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Weak forms of ω-open sets and decompositions of continuity

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Abstract. In this paper, we introduce some generalizations of ω-open sets and investigate some properties of the sets. Moreover, we use them to obtain decompositions of continuity.

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1. introduction

Throughout this paper, (X,τ) and (Y,σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let (X,τ) be a space and A a subset of X. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is said to be ω-closed [8] if it contains all its condensation points. The complement of an ω-closed set is said to be ω-open. It is well known that a subset W of a space (X,τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. The family of all ω-open sets of a space (X,τ), denoted by τ_ω or $\omega O(X)$, forms a topology on X finer than τ. The ω-closure and ω-interior, that can be defined

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in the same way as $Cl(A)$ and $Int(A)$, respectively, will be denoted by $Cl_{\omega}(A)$ and $Int_{\omega}(A)$, respectively. Several characterizations of ω-closed sets were provided in [2,3,8,9,13].

Definition 1.1. A subset A of a space X is said to be

1. α-open [12] if $A \subseteq Int(Cl(Int(A)))$;
2. semi-open [10] if $A \subseteq Cl(Int(A))$;
3. pre-open [11] if $A \subseteq Int(Cl(A))$;
4. β-open [1] if $A \subseteq Cl(Int(Cl(A)))$;
5. b-open [5] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

In this paper we introduce and investigate the new notions called b-ω-open sets , pre-ω-open sets and α-ω-open sets which are weaker than ω-open. Moreover, we use these notions to obtain decompositions of continuity.

2. Weak forms of ω-open sets

In this section we introduce the following notions.

Definition 2.1. A subset A of a space X is said to be

1. α-ω-open if $A \subseteq Int_{\omega}(Cl(Int_{\omega}(A)))$;
2. pre-ω-open if $A \subseteq Int_{\omega}(Cl(A))$;
3. β-ω-open if $A \subseteq Cl(Int_{\omega}(Cl(A)))$;
4. b-ω-open if $A \subseteq Int_{\omega}(Cl(A)) \cup Cl(Int_{\omega}(A))$.

Lemma 2.2. *Let (X,τ) be a topological space, then the following properties hold:*

1. *every ω-open set is α-ω-open.*
2. *every α-ω-open set is pre-ω-open.*

3. *every pre-ω-open set is b-ω-open.*

4. *every b-ω-open set is β-ω-open.*

Proof. (1) If A is an ω-open set, then $A = Int_{\omega}(A)$. Since $A \subseteq Cl(A)$, then $A \subseteq Cl(Int_{\omega}(A))$ and $A \subseteq Int_{\omega}(Cl(Int_{\omega}(A)))$. Therefore A is α-ω-open.
(2) If A is an α-ω-open set, then $A \subseteq Int_{\omega}(Cl(Int_{\omega}(A))) \subseteq Int_{\omega}(Cl(A))$. Therefore A is pre-ω-open.
(3) If A is pre-ω-open, then $A \subseteq Int_{\omega}(Cl(A)) \subseteq Int_{\omega}(Cl(A)) \cup Cl(Int_{\omega}(A))$. Therefore, A is b-ω-open.
(4) If A is b-ω-open, then $A \subseteq Int_{\omega}(Cl(A)) \cup Cl(Int_{\omega}(A)) \subseteq Cl(Int_{\omega}(Cl(A))) \cup Cl(Int_{\omega}(A)) \subseteq Cl(Int_{\omega}(Cl(A)))$. Therefore A is β-ω-open.

Since every open set is ω-open, then we have the following diagram for properties of subsets.

$$\begin{array}{ccccccccc}
\text{open} & \longrightarrow & \alpha\text{-open} & \longrightarrow & \text{preopen} & \longrightarrow & b\text{-open} & \longrightarrow & \beta\text{-open} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\omega\text{-open} & \longrightarrow & \alpha\text{-}\omega\text{-open} & \longrightarrow & \text{pre } \omega\text{-open} & \longrightarrow & b\text{-}\omega\text{-open} & \longrightarrow & \beta\text{-}\omega\text{-open}
\end{array}$$

The converses need not be true as shown by the following examples.

