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Pseudo Conharmonically Symmetric Manifolds

Füsun Özen Zengin [1,*], Ayşe Yavuz Taşcı [2]

[1] *Department of Mathematics, Faculty of Sciences and Letters, Istanbul Technical University, Istanbul, Turkey*
[2] *Department of Mathematics, Faculty of Sciences and Letters, Piri Reis University, Istanbul, Turkey*

Abstract. The object of the present paper is to study pseudo conharmonically symmetric manifold which is a type of non-flat Riemannian manifold. In the first section, we give the definition of a pseudo conharmonically symmetric manifold. In the second section, some theorems about this manifold are proved. In the last section, we give an example for the existence of this manifold.

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1. Introduction

As we know, in differential geometry, symmetric spaces play an important role. In the late twenties, Cartan [3] initiated Riemannian symmetric spaces and obtained a classification of those spaces. Let (M, g) be an n-dimensional Riemannian manifold with the Riemannian metric g and the Levi-Civita connection ∇. If the Riemannian curvature tensor of a Riemannian manifold satisfies the condition $\nabla R = 0$ then this manifold is called locally symmetric [3]. For every point P of this manifold, this symmetry condition is equivalent to the fact that the local geodesic symmetry $F(P)$ is an isometry [13]. The class of Riemannian symmetric manifolds is very natural generalization of the class of manifolds of constant curvature. Many authors have been studied the notion of locally symmetric manifolds extending several manifolds such as conformally symmetric manifolds [5], recurrent manifolds [24], conformally recurrent manifolds [2], conformally symmetric Ricci-recurrent spaces [18], pseudo-Riemannian manifold with recurrent concircular curvature tensor [12], semi-symmetric manifolds [22], pseudo symmetric manifolds [4, 14, 15], weakly symmetric manifolds [23], projective symmetric manifolds [21], almost pseudo concircularly symmetric manifolds [9], decomposable

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*Corresponding author.

Email addresses: `fozen@itu.edu.tr` (F. Zengin), `aytasci@pirireis.edu.tr` (A. Taşcı)

almost pseudo conharmonically symmetric manifolds [25], etc. A non-flat Riemannian manifold (M,g) $(n>2)$ is said to be a pseudo symmetric manifold [4] if its curvature tensor R satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z)W =& 2A(X)R(Y,Z)W + A(Y)R(X,Z)W + A(Z)R(Y,X)W \\ &+ A(W)R(Y,Z)X + g(R(Y,Z)W,X)\rho \end{aligned} \tag{1}$$

where A is a non-zero 1-form, ρ is a vector field defined by

$$g(X,\rho) = A(X) \tag{2}$$

for all X and ∇ denotes the operator of the covariant differentiation with respect to the metric tensor g. The 1-form A is called the associated 1-form of the manifold. If $A=0$, then the manifold reduces to a symmetric manifold in the sense of E.Cartan. An n-dimensional pseudo symmetric manifold is denoted by $(PS)_n$. This is to be noted that the notion of pseudo symmetric manifold studied in particular by Deszcz [10] is different from that Chaki [4]. The notion of weakly symmetric manifolds was introduced by Tamassy and Binh [23]. If the curvature tensor R of type (1,3) of an n-dimensional Riemannian manifold $(n>2)$ satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z)W =& A(X)R(Y,Z)W + B(Y)R(X,Z)W + D(Z)R(Y,X)W \\ &+ E(W)R(Y,Z)X + g(R(Y,Z)W,X)\rho \end{aligned} \tag{3}$$

where ∇ denotes the Levi-Civita connection on (M,g) and A,B,D,E and ρ are 1-forms and a vector field respectively, which are non-zero simultaneously, then this manifold is denoted by $(WS)_n$. Many authors have been studied weakly symmetric manifolds [6, 7, 11, 16, 17], etc.

Conformal transformation of a Riemannian structure is an important object of study in differential geometry. The conharmonic transformation which is a special type of conformal transformations preserves the harmonicity of smooth functions. Such transformation has an invariant tensor which is called the conharmonic curvature tensor. It is easy to verify that this tensor is an algebraic curvature tensor, that is, it possesses the classical symmetry properties of the Riemannian curvature tensor.

