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Genetic Approach for Radio Labeling of Networks

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Abstract. Effective frequency assignment is critical for modern wireless and communication systems, where limited spectrum must be allocated in a way that minimizes interference and maximizes service reliability. Radio labeling of graphs provides a mathematical framework for modeling frequency assignment in wireless and communication networks, where channel (frequency) indices must be allocated to network nodes with sufficient separation to avoid interference. For a connected graph G, a radio labeling assigns integer frequencies to vertices such that

$$|f(u) - f(v)| \geq d(G) + 1 - d(u, v) \quad \text{for all } u, v \in V(G),$$

where $d(u, v)$ denotes the length of the shortest path between u and v, and $d(G)$ denotes the diameter of G. The minimum span of such a labeling defines the radio number $\mathrm{rn}(G)$. Finding an optimal labeling is computationally hard for many graph families, thus motivating the use of heuristic methods. This paper investigates the use of a Genetic Algorithm (GA) as a heuristic optimization method for computing efficient radio labeling and estimating the radio number of several graphs on path, cycle, and friendship graphs. Seven crossover operators—PMX, CX, OX1, OX2, POX, MPX, and EMPX—are evaluated. The paper also provides new upper bounds of radio number for various classes of k-partite networks.

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1. Introduction

The rapid development of wireless communication technologies has resulted in a rising demand for the efficient management of the limited radio frequency spectrum. One of the most significant and challenging issues in this area is the Frequency Assignment Problem (FAP), which involves assigning frequencies to communication transmitters or base stations in a way that minimizes interference, maintains service quality, and optimally utilizes the available spectrum resources [1]. This problem arises in a variety of real-world applications, such as cellular networks, satellite communication systems, military operations, and emergency communication infrastructures. As wireless networks become increasingly complex, traditional manual or static assignment methods are proving inadequate, highlighting the need for effective mathematical and algorithmic solutions.

Graph theory is a branch of mathematics that concentrates on studying and analyzing graphs—structures composed of nodes (or vertices) and edges connecting pairs of nodes. It was first introduced by Leonhard Euler in the 18th century to tackle the well-known Seven Bridges of Königsberg problem and has since transformed into a crucial field with significant applications in computer science, communication systems, biology, sociology, and many others.

Graph theory has provided a robust tool for modeling and solving the FAP. In this modeling, transmitters or communication stations are represented as vertices of a graph, while the potential for interference between any two transmitters is represented as an edge connecting the corresponding vertices. The resulting graph is called the *interference graph*. This abstraction facilitates the implementation of graph labeling techniques to effectively model and resolve the frequency assignment problem. A particularly effective method in this context is radio labeling, which was introduced to model channel assignment problems in a simplified yet precise way.

Most graph labeling methods trace their origin to one introduced by Rosa [2] in 1967, or one given by Graham and Sloane [3] in 1980. In 1992, Griggs and Yeh [4] introduced labeling of graphs with a condition at distance 2, known as $L(2,1)$-labeling or distance-two labeling, to address the frequency assignment problem. The graph labeling $L(2,1)$ of a simple graph G is a non-negative real-valued function $L : V(G) \to [0,\infty)$ such that $|L(u) - L(v)| \geq 2$ whenever $d(u,v) = 1$, and $|L(u) - L(v)| \geq 1$ whenever $d(u,v) = 2$.

Chartrand et al. [5] presented an effective graph labeling technique called radio labeling, which models frequency assignment problems in a simplified and precise way. The radio labeling problem of a simple, finite and connected graph $G = (V(G), E(G))$ is defined as follows. Let $u, v \in V(G)$. The distance $d(u,v)$ denotes the length of the shortest path between u and v. The diameter $d(G)$ of G is defined as

$$d(G) = \max\{d(u,v) : u, v \in V(G)\}.$$

A *radio labeling* of G is an injective function $f : V(G) \to \mathbb{N} = \{0, 1, 2, 3, \ldots\}$ satisfying

$$|f(u) - f(v)| \geq d(G) + 1 - d(u,v) \quad \text{for all } u, v \in V(G). \tag{1}$$

The *span* of f is

$$\operatorname{span}(f) = \max\{|f(u) - f(v)| : u, v \in V(G)\}.$$

The *radio number* of G is defined as

$$rn(G) = \min\{\operatorname{span}(f)\}$$

over all radio labelings f of G. From a complexity viewpoint, computing the exact value of $rn(G)$ is NP-complete [6]. Consequently, many researchers have concentrated on obtaining upper bounds of $rn(G)$.

