mathfrak{R}_3$	$\left(0.57e^{i2\pi(0.61)}, 0.43e^{i2\pi(0.47)}, 0.25e^{i2\pi(0.21)}\right)$	$\left(0.79e^{i2\pi(0.67)}, 0.29e^{i2\pi(0.24)}, 0.29e^{i2\pi(0.33)}\right)$
$\mathfrak{R}_4$	$\left(0.67e^{i2\pi(0.72)}, 0.31e^{i2\pi(0.24)}, 0.22e^{i2\pi(0.15)}\right)$	$\left(0.65e^{i2\pi(0.53)}, 0.51e^{i2\pi(0.43)}, 0.51e^{i2\pi(0.30)}\right)$

Table 6: CSFSPM of decision-matrix $\mathfrak{Q}_3$

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$(0.40e^{i2\pi(0.63)}, 0.57e^{i2\pi(0.53)}, 0.65e^{i2\pi(0.30)})$	$(0.83e^{i2\pi(0.71)}, 0.33e^{i2\pi(0.31)}, 0.25e^{i2\pi(0.23)})$
$\mathfrak{R}_2$	$(0.79e^{i2\pi(0.48)}, 0.43e^{i2\pi(0.45)}, 0.25e^{i2\pi(0.29)})$	$(0.33e^{i2\pi(0.73)}, 0.44e^{i2\pi(0.47)}, 0.73e^{i2\pi(0.33)})$
$\mathfrak{R}_3$	$(0.54e^{i2\pi(0.36)}, 0.61e^{i2\pi(0.57)}, 0.53e^{i2\pi(0.21)})$	$(0.53e^{i2\pi(0.79)}, 0.31e^{i2\pi(0.21)}, 0.37e^{i2\pi(0.43)})$
$\mathfrak{R}_4$	$(0.85e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.42)}, 0.10e^{i2\pi(0.18)})$	$(0.63e^{i2\pi(0.71)}, 0.37e^{i2\pi(0.27)}, 0.53e^{i2\pi(0.53)})$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$(0.70e^{i2\pi(0.78)}, 0.47e^{i2\pi(0.44)}, 0.45e^{i2\pi(0.34)})$	$(0.83e^{i2\pi(0.89)}, 0.35e^{i2\pi(0.29)}, 0.25e^{i2\pi(0.13)})$
$\mathfrak{R}_2$	$(0.71e^{i2\pi(0.80)}, 0.33e^{i2\pi(0.27)}, 0.53e^{i2\pi(0.23)})$	$(0.51e^{i2\pi(0.90)}, 0.39e^{i2\pi(0.21)}, 0.73e^{i2\pi(0.26)})$
$\mathfrak{R}_3$	$(0.72e^{i2\pi(0.73)}, 0.23e^{i2\pi(0.28)}, 0.37e^{i2\pi(0.29)})$	$(0.51e^{i2\pi(0.76)}, 0.44e^{i2\pi(0.24)}, 0.73e^{i2\pi 0.21})$
$\mathfrak{R}_4$	$(0.65e^{i2\pi(0.76)}, 0.33e^{i2\pi(0.25)}, 0.53e^{i2\pi(0.20)})$	$(0.57e^{i2\pi(0.47)}, 0.49e^{i2\pi(0.26)}, 0.17e^{i2\pi(0.21)})$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$(0.45e^{i2\pi(0.73)}, 0.44e^{i2\pi(0.49)}, 0.73e^{i2\pi 0.28)})$	$(0.51e^{i2\pi(0.72)}, 0.45e^{i2\pi(0.34)}, 0.10e^{i2\pi(0.26)})$
$\mathfrak{R}_2$	$(0.57e^{i2\pi(0.71)}, 0.32e^{i2\pi(0.34)}, 0.45e^{i2\pi(0.20)})$	$(0.65e^{i2\pi(0.82)}, 0.53e^{i2\pi(0.47)}, 0.25e^{i2\pi(0.30)})$
$\mathfrak{R}_3$	$(0.69e^{i2\pi(0.72)}, 0.37e^{i2\pi(0.31)}, 0.53e^{i2\pi(0.19)})$	$(0.57e^{i2\pi(0.71)}, 0.61e^{i2\pi(0.55)}, 0.17e^{i2\pi(0.41)})$
$\mathfrak{R}_4$	$(0.51e^{i2\pi(0.70)}, 0.31e^{i2\pi(0.21)}, 0.73e^{i2\pi(0.14)})$	$(0.73e^{i2\pi(0.70)}, 0.18e^{i2\pi(0.41)}, 0.45e^{i2\pi(0.46)})$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$(0.65e^{i2\pi(0.68)}, 0.61e^{i2\pi(0.55)}, 0.45e^{i2\pi(0.47)})$	$(0.47e^{i2\pi(0.73)}, 0.44e^{i2\pi(0.49)}, 0.65e^{i2\pi(0.28)})$
$\mathfrak{R}_2$	$(0.67e^{i2\pi(0.57)}, 0.53e^{i2\pi(0.51)}, 0.37e^{i2\pi(0.45)})$	$(0.72e^{i2\pi(0.73)}, 0.23e^{i2\pi(0.28)}, 0.45e^{i2\pi(0.29)})$
$\mathfrak{R}_3$	$(0.69e^{i2\pi(0.71)}, 0.46e^{i2\pi(0.43)}, 0.37e^{i2\pi(0.40)})$	$(0.57e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.42)}, 0.37e^{i2\pi(0.18)})$
$\mathfrak{R}_4$	$(0.72e^{i2\pi(0.77)}, 0.39e^{i2\pi(0.37)}, 0.45e^{i2\pi(0.30)})$	$(0.48e^{i2\pi(0.76)}, 0.44e^{i2\pi(0.24)}, 0.34e^{i2\pi(0.21)})$

Step 08: The CN level ς, interpolated by Eq. 11, is 0.1011 calculated as

$$\varsigma = \frac{1}{\mathfrak{r}(\mathfrak{r}-1)} \max[Sc(\mathfrak{T}_{\kappa\tau})] = \frac{1}{4(4-1)} 1.2140 = 0.1011$$

and the CSFS CN entities' score values are recorded in Matrix 17, which serves as a basis for establishing the CSFS effective CN matrix, as shown below:

$$\varrho = \begin{pmatrix} - & 1 & 1 & 0 \\ 1 & - & 1 & 0 \\ 1 & 0 & - & 0 \\ 1 & 1 & 1 & - \end{pmatrix}$$