Let M and N be two Riemannian manifolds with the metrics g and $\overline{g}$, respectively related by

$$\bar{g} = e^{2\sigma} g \tag{4}$$

where σ is a real function. Then M and N are called conformally related manifolds, and the correspondence M and N is known as conformal transformation [20]. It is known that a harmonic function is defined as a function whose Laplacian vanishes. In generally, the harmonic function is not invariant. In 1957, Ishii obtained the conditions at which a harmonic function remains invariant and he introduced the conharmonic transformation as a subgroup of the conformal transformation (4) satisfying the condition

$$\sigma^{h}_{,h} + \sigma_{,h}\sigma^{h}_{,} = 0 \tag{5}$$

where comma denotes the covariant differentiation with respect to the metric g. A rank-four tensor H that remains invariant under conharmonic transformation of a Riemannian manifold (M,g) is given by

$$\begin{aligned} H(X,Y,Z,U) =& R(X,Y,Z,U) - \frac{1}{n-2}[g(Y,Z)S(X,U) - g(X,Z)S(Y,U) \\ &+ g(X,U)S(Y,Z) - g(Y,U)S(X,Z)] \end{aligned} \tag{6}$$

where R and S denote the Riemannian curvature tensor of type (0,4) defined by $R(X,Y,Z,U) = g(R(X,Y)Z,U)$ and the Ricci tensor of type (0,2), respectively. The curvature tensor defined by (6) is known as conharmonic curvature tensor. A manifold whose conharmonic curvature tensor vanishes at every point of the manifold is called conharmonically flat. Thus, this tensor represents the deviation of the manifold from conharmonic flatness. Many authors have been studied the conharmonic curvature tensor [1, 20]. The present paper deals with an n-dimensional pseudo conharmonically symmetric Riemannian manifold (M,g) (non-conharmonically flat) whose conharmonic curvature tensor H satisfies the condition

$$\begin{aligned} (\nabla_X H)(Y,Z,U,V) =& 2A(X)H(Y,Z,U,V) + A(Y)H(X,Z,U,V) \\ &+ A(Z)H(Y,X,U,V) + A(U)H(Y,Z,X,V) \\ &+ A(V)H(Y,Z,U,X) \end{aligned} \tag{7}$$

where A has the meaning already mentioned in (2). Such a manifold is called a pseudo conharmonically symmetric manifold [4] and denoted by $(PCHS)_n$. Since the conformal curvature tensor vanishes identically for $n = 3$, we assume that $n > 3$ throughout the paper. This paper is organized as follows:

Section 2 deals with some properties of $(PCHS)_n$. Considering special case of conharmonic curvature tensor of this manifold, some theorems are proved. In section 3, an example is given for the existence to this manifold.

2. Pseudo Conharmonically Symmetric Manifold

L denotes the symmetric endomorphism of the tangent space at each point of the manifold corresponding to the Ricci tensor S of type (0,2), that is

$$g(LX,Y) = S(X,Y). \tag{8}$$

Let e_i, $(1 \le i \le n)$ be an orthonormal basis of the tangent space at any point of the manifold. From (6), we have

$$\begin{aligned} \overline{H}(X,Y) =& \sum_{i=1}^{n} H(X,e_i,e_i,Y) = \sum_{i=1}^{n} H(e_i,X,Y,e_i) \\ =& -\frac{r}{n-2} g(X,Y) \end{aligned} \tag{9}$$

and

$$\sum_{i=1}^{n} H(e_i, e_i, X, Y) = \sum_{i=1}^{n} H(X, Y, e_i, e_i) = 0 \tag{10}$$

where r is the scalar curvature of the manifold. Also, from (6) it follows that [19]

$$\begin{aligned} H(X,Y,Z,U) &= -H(Y,X,Z,U) \\ H(X,Y,Z,U) &= -H(X,Y,U,Z) \\ H(X,Y,Z,U) &= H(Z,U,X,Y) \\ H(X,Y,Z,U) + H(X,Z,U,Y) &+ H(X,U,Y,Z) = 0. \end{aligned} \tag{11}$$

We assume that our manifold is $(PCHS)_n$. Thus, the relation (7) holds.

Proposition 1 ([19]). *In a Riemannian manifold V_n ($n > 3$), the conharmonic curvature tensor satisfies the second Bianchi Identity, i.e., the following relation*

$$(\nabla_X H)(Y,Z,U,W) + (\nabla_U H)(Y,Z,W,X) + (\nabla_W H)(Y,Z,X,U) = 0$$

holds if and only if the Ricci tensor is of Codazzi type.

