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An Unconventional Splitting for Korteweg de Vries–Burgers Equation

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Abstract. Numerical solutions of the Korteweg de Vries–Burgers (KdVB) equation based on splitting is studied. We put a real parameter into a KdVB equation and split the equation into two parts. The real parameter that is inserted into the KdVB equation enables us to play with the splitted parts. The real parameter enables to write the each splitted equation as close to the Korteweg de Vries (KdV) equation as we wish and as far from the Burgers equation as we wish or vice a versa. Then we solve the splitted parts numerically and compose the solutions to obtained the integrator for the KdVB equation. Finally we present some numerical experiments for the solution of the KdV, Burger's and KdVB equations. The numerical experiments shows that the new splitting gives feasible and valid results.

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Key Words and Phrases: Korteweg de Vries–Burgers equations, splitting methods, finite differences

1. Introduction

In this paper we consider the Korteweg de Vries–Burgers (KdVB) equation derived by Su and Gardner [17]

$$u_t + \varepsilon u u_x - \nu u_{xx} + \mu u_{xxx} = 0, \quad x \in \Omega, \quad t \in [0, T], \tag{1}$$

with the initial condition

$$u(x, 0) = f(x), \qquad x \in \Omega, \tag{2}$$

and the boundary condition

$$u(x_L, t) = u(x_R, t) = 0, \qquad t \in [0, T] \tag{3}$$

where $\Omega = [x_L, x_R]$, ε, ν and μ are positive parameters with $\varepsilon\nu\mu \neq 0$ and the subscripts t and x denote the differentiation with respect to time and space. It is one of the important mathematical models which has many applications in science and engineering such as fluid

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dynamics, plasma physics, nonlinear circuit theory and astrophysics (see, for example [18] and reference therein). It is also a model equation for wave propagation in fluid-filled elastic or viscoelastic tubes [3, 13].

The equation (1) is interesting because of several reason. First of all, it contains the nonlinearity (uu_x), dissipation (u_{xx}) and dispersion (u_{xxx}) terms. In particular, a progressive wave solution of the KdVB equation (1) by use of hyperbolic tangent method have been presented in [4]. When the dissipative parameter ν is varied, the solution is altered from an oscillatory profile to a monotonic one [7] and the solution vectors end up behaving like traveling waves for which the amplitudes are damped [21]. Also, as ν varied shock wave becomes more oscillatory [21]. Moreover, the choices $\varepsilon \neq 0$, $\nu \neq 0$ and $\mu = 0$ reduces the evolution equation (1) to the Burgers' equation

$$u_t + \varepsilon u u_x - \nu u_{xx} = 0, \tag{4}$$

and the choices $\varepsilon \neq 0$, $\mu \neq 0$ and $\nu = 0$ lead the equation (1) to the Korteweg de Vries (KdV) equation

$$u_t + \varepsilon u u_x + \mu u_{xxx} = 0, \tag{5}$$

which has at least the following three invariants

$$I_1 = \int_{\infty}^{\infty} u \, \mathrm{d}x, \quad I_1 = \int_{\infty}^{\infty} u^2 \, \mathrm{d}x, \quad I_3 = \int_{\infty}^{\infty} \left(u^3 - \frac{3\mu}{\varepsilon} \left(u'\right)^2 \right) \mathrm{d}x \tag{6}$$

corresponding to conservations of mass, momentum and energy respectively. It is well known that the Burgers' equation (4) and the KdV equation (5) are exactly solvable and have traveling wave solutions of the form

$$u(x,t) = \frac{2k}{\varepsilon} + \frac{2\nu k}{\varepsilon} \tanh[k(x - 2kt)] \tag{7}$$

and

$$u(x,t) = \frac{2\mu k^2}{\varepsilon} \operatorname{sech}^2\left[k(x - 4\mu k^2 t)\right] \tag{8}$$

respectively, where $k = \nu/(10\mu)$. The KdVB equation (1) has an exact solution [19]

$$u(x,t) = -\frac{3\nu^2}{25\varepsilon\mu}\left(-\operatorname{sech}^2(k(x - x_0) - ct) + 2\tanh(k(x - x_0) - ct) + 2\right) \tag{9}$$