Example 2.3. *Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $\{c\}$ is an ω-open (since X is a countable set) set but it is not β-open.*

Example 2.4. *Let $X = \mathbb{R}$ with the usual topology τ. Let $A = \mathbb{Q} \cap [0, 1]$. Then A is a β-open set which is not b-ω-open.*

Example 2.5. *Let $X = \mathbb{R}$ with the usual topology τ. Let $A = (0, 1]$. Then A is a b-open set which is not pre-ω-open.*

Example 2.6. *Let $X = \mathbb{R}$ with the usual topology τ. Let $A = \mathbb{Q}$ be the set of all rational numbers. Then A is a preopen set which is not α-ω-open.*

Example 2.7. *Let X be an uncountable set and let A, B, C and D be subsets of X such that each of them is uncountable and the family $\{A, B, C, D\}$ is a partition of X. We defined the topology*

$\tau=\{\phi, X, \{A\},$
$\{B\}, \{A, B\}, \{A, B, C\}\}$. *Then* $\{A, B, D\}$ *is an* α*-open set which is not* ω*-open.*

Lemma 2.8. *[7] If* U *is an open set, then* $Cl(U \cap A) = Cl(U \cap Cl(A))$ *and hence* $U \cap Cl(A) \subseteq Cl(U \cap A)$ *for any subset* A*.*

Theorem 2.9. *If* A *is a pre-*ω*-open subset of a space* (X, τ) *such that* $U \subseteq A \subseteq Cl(U)$ *for a subset* U *of* X*, then* U *is a pre-*ω*-open set.*

Proof. Since $A \subseteq Int_{\omega}(Cl(A))$, $U \subseteq Int_{\omega}(Cl(A))$. Also $Cl(A) \subseteq Cl(U)$ implies that $Int_{\omega}(Cl(A)) \subseteq Int_{\omega}(Cl(U))$. Thus $U \subseteq Int_{\omega}(Cl(A))$
$\subseteq Int_{\omega}(Cl(U))$ and hence U ia a pre-ω-open set.

Theorem 2.10. *A subset* A *of a space* (X, τ) *is semi-open if and only if* A *is* β*-*ω*-open and* $Int_{\omega}(Cl(A)) \subseteq Cl(Int(A))$*.*

Proof. Let A *be semi-open. Then* $A \subseteq Cl(Int(A)) \subseteq Cl(Int_{\omega}(Cl(A)))$ *and hence* A *is* β*-*ω*-open. In addition* $Cl(A) \subseteq Cl(Int(A))$ *and hence* $Int_{\omega}(Cl(A)) \subseteq Cl(Int(A))$*. Conversely let* A *be* β*-*ω*-open and* $Int_{\omega}(Cl(A))$
$\subseteq Cl(Int(A))$*. Then* $A \subseteq Cl(Int_{\omega}(Cl(A))) \subseteq Cl(Cl(Int(A))) = Cl(Int(A))$*. And hence* A *is semi-open.*

Proposition 2.11. *The intersection of a pre-*ω*-open set and an open set is pre-*ω*-open.*

Proof. Let A *be a pre-*ω*-open set and* U *be an open set in* X*. Then* $A \subseteq Int_{\omega}(Cl(A))$ *and* $Int_{\omega}(U) = U$*, by Lemma 2.8, we have* $U \cap A \subseteq Int_{\omega}(U) \cap Int_{\omega}(Cl(A)) \subseteq Int_{\omega}(U \cap Cl(A)) \subseteq Int_{\omega}(Cl(U \cap A))$*. Therefore,* $A \cap U$ *is pre-*ω*-open.*

Proposition 2.12. *The intersection of a* β*-*ω*-open set and an open set is* β*-*ω*-open.*

Proof. Let U be an open set and A a β-ω-open set. Since every open set is ω-open, by

Lemma 2.8, we have

$$\begin{aligned} U \cap A &\subseteq U \cap Cl(Int_\omega(Cl(A))) \\ &\subseteq Cl(U \cap Int_\omega(Cl(A))) \\ &= Cl(Int_\omega(U) \cap Int_\omega(Cl(A))) \\ &= Cl(Int_\omega(U \cap Cl(A))) \\ &\subseteq Cl(Int_\omega(Cl(U \cap A))). \end{aligned}$$

This shows that $U \cap A$ is β-ω-open.

We note that the intersection of two pre-ω-open (resp. b-ω-open, β-ω-open) sets need not be pre-ω-open (resp. b-ω-open, β-ω-open) as can be seen from the following example:

Example 2.13. *Let $X = \mathbb{R}$ with the usual topology τ. Let $A = \mathbb{Q}$ and $B = (\mathbb{R} \backslash \mathbb{Q}) \cup \{1\}$, then A and B are pre-ω-open, but $A \cap B = \{1\}$ which is not β-ω-open since $Cl(Int_\omega(Cl(\{1\}))) = Cl(Int_\omega(\{1\}) = Cl(\{\phi\}) = \phi$.*

Proposition 2.14. *The intersection of a b-ω-open set and an open set is b-ω-open.*