Theorem 1. *In a pseudo conharmonically symmetric Riemannian manifold, the conharmonic curvature tensor satisfies the second Bianchi Identity, i.e.,*

$$(\nabla_X H)(Y,Z,U,W) + (\nabla_U H)(Y,Z,W,X) + (\nabla_W H)(Y,Z,X,U) = 0$$

Proof. Permutating X,U,W in (7) and adding these three equations, we obtain

$$\begin{aligned} &(\nabla_X H)(Y,Z,U,W) + (\nabla_U H)(Y,Z,W,X) + (\nabla_W H)(Y,Z,X,U) \\ &\quad = A(X)[2H(Y,Z,U,W) + H(Y,Z,W,U) + H(Y,Z,W,U)] \\ &\qquad + A(Y)[H(X,Z,U,W) + H(U,Z,W,X) + H(W,Z,X,U)] \\ &\qquad + A(Z)[H(Y,X,U,W) + H(Y,U,W,X) + H(Y,W,X,U)] \\ &\qquad + A(U)[H(Y,Z,X,W) + 2H(Y,Z,W,X) + H(Y,Z,X,W)] \\ &\qquad + A(W)[2H(Y,Z,X,U) + H(Y,Z,U,X) + H(Y,Z,U,X)]. \end{aligned} \tag{12}$$

Thus, from (11), (12) reduces to

$$(\nabla_X H)(Y,Z,U,W) + (\nabla_U H)(Y,Z,W,X) + (\nabla_W H)(Y,Z,X,U) = 0 \tag{13}$$

i.e. the conharmonic curvature tensor satisfies the second Bianchi Identity. □

Theorem 2. *A pseudo conharmonically symmetric manifold with non-zero scalar curvature is of closed associated 1-form.*

Proof. Contracting on Y and V in (7), we find

$$(\nabla_X \overline{H})(Z,U) = 2A(X)\overline{H}(Z,U) + A(H(U,Z)X) + A(Z)\overline{H}(X,U) + A(U)\overline{H}(Z,X) + A(H(U,Z)X) \tag{14}$$

where $\overline{H}$ is in the form (9). Putting $Y = Z = e_i$ in (14), where e_i is an orthonormal basis of tangent space at each point of the manifold and i is summed for $1 \leq i \leq n$, we get

$$(\nabla_X h) = 2A(X)h + 4A(LX) \tag{15}$$

where $h = \overline{H}(LX)$ and L denotes the symmetric endomorphism of the tangent space at each point corresponding to the tensor $\overline{H}(X,Y)$. If we take the covariant derivative of (15), we find

$$\nabla_Y \nabla_X h = 2(\nabla_Y A)(X)h + 2A(X)(\nabla_Y h) + 4(\nabla_Y A)(LX) + 4A(\nabla_Y \overline{H})(LX). \tag{16}$$

Changing X and Y in (16) and subtracting these two equations, we obtain from (6), (11), (14) and (15), assuming that our manifold admits non-zero scalar curvature then

$$(\nabla_Y A)(X) - (\nabla_X A)(Y) = 0. \tag{17}$$

By the aid of (17), we can say that the associated 1-form of this manifold is closed. Thus, the proof is completed. □

Theorem 3. *A Riemannian manifold admits divergence-free conharmonic curvature tensor is of constant scalar curvature.*

Proof. In local coordinates, from the second Bianchi Identity, we have

$$R^h_{ijk,h} = S_{ij,k} - S_{ik,j} \tag{18}$$

and then

$$S^h_{k,h} = \frac{1}{2} r_{,k} \tag{19}$$

where r is the scalar curvature and S_{ij} is the Ricci tensor of this manifold. Thus from (6),

$$H^h_{ijk,l} = R^h_{ijk,l} - \frac{1}{n-2}(g^{hm} g_{ij} S_{mk,l} - \delta^h_j S_{ik,l} + \delta^h_k S_{ij,l} - g^{hm} g_{ik} S_{mj,l}). \tag{20}$$

Contracting on h and l in (20), we find

$$H^h_{ijk,h} = R^h_{ijk,h} - \frac{1}{n-2}(g_{ij} S^h_{k,h} - S_{ik,j} + S_{ij,k} + g_{ik} S^h_{j,h}). \tag{21}$$