where $c = 3\nu^3/(125\mu^2)$. Since most of the nonlinear partial differential equations (PDEs) that contain dissipation and/or dispersion does not have exact solution, developing some new techniques for the numerical solution of these types of nonlinear PDEs are essential to understand solution behavior. For the equation (1) many works have been done analytically and numerically. In particular, a progressive wave solution of the equation KdVB equation by use of hyperbolic tangent method(1) have been presented in [4]. By introducing a new potential function and by using the hyperbolic tangent method and an exponential rational function approach, a traveling wave solution to the KdVB equation (1) has been obtained in [5]. An exact solution to the equation (1) is presented in [6] by using the first-integral method. The numerical investigation of the problem has been carried out by many authors. In paricular, the numerical solution of the KdVB equation has been studied by using a radial basis functions (RBFs) collocation (Kansa) method in [9] . The collocation method with quintic B-spline finite element has been used in [21] to simulate the solutions of the equation (1). In [2] KdVB equation is solved numericaly by means of spectral collocation method. Variational iteration method has been implemented in [16] to solve the KdVB equation. In [10], a finite differences with variable mesh and the semi-analytic Adomian decomposition method are used to solve KdVB equation. In [19] the KdVB equations is solved by using the local discontinuous Galerkin method.

Splitting methods are used frequently to recapture the dynamics of different parts of differential equations and applying appropriate numerical integrators for each part (see [11, 15] and references therein). In [1] a KdVB type equation with fast and slow dynamics is solved by using operator splitting. The KdVB equation can be splitted as KdV equation and the Burgers's equation or vice a versa. However, since the KdVB equation is a nonlinear diffusive dispersive equation, the numerical solution of this equation should represent all qualitative behavior of both the KdV equation and the Burger's equation. In this paper, to capture this feature of the KdVB equation a new kind of splitting is introduced. For this we put a real parameter into a KdV-Burgers equation and split the vector field of the equations into two parts both of which are KdVB equation. The real parameter that is inserted into the KdVB equation enables us to play with the splitted parts. By varying the real parameter, we are able to write the KdVB equation as close to the KdV equation as we wish and as far from the Burgers equation as we wish or vice a versa. We solve then the splitted parts numerically and compose the solutions to obtained the integrator for the KdVB equation. The rest of the paper is organized as follows: In Section 2, we described the proposed unconventional splitting for the KdVB equation. The numerical method for the proposed splitting is presented in Section 3. In Section 4 we tested the performance of the splitting in KdV simulation, Burgers's simulation and KdV-Burgers simulation. Finally the conclusion is given in Section 5.

2. An Unconventional Splitting for the KdVB Equation

Numerical solution of differential equations by using splitting methods has been frequently used in the literature especially after introducing higher-order composition formulae [15, 20]. The main idea of the composition method is to split the vector field associated with the differential equation into sum of two or more pieces that can be solved exactly or integrated easily than the original equation. In this section we will presents a new splitting to integrate the KdVB equation (1) numerically. The KdVB equation can be splitted as KdV equation

$$u_t + \frac{1}{2}\varepsilon u u_x + \mu u_{xxx} = 0 \tag{10}$$

and the Burgers's equation

$$u_t + \frac{1}{2}\varepsilon u u_x - \nu u_{xx} = 0 \tag{11}$$

or vice a versa. However, since the KdVB equation (1) is a nonlinear diffusive dispersive equation, the numerical solution of this equation should represent all qualitative behavior of both the KdV equation and the Burger's equation. To capture this feature we rewrite the equation (1) as

$$u_t + \frac{1}{2}\varepsilon u u_x + \frac{1}{2}\varepsilon u u_x - \nu u_{xx} + \mu u_{xxx} = 0,$$

and add then subtract the terms $\nu\alpha u_{xx}$ and $\mu\beta u_{xxx}$ to obtain

$$u_t + \frac{1}{2}\varepsilon u u_x + \frac{1}{2}\varepsilon u u_x - \nu u_{xx} + \mu u_{xxx} - \nu(\alpha - \alpha)u_{xx} + \mu(\beta - \beta)u_{xxx} = 0 \tag{12}$$

where $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ are real parameters. Then the equation (12) is rewritten as

$$u_t + \frac{1}{2}\varepsilon u u_x + \frac{1}{2}\varepsilon u u_x - \nu(1-\alpha)u_{xx} - \nu\alpha u_{xx} + \mu(1-\beta)u_{xxx} + \mu\beta u_{xxx} = 0. \tag{13}$$

Note that the equation (13) is reduced to the KdVB equation (1) for the choices of $\alpha = \beta = 0$ and $\alpha = \beta = 1$.