Table 7: Aggregated CSFS N-soft performance matrix

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$\left(0.7280e^{i2\pi(0.6983)}, 0.4122e^{i2\pi(0.3157)}, 0.3433e^{i2\pi(0.2388)}\right)$	$\left(0.5948e^{i2\pi(0.6127)}, 0.5097e^{i2\pi(0.4784)}, 0.2698e^{i2\pi(0.2240)}\right)$
$\mathfrak{R}_2$	$\left(0.7525e^{i2\pi(0.5768)}, 0.3908e^{i2\pi(0.4271)}, 0.2903e^{i2\pi(0.2818)}\right)$	$\left(0.5870e^{i2\pi(0.6631)}, 0.5749e^{i2\pi(0.5620)}, 0.3936e^{i2\pi(0.2926)}\right)$
$\mathfrak{R}_3$	$\left(0.7072e^{i2\pi(0.6409)}, 0.4270e^{i2\pi(0.4492)}, 0.2332e^{i2\pi(0.1657)}\right)$	$\left(0.5099e^{i2\pi(0.6575)}, 0.5638e^{i2\pi(0.4981)}, 0.4164e^{i2\pi(0.3884)}\right)$
$\mathfrak{R}_4$	$\left(0.6701e^{i2\pi(0.6620)}, 0.3250e^{i2\pi(0.3396)}, 0.2060e^{i2\pi(0.2510)}\right)$	$\left(0.6484e^{i2\pi(0.7243)}, 0.3839e^{i2\pi(0.3786)}, 0.4208e^{i2\pi(0.3696)}\right)$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$\left(0.6906e^{i2\pi(0.6735)}, 0.4122e^{i2\pi(0.4119)}, 0.4615e^{i2\pi(0.3713)}\right)$	$\left(0.7490e^{i2\pi(0.7450)}, 0.4623e^{i2\pi(0.3976)}, 0.3570e^{i2\pi(0.2875)}\right)$
$\mathfrak{R}_2$	$\left(0.7918e^{i2\pi(0.7198)}, 0.3611e^{i2\pi(0.2894)}, 0.2625e^{i2\pi(0.1886)}\right)$	$\left(0.7469e^{i2\pi(0.7997)}, 0.4079e^{i2\pi(0.3230)}, 0.2595e^{i2\pi(0.2123)}\right)$
$\mathfrak{R}_3$	$\left(0.6955e^{i2\pi(0.6895)}, 0.4256e^{i2\pi(0.4114)}, 0.2901e^{i2\pi(0.2585)}\right)$	$\left(0.5574e^{i2\pi(0.6797)}, 0.2713e^{i2\pi(0.2679)}, 0.3918e^{i2\pi(0.2461)}\right)$
$\mathfrak{R}_4$	$\left(0.6684e^{i2\pi(0.6581)}, 0.3929e^{i2\pi(0.3451)}, 0.3068e^{i2\pi(0.2165)}\right)$	$\left(0.6088e^{i2\pi(0.6738)}, 0.4523e^{i2\pi(0.3481)}, 0.2538e^{i2\pi(0.2348)}\right)$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$\left(0.6406e^{i2\pi(0.6971)}, 0.4360e^{i2\pi(0.4262)}, 0.3696e^{i2\pi(0.2640)}\right)$	$\left(0.6246e^{i2\pi(0.7029)}, 0.5354e^{i2\pi(0.4596)}, 0.2478e^{i2\pi(0.3427)}\right)$
$\mathfrak{R}_2$	$\left(0.6416e^{i2\pi(0.7985)}, 0.4858e^{i2\pi(0.4262)}, 0.3368e^{i2\pi(0.2134)}\right)$	$\left(0.7248e^{i2\pi(0.7495)}, 0.5051e^{i2\pi(0.4448)}, 0.2563e^{i2\pi(0.2595)}\right)$
$\mathfrak{R}_3$	$\left(0.7332e^{i2\pi(0.7538)}, 0.4481e^{i2\pi(0.4196)}, 0.4449e^{i2\pi(0.2824)}\right)$	$\left(0.7150e^{i2\pi(0.7056)}, 0.4164e^{i2\pi(0.3247)}, 0.3043e^{i2\pi(0.3788)}\right)$
$\mathfrak{R}_4$	$\left(0.6767e^{i2\pi(0.6284)}, 0.4109e^{i2\pi(0.3922)}, 0.2807e^{i2\pi(0.2045)}\right)$	$\left(0.6724e^{i2\pi(0.7148)}, 0.4031e^{i2\pi(0.4545)}, 0.2899e^{i2\pi(0.2797)}\right)$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$\left(0.6900e^{i2\pi(0.7858)}, 0.4684e^{i2\pi(0.3946)}, 0.1929e^{i2\pi(0.2152)}\right)$	$\left(0.6908e^{i2\pi(0.7146)}, 0.3885e^{i2\pi(0.4279)}, 0.3547e^{i2\pi(0.2049)}\right)$
$\mathfrak{R}_2$	$\left(0.6790e^{i2\pi(0.6844)}, 0.4582e^{i2\pi(0.4814)}, 0.2940e^{i2\pi(0.3135)}\right)$	$\left(0.7170e^{i2\pi(0.6401)}, 0.3662e^{i2\pi(0.3600)}, 0.3853e^{i2\pi(0.3125)}\right)$
$\mathfrak{R}_3$	$\left(0.6912e^{i2\pi(0.6706)}, 0.3851e^{i2\pi(0.3807)}, 0.2427e^{i2\pi(0.2490)}\right)$	$\left(0.7010e^{i2\pi(0.6656)}, 0.4011e^{i2\pi(0.3602)}, 0.3619e^{i2\pi(0.3052)}\right)$
$\mathfrak{R}_4$	$\left(0.7031e^{i2\pi(0.7420)}, 0.3361e^{i2\pi(0.2776)}, 0.2577e^{i2\pi(0.2231)}\right)$	$\left(0.6507e^{i2\pi(0.6438)}, 0.4295e^{i2\pi(0.3274)}, 0.2796e^{i2\pi(0.2201)}\right)$

Step 09: The entries of the CSFS DCN matrix are compared with the DCN threshold value $\mathfrak{P} = .657$ (calculated by Eq. 12) for the composition of CSFS effective DCN matrix $\sqsupset$, be seen below:

$$\sqsupset = \begin{pmatrix} - & 1 & 0 & 0 \\ 0 & - & 1 & 1 \\ 1 & 0 & - & 1 \\ 0 & 0 & 1 & - \end{pmatrix}$$

Step 10: The CSFS Boolean CN and DCN matrices are combined to generate the aggregated outranking Boolean matrix $\mho$. The collective outranking Boolean matrix, $\mho$, can be expressed in the following fashion:

$$\mho = \begin{pmatrix} - & 1 & 0 & 0 \\ 0 & - & 1 & 0 \\ 1 & 0 & - & 0 \\ 1 & 1 & 1 & - \end{pmatrix}$$

Step 11: The outranking graph for the MAGDM problem is depicted in Figure 4, delineates that the alternative $\mathfrak{R}_4$ outranks all the other alternatives and the partial preferences of the alternatives are pinned up in Table 16. It is note worthy from Figure 4 that the hospital $\mathfrak{R}_4$ is outranking all of the other hospitals $\mathfrak{R}_1$, $\mathfrak{R}_2$, and $\mathfrak{R}_3$.

Step 12: Table 14 presents the CN outranking coefficient $\overbrace{\Lambda_{\mathfrak{d}}}$, DCN outranking coefficient $\underbrace{\Lambda_{\mathfrak{d}}}$, and net outranking index $\maltese_{\mathfrak{d}}$. According to the table, it is evident that $\mathfrak{R}_4$ is the most favorable choice as optimal hospital. The linear ranking of the alternatives is as follows: $\mathfrak{R}_4 > \mathfrak{R}_2 > \mathfrak{R}_1 > \mathfrak{R}_3$.

Table 8: Linguistic terms to the rate the hospital facilities

Criteria	$\mathfrak{Q}_1$	$\mathfrak{Q}_2$	$\mathfrak{Q}_3$	CSFS-weights
$\mathfrak{Y}_1$	Ex Gd	Md	Gd	$\left(0.8246e^{i2\pi(0.8011)}, 0.2322e^{i2\pi(0.2537)}, 0.3480e^{i2\pi(0.3687)}\right)$
$\mathfrak{Y}_2$	Ex Gd	Ex Gd	Gd	$\left(0.9032e^{i2\pi(0.8822)}, 0.1570e^{i2\pi(0.1774)}, 0.2686e^{i2\pi(0.2791)}\right)$
$\mathfrak{Y}_3$	Gd	Md	Md	$\left(0.7294e^{i2\pi(0.7126)}, 0.3014e^{i2\pi(0.3217)}, 0.4125e^{i2\pi(0.4327)}\right)$
$\mathfrak{Y}_4$	Md	Md	Bd	$\left(0.6585e^{i2\pi(0.6585)}, 0.3592e^{i2\pi(0.3793)}, 0.4697e^{i2\pi(0.4897)}\right)$
$\mathfrak{Y}_5$	Gd	Gd	Md	$\left(0.7797e^{i2\pi(0.7797)}, 0.2585e^{i2\pi(0.2787)}, 0.3692e^{i2\pi(0.3893)}\right)$
$\mathfrak{Y}_6$	Md	Bd	Bd	$\left(0.6077e^{i2\pi(0.6077)}, 0.4024e^{i2\pi(0.4226)}, 0.5131e^{i2\pi(0.5332)}\right)$
$\mathfrak{Y}_7$	Bd	Ex Bd	Bd	$\left(0.5129e^{i2\pi(0.5129)}, 0.4816e^{i2\pi(0.5017)}, 0.5920e^{i2\pi(0.6040)}\right)$
$\mathfrak{Y}_8$	Md	Bd	Ex Bd	$\left(0.5893e^{i2\pi(0.5893)}, 0.4204e^{i2\pi(0.4373)}, 0.5317e^{i2\pi(0.5519)}\right)$

Table 9: CSFS-weights of attributes

	$\mathfrak{Q}_1$	$\mathfrak{Q}_2$	$\mathfrak{Q}_3$
$\mathfrak{Y}_1$	$\begin{pmatrix} 0.92e^{i2\pi(0.90)}, \\ 0.14e^{i2\pi(0.16)}, 0.25e^{i2\pi 0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.80e^{i2\pi(0.78)}, \\ 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi 0.37)} \end{pmatrix}$
$\mathfrak{Y}_2$	$\begin{pmatrix} 0.92e^{i2\pi(0.90)}, \\ 0.14e^{i2\pi(0.16)}, 0.25e^{i2\pi 0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.92e^{i2\pi(0.90)}, \\ 0.14e^{i2\pi(0.16)}, 0.25e^{i2\pi 0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.80e^{i2\pi(0.78)}, \\ 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi 0.37)} \end{pmatrix}$
$\mathfrak{Y}_3$	$\begin{pmatrix} 0.80e^{i2\pi(0.78)}, \\ 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi 0.37)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$
$\mathfrak{Y}_4$	$\begin{pmatrix} 0.92e^{i2\pi(0.90)}, \\ 0.14e^{i2\pi(0.16)}, 0.25e^{i2\pi 0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$
$\mathfrak{Y}_5$	$\begin{pmatrix} 0.80e^{i2\pi(0.78)}, \\ 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi 0.37)} \end{pmatrix}$	$\begin{pmatrix} 0.80e^{i2\pi(0.78)}, \\ 0.24e^{i2\pi(0.26)}, 0.35e^{i2\pi 0.37)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$
$\mathfrak{Y}_6$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$
$\mathfrak{Y}_7$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$	$\begin{pmatrix} 0.44e^{i2\pi(0.42)}, \\ 0.54e^{i2\pi(0.56)}, 0.65e^{i2\pi 0.67)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$
$\mathfrak{Y}_8$	$\begin{pmatrix} 0.68e^{i2\pi(0.66)}, \\ 0.34e^{i2\pi(0.36)}, 0.45e^{i2\pi 0.47)} \end{pmatrix}$	$\begin{pmatrix} 0.56e^{i2\pi(0.54)}, \\ 0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi 0.57)} \end{pmatrix}$	$\begin{pmatrix} 0.44e^{i2\pi(0.42)}, \\ 0.54e^{i2\pi(0.56)}, 0.65e^{i2\pi 0.67)} \end{pmatrix}$

Table 10: Aggregated weighted CSFS N-soft performance matrix

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$\left(0.6003e^{i2\pi(0.5594)}, 0.4633e^{i2\pi(0.3970)}, 0.4740e^{i2\pi(0.4304)}\right)$	$\left(0.5372e^{i2\pi(0.5405)}, 0.5273e^{i2\pi(0.5031)}, 0.3737e^{i2\pi(0.3524)}\right)$
$\mathfrak{R}_2$	$\left(0.6205e^{i2\pi(0.4621)}, 0.4454e^{i2\pi(0.4848)}, 0.4418e^{i2\pi(0.4523)}\right)$	$\left(0.5302e^{i2\pi(0.5850)}, 0.5891e^{i2\pi(0.5808)}, 0.4646e^{i2\pi(0.3960)}\right)$
$\mathfrak{R}_3$	$\left(0.5832e^{i2\pi(0.5134)}, 0.4758e^{i2\pi(0.5031)}, 0.4110e^{i2\pi(0.3996)}\right)$	$\left(0.4605e^{i2\pi(0.5800)}, 0.5785e^{i2\pi(0.5213)}, 0.4827e^{i2\pi(0.4658)}\right)$
$\mathfrak{R}_4$	$\left(0.5526e^{i2\pi(0.5303)}, 0.3922e^{i2\pi(0.4151)}, 0.3980e^{i2\pi(0.4363)}\right)$	$\left(0.5856e^{i2\pi(0.6390)}, 0.4104e^{i2\pi(0.4127)}, 0.4863e^{i2\pi(0.4515)}\right)$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$\left(0.5037e^{i2\pi(0.4799)}, 0.4953e^{i2\pi(0.5056)}, 0.5890e^{i2\pi(0.5471)}\right)$	$\left(0.4932e^{i2\pi(0.4906)}, 0.5614e^{i2\pi(0.5284)}, 0.5656e^{i2\pi(0.5501)}\right)$
$\mathfrak{R}_2$	$\left(0.5775e^{i2\pi(0.5129)}, 0.4576e^{i2\pi(0.4226)}, 0.4768e^{i2\pi(0.4649)}\right)$	$\left(0.4918e^{i2\pi(0.5266)}, 0.5234e^{i2\pi(0.4829)}, 0.5226e^{i2\pi(0.5235)}\right)$
$\mathfrak{R}_3$	$\left(0.5073e^{i2\pi(0.4913)}, 0.5055e^{i2\pi(0.5052)}, 0.4899e^{i2\pi(0.4915)}\right)$	$\left(0.3670e^{i2\pi(0.4476)}, 0.4395e^{i2\pi(0.4531)}, 0.5833e^{i2\pi(0.5346)}\right)$
$\mathfrak{R}_4$	$\left(0.4875e^{i2\pi(0.4690)}, 0.4808e^{i2\pi(0.4585)}, 0.4983e^{i2\pi(0.4747)}\right)$	$\left(0.4009e^{i2\pi(0.4437)}, 0.5543e^{i2\pi(0.4976)}, 0.5204e^{i2\pi(0.5308)}\right)$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$\left(0.4995e^{i2\pi(0.5435)}, 0.4942e^{i2\pi(0.4952)}, 0.5043e^{i2\pi(0.4590)}\right)$	$\left(0.3796e^{i2\pi(0.4272)}, 0.6342e^{i2\pi(0.5934)}, 0.5554e^{i2\pi(0.6069)}\right)$
$\mathfrak{R}_2$	$\left(0.5003e^{i2\pi(0.6226)}, 0.5358e^{i2\pi(0.4952)}, 0.4840e^{i2\pi(0.4361)}\right)$	$\left(0.4405e^{i2\pi(0.4555)}, 0.6130e^{i2\pi(0.5840)}, 0.5583e^{i2\pi(0.5766)}\right)$
$\mathfrak{R}_3$	$\left(0.5717e^{i2\pi(0.5877)}, 0.5042e^{i2\pi(0.4900)}, 0.5543e^{i2\pi(0.4682)}\right)$	$\left(0.4345e^{i2\pi(0.4288)}, 0.5543e^{i2\pi(0.5150)}, 0.5758e^{i2\pi(0.6221)}\right)$
$\mathfrak{R}_4$	$\left(0.5276e^{i2\pi(0.4900)}, 0.4737e^{i2\pi(0.4686)}, 0.4521e^{i2\pi(0.4325)}\right)$	$\left(0.4086e^{i2\pi(0.4344)}, 0.5460e^{i2\pi(0.5901)}, 0.5703e^{i2\pi(0.5833)}\right)$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$\left(0.3539e^{i2\pi(0.4030)}, 0.6328e^{i2\pi(0.6068)}, 0.6121e^{i2\pi(0.6279)}\right)$	$\left(0.4071e^{i2\pi(0.4211)}, 0.5486e^{i2\pi(0.5825)}, 0.6107e^{i2\pi(0.5777)}\right)$
$\mathfrak{R}_2$	$\left(0.3483e^{i2\pi(0.3510)}, 0.6270e^{i2\pi(0.6520)}, 0.6377e^{i2\pi(0.6536)}\right)$	$\left(0.4225e^{i2\pi(0.3772)}, 0.5359e^{i2\pi(0.5441)}, 0.6239e^{i2\pi(0.6103)}\right)$
$\mathfrak{R}_3$	$\left(0.3545e^{i2\pi(0.3440)}, 0.5881e^{i2\pi(0.6001)}, 0.6235e^{i2\pi(0.6358)}\right)$	$\left(0.4131e^{i2\pi(0.3922)}, 0.5560e^{i2\pi(0.5442)}, 0.6137e^{i2\pi(0.6078)}\right)$
$\mathfrak{R}_4$	$\left(0.3606e^{i2\pi(0.3806)}, 0.5645e^{i2\pi(0.5562)}, 0.6274e^{i2\pi(0.6296)}\right)$	$\left(0.3835e^{i2\pi(0.3794)}, 0.5732e^{i2\pi(0.5272)}, 0.5820e^{i2\pi(0.5816)}\right)$

Table 11: Score, accuracy, and refusal degree of AWCSFSPM

	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$
$\mathfrak{R}_1$	$(-0.1089, 1.4555, 1.0342)$	$(-0.2142, 1.3758, 1.1168)$
$\mathfrak{R}_2$	$(-0.2346, 1.4317, 1.0595)$	$(-0.4338, 1.6804, 0.7994)$
$\mathfrak{R}_3$	$(-0.2045, 1.4118, 1.0832)$	$(-0.5079, 1.6050, 0.8873)$
$\mathfrak{R}_4$	$(-0.0883, 1.2615, 1.2152)$	$(-0.0277, 1.5303, 0.9686)$
	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$
$\mathfrak{R}_1$	$(-0.4095, 1.4993, 1.0007)$	$(-1.1046, 1.7577, 0.6927)$
$\mathfrak{R}_2$	$(-0.3189, 1.5946, 0.8986)$	$(-0.9595, 1.7625, 0.6892)$
$\mathfrak{R}_3$	$(-0.3485, 1.6930, 0.7762)$	$(-0.9183, 1.6636, 0.8202)$
$\mathfrak{R}_4$	$(-0.3169, 1.3538, 1.1356)$	$(-0.9562, 1.6675, 0.8083)$
	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathfrak{R}_1$	$(-0.4095, 1.4993, 1.0007)$	$(-1.1046, 1.7577, 0.6927)$
$\mathfrak{R}_2$	$(-0.3189, 1.5946, 0.8986)$	$(-0.9595, 1.7625, 0.6892)$
$\mathfrak{R}_3$	$(-0.3485, 1.6930, 0.7762)$	$(-0.9183, 1.6636, 0.8202)$
$\mathfrak{R}_4$	$(-0.3169, 1.3538, 1.1356)$	$(-0.9562, 1.6675, 0.8083)$
	$\mathfrak{Y}_7$	$\mathfrak{Y}_8$
$\mathfrak{R}_1$	$(-1.2499, 1.8252, 0.5898)$	$(-1.0040, 1.6901, 0.7871)$
$\mathfrak{R}_2$	$(-1.4077, 1.8967, 0.4372)$	$(-1.0241, 1.6657, 0.8159)$
$\mathfrak{R}_3$	$(-1.2549, 1.7429, 0.7164)$	$(-1.0269, 1.6759, 0.8038)$
$\mathfrak{R}_4$	$(-1.1432, 1.6930, 0.7835)$	$(-0.9926, 1.5746, 0.9205)$

Table 12: CSFS-Concordance Matrix

	$\mathfrak{R}_1$	$\mathfrak{R}_2$
$\mathfrak{R}_1$	$-$	$\left(\begin{array}{c} 0.7790e^{i2\pi(0.7855)}, 0.1763e^{i2\pi(0.1557)}, \\ 0.1755e^{i2\pi(0.1504)} \end{array}\right)$
$\mathfrak{R}_2$	$\left(\begin{array}{c} 0.7894e^{i2\pi(0.8361)}, 0.2040e^{i2\pi(0.1703)}, \\ 0.1700e^{i2\pi(0.1570)} \end{array}\right)$	$-$
$\mathfrak{R}_3$	$\left(\begin{array}{c} 0.8582e^{i2\pi(0.8445)}, 0.2508e^{i2\pi(0.2403)}, \\ 0.3037e^{i2\pi(0.2560)} \end{array}\right)$	$\left(\begin{array}{c} 0.6045e^{i2\pi(0.5983)}, 0.5373e^{i2\pi(0.5377)}, \\ 0.5284e^{i2\pi(0.5407)} \end{array}\right)$
$\mathfrak{R}_4$	$\left(\begin{array}{c} 0.6695e^{i2\pi(0.6209)}, 0.1029e^{i2\pi(0.1027)}, \\ 0.1290e^{i2\pi(0.1294)} \end{array}\right)$	$\left(\begin{array}{c} 0.6398e^{i2\pi(0.6366)}, 0.2379e^{i2\pi(0.2404)}, \\ 0.2628e^{i2\pi(0.2630)} \end{array}\right)$
	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\mathfrak{R}_1$	$\left(\begin{array}{c} 0.7637e^{i2\pi(0.8298)}, 0.1299e^{i2\pi(0.1116)}, \\ 0.1276e^{i2\pi(0.1103)} \end{array}\right)$	$\left(\begin{array}{c} 0.4932e^{i2\pi(0.4906)}, 0.5614e^{i2\pi(0.5284)}, \\ 0.5656e^{i2\pi(0.5501)} \end{array}\right)$
$\mathfrak{R}_2$	$\left(\begin{array}{c} 0.7219e^{i2\pi(0.7409)}, 0.2918e^{i2\pi(0.2703)}, \\ 0.2144e^{i2\pi(0.1959)} \end{array}\right)$	$\left(\begin{array}{c} 0.5775e^{i2\pi(0.5129)}, 0.4576e^{i2\pi(0.4226)}, \\ 0.4768e^{i2\pi(0.4649)} \end{array}\right)$
$\mathfrak{R}_3$	$-$	$\left(\begin{array}{c} 0.3916e^{i2\pi(0.4338)}, 0.6245e^{i2\pi(0.6156)}, \\ 0.6951e^{i2\pi(0.6928)} \end{array}\right)$
$\mathfrak{R}_4$	$\left(\begin{array}{c} 0.6338e^{i2\pi(0.6338)}, 0.2281e^{i2\pi(0.2210)}, \\ 0.2512e^{i2\pi(0.2455)} \end{array}\right)$	$-$

Table 13: Normalized Euclidean Distance $d(\overline{\mathfrak{S}}_{\kappa\mathfrak{t}}, \overline{\mathfrak{S}}_{\tau\mathfrak{t}})$

	$\overline{\mathfrak{S}}_{11}$	$\overline{\mathfrak{S}}_{21}$	$\overline{\mathfrak{S}}_{31}$	$\overline{\mathfrak{S}}_{41}$		$\overline{\mathfrak{S}}_{12}$	$\overline{\mathfrak{S}}_{22}$	$\overline{\mathfrak{S}}_{32}$	$\overline{\mathfrak{S}}_{42}$
$\overline{\mathfrak{S}}_{11}$	-	-	-	-	$\overline{\mathfrak{S}}_{12}$	-	-	-	-
$\overline{\mathfrak{S}}_{21}$	0.0774	-	-	-	$\overline{\mathfrak{S}}_{22}$	0.0842	-	-	-
$\overline{\mathfrak{S}}_{31}$	0.0727	0.0527	-	-	$\overline{\mathfrak{S}}_{32}$	0.0978	0.0663	-	-
$\overline{\mathfrak{S}}_{41}$	0.0642	0.0783	0.0692	-	$\overline{\mathfrak{S}}_{42}$	0.1304	0.1534	0.1419	-
	$\overline{\mathfrak{S}}_{13}$	$\overline{\mathfrak{S}}_{23}$	$\overline{\mathfrak{S}}_{33}$	$\overline{\mathfrak{S}}_{43}$		$\overline{\mathfrak{S}}_{14}$	$\overline{\mathfrak{S}}_{24}$	$\overline{\mathfrak{S}}_{34}$	$\overline{\mathfrak{S}}_{44}$
$\overline{\mathfrak{S}}_{13}$	-	-	-	-	$\overline{\mathfrak{S}}_{14}$	-	-	-	-
$\overline{\mathfrak{S}}_{23}$	0.1094	-	-	-	$\overline{\mathfrak{S}}_{24}$	0.0522	-	-	-
$\overline{\mathfrak{S}}_{33}$	0.0707	0.0709	-	-	$\overline{\mathfrak{S}}_{34}$	0.1072	0.0989	-	-
$\overline{\mathfrak{S}}_{43}$	0.0771	0.0659	0.0356	-	$\overline{\mathfrak{S}}_{44}$	0.0649	0.0694	0.0823	-
	$\overline{\mathfrak{S}}_{15}$	$\overline{\mathfrak{S}}_{25}$	$\overline{\mathfrak{S}}_{35}$	$\overline{\mathfrak{S}}_{45}$		$\overline{\mathfrak{S}}_{16}$	$\overline{\mathfrak{S}}_{26}$	$\overline{\mathfrak{S}}_{36}$	$\overline{\mathfrak{S}}_{46}$
$\overline{\mathfrak{S}}_{15}$	-	-	-	-	$\overline{\mathfrak{S}}_{16}$	-	-	-	-
$\overline{\mathfrak{S}}_{25}$	0.0610	-	-	-	$\overline{\mathfrak{S}}_{26}$	0.0418	-	-	-
$\overline{\mathfrak{S}}_{35}$	0.0619	0.0705	-	-	$\overline{\mathfrak{S}}_{36}$	0.0805	0.0693	-	-
$\overline{\mathfrak{S}}_{45}$	0.0516	0.0967	0.0937	-	$\overline{\mathfrak{S}}_{46}$	0.0645	0.0497	0.0569	-
	$\overline{\mathfrak{S}}_{17}$	$\overline{\mathfrak{S}}_{27}$	$\overline{\mathfrak{S}}_{37}$	$\overline{\mathfrak{S}}_{47}$		$\overline{\mathfrak{S}}_{18}$	$\overline{\mathfrak{S}}_{28}$	$\overline{\mathfrak{S}}_{38}$	$\overline{\mathfrak{S}}_{48}$
$\overline{\mathfrak{S}}_{17}$	-	-	-	-	$\overline{\mathfrak{S}}_{18}$	-	-	-	-
$\overline{\mathfrak{S}}_{27}$	0.0481	-	-	-	$\overline{\mathfrak{S}}_{28}$	0.0417	-	-	-
$\overline{\mathfrak{S}}_{37}$	0.0420	0.0495	-	-	$\overline{\mathfrak{S}}_{38}$	0.0356	0.0168	-	-
$\overline{\mathfrak{S}}_{47}$	0.0601	0.0829	0.0371	-	$\overline{\mathfrak{S}}_{48}$	0.0489	0.0474	0.0355	-

Table 14: Outranking indices of the alternatives

Alternatives	Concordance OI	DCN OI	Net OI	Ranking
$\mathfrak{R}_1$	-1.5523	0.5641	-2.1164	3
$\mathfrak{R}_2$	0.5395	-0.9718	1.5113	2
$\mathfrak{R}_3$	-3.2660	-0.259	-3.007	4
$\mathfrak{R}_4$	4.2788	0.6667	3.6121	1

The ranking of CSFS-ELECTRE I method is given is Figure 3. Ranking of alternative by CSFS-ELECTRE I method is given in Table 15.

Table 15: Ranking of alternative by CSFS-ELECTRE I method

Alternative	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
Ranking	3	2	4	1

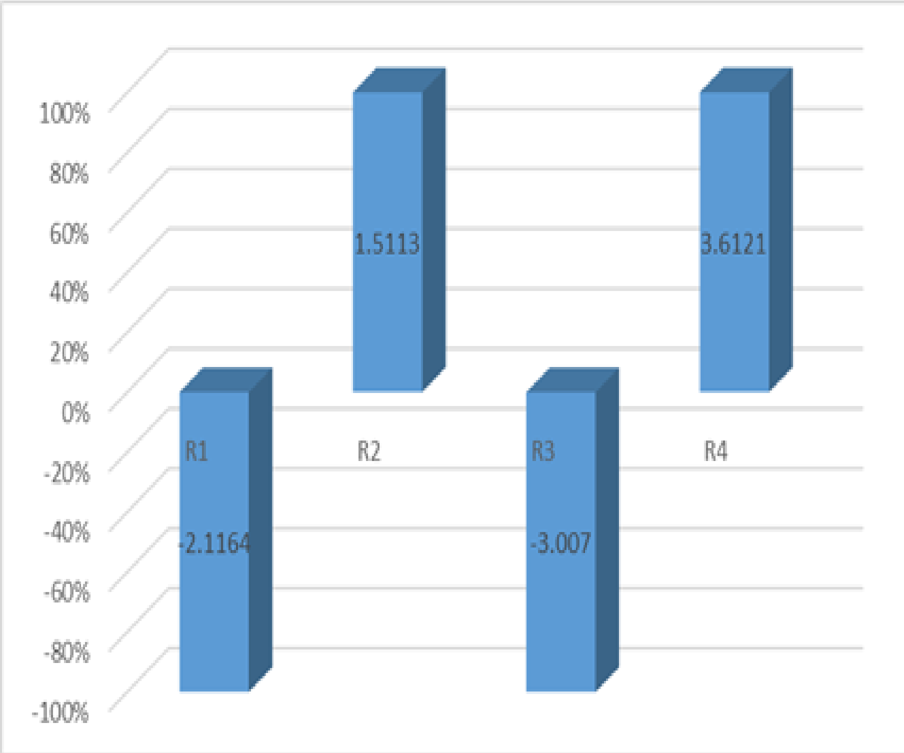

Figure 3: Ranking of alternatives using the CSFS-ELECTRE I method

Table 16: Exploration of decision graph

Alternatives	Submissive alternatives	Incomparable alternatives
$\mathfrak{R}_1$	$\mathfrak{R}_2$	-
$\mathfrak{R}_2$	$\mathfrak{R}_3$	-
$\mathfrak{R}_3$	$\mathfrak{R}_1$	-
$\mathfrak{R}_4$	$\mathfrak{R}_1$, $\mathfrak{R}_2$, $\mathfrak{R}_3$	-

6.1.4. Analysis of the results

The conclusive findings of the case study reveal that among all the chosen alternatives, $\mathfrak{R}_4$ stands out as the optimal selection for the hospital.

7. Comparative Analysis

In this section, we have introduced a novel technique, specifically the CSFS-TOPSIS method. Subsequently, we carried out a comparison using the previously mentioned example to assess hospital performance by utilizing this innovative technique, thereby confirming the credibility of the proposed approach.

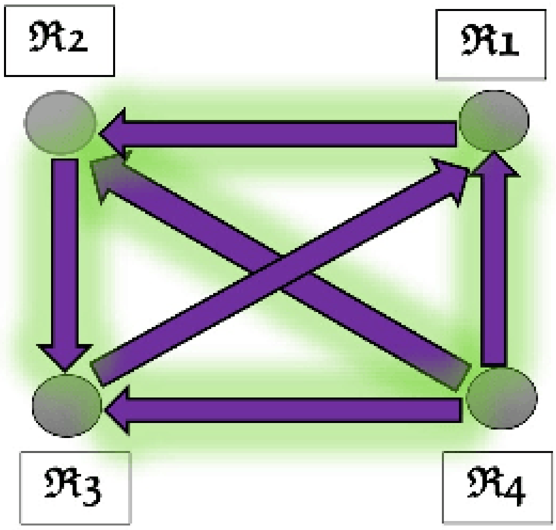

Figure 4: Outranking decision graph based on the aggregated CSFS-ELECTRE I results

7.1. Complex Spherical Fuzzy Soft Technique for Order Preference by Similarity to Ideal Solution Approach

This novel approach in the realm of MAGDM, referred to as the CSFS-TOPSIS technique, amalgamates the methodologies of CSFS and TOPSIS. In this context, CSFSNs fulfill a dual role by signaling the alternative allocations with regard to the criteria and by ascertaining the criteria weights. The CSFS-TOPSIS method represents an innovative and distinctive strategy for tackling MAGDM. The subsequent passage serves as the MAGDM description for the CSFS-TOPSIS approach:

Step 01 Construct the CSFS performance matrix (CSFSPM).
Let $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3, ..., \mathfrak{R}_\mathfrak{r}$ resemble the alternatives and $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3, ..., \mathfrak{Y}_\mathfrak{y}$ represent the decision attributes based on which the consultants $\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3, ..., \mathfrak{Q}_\mathfrak{q}$ allocate CSFSNs representing language phrases given by the MAGDM delegation. The CSFSPM $\mathfrak{S}^{(\mathfrak{s})}$ with the entries $\mathfrak{S}_{\mathfrak{dt}}^{(s)} = (\chi_{\mathfrak{dt}}^{(s)}, \vartheta_{\mathfrak{dt}}^{(s)}, \xi_{\mathfrak{dt}}^{(s)}) = (\mu_{\mathfrak{dt}}^{\mathfrak{s}} e^{i2\pi f_{\mathfrak{dt}}^{\mathfrak{s}}}, \nu_{\mathfrak{dt}}^{\mathfrak{s}} e^{i2\pi g_{\mathfrak{dt}}^{\mathfrak{s}}}, \omega_{\mathfrak{dt}}^{\mathfrak{s}} e^{i2\pi h_{\mathfrak{dt}}^{\mathfrak{s}}})$ are in accordance with expert $\mathfrak{Q}_\mathfrak{s}$. The MAGDM concerns can be delineated by the proceeding decision-matrix

$$\mathfrak{S}^{(\mathfrak{s})} = \begin{pmatrix} (\chi_{11}^{(\mathfrak{s})}, \vartheta_{11}^{(\mathfrak{s})}, \xi_{11}^{(\mathfrak{s})}) & (\chi_{12}^{(\mathfrak{s})}, \vartheta_{12}^{(\mathfrak{s})}, \xi_{12}^{(\mathfrak{s})}) & . \quad . \quad . & (\chi_{1\mathfrak{y}}^{(\mathfrak{s})}, \vartheta_{1\mathfrak{y}}^{(\mathfrak{s})}, \xi_{1\mathfrak{y}}^{(\mathfrak{s})}) \\ (\chi_{21}^{(\mathfrak{s})}, \vartheta_{21}^{(\mathfrak{s})}, \xi_{21}^{(\mathfrak{s})}) & (\chi_{22}^{(\mathfrak{s})}, \vartheta_{22}^{(\mathfrak{s})}, \xi_{22}^{(\mathfrak{s})}) & . \quad . \quad . & (\chi_{2\mathfrak{y}}^{(\mathfrak{s})}, \vartheta_{2\mathfrak{y}}^{(\mathfrak{s})}, \xi_{2\mathfrak{y}}^{(\mathfrak{s})}) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ (\chi_{\mathfrak{r}1}^{(\mathfrak{s})}, \vartheta_{\mathfrak{r}1}^{(\mathfrak{s})}, \xi_{\mathfrak{r}1}^{(\mathfrak{s})}) & (\chi_{\mathfrak{r}2}^{(\mathfrak{s})}, \vartheta_{\mathfrak{r}2}^{(\mathfrak{s})}, \xi_{\mathfrak{r}2}^{(\mathfrak{s})}) & . \quad . \quad . & (\chi_{\mathfrak{r}\mathfrak{y}}^{(\mathfrak{s})}, \vartheta_{\mathfrak{r}\mathfrak{y}}^{(\mathfrak{s})}, \xi_{\mathfrak{r}\mathfrak{y}}^{(\mathfrak{s})}) \end{pmatrix}$$

where $\mathfrak{d} \in \{1, 2, ..., \mathfrak{r}\}$, $\mathfrak{t} \in \{1, 2, ..., \mathfrak{y}\}$ and $\mathfrak{s} \in \{1, 2, ..., \mathfrak{q}\}$.

Step 02 Construct the aggregated CSFS performance matrix.

In the MAGDM context, each consultant $\mathfrak{Q}_\mathfrak{s}$ has self-endorsement, so the MAGDM deputation assigned linear and normalized w.v $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_l)^T$ to the panel of consultants. The opinions of the consultants are assembled using CSFSWA operator in the subsequent way:

$$\begin{aligned}
\mathfrak{S}_{\mathfrak{d}\mathfrak{t}} &= CSFSWA_\tau(\mathfrak{S}^1_{\mathfrak{d}\mathfrak{t}}, \mathfrak{S}^2_{\mathfrak{d}\mathfrak{t}}, ..., \mathfrak{S}^\mathfrak{q}_{\mathfrak{d}\mathfrak{t}}) \\
&= \tau_1\mathfrak{S}^1_{\mathfrak{d}\mathfrak{t}} \oplus \tau_2\mathfrak{S}^2_{\mathfrak{d}\mathfrak{t}} \oplus ... \oplus \tau_\mathfrak{q}\mathfrak{S}^\mathfrak{q}_{\mathfrak{d}\mathfrak{t}} \\
&= \begin{pmatrix} \sqrt{1-\Pi^\mathfrak{q}_{\mathfrak{s}=1}(1-(\mu(^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}})^2)^{\tau_\mathfrak{s}}}e^{i2\pi\sqrt{1-\Pi^\mathfrak{q}_{\mathfrak{s}=1}(1-(f(^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}})^2)^{\tau_\mathfrak{s}}}}, \\ \Pi^\mathfrak{q}_{\mathfrak{s}=1}[\nu^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_\mathfrak{s}}e^{i2\pi\Pi^\mathfrak{q}_{\mathfrak{s}=1}[g^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_\mathfrak{s}}}, \Pi^\mathfrak{q}_{\mathfrak{s}=1}[\omega^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_\mathfrak{s}}e^{i2\pi\Pi^\mathfrak{q}_{\mathfrak{s}=1}[h^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_\mathfrak{s}}} \end{pmatrix} \\
&= (\mu_{\mathfrak{d}\mathfrak{t}}e^{i2\pi f_{\mathfrak{d}\mathfrak{t}}}, \nu_{\mathfrak{d}\mathfrak{t}}e^{i2\pi g_{\mathfrak{d}\mathfrak{t}}}, \omega_{\mathfrak{d}\mathfrak{t}}e^{i2\pi h_{\mathfrak{d}\mathfrak{t}}})
\end{aligned}$$

The aggregated CSFSPM (ACSFSPM) is constructed as follows:

$$\mathfrak{S} = \begin{pmatrix}
(\chi_{11}, \vartheta_{11}, \xi_{11}) & (\chi_{12}, \vartheta_{12}, \xi_{12}) & . & . & . & (\chi_{1\mathfrak{y}}, \vartheta_{1\mathfrak{y}}, \xi_{1\mathfrak{y}}) \\
(\chi_{21}, \vartheta_{21}, \xi_{21}) & (\chi_{22}, \vartheta_{22}, \xi_{22}) & . & . & . & (\chi_{2\mathfrak{y}}, \vartheta_{2\mathfrak{y}}, \xi_{2\mathfrak{y}}) \\
. & . & . & & & . \\
. & . & & . & & . \\
. & . & & & . & . \\
(\chi_{\mathfrak{r}1}, \vartheta_{\mathfrak{r}1}, \xi_{\mathfrak{r}1}) & (\chi_{\mathfrak{r}2}, \vartheta_{\mathfrak{r}2}, \xi_{\mathfrak{r}2}) & . & . & . & (\chi_{\mathfrak{r}\mathfrak{y}}, \vartheta_{\mathfrak{r}\mathfrak{y}}, \xi_{\mathfrak{r}\mathfrak{y}})
\end{pmatrix}$$

Step 03 Assess the weights for the attributes. The relevance of the characteristics is determined by their respective linguistic weights, which are then assessed by the consultants and given CSFSNs. The judgements made by the consultants are compiled by the CSFSWA operator, which is then used in the process of formulating the CSFS-weights of the characteristics. $\mathfrak{p}^{(s)}_\mathfrak{t} = (\mu^\mathfrak{s}_\mathfrak{t}e^{i2\pi f^\mathfrak{s}_\mathfrak{t}}, \nu^\mathfrak{s}_\mathfrak{t}e^{i2\pi g^\mathfrak{s}_\mathfrak{t}}, \omega^\mathfrak{s}_\mathfrak{t}e^{i2\pi h^\mathfrak{s}_\mathfrak{t}})$ outlined below:

$$\begin{aligned}
\mathfrak{p}_\mathfrak{t} &= CSFSWA_\tau(\mathfrak{p}^1_\mathfrak{t}, \mathfrak{p}^2_\mathfrak{t}, ..., \mathfrak{p}^\mathfrak{q}_\mathfrak{t}) \\
&= \tau_1\mathfrak{p}^1_\mathfrak{t} \oplus \tau_2\mathfrak{p}^2_\mathfrak{t} \oplus ... \oplus \tau_\mathfrak{q}\mathfrak{p}^\mathfrak{q}_\mathfrak{t} \\
&= \begin{pmatrix} \sqrt{1-\Pi^\mathfrak{q}_{\mathfrak{s}=1}(1-(\mu(^{\mathfrak{s})}_{\mathfrak{t}})^2)^{\tau_\mathfrak{s}}}e^{i2\pi\sqrt{1-\Pi^\mathfrak{q}_{\mathfrak{s}=1}(1-(f(^{\mathfrak{s})}_{\mathfrak{t}})^2)^{\tau_\mathfrak{s}}}}, \\ \Pi^\mathfrak{q}_{\mathfrak{s}=1}[\nu^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_\mathfrak{s}}e^{i2\pi\Pi^\mathfrak{q}_{\mathfrak{s}=1}[g^{(\mathfrak{s})}_{\mathfrak{d}\mathfrak{t}}]^{\tau_\mathfrak{s}}}, \Pi^\mathfrak{q}_{\mathfrak{s}=1}[\omega^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_\mathfrak{s}}e^{i2\pi\Pi^\mathfrak{q}_{\mathfrak{s}=1}[h^{(\mathfrak{s})}_{\mathfrak{t}}]^{\tau_\mathfrak{s}}} \end{pmatrix} \\
&= (\mu_\mathfrak{t}e^{i2\pi f_\mathfrak{t}}, \nu_\mathfrak{t}e^{i2\pi g_\mathfrak{t}}, \omega_\mathfrak{t}e^{i2\pi h_\mathfrak{t}})
\end{aligned}$$

Step 04 Instigate the aggregated weighted CSFSPM (AWCSFSPM).

The entities of AWCSFSPM $\overline{\mathfrak{S}} = (\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}})_{\mathfrak{r}\times\mathfrak{y}} = (\overline{\mu}_{\mathfrak{d}\mathfrak{t}}e^{i2\pi\overline{f}_{\mathfrak{d}\mathfrak{t}}}, \overline{\nu}_{\mathfrak{d}\mathfrak{t}}e^{i2\pi\overline{g}_{\mathfrak{d}\mathfrak{t}}}, \overline{\omega}_{\mathfrak{d}\mathfrak{t}}e^{i2\pi\overline{h}_{\mathfrak{d}\mathfrak{t}}})$ are assessed by employing the ACSFSPM and weightage of attributes $\mathfrak{p}_\mathfrak{t}$ are outlined as:

$$\begin{aligned}
\overline{\mathfrak{S}}_{\mathfrak{d}\mathfrak{t}} &= \mathfrak{S}_{\mathfrak{d}\mathfrak{t}} \otimes \mathfrak{p}_\mathfrak{t} \\
&= (\mu_{\mathfrak{d}\mathfrak{t}}\mu_\mathfrak{t}e^{i2\pi f_{\mathfrak{d}\mathfrak{t}}f_\mathfrak{t}}, \sqrt{\nu^2_{\mathfrak{d}\mathfrak{t}}+\nu^2_\mathfrak{t}-\nu^2_{\mathfrak{d}\mathfrak{t}}\nu^2_\mathfrak{t}}e^{i2\pi\sqrt{g^2_{\mathfrak{d}\mathfrak{t}}+g^2_\mathfrak{t}-g^2_{\mathfrak{d}\mathfrak{t}}g^2_\mathfrak{t}}}, \sqrt{\omega^2_{\mathfrak{d}\mathfrak{t}}+\omega^2_\mathfrak{t}-\omega^2_{\mathfrak{d}\mathfrak{t}}\omega^2_\mathfrak{t}}e^{i2\pi\sqrt{h^2_{\mathfrak{d}\mathfrak{t}}+h^2_\mathfrak{t}-h^2_{\mathfrak{d}\mathfrak{t}}h^2_\mathfrak{t}}})
\end{aligned} \quad (19)$$

$$= (\overline{\mu}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi \overline{f}_{\mathfrak{d}\mathfrak{t}}}, \overline{\nu}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi \overline{g}_{\mathfrak{d}\mathfrak{t}}}, \overline{\omega}_{\mathfrak{d}\mathfrak{t}} e^{i2\pi \overline{h}_{\mathfrak{d}\mathfrak{t}}})$$
$$= (\overline{\chi}_{\mathfrak{d}\mathfrak{t}}, \overline{\vartheta}_{\mathfrak{d}\mathfrak{t}}, \overline{\xi}_{\mathfrak{d}\mathfrak{t}})$$

Then the matrix can be formed as following:

$$\overline{\mathfrak{S}} = \begin{pmatrix} (\overline{\chi}_{11}, \overline{\vartheta}_{11}, \overline{\xi}_{11}) & (\overline{\chi}_{12}, \overline{\vartheta}_{12}, \overline{\xi}_{12}) & . & . & . & (\overline{\chi}_{1\mathfrak{y}}, \overline{\vartheta}_{1\mathfrak{y}}, \overline{\xi}_{1\mathfrak{y}}) \\ (\overline{\chi}_{21}, \overline{\vartheta}_{21}, \overline{\xi}_{21}) & (\overline{\chi}_{22}, \overline{\vartheta}_{22}, \overline{\xi}_{22}) & . & . & . & (\overline{\chi}_{2\mathfrak{y}}, \overline{\vartheta}_{2\mathfrak{y}}, \overline{\xi}_{2\mathfrak{y}}) \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ (\overline{\chi}_{\mathfrak{r}1}, \overline{\vartheta}_{\mathfrak{r}1}, \overline{\xi}_{\mathfrak{r}1}) & (\overline{\chi}_{\mathfrak{r}2}, \overline{\vartheta}_{\mathfrak{r}2}, \overline{\xi}_{\mathfrak{r}2}) & . & . & . & (\overline{\chi}_{\mathfrak{r}\mathfrak{y}}, \overline{\vartheta}_{\mathfrak{r}\mathfrak{y}}, \overline{\xi}_{\mathfrak{r}\mathfrak{y}}) \end{pmatrix}$$

Step 05 The identification of CSFS-PIS and CSFS-NIS is carried out with regard to each criterion, relying on the score degree and accuracy degree of CSFSNs in the following manner:

$$CSFS - PIS = \tau^{\diamond} = \{\mathfrak{F}_{\mathfrak{t}}^{\diamond} = (\chi_{\mathfrak{t}}^{\diamond}, \vartheta_{\mathfrak{t}}^{\diamond}, \xi_{\mathfrak{t}}^{\diamond}) | \mathfrak{t} = 1, 2, ..., l\}$$
$$CSFS - NIS = \tau^{\circ} = \{\mathfrak{F}_{\mathfrak{t}}^{\circ} = (\chi_{\mathfrak{t}}^{\circ}, \vartheta_{\mathfrak{t}}^{\circ}, \xi_{\mathfrak{t}}^{\circ}) | \mathfrak{t} = 1, 2, ..., l\}$$

The CSFS-positive ideal solution (CSFS-PIS) and CSFS-negative ideal solution (CSFS-NIS) are opted regarding the nature of criteria. Let $\mathfrak{Y}_{\mathfrak{B}}$ and $\mathfrak{Y}_{\mathfrak{C}}$ be the collection of benefit-type criteria and cost-type criteria, respectively. Then CSFS-PIS $\mathfrak{F}_{\mathfrak{t}}^{\diamond} = (\chi_{\mathfrak{t}}^{\diamond}, \vartheta_{\mathfrak{t}}^{\diamond}, \xi_{\mathfrak{t}}^{\diamond}) = (\mu_{\mathfrak{t}}^{\diamond} e^{i2\pi f_{\mathfrak{t}}^{\diamond}}, \nu_{\mathfrak{t}}^{\diamond} e^{i2\pi g_{\mathfrak{t}}^{\diamond}}, \omega_{\mathfrak{t}}^{\diamond} e^{i2\pi h_{\mathfrak{t}}^{\diamond}})$ relative to criteria $\mathfrak{Y}_{\mathfrak{t}}$, the selection can be made in the following manner:

$$\mathfrak{F}_{\mathfrak{t}}^{\diamond} = \begin{cases} \max\limits_{1 \leq \mathfrak{d} \leq \mathfrak{r}} \mathfrak{F}_{\mathfrak{d}\mathfrak{t}}^{\star} & , \text{ if } \mathfrak{Y}_{\mathfrak{t}} \in \mathfrak{Y}_{\mathfrak{B}}; \\ \min\limits_{1 \leq \mathfrak{d} \leq \mathfrak{r}} \mathfrak{F}_{\mathfrak{d}\mathfrak{t}}^{\star} & , \text{ if } \mathfrak{Y}_{\mathfrak{t}} \in \mathfrak{Y}_{\mathfrak{C}}. \end{cases} \tag{20}$$

The CSFS-NIS $\mathfrak{F}_{\mathfrak{t}}^{\circ} = (\chi_{\mathfrak{t}}^{\circ}, \vartheta_{\mathfrak{t}}^{\circ}, \xi_{\mathfrak{t}}^{\circ}) = (\mu_{\mathfrak{t}}^{\circ} e^{i2\pi f_{\mathfrak{t}}^{\circ}}, \nu_{\mathfrak{t}}^{\circ} e^{i2\pi g_{\mathfrak{t}}^{\circ}}, \omega_{\mathfrak{t}}^{\circ} e^{i2\pi h_{\mathfrak{t}}^{\circ}})$ relative to criteria $\mathfrak{Y}_{\mathfrak{t}}$, the selection can be made in the following manner:

$$\mathfrak{F}_{\mathfrak{t}}^{\circ} = \begin{cases} \min\limits_{1 \leq \mathfrak{d} \leq \mathfrak{r}} \mathfrak{F}_{\mathfrak{d}\mathfrak{t}}^{\star} & , \text{ if } \mathfrak{Y}_{\mathfrak{t}} \in \mathfrak{Y}_{\mathfrak{B}}; \\ \max\limits_{1 \leq \mathfrak{d} \leq \mathfrak{r}} \mathfrak{F}_{\mathfrak{d}\mathfrak{t}}^{\star} & , \text{ if } \mathfrak{Y}_{\mathfrak{t}} \in \mathfrak{Y}_{\mathfrak{C}}. \end{cases} \tag{21}$$

Step 06 For each scenario, we compute the normalized Euclidean distance, represented by the symbol $\mathfrak{H}_{\mathfrak{d}}$. The CSFS-PIS and the CSFS-NIS are both incorporated into the computation of this distance. This is done to determine which option gives the most separation from CSFS-NIS while remaining geographically near to CSFS-PIS. This is done so that we may choose the alternative that places us geographically closest to CSFS-PIS. To calculate the normalized Euclidean distance between any given alternative $\mathfrak{H}_{\mathfrak{d}}$ and the CSFS-PIS, use the following formula:

$$d(\mathfrak{H}_{\mathfrak{d}}, \tau^{\diamond}) =$$

$$\sqrt{\frac{\sum_{\mathfrak{d}=1}^{\mathfrak{r}}\left[(\mu^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\mu^{\diamond 2}_{\mathfrak{d}})+(\nu^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\nu^{\diamond 2}_{\mathfrak{d}})+(\omega^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\omega^{\diamond 2}_{\mathfrak{d}})+(f^{\star 2}_{\mathfrak{d}\mathfrak{t}}-f^{\diamond 2}_{\mathfrak{d}})+(g^{\star 2}_{\mathfrak{d}\mathfrak{t}}-g^{\diamond 2}_{\mathfrak{d}})+(h^{\star 2}_{\mathfrak{d}\mathfrak{t}}-h^{\diamond 2}_{\mathfrak{d}})\right]}{3l}} \tag{22}$$

Likewise, the normalized Euclidean distance between an alternative $\mathfrak{H}_{\mathfrak{d}}$ and CSFS-NIS can be determined in the following manner:

$$d(\mathfrak{H}_{\mathfrak{d}},\tau^{\circ}) = \sqrt{\frac{\sum_{\mathfrak{d}=1}^{\mathfrak{r}}\left[(\mu^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\mu^{\circ 2}_{\mathfrak{d}})+(\nu^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\nu^{\circ 2}_{\mathfrak{d}})+(\omega^{\star 2}_{\mathfrak{d}\mathfrak{t}}-\omega^{\circ 2}_{\mathfrak{d}})+(f^{\star 2}_{\mathfrak{d}\mathfrak{t}}-f^{\circ 2}_{\mathfrak{d}})+(g^{\star 2}_{\mathfrak{d}\mathfrak{t}}-g^{\circ 2}_{\mathfrak{d}})+(h^{\star 2}_{\mathfrak{d}\mathfrak{t}}-h^{\circ 2}_{\mathfrak{d}})\right]}{3l}} \tag{23}$$

Step 07 Subsequently, we calculate the closeness index for positive outcomes in the following manner:

$$\varpi(\mathfrak{H}_{\mathfrak{d}}) = \frac{d(\mathfrak{H}_{\mathfrak{d}},\tau^{\diamond})}{d_{\min}(\mathfrak{H}_{\mathfrak{d}},\tau^{\diamond})} - \frac{d(\mathfrak{H}_{\mathfrak{d}},\tau^{\circ})}{d_{\max}(\mathfrak{H}_{\mathfrak{d}},\tau^{\circ})}. \tag{24}$$

Where

$$d_{\min}(\mathfrak{H}_{\mathfrak{d}},\tau^{\diamond}) = \min_{1\le\mathfrak{d}\le\mathfrak{r}} d_{\min}(\mathfrak{H}_{\mathfrak{d}},\tau^{\diamond}), d_{\max}(\mathfrak{H}_{\mathfrak{d}},\tau^{\circ}) = \max_{1\le\mathfrak{d}\le\mathfrak{r}} d_{\min}(\mathfrak{H}_{\mathfrak{d}},\tau^{\circ}).$$

Step 08 Once the closeness index has been evaluated, the available choices are then ranked based on this index, with the highest scores placed at the top of the list. The most optimal solution to the MAGDM problem is the one that selects the option leading to the lowest possible closeness index value.

7.2. Comparison with Decision Alternatives in Hospital Performance

In the CSFS-TOPSIS method, **Step 01** to **Step 04** are identical to those in the CSFS-ELECTRE I method.

Step 05 The decision criteria take into account all the different structures, and in order to be fair to those structures, we will refer to all the criteria as benefit-type criteria. Based on the type of decision-making criteria, Table 17 shows which CSFS-PIS and CSFS-NIS were chosen for each one. This was done by using Equations 20 and 21.

Table 17: CSFS-PIS and CSFS-NIS

Criteria	CSFS-PIS ($\mathfrak{H}^{\diamond}$)	CSFS-NIS ($\mathfrak{H}^{\circ}$)
$\mathfrak{Y}_1$	$\left(0.5526e^{i2\pi(0.4621)}, 0.3922e^{i2\pi(0.3970)}, 0.3980e^{i2\pi(0.3996)}\right)$	$\left(0.6003e^{i2\pi(0.5594)}, 0.4758e^{i2\pi(0.5031)}, 0.4740e^{i2\pi(0.4523)}\right)$
$\mathfrak{Y}_2$	$\left(0.4605e^{i2\pi(0.5405)}, 0.4104e^{i2\pi(0.4127)}, 0.3737e^{i2\pi(0.3524)}\right)$	$\left(0.5856e^{i2\pi(0.6390)}, 0.5891e^{i2\pi(0.5808)}, 0.4863e^{i2\pi(0.4658)}\right)$
$\mathfrak{Y}_3$	$\left(0.4875e^{i2\pi(0.4690)}, 0.4576e^{i2\pi(0.4226)}, 0.4768e^{i2\pi(0.4649)}\right)$	$\left(0.5775e^{i2\pi(0.5129)}, 0.5055e^{i2\pi(0.5056)}, 0.5890e^{i2\pi(0.5471)}\right)$
$\mathfrak{Y}_4$	$\left(0.3690e^{i2\pi(0.4437)}, 0.4395e^{i2\pi(0.4531)}, 0.5204e^{i2\pi(0.5235)}\right)$	$\left(0.4932e^{i2\pi(0.5266)}, 0.5614e^{i2\pi(0.5284)}, 0.5833e^{i2\pi(0.5501)}\right)$
$\mathfrak{Y}_5$	$\left(0.4995e^{i2\pi(0.4900)}, 0.4737e^{i2\pi(0.4686)}, 0.4521e^{i2\pi(0.4325)}\right)$	$\left(0.5717e^{i2\pi(0.6262)}, 0.5358e^{i2\pi(0.4952)}, 0.5543e^{i2\pi(0.4952)}\right)$
$\mathfrak{Y}_6$	$\left(0.3796e^{i2\pi(0.4272)}, 0.5460e^{i2\pi(0.5150)}, 0.5554e^{i2\pi(0.5766)}\right)$	$\left(0.4405e^{i2\pi(0.4555)}, 0.6342e^{i2\pi(0.5939)}, 0.5758e^{i2\pi(0.6221)}\right)$
$\mathfrak{Y}_7$	$\left(0.3483e^{i2\pi(0.3440)}, 0.5645e^{i2\pi(0.5562)}, 0.6121e^{i2\pi(0.6279)}\right)$	$\left(0.3606e^{i2\pi(0.4030)}, 0.6328e^{i2\pi(0.6520)}, 0.6377e^{i2\pi(0.6536)}\right)$
$\mathfrak{Y}_8$	$\left(0.3835e^{i2\pi(0.3772)}, 0.5359e^{i2\pi(0.5272)}, 0.5820e^{i2\pi(0.5777)}\right)$	$\left(0.4225e^{i2\pi(0.4211)}, 0.5732e^{i2\pi(0.5825)}, 0.6239e^{i2\pi(0.6103)}\right)$

Step 06 The distance of each alternative from the CSFS-PIS and CSFS-NIS is calculated using Equations 22 and 23, and these distance measurements are then recorded in Table 18.

Table 18: Normalized Euclidean distance of each alternative from ideal solutions

Alternative	$d(\mathfrak{H}_{\mathfrak{d}}, \tau^{\diamond})$	$d(\mathfrak{H}_{\mathfrak{d}}, \tau^{\circ})$
$\mathfrak{R}_1$	0.1325	0.1046
$\mathfrak{R}_2$	0.1478	0.0989
$\mathfrak{R}_3$	0.1218	0.1194
$\mathfrak{R}_4$	0.0949	0.1558

Step 07 The updated values of the closeness index for each alternative are presented in Table 19, which was generated by applying Equation 24.

Table 19: Closeness index of each alternative

Alternative	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
closeness index (ϖ)	0.7382	0.9367	0.5283	0.0096

Step 08 Table 20 lists the hospitals ranked by their performance according to the closeness index.

Table 20: Ranking of alternative

Alternative	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$
Ranking	3	4	2	1

Table 21: Comparative analysis

Methods	Ranking of Hospital Performance	Best Strategy
CSFS-ELECTRE I method	$\mathfrak{R}_4 > \mathfrak{R}_3 > \mathfrak{R}_1 > \mathfrak{R}_2$	$\mathfrak{R}_4$
CSFS-TOPSIS method	$\mathfrak{R}_4 > \mathfrak{R}_2 > \mathfrak{R}_1 > \mathfrak{R}_3$	$\mathfrak{R}_4$

8. Conclusion

In MAGDM challenges, addressing uncertain data is made feasible using the important framework of CSFSS. This structure provides a convincing two-dimensional setting in which happiness, neutrality, and discontent may be represented. The well-established theory and practical capabilities of ELECTRE techniques have also provided substantial inspiration for the adaptation and creation of a modified version of the ELECTRE I approach specific to CSFSSs. This modification maintains CSFSSs beneficial structure while embracing the ELECTRE I method's core ideas. The suggested approach works well for MAGDM issues, especially those with imprecise and/or periodic data. Within a two-dimensional setting, it excels at supporting appropriate and consistent decision-making.

We have provided a clear and understandable presentation of the CSFS-ELECTRE I method's implementation via extensive explanation and a flowchart. The methodology has many benefits, one of which is its superiority over methods developed for more conventional settings of decision-making. To do this, it takes into account not only the happiness and discontent levels, but also the neutral membership level. With this larger view, there is more room for evaluating options with potentially competing characteristics. In the realm of MAGDM problems, the utilization of the influential CSFS environment proves beneficial in clarifying obscure data. This approach presents satisfaction, neutrality, and dissatisfaction degrees in a convincing two-dimensional framework. It draws inspiration from the well-established and effective TOPSIS method. As a result, an adapted form of the TOPSIS methodology has been tailored to suit CSFSSs. This adaptation amalgamates the favorable structure of CSFSSs with the foundational principles of the TOPSIS approach. As a result, the proposed methodology serves as an excellent solution for MAGDM issues, especially when dealing with vague data of periodic nature, facilitating sound and harmonious DM within a two-dimensional context.
A prominent advantage of the presented techniques lies in its superiority over existing methodologies designed for conventional environments. By taking into account neutral membership degrees in conjunction with satisfaction and dissatisfaction degrees, it significantly broadens the DM space when rating alternatives based on their conflicting attributes. To implement the proposed technique, a step-by-step process is followed. It involves fusing CSFS opinions and attribute weights into a group opinion matrix using the CSFSWA operator. Subsequently, an ELECTRE and TOPSIS-based outranking method, AWCSFSPM, is developed to thoroughly analyze this group opinion matrix. CSFS CN and DCN indices are meticulously formulated for each pair of alternatives, relying on the score degree, accuracy degree, and refusal degree of CSFSNs present in AWCSFSPM in ELECTRE I while CSFS-PIS, and CSFS-NIS are find for TOPSIS method on the basis of AWCSFSPM matrix. The decision graph, derived from the aggregated outranking matrix, is then harnessed to derive the ultimate ELECTRE and TOPSIS based solutions. A numerical illustration has been conducted using the proposed MAGDM techniques. The results from a comparative study conducted with the CSFS-TOPSIS method and those obtained through the CSFS-ELECTRE I approach exhibit a high degree of consistency. In addition to the numerical comparison, we have provided a comprehensive explanation to underscore the superiority and benefits of the suggested approach when contrasted with previously employed methods.

8.1. Limitation and Future work

- Although the present crop of research has a large number of efficient decision-making techniques intended to manage uncertain information within the framework of FS theory, these methods often lack the efficiency that is necessary to efficiently manage two-dimensional data with neutral membership. The robust theory of CSFSs, on the other hand, makes the approach that has been developed exceptionally well-equipped to appropriately meet the issues that are provided by two-dimensional information.

- In comparison to other techniques to DM that are already in use, the structure and framework of the approach that is being suggested here are noticeably more all-encompassing. This more comprehensive strategy is well-suited to properly address the uncertainty that is inherent in ambiguous information, particularly with regard to the neutral membership function.

- The CSFS-ELECTRE I and CSFS-TOPSIS approaches that are described here are superior to other, more sophisticated fuzzy models and their extensions. The reason for this is because the CSFS model may effectively be applied to CIFSS and CPyFSS if the neutral membership is set to equal to zero. As a result, the CSFS model is an extension of these two models that is more comprehensive than the others.

- Both the CSFS-ELECTRE I method and the CSFS-TOPSIS method prove to be insufficient when faced with scenarios in which the square sum of truth membership, neutral membership, and falsity membership for amplitude terms (or phase terms) exceeds a value of 1.

- When dealing with a considerable number of alternatives or characteristics using the established approaches, complexity, and convolution emerge during the assessment of the three different forms of concordance and discordance sets. This is because of the nature of the situation.

- The suggested framework may be further developed in future work with the hybridization with the technologies of artificial intelligence and machine learning. As an example, machine learning algorithms may be introduced to automatically acquire weights of attributes using extensive healthcare datasets, whereas pattern recognition algorithms using AI may increase the accuracy and versatility of the decisions.

- Additionally, we aim to explore the extension of other variations of the ELECTRE approach, including ELECTRE II, ELECTRE III, and ELECTRE IV, within the suitable contexts of CSFSSs.

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