Proof. Let A be b-ω-open and U be open, then $A \subseteq Int_\omega(Cl(A)) \cup Cl(Int_\omega(A))$ and $U = Int_\omega(U)$. Then we have

$$\begin{aligned} U \cap A &\subseteq U \cap [Int_\omega(Cl(A)) \cup Cl(Int_\omega(A))] \\ &= [U \cap Int_\omega(Cl(A))] \cup [U \cap Cl(Int_\omega(A))] \\ &= [Int_\omega(U) \cap Int_\omega(Cl(A))] \cup [U \cap Cl(Int_\omega(A))] \\ &\subseteq [Int_\omega(U \cap Cl(A))] \cup [Cl(U \cap Int_\omega(A))] \\ &\subseteq [Int_\omega(Cl(U \cap A))] \cup [Cl(Int_\omega(U \cap A))]. \end{aligned}$$

This shows that $U \cap A$ is b-ω-open.

Proposition 2.15. *The intersection of an α-ω-open set and an open set is α-ω-open.*

Theorem 2.16. *If $\{A_\alpha : \alpha \in \Delta\}$ is a collection of b-ω-open (resp. pre-ω-open, β-ω-open) sets of a space (X, τ), then $\cup_{\alpha \in \Delta} A_\alpha$ is b-ω-open (resp. pre-ω-open, β-ω-open).*

Proof. We prove only the first case since the other cases are similarly shown. Since $A_\alpha \subseteq Int_\omega(Cl(A_\alpha)) \cup Cl(Int_\omega(A_\alpha))$ for every $\alpha \in \Delta$, we have

$$\begin{aligned} \cup_{\alpha\in\Delta} A_\alpha &\subseteq \cup_{\alpha\in\Delta}[Int_\omega(Cl(A_\alpha)) \cup Cl(Int_\omega(A_\alpha))] \\ &\subseteq [\cup_{\alpha\in\Delta} Int_\omega(Cl(A_\alpha))] \cup [\cup_{\alpha\in\Delta} Cl(Int_\omega(A_\alpha))] \\ &\subseteq [Int_\omega(\cup_{\alpha\in\Delta} Cl(A_\alpha))] \cup [Cl(\cup_{\alpha\in\Delta} Int_\omega(A_\alpha))] \\ &\subseteq [Int_\omega(Cl(\cup_{\alpha\in\Delta} A_\alpha))] \cup [Cl(Int_\omega(\cup_{\alpha\in\Delta} A_\alpha))]. \end{aligned}$$

Therefore, $\cup_{\alpha\in\Delta} A_\alpha$ is b-ω-open.

Proposition 2.17. *Let A be a b-ω-open set such that $Int_\omega(A) = \phi$. Then A is pre-ω-open.*

A space (X, τ) is called a door space if every subset of X is open or closed.

Proposition 2.18. *If (X, τ) is a door space, then every pre-ω-open set is ω-open.*

Proof. Let A be a pre-ω-open set. If A is open, then A is ω-open. Otherwise, A is closed and hence $A \subseteq Int_\omega(Cl(A)) = Int_\omega(A) \subseteq A$. Therefore, $A = Int_\omega(A)$ and thus A is an ω-open set.

A topological space X is said to be anti-locally countable [4] if every non-empty open set is uncountable.

Lemma 2.19. *[4] If (X, τ) is an anti-locally countable space, then $Int_\omega(A) = Int(A)$ for every ω-closed set A of X and $Cl_\omega(A) = Cl(A)$ for every ω-open set A of X.*

Theorem 2.20. *Let (X, τ) be an anti-locally countable space and A a subset of X. Then, the following properties hold:*

1. *if A is pre-ω-open, then it is pre-open.*
2. *if A is b-ω-open and ω-closed, then it is b-open.*
3. *if A is β-ω-open, then it is β-open.*

Proof. (1) Let A be a pre-ω-open set. Then by Lemma 2.19 $A \subseteq Int_\omega(Cl(A)) = Int(Cl(A))$ since every closed set is ω-closed.

(2) Let A be a b-ω-open and ω-closed set. By Lemma 2.19, we have $Int_\omega(Cl(A)) = Int(Cl(A))$, $Cl(Int_\omega(A)) = Cl(Int(A))$ and hence $A \subseteq Int_\omega(Cl(A)) \cup Cl(Int_\omega(A)) = Int(Cl(A)) \cup Cl(Int(A))$. This shows that A is b-open.

(3) Let A be a β-ω-open set. Then, by Lemma 2.19, we have $A \subseteq Cl(Int_\omega(Cl(A))) = Cl(Int(Cl(A)))$ and hence A is β-open.