By using (18) and (19), (21) reduces to

$$H^h_{ijk,h} = \frac{n-3}{n-2}(S_{ij,k} - S_{ik,j}) - \frac{1}{2(n-2)}(g_{ij} r_{,k} - g_{ik} r_{,j}). \tag{22}$$

If we assume that the conharmonic curvature tensor of this manifold is divergence-free then we find from (22)

$$(n-3)(S_{ij,k}-S_{ik,j})-\frac{1}{2}(g_{ij}r_{,k}-g_{ik}r_{,j})=0. \tag{23}$$

Multiplying (23) by g^{ij} and putting (19) in (23), we obtain

$$r_{,k}=0.$$

Thus, we get finally that the scalar curvature of this manifold r is constant. This completes the proof. □

Theorem 4. *A $(PCHS)_n$ admits divergence-free conharmonic curvature tensor is of zero scalar curvature tensor.*

Proof. Assuming that the conharmonic curvature tensor of $(PCHS)_n$ is divergence-free, from (7), we get

$$2A_h H^h_{ijk}+A^h H_{hijk}-A_j\overline{H}_{ik}+A_k\overline{H}_{ij}=0. \tag{24}$$

Multiplying (24) by g^{ij} we obtain

$$2A^h\overline{H}_{hk}=-hA_k. \tag{25}$$

It follows from (6) and (25)

$$rA_k=0. \tag{26}$$

Since A_k is not zero for $(PCHS)_n$, we get r must be zero. The proof is completed. □

Theorem 5. *If $(PCHS)_n$ is recurrent then either the scalar curvature of this manifold is zero or the recurrence vector field and the associated 1-form are related by*

$$\lambda_l=\frac{2(n+2)}{n}A_l.$$

Proof. By taking the covariant derivative of (6), we get

$$H_{hijk,l}=R_{hijk,l}-\frac{1}{n-2}(g_{ij}S_{hk,l}-g_{hj}S_{ik,l}+g_{hk}S_{ij,l}-g_{ik}S_{hj,l}). \tag{27}$$

Comparing (27) with (7), using (6) and assuming our manifold is recurrent, i.e, we have that

$$R_{hijk,l}=\lambda_l R_{hijk} \tag{28}$$

and

$$S_{ij,l}=\lambda_l S_{ij}, \quad r_{,l}=\lambda_l r \tag{29}$$

are satisfied. Then from (6), (7), (28) and (29), we obtain

$$\begin{aligned}(\lambda_l-2A_l)R_{hijk}-A_hR_{lijk}-A_iR_{hljk}-A_jR_{hilk}+A_kR_{hijl}\\ +\frac{1}{n-2}[S_{hk}(2A_lg_{ij}+A_ig_{lj}+A_jg_{il}-\lambda_lg_{ij})\end{aligned}$$

$$
\begin{aligned}
& - S_{ik}(2A_l g_{hj} + A_h g_{lj} + A_j g_{hl} - \lambda_l g_{hj}) \\
& + S_{ij}(2A_l g_{hk} + A_h g_{lk} + A_k g_{hl} - \lambda_l g_{hk}) \\
& - S_{hj}(2A_l g_{ik} + A_i g_{lk} + A_k g_{il} - \lambda_l g_{ik}) \\
& + S_{lk}(A_h g_{ij} - A_i g_{hj}) - S_{lj}(A_h g_{ik} - A_i g_{hk}) \\
& + S_{il}(A_j g_{hk} - A_k g_{hj}) - S_{hl}(A_j g_{ik} - A_k g_{ij})] \\
=& 0 \qquad (30)
\end{aligned}
$$

Multiplying (30) by g^{hk} and g^{ij}, we get

$$r(-n(\lambda_l - 2A_l) + 4A_l) = 0. \qquad (31)$$

It follows from (31) that either r is zero or $\lambda_l = \frac{2(n+2)}{n} A_l$. Thus, this completes the proof. □