Now the new splitting method for the KdVB equation (13) is defined by solving first

$$u_t + \frac{1}{2}\varepsilon u u_x - \nu(1-\alpha)u_{xx} + \mu\beta u_{xxx} = 0, \tag{14}$$

followed by

$$u_t + \frac{1}{2}\varepsilon u u_x - \nu\alpha u_{xx} + \mu(1-\beta)u_{xxx} = 0. \tag{15}$$

Note that the equations (14) and (15) are both KdVB equations for $0 < \alpha, \beta < 1$. With this point of view the splitting (14) and (15) are different from the splittings (10) and (11).

The equations (14) and (15) are reduced to the Burgers' equation and the KdV equation for $\alpha = \beta = 0$ respectively. In addition, the choices $\alpha = \beta = 1$ reverse the situation that is the equation (14) becomes the KdV equation and (15) becomes the Burgers' equation. Therefore, the choices $\alpha = \beta = 0$ or $\alpha = \beta = 1$ reduces the splitting (14) and (15) to the splittings (10) and (11).

3. Numerical Method

In order to obtain numerical solutions of the KdVB equation (13), the space interval $[x_L, x_R]$ is discretized by the uniform N grid

$$x_i = x_L + ih, \qquad i = 1, 2, \ldots N$$

where the grid spacing h is given by $h = (x_L - x_R)/N$. The solution is assumed to be negligible outside the interval $[x_L, x_R]$. Let $U_i(t)$ denotes the approximate solution to the exact solution $u(x_i, t)$. We discretize the space variable of the splitted equation (14) and (15) using the central difference approximation and get the semi–discrete systems

$$\frac{d}{dt}U_i + \frac{\varepsilon}{2}U_i\left(\frac{U_{i+1} - U_{i-1}}{2h}\right) - \nu(1-\alpha)\left(\frac{U_{i+1} - 2U_i + U_{i+1}}{h^2}\right)$$

$$+\mu\beta\left(\frac{U_{i+2}-2U_{i+1}+2U_{i-1}-U_{i-2}}{2h^3}\right)=0 \tag{16}$$

$$\frac{d}{dt}U_i+\frac{\varepsilon}{2}U_i\left(\frac{U_{i+1}-U_{i-1}}{2h}\right)-\nu\alpha\left(\frac{U_{i+1}-2U_i+U_{i+1}}{h^2}\right)$$
$$+\mu(1-\beta)\left(\frac{U_{i+2}-2U_{i+1}+2U_{i-1}-U_{i-2}}{2h^3}\right)=0 \tag{17}$$

where $i=1,2,\ldots N$ and

$$U_{-2}=U_{-1}=0, \qquad U_{N+1}=U_{N+2}=0.$$

The systems (16) and (17) can be written as

$$\frac{d}{dt}U_i=F_1(U_i) \qquad \frac{d}{dt}U_i=F_2(U_i) \tag{18}$$

where

$$F_1(U_i)=aA(U_i)U_i+b_1BU_i+c_1CU_i \tag{19}$$
$$F_2(U_i)=aA(U_i)U_i+b_2BU_i+c_2CU_i \tag{20}$$

respectively with

$$a=-\frac{\varepsilon}{4h}, \qquad b_1=\frac{\nu(1-\alpha)}{h^2}, \qquad c_1=-\frac{\mu\beta}{2h^3},$$
$$b_2=\frac{\nu\alpha}{h^2}, \qquad c_2=-\frac{\mu(1-\beta)}{2h^3}$$

and

$$A=\begin{pmatrix} U_2 & 0 & \cdots & 0 & 0 \\ 0 & U_3-U_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -U_N-U_{N-2} & 0 \\ 0 & 0 & \cdots & 0 & -U_{N-1} \end{pmatrix}_{N\times N},$$

$$B=\begin{pmatrix} -2 & 1 & 0 & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & 0 & 1 & -2 \end{pmatrix}_{N\times N}$$

$$C=\begin{pmatrix} 0 & -2 & 1 & 0 & 0 & & & & \\ 2 & 0 & -2 & 1 & 0 & & & & \\ -1 & 2 & 0 & -2 & 1 & & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ & & & & -1 & 2 & 0 & -2 & 1 \\ & & & & 0 & -1 & 2 & 0 & -2 \\ & & & & 0 & 0 & -1 & 2 & 0 \end{pmatrix}.$$

Because the right-hand sides of (18) are quadratic, the reflexive method of Kahan [14] can be applied for the time discretization. Application of the reflexive method to the systems of equations in (18) yields

$$\left(I - \frac{\Delta t}{2} J_{F_1}(U^n)\right)\left(U^{n+1} - U^n\right) = \Delta t F_1(U^n), \tag{21}$$

$$\left(I - \frac{\Delta t}{2} J_{F_2}(U^n)\right)\left(U^{n+1} - U^n\right) = \Delta t F_2(U^n) \tag{22}$$

respectively, where J_{F_1} and J_{F_2} denote the Jacobians of the right-hand sides of (21) and (22). Denoting the solution U^{n+1} of (21) and (22) by $\widetilde{S1}$ and $\widetilde{S2}$ respectively, the numerical solution for the KdVB equation (13) is obtained by the time reversible second–order Strang splitting [8]

$$\mathscr{S}(t) = \exp\left(\widetilde{S1}\left(\frac{\Delta t}{2}\right)\right)\exp\left(\widetilde{S2}(\Delta t)\right)\exp\left(\widetilde{S1}\left(\frac{\Delta t}{2}\right)\right). \tag{23}$$

4. Numerical Experiment

In this section, some numerical example will be demonstrated to examine the robustness and accuracy of the proposed splitting (14)-(15) by using (23). The discrete values of (1) will be computed via the (23). The accuracy of the method (23) is measured by calculating the L_2 and L_∞ error norms of the solution defined by

$$L_2 = \|U^n - u^n\|_2 = \left(h\sum_{i=1}^{N}\left|U(x_i,t_n) - u(x_i,t_n)\right|^2\right)^{1/2}$$

$$L_\infty = \|U^n - u^n\|_\infty = \max_i \left|U(x_i,t_n) - u(x_i,t_n)\right|, \quad i = 1,2,\ldots,N.$$

4.1. KdV Simulation

To examine the performance of the proposed scheme for the KdV equation, we set the parameter $\nu = 0$ in (1). The performance of the splittings (14)-(15) with $\alpha = \beta = 0.5$ by using (23) will be check for two types of initial conditions namely a soliton solution and a Maxwellian initial condition. Moreover, the conservation of the numerical scheme will be examined by looking the invariants (6) where the integrals are approximated by trapezoidal rule.

Experiment A

The KdV equation has an analytic solution of the form

$$u(x,t) = 3C\text{sech}^2(Ax - Bt + D) \tag{24}$$

where $A = \frac{1}{2}\sqrt{\varepsilon C\mu^{-1}}$ and $B = \varepsilon AC$. Initial condition

$$u(x,0) = 3C\text{sech}^2(Ax + D) \tag{25}$$

is obtained from the exact solution (24). Boundary conditions are taken as $u(0,t) = 0$ and $u(2,t) = 0$. We take $\varepsilon = 1.0$, $\mu = 4.84 \times 10^{-4}$, $C = 0.3$, $D = -6.0$. For these parameters the exact values of the invariants I_1, I_2 and I_3 in (6) are 0.144597866741365, 0.0867592530989926 and 0.0468494613399944 respectively. To see the conservation of the invariants of the proposed scheme, we performed a numerical experiment with the time length $\Delta t = 0.005$ and the space length $\Delta x = h = 0.001$ up to time $T = 3.0$ in the region $x \in [0,2]$. Moreover, the L_2-errors and the L_∞-errors are presented to see the accuracy of the proposed scheme. The results are shown in Table 1. We see that the