3. Decompositions of continuity

Definition 3.1. A subset A of a space X is called

1. an ω-t-set if $Int(A) = Int_\omega(Cl(A))$;
2. an ω-B-set if $A = U \cap V$, where $U \in \tau$ and V is an ω-t-set.

Proposition 3.2. *Let A and B be subsets of a space (X, τ). If A and B are ω-t-sets, then $A \cap B$ is an ω-t-set.*

Proof. Let A and B be ω-t-sets. Then we have

$$\begin{aligned} Int(A\cap B) &\subseteq Int_\omega(Cl(A\cap B)) \\ &\subseteq (Int_\omega(Cl(A)) \cap (Cl(B))) \\ &= Int_\omega(Cl(A) \cap Int_\omega(Cl(B)) \\ &= Int(A) \cap Int(B) \\ &= Int(A\cap B). \end{aligned}$$

Then $Int(A\cap B) = Int_\omega(Cl(A\cap B))$ and hence $A\cap B$ is an ω-t-set.

From the following examples one can deduce that a pre-ω-open set and an ω-B-set are independent.

Example 3.3. *Let $X = \mathbb{R}$ with the usual topology τ. Then $\mathbb{R}\backslash\mathbb{Q}$ is pre-ω-open but it is not an ω-B-set and $(0, 1]$ is an ω-B-set which is not pre-ω-open.*

Proposition 3.4. *For a subset A of a space (X, τ), the following properties are equivalent:*

1. *A is open;*

2. *A is pre-ω-open and an ω-B-set.*

Proof. (1) ⇒ (2): Let A be open. Then $A = Int(A) \subseteq Int_\omega(Cl(A))$ and A is pre-ω-open. Also $A = A \cap X$ and hence A is an ω-B-set.
(2) ⇒ (1): Since A is an ω-B-set, we have $A = U \cap V$, where U is an open set and $Int(V) = Int_\omega(Cl(V))$. By the hypothesis, A is also pre-ω-open and we have

$$\begin{aligned} A &\subseteq Int_\omega(Cl(A)) \\ &= Int_\omega(Cl(U \cap V)) \\ &\subseteq Int_\omega(Cl(U) \cap Cl(V)) \\ &= Int_\omega(Cl(U)) \cap Int_\omega(Cl(V)) \\ &= Int_\omega(Cl(U)) \cap Int(V). \end{aligned}$$

Hence

$$\begin{aligned} A = U \cap V &= (U \cap V) \cap U \\ &\subseteq (Int_\omega(Cl(U)) \cap Int(V)) \cap U \\ &= (Int_\omega(Cl(U)) \cap U) \cap Int(V) \\ &= U \cap Int(V). \end{aligned}$$

Therefore, $A = (U \cap V) = (U \cap Int(V))$ and A is open.

Definition 3.5. A subset A of a space X is called

1. an ω-t_α-set if $Int(A) = Int_\omega(Cl(Int_\omega(A)))$;

2. an ω-B_α-set if $A = U \cap V$, where $U \in \tau$ and V is an ω-t_α-set.

Proposition 3.6. *Let A and B be subsets of a space (X, τ). If A and B are ω-t_α-sets, then $A \cap B$ is an ω-t_α-set.*

Proof. Let A and B be ω-t_α-sets. Then we have

$$\begin{aligned} Int(A\cap B) &\subseteq Int_\omega(Cl(Int_\omega(A\cap B))) \\ &\subseteq (Int_\omega(Cl(Int_\omega(A)))\cap(Cl(Int_\omega(B)))) \\ &= Int_\omega(Cl(Int_\omega(A))\cap Int_\omega(Cl(Int_\omega(B))) \\ &= Int(A)\cap Int(B) \\ &= Int(A\cap B). \end{aligned}$$

Then $Int(A\cap B) = Int_\omega(Cl(Int_\omega(A\cap B)))$ and hence $A\cap B$ is an ω-t_α-set.

From the following examples one can deduce that an α-ω-open set and an ω-B_α-set are independent.