3. An Example of $(PCHS)_n$

In this section we will give an example for $(PCHS)_n$ satisfying the conditions (6) and (7). We define a Riemannian metric on $\mathbb{R}^n$ ($n \geq 4$) by the formula,[18]

$$ds^2 = \varphi(dx^1)^2 + k_{\alpha\beta} dx^\alpha dx^\beta + 2dx^1 dx^n \qquad (32)$$

where $[k_{\alpha\beta}]$ is a symmetric and non-singular matrix consisting of constant and φ is a function of $x^1, x^2, \ldots, x^{n-1}$ and independent of x^n. Let each Latin index runs over $1, 2, \ldots, n$ and each Greek index runs over $2, 3, \ldots, (n-1)$.
In the metric considered, the only non-vanishing components of Christoffel symbols, the curvature tensor and the Ricci tensor are, according to [18]

$$
\begin{gathered}
\Gamma^{\beta}_{11} = -\frac{1}{2} k^{\alpha\beta} \varphi_{,\alpha}, \quad \Gamma^{n}_{11} = \frac{1}{2}\varphi_{,1} \quad \Gamma^{n}_{1\alpha} = \frac{1}{2}\varphi_{,\alpha} \\
R_{1\alpha\beta 1} = \frac{1}{2}\varphi_{,\alpha\beta}, \quad S_{11} = \frac{1}{2} k^{\alpha\beta}\varphi_{,\alpha\beta} \qquad (33)
\end{gathered}
$$

where "," denotes the partial differentiation with respect to the coordinates and $k^{\alpha\beta}$ are the elements of the matrix inverse to $[k_{\alpha\beta}]$.
We consider $k_{\alpha\beta}$ as the kronecker symbol $\delta_{\alpha\beta}$ and φ as, [8]

$$\varphi = (M_{\alpha\beta} + \delta_{\alpha\beta}) x^\alpha x^\beta e^{(x^1)^2} \qquad (34)$$

where $M_{\alpha\beta}$ are constant and satisfy the relations

$$
\begin{gathered}
M_{\alpha\beta} = 0 \text{ for } \alpha \neq \beta \\
M_{\alpha\beta} \neq 0 \text{ for } \alpha = \beta \\
\sum_{\alpha=2}^{n-1} M_{\alpha\alpha} = 0. \qquad (35)
\end{gathered}
$$

In this case, we have the following relations

$$\begin{aligned} \varphi_{,\alpha\beta} =& 2(M_{\alpha\beta} + \delta_{\alpha\beta})e^{(x^1)^2} \\ \delta_{\alpha\beta}\delta^{\alpha\beta} =& n-2 \\ \delta^{\alpha\beta}M_{\alpha\beta} =& \Sigma M_{\alpha\alpha} = 0. \end{aligned} \tag{36}$$

Thus, from (34) and (36), we have

$$\delta^{\alpha\beta}\varphi_{,\alpha\beta} = 2(n-2)e^{(x^1)^2}. \tag{37}$$

By using (33), we find the only non-zero components for R_{hijk} and S_{ij} as

$$\begin{aligned} R_{1\alpha\alpha 1} =& \frac{1}{2}\varphi_{,\alpha\alpha} = (1 + M_{\alpha\alpha})e^{(x^1)^2} \\ S_{11} =& \frac{1}{2}\varphi_{,\alpha\beta}\delta^{\alpha\beta} = (n-2)e^{(x^1)^2}. \end{aligned} \tag{38}$$

Hence, the only non-zero components of the conharmonic curvature tensor H_{hijk} are

$$\begin{aligned} H_{1\alpha\alpha 1} =& R_{1\alpha\alpha 1} - \frac{1}{n-2}(g_{\alpha\alpha}S_{11}) \\ =& (1 + M_{\alpha\alpha})e^{(x^1)^2} - \frac{1}{n-2}(n-2)e^{(x^1)^2} \\ =& M_{\alpha\alpha}e^{(x^1)^2} \end{aligned} \tag{39}$$

which never vanish. In this case, from (39), the only non-zero components of the derivative of H_{hijk} are found as

$$\begin{aligned} H_{1\alpha\alpha 1,1} =& 2x^1 M_{\alpha\alpha}e^{(x^1)^2} \\ =& 2x^1 H_{1\alpha\alpha 1}. \end{aligned} \tag{40}$$

Let us consider the associated 1-form as

$$A_i(x) = \begin{cases} \frac{x^1}{2}, & \text{for } i = 1 \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

at any point $x \in \mathbb{R}^n$.
To verify the relation (7) it is sufficient to prove that the equation

$$H_{1\alpha\alpha 1,1} = 4A_1 H_{1\alpha\alpha 1}. \tag{42}$$

By the aid of (40) and (41), we can easily see that (42) is satisfied. The other components of each term of (7) vanish identically and the relation (7) holds trivially.
Under our assumptions (32), (34) and (35), this manifold is a $(PCHS)_n$.

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References

[1] D B Abdussatter. On conharmonic transformations in general relativity. *Bulletin of Calcutta Mathematical Society*, 41:409–416, 1966.

[2] T Adati and T Miyazawa. On a riemannian space with recurrent conformal curvature. *The Tensor Society. Tensor. New Series*, 18:348–354, 1967.

[3] E Cartan. Surune classe remarquable d' espaces de riemannian. *Bulletin de la Societe Mathematique de France*, 54:214–264, 1926.

[4] M C Chaki. On pseudo symmetric manifolds. *Analele Stiintifice ale Universitatii Al. I. Cuza din Iasi*, 33:53–58, 1987.

[5] M C Chaki and B Gupta. On conformally symmetric spaces. *Indian Journal of Mathematics*, 5:113–295, 1963.

[6] U C De and S Bandyopadhyay. On weakly symmetric riemannian spaces. *Publicationes Mathematicae Debrecen*, 54:371–381, 1999.

[7] U C De and S Bandyopadhyay. On weakly symmetric spaces. *Acta Mathematics Hungarica*, 83:205–212, 2000.

[8] U C De and A De. On almost pseudo-conformally symmetric ricci-recurrent manifolds with applications to relativity. *Czechoslovak Mathematical Journal*, 62(137):1055–1072, 2012.

[9] U C De and S Mallick. On almost pseudo concircularly symmetric manifolds. *The Journal of Mathematics and Computer Science*, 4(3):317–330, 2012.

[10] R Deszcz and W Grycak. On some class of warped product manifolds. *Bulletin of the Institute of Mathematics. Academia Sinica*, 15:311–322, 1987.

[11] S K Hui, A A Shaikh, and I Roy. On totaly umbilical hypersurfaces of weakly conharmonically symmetric spaces. *Indian Journal of Pure and Applied Mathematics*, 10(4):28–31, 2010.

[12] K Olszak and Z Olszak. On pseudo-riemannian manifolds with recurrent concircular curvature tensor. *Acta Mathematica Hungar*, 137(1-2):64–71, 2012.

[13] B O'Neill. Semi-riemannian geometry with applications to the relativity. *Academic Press, New York-London*, 1983.

[14] F Özen and S Altay. On weakly and pseudo symmetric riemannian spaces. *Indian Journal of Pure and Applied Mathematics*, 33(10):1477–1488, 2001.

[15] F Özen and S Altay. On weakly and pseudo concircular symmetric structures on a riemannian manifold. *Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica*, 47:129–138, 2008.

[16] M Prvanovic. On weakly symmetric riemannian manifolds. *Publicationes Mathematicae Debrecen*, 46:19–25, 1995.

[17] M Prvanovic. On totally umbilical submanifolds immersed in a weakly symmetric riemannian manifolds. *Publicationes Mathematicae Debrecen*, 6:54–64, 1998.

[18] W Roter. On conformally symmetric ricci-recurrent spaces. *Colloquium Mathematicum*, 31:87–96, 1974.

[19] A A Shaikh and S K Hui. On weakly conharmonically symmetric manifolds. *The Tensor Society. Tensor. New Series*, 70:119–134, 2008.

[20] S A Siddiqui and Z Ahsan. Conharmonic curvature tensor and the spacetime of general relativity. *Differential Geometry–Dynamical Systems*, 12:213–220, 2010.

[21] G Soos. Uber die geodatischen abbildungen von riemannschen raumen auf projektiv symmetrische remannsche raume. *Acta Mathematica Academiae Scientiarum Hungaricae*, 9:359–361, 1958.

[22] Z I Szabo. Structure theorems on riemannian spaces satisfying r(x,y)r=0. *Journal of Differential Geometry*, 17:531–582, 1982.

[23] L Tamassy and T Q Binh. On weakly symmetric and weakly projectively symmetric riemannian manifolds. *Colloquia Mathematica Societatis Janos Bolyai*, 56:663–670, 1989.

[24] A G Walker. On ruse' s space of recurrent curvature. *Proceedings of the London Mathematical Society*, 52:36–64, 1951.

[25] H B Yilmaz. On decomposable almost pseudo conharmonically symmetric manifolds. *Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica*, 51(1):111–124, 2012.