Saha and Panigrahi [7] presented an algorithm that finds upper bounds of $rn(G)$ for a given graph G. In [8], Badr and Moussa suggested an improved algorithm and provided an integer linear programming model. Authors of [9] presented a new algorithm based on the centroid of the given graph. Recently, radio labeling has been investigated for many graph families [9–19].

The complexity of radio labeling motivates the use of heuristic and metaheuristic algorithms to find near-optimal values of $rn(G)$ within reasonable time. One of the most effective heuristic approaches for the radio labeling problem is the Genetic Algorithm (GA). This paper investigates the use of GA as a heuristic optimization method for computing efficient radio labeling and estimating the radio number of several graphs. Across all cases, GA delivers frequency assignments with significantly reduced span compared to existing techniques, demonstrating improved spectral efficiency and more effective use of available bandwidth. The paper also provides new upper bounds of the radio number for various classes of k-partite networks, offering practical guidance for planning and designing multi-cluster or multi-service wireless systems.

The rest of the paper is organized as follows. Section 2 presents the radio number for k-partite graphs. Section 3 reviews related work. Section 4 describes the Genetic Algorithm methodology. Section 5 presents the computational study. Section 6 concludes the paper.

2. Radio Number for k-Partite Graphs

Definition 1. *A* k-partite graph *is a graph whose vertex set can be partitioned into k disjoint sets (partitions) so that no two vertices within the same set are adjacent.*

Definition 2. *A* complete k-partite graph *is a k-partite graph such that every pair of vertices in distinct sets are adjacent. If there are $n_1, n_2, \ldots, n_k$ vertices in the k sets, the complete k-partite graph is denoted $K_{n_1,n_2,\ldots,n_k}$.*

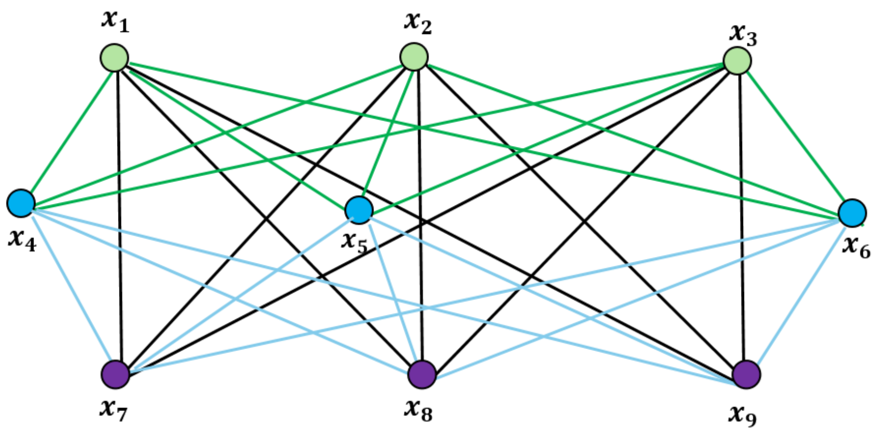

Figure 1: Flowchart of the proposed Genetic Algorithm.

Definition 3. *For a given positive integer k, a* friendship graph $F_{3,k}$ *consists of k cycles (blocks), each of size 3, all sharing exactly one common vertex.*

Theorem 1. *Let $G = (V, E)$ be the complete k-partite graph $K_{\underbrace{n, n, \ldots, n}_{k \text{ times}}}$ where each partite set has $n \geq 3$ vertices. Then*

$$rn\left(K_{\underbrace{n, n, \ldots, n}_{k}}\right) \leq nk + k - 2 \quad \textit{for } k \geq 3.$$

Proof. Let β be a non-negative integer with $0 \leq \beta \leq k - 2$. Define the map f as follows:

$$f(x_i) = \begin{cases} i - 1, & 1 \leq i \leq n, \\ i + \beta, & (\beta + 1)n + 1 \leq i \leq (\beta + 2)n. \end{cases}$$

Since $\operatorname{diam}(G) = 2$, we verify $|f(x_i) - f(x_t)| \geq 3 - d(x_i, x_t)$ for all distinct $x_i, x_t \in V(G)$.

Subcase 1. For $s \in \{1, 2\}$, let $i_s \in \{1, \ldots, n\}$. Since $d(x_{i_1}, x_{i_2}) = 2$,

$$|f(x_{i_1}) - f(x_{i_2})| = |i_1 - i_2| \geq 1 = 3 - d(x_{i_1}, x_{i_2}).$$

Subcase 2. Let $(\beta + 1)n + 1 \leq t_s \leq (\beta + 2)n$ for $s \in \{1, 2\}$. Since $d(x_{t_{\beta_1}}, x_{t_{\beta_2}}) = 2$ if $\beta_1 = \beta_2$ and $= 1$ if $\beta_1 \neq \beta_2$, in both cases $|f(x_{t_1}) - f(x_{t_2})| \geq 3 - d(x_{t_{\beta_1}}, x_{t_{\beta_2}})$.
Subcase 3. Let $i \in \{1, \ldots, n\}$ and $(\beta + 1)n + 1 \leq j \leq (\beta + 2)n$. Since $d(x_i, x_j) = 1$, $|f(x_i) - f(x_j)| = j - i + \beta + 1 \geq 2 = 3 - d(x_i, x_j)$.

Theorem 2. *Let $G = (V, E)$ be a k-partite graph $K_{n_1, n_2, \ldots, n_k}$ with $|V| = N$, where only vertices in successive partitions are adjacent. Then*

$$rn(G) \leq N(k - 2) + 1 \quad \textit{for } k \geq 3.$$

Proof. The diameter of G is $d = k - 1$. Label the vertices of the first partition as 0, $(d-1)$, $2(d-1)$, $\dots$, $(n_1 - 1)(d-1)$. The first vertex of the ℓ-th partition ($\ell \geq 2$) receives label $(n_1 - 1)(d-1) + (\ell - 1)d$, and subsequent vertices in that partition increment by $d - 1$. Summing all contributions gives

$$\begin{aligned} rn(G) &\leq \sum_{i=1}^{k}(n_i - 1)(d-1) + (k-1)d \\ &= (k-1)d - k(d-1) + (d-1)\sum_{i=1}^{k} n_i \\ &= N(k-2) + 1. \end{aligned}$$

3. Genetic Algorithms and Graph Labeling Problems

Genetic Algorithms represent stochastic search strategies that are extensively employed in fields such as engineering, medicine, and machine learning [20]. Hindi and Yampolskiy [21] used Genetic Algorithms to address Graph Coloring Problems. Barod, Hawanna, and Jagtap [22] combined Genetic Algorithms with Memetic Algorithms for Graph Coloring tasks. Assi, Halawi, and Haraty [23] applied the Graph Coloring Method to solve the University Timetable Problem. Anggraini, Rosyida, and Asih [24] tackled the Graph Coloring Problem using Genetic Algorithms. Based on these findings, and given that the radio labeling problem is NP-Hard, we employ a Genetic Algorithm to find upper bounds of $\mathrm{rn}(G)$ utilizing crossover and mutation techniques.

4. Methodology and Material

A Genetic Algorithm (GA), inspired by the process of natural selection, is a population-based search algorithm that explores the solution space through iterative evolution. The following steps are implemented as outlined in Algorithm 1:

- **Encoding:** Each candidate radio labeling is represented as a chromosome.
- **Initialization:** An initial population of feasible labelings is generated randomly.
- **Fitness Function:** Evaluates each labeling by minimizing the span. Lower $\mathrm{rn}(G)$ corresponds to higher fitness.
- **Selection:** Better solutions are selected for reproduction based on their fitness values.
- **Crossover and Mutation:** We implemented and compared PMX, MPX, OX1, OX2, POX, CX, and EMPX. For mutation, we applied the Segment Reversal Mutation (SIM) operator, which reverses the sequence of vertices between two randomly selected positions. Table 1 compares the crossover operators used in this study.

- **Termination:** The algorithm stops when a predefined number of generations is reached.

Table 1: Comparison among different crossover operators.

Crossover Operator	**Mechanism**
PMX (Partially Mapped)	Exchanges mapped segments between two parents while preserving position-based relationships.
MPX (Modified PMX)	Enhances PMX by adjusting the mapping process to reduce positional conflicts.
OX1 (Order Crossover 1)	Copies a substring from one parent and fills remaining positions following the order in the second parent.
OX2 (Order Crossover 2)	Selects positions from one parent and fills remaining positions based on the relative order from the other parent.
POX (Position-Based)	Randomly selects positions from one parent and fills unselected positions from the other parent while maintaining order.
CX (Cycle Crossover)	Builds offspring by identifying cycles of position-value relationships between two parents.
EMPX (Enhanced MPX)	Further refines MPX by improving the exchange mapping to enhance diversity and maintain feasibility.

Algorithm 1 Upper bound of the radio number of a given graph G.

Input: $G(V, E)$; `pop_size`; `generations`; `is_cycle`.

Output: `best_order`; `best_value` (upper bound of the radio number).

vertices $\leftarrow$ list of nodes in G; population $\leftarrow$ `pop_size` random permutations of vertices
global_best_score $\leftarrow -\infty$; global_best_individual $\leftarrow$ `None`
for gen $= 1$ **to** `generations` **do**
  for each `individual` in population **do**
    Set labels $\leftarrow \{\}$; $k \leftarrow \text{diam}(G)$
    for each vertex v in ordering **do**
      label $\leftarrow 0$
      while $\exists\, u \in$ labels s.t. $|\text{label} - \text{labels}[u]| < k + 1 - d(u, v)$ **do**
        label $\leftarrow$ label $+ 1$
      end while
      labels$[v] \leftarrow$ label
    end for
    fitness $\leftarrow -\max(\text{labels.values}())$; Store (fitness, individual)
  end for
  Sort all individuals by fitness (descending); update global best if improved
  next_generation $\leftarrow$ top 50% of population (elitism)
  while $|\text{next\_generation}| <$ `pop_size` **do**
    Select p_1, p_2 from `next_generation`
    child $\leftarrow$ `Crossover`(p_1, p_2) with prob. 0.9, else `copy`(p_1)
    Apply `SIM`(child) with prob. 0.05; add child to `next_generation`
  end while
  population $\leftarrow$ next_generation
end for
return `global_best_individual`, $-$global_best_score

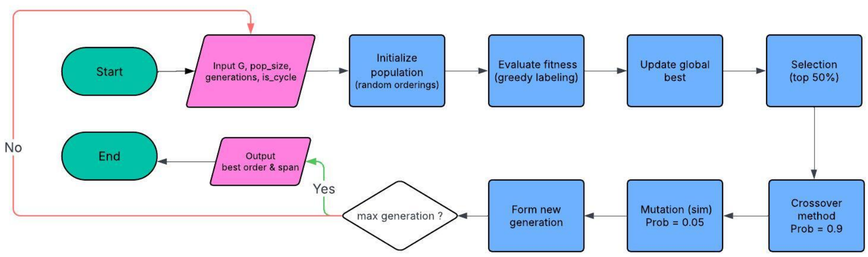

Figure 2: Flowchart of the proposed Genetic Algorithm.

Figure 1 presents the main steps of the Genetic Algorithm. The algorithm receives the input parameters, generates an initial population of random vertex orderings, then iteratively evaluates fitness, applies selection, crossover, and mutation to produce new generations, and terminates when the maximum number of generations is reached.

5. Computational Study

This section evaluates the proposed Genetic Algorithm on path, cycle, and friendship graphs. The algorithm was executed in 10 independent runs using Python 3.11 on a MacBook Air with Apple M1 chip (8-core CPU, 7-core GPU, 8 GB RAM). Seven crossover operators were tested: PMX, CX, OX1, OX2, POX, MPX, and EMPX. Evaluation criteria: **Best value**, **Average value**, and **SD**.

5.1. Results and Discussion

Tables 2, 3, and 4 report the performance of seven genetic crossover operators (PMX, CX, OX1, OX2, POX, MPX, and EMPX) across path, cycle, and friendship graphs of increasing size $N \in \{3, 6, 9, 12, 15, 18\}$, where N denotes the number of vertices. For each instance size and operator, four performance indicators are presented: best solution value, average solution value, and standard deviation (SD) over multiple runs.

Path graphs (Table 2). For small path sizes ($N = 3$ and $N = 6$), all crossover operators consistently achieve identical best and average solutions with zero standard deviation, indicating that the search space is sufficiently small for the GA to reliably converge to the global optimum regardless of the crossover strategy. As the size increases to moderate levels ($N = 9$ and $N = 12$), the best solutions remain identical across all operators, but slight deviations begin to appear in the average results for some operators (e.g., CX, OX1, POX, EMPX), suggesting early signs of variability in convergence behavior. For larger sizes ($N = 15$ and $N = 18$), more pronounced differences emerge in average solution quality. While best values remain nearly identical (with EMPX slightly underperforming at $N = 15$), operators such as PMX, OX2, and MPX tend to produce lower average values, indicating more stable convergence. In contrast, CX and EMPX show higher average values, reflecting a greater tendency to be trapped in suboptimal regions.

Standard deviation values provide further insight into robustness. For $N \leq 6$, all SD values are zero, confirming deterministic behavior across runs. From $N = 9$ onward, variability increases, particularly for CX, OX2, POX, and EMPX. PMX and MPX consistently exhibit lower SD values, especially for larger instances, whereas OX2 and EMPX show the highest SD at $N = 18$, suggesting less stable performance as problem complexity increases. Overall, PMX and MPX demonstrate superior stability across all tested sizes, providing the best trade-off between solution quality and stability, especially for large problem instances.

Cycle graphs (Table 3). For small cycle graphs ($N \leq 9$), all crossover operators converge to identical optimal radio numbers with zero or negligible variance. As the graph size increases, performance differences become more pronounced. Order-based crossover operators, particularly OX1 and OX2, consistently achieve lower average radio numbers and superior best solutions, especially for $N = 18$. These results indicate that preserving relative vertex ordering plays a key role in efficiently solving the radio labeling problem on cycle graphs.

Friendship graphs (Table 4). A friendship graph $F_{3,k}$ consists of k cycles of size 3 sharing a single common vertex, giving $N = 2k + 1$ vertices in total. For all tested values of N, all crossover operators achieved identical best and average objective values with zero standard deviation, indicating fully stable convergence and the complete absence of stochastic variability across runs. This behavior suggests that the structural regularity of friendship graphs renders the radio labeling problem relatively tractable, allowing every operator to reach the optimal solution deterministically regardless of crossover strategy.

Table 2: Radio number of **path** graphs: best, average, and SD over 10 executions.

N	Metric	PMX	CX	OX1	OX2	POX	MPX	EMPX
	Best	3	3	3	3	3	3	3
3	Average	3	3	3	3	3	3	3
	SD	0	0	0	0	0	0	0
	Best	13	13	13	13	13	13	13
6	Average	13	13	13	13	13	13	13
	SD	0	0	0	0	0	0	0
	Best	34	34	34	34	34	34	34
9	Average	34	34.3	34.1	34.1	34.3	34	34.1
	SD	0	0.4	0.3	0.3	0.4	0	0.3
	Best	61	61	61	61	61	61	61
12	Average	61	61.1	61	61	61.1	61.1	61
	SD	0	0.3	0	0	0.3	0.3	0
	Best	100	100	100	100	100	100	101
15	Average	100.6	101.7	100.7	100.2	100.6	100.3	101.7
	SD	0.5	0.8	0.8	0.4	0.8	0.6	0.8
	Best	145	145	145	145	145	145	145
18	Average	145.2	145.6	145.3	145.7	145.4	145.4	145.8
	SD	0.4	0.8	0.6	1.3	0.6	0.5	1

Table 3: Radio number of **cycle** graphs: best, average, and SD over 10 executions.

N	Metric	PMX	CX	OX1	OX2	POX	MPX	EMPX
	Best	2	2	2	2	2	2	2
3	Average	2	2	2	2	2	2	2
	SD	0	0	0	0	0	0	0
	Best	7	7	7	7	7	7	7
6	Average	7	7	7	7	7	7	7
	SD	0	0	0	0	0	0	0
	Best	12	12	12	12	12	12	12
9	Average	12	12.1	12	12	12	12	12
	SD	0	0.3	0	0	0	0	0
	Best	26	26	26	26	26	26	26
12	Average	26.7	26.9	26.6	26.4	26.6	26.4	27
	SD	0.6	0.7	0.5	0.5	0.5	0.5	0.9
	Best	35	35	35	35	35	35	35
15	Average	35.7	36.8	35.5	35.7	35.8	35.8	35.9
	SD	0.6	0.7	0.5	0.4	0.7	0.6	0.5
	Best	49	53	49	49	52	53	51
18	Average	54	55.7	51.6	51.5	53.8	54	54.8
	SD	2.4	2	1.6	1.9	1	0.9	1.8

Table 4: Radio number of **friendship** graphs $F_{3,k}$: best, average, and SD over 5 executions.

N	Metric	PMX	CX	OX1	OX2	POX	MPX	EMPX
	Best	5	5	5	5	5	5	5
2	Average	5	5	5	5	5	5	5
	SD	0	0	0	0	0	0	0
	Best	7	7	7	7	7	7	7
3	Average	7	7	7	7	7	7	7
	SD	0	0	0	0	0	0	0
	Best	9	9	9	9	9	9	9
4	Average	9	9	9	9	9	9	9
	SD	0	0	0	0	0	0	0
	Best	11	11	11	11	11	11	11
5	Average	11	11	11	11	11	11	11
	SD	0	0	0	0	0	0	0
	Best	13	13	13	13	13	13	13
6	Average	13	13	13	13	13	13	13
	SD	0	0	0	0	0	0	0
	Best	15	15	15	15	15	15	15
7	Average	15	15	15	15	15	15	15
	SD	0	0	0	0	0	0	0

6. Conclusion

This paper studies the use of a Genetic Algorithm (GA) for computing the radio number of path, cycle, and friendship graphs. The results demonstrate that PMX and MPX exhibit greater robustness, whereas CX and EMPX show less stable convergence. For cycle graphs, OX1 and OX2 consistently outperform alternative strategies. In future research, we aim to extend this approach to more complex graph structures and explore hybrid evolutionary techniques.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] W. K. Hale. Frequency assignment: Theory and applications. *Proc. IEEE*, 68(12):1497–1514, 1980.

[2] A. Rosa. On certain valuations of the vertices of a graph. In *Theory of Graphs (Internat. Symposium, Rome, 1966)*, pages 349–355, New York, 1967. Gordon and Breach.

[3] R. L. Graham and N. J. A. Sloane. On additive bases and harmonious graphs. *SIAM J. Alg. Discrete Methods*, 1:382–404, 1980.

[4] J. Griggs and R. Yeh. Labelling graphs with a condition at distance 2. *SIAM Journal on Discrete Mathematics*, 5(4):586–595, 1992.

[5] G. Chartrand, D. Erwin, P. Zhang, and F. Harary. Radio labeling of graphs. *Bulletin of the Institute of Combinatorics and its Applications*, 33:77–85, 2001.

[6] M. Kchikech, R. Khennoufa, and O. Togni. Linear and cyclic radio k-labelings of trees. *Discussiones Mathematicae Graph Theory*, 27(1):105–123, 2007.

[7] L. Saha and P. Panigrahi. A graph radio k-coloring algorithm. In *Lecture Notes in Computer Science*, volume 7643, pages 125–129, 2012.

[8] E. M. Badr and M. I. Moussa. An upper bound of radio k-coloring problem and its integer linear programming model. *Wireless Networks*, 26(7):4955–4964, 2020.

[9] N. M. NourEldeen, E. Badr, I. M. Hagag, and H. Shabana. Graph-based approach to the radio assignment problem in wireless communication networks with applications in cryptography. *Eur. J. Pure Appl. Math.*, 18(1):5614, 2025.

[10] M. Morris-Rivera, M. Tomova, C. Wyels, and Y. Yeager. The radio number of $c_n \times c_n$. *Ars Combinatoria*, 103:81–96, 2012.

[11] J. P. Ortiz, P. Martinez, M. Tomova, and C. Wyels. Radio numbers of some generalized prism graphs. *Discussiones Mathematicae Graph Theory*, 31(1):45–62, 2011.

[12] V. S. Reddy and V. K. Iyer. Upper bounds on the radio number of some trees. *Int. J. Pure Applied Math.*, 71(2):207–215, 2011.

[13] L. Saha and P. Panigrahi. On the radio number of toroidal grids. *Australasian Journal of Combinatorics*, 55:273–288, 2013.

[14] L. Saha and P. Panigrahi. A lower bound for radio k-chromatic number. *Discrete Applied Mathematics*, 192:87–100, 2015.

[15] N. M. NourEldeen, E. Badr, H. M. Shabana, and M. Abdel-Aal. Upper bound of radio span for shadow-path network and its mathematical modeling. *European Journal of Pure and Applied Mathematics*, 18(2):5569, 2025.

[16] P. Zhang. Radio labellings of cycles. *Ars Combinatoria*, 65:21–32, 2002.

[17] E. Badr, S. Almotairi, A. Elrokh, A. Abdel-Hady, and B. Almutairi. An integer linear programming model for solving radio mean labeling problem. *IEEE Access*, 8:162343–162349, 2020.

[18] A. H. Alkasasbeh, E. Badr, H. Attiya, and H. M. Shabana. Radio number for friendship communication networks. *Mathematics*, 11:4232, 2023.

[19] E. Badr, S. Nada, M. M. A. Al-Shamiri, A. Abdel-Hay, and A. ELrokh. A novel mathematical model for radio mean square labeling problem. *Journal of Mathematics*, page Article ID 3303, 2022.

[20] H. Farooq, M. N. Zakaria, M. F. Hassan, and S. Sulaiman. An interactive visualization of genetic algorithm on 2-d graph. *Int. J. Software Science and Computational Intelligence*, 4(1):34–54, 2020.

[21] M. M. Hindi and R. V. Yampolskiy. Genetic algorithm applied to the graph coloring problem. In *Proc. Twenty-third Midwest AI and Cognitive Science Conference*, pages 60–66, University of Cincinnati, Ohio, 2012.

[22] P. Barod, V. Hawanna, and V. Jagtap. Genetic algorithm and memetic algorithm on graph colouring. *Scientific Journal of Impact Factor (SJIF)*, 1:269–276, 2014.

[23] M. Assi, B. Halawi, and R. A. Haraty. Genetic algorithm analysis using the graph coloring method for solving the university timetable problem. *Procedia Computer Science*, 126:899–906, 2018.

[24] L. A. Anggraini, I. Rosyida, and T. S. N. Asih. Graph coloring problem solution with genetics algorithm. *UNNES Journal of Mathematics*, 8(1):30–39, 2019.