Example 3.7. *Let $X = \mathbb{R}$ with the usual topology τ. Then $\mathbb{R}\backslash\mathbb{Q}$ is α-ω-open but it is not an ω-B_α-set and $(0,1]$ is an ω-B_α-set which is not α-ω-open.*

Proposition 3.8. *For a subset A of a space (X,τ), the following properties are equivalent:*

1. *A is open;*
2. *A is α-ω-open and an ω-B_α-set.*

Proof. (1) ⇒ (2): Let A be open. Then $A = Int_\omega(A) \subseteq Cl(Int_\omega(A))$ and $A = Int_\omega(A) \subseteq Int_\omega(Cl(Int_\omega(A))$. Therefore A is α-ω-open. Also $A = A\cap X$ and hence A is an ω-B_α-set.
(2) ⇒ (1): Since A is an ω-B_α-set, we have $A = U\cap V$, where U is an open set and $Int(V) = Int_\omega(Cl(Int_\omega(V))$. By the hypothesis, A is also α-ω-open, and we have

$$\begin{aligned} A &\subseteq Int_\omega(Cl(Int_\omega(A))) \\ &= Int_\omega(Cl(Int_\omega(U\cap V)) \\ &\subseteq Int_\omega(Cl(Int_\omega(U)\cap Cl(Int_\omega(V)))) \\ &= Int_\omega(Cl(U))\cap Int_\omega(Cl(Int_\omega(V))) \\ &= Int_\omega(Cl(U))\cap Int(V). \end{aligned}$$

Hence,

$$\begin{aligned} A = U \cap V &= (U \cap V) \cap U \\ &\subseteq (Int_{\omega}(Cl(U)) \cap Int(V)) \cap U \\ &= (Int_{\omega}(Cl(U)) \cap U) \cap Int(V) \\ &= U \cap Int(V). \end{aligned}$$

Therefore, $A = (U \cap V) = (U \cap Int(V))$ and A is open.

Definition 3.9. A subset A of a space X is called an ω-set if $A = U \cap V$, where $U \in \tau$ and $Int(V) = Int_{\omega}(V)$.

From the following examples one can deduce that an ω-open set and an ω-set are independent.

Example 3.10. *Let $X = \mathbb{R}$ with the usual topology τ. Then $\mathbb{R} \backslash \mathbb{Q}$ is ω-open but it is not an ω-set and $A = (0,1) \cap \mathbb{Q}$ is an ω–set which is not ω-open.*

Proposition 3.11. *For a subset A of a space (X, τ), the following properties are equivalent:*

1. *A is open;*
2. *A is ω-open and an ω-set.*

Proof. (1) $\Rightarrow$ (2): This is obvious.
(2) $\Rightarrow$ (1): Since A is an ω-set, we have $A = U \cap V$, where U is an open set and $Int(V) = Int_{\omega}(V)$. By the hypothesis, A is also ω-open and we have $A = Int_{\omega}(A) = Int_{\omega}(U \cap V) = Int_{\omega}(U) \cap Int_{\omega}(V) = U \cap Int(V)$. Therefore, A is open.

Definition 3.12. A function $f : X \to Y$ is said to be ω-continuous [9] (resp. pre-ω-continuous, ω-B-continuous, α-ω-continuous, ω-B_{α}-continuous, ω^*-continuous) if $f^{-1}(V)$ is ω-open (resp. pre-ω-open, an ω-B-set, α-ω-open, an ω-B_{α}-set, an ω-set) for each open set V in Y.

By Propositions 3.4, 3.8 and 3.11 we have an immediate result.

Theorem 3.13. *For a function $f : X \to Y$, the following properties are equivalent:*

1. *f is continuous;*
2. *f is pre-ω-continuous and ω-B-continuous;*
3. *f is α-ω-continuous and ω-B_α-continuous;*
4. *f is ω-continuous and ω^*-continuous.*

Proposition 3.14. *For a subset A of an anti-locally countable space (X, τ), the following properties are equivalent:*

1. *A is regular open;*
2. *$A = Int_\omega(Cl(A))$;*
3. *A is pre-ω-open and an ω-t-set.*

Proof. (1) ⇒ (2): Let A be regular open. Then by Lemma 2.19, we have $Int_\omega(Cl(A)) = Int(Cl(A)) = A$.
(2) ⇒ (3): The proof is obvious.
(3)⇒ (1): Let A be pre-ω-open and an ω-t-set. Then $A \subseteq Int_\omega(Cl(A)) = Int(A) \subseteq A$ and hence $A = Int_\omega(Cl(A)) = Int(Cl(A))$.

Definition 3.15. A function $f : X \to Y$ is said to be completely continuous [6] (resp. ω-t-continuous) if $f^{-1}(V)$ is regular open (resp. an ω-t-set) in X for each open set V of Y.

Theorem 3.16. *Let (X, τ) be an anti-locally countable space. A function $f : X \to Y$ is completely continuous if and only if f is pre-ω-continuous and ω-t-continuous.*

Proof. This is an immediate consequence of Proposition 3.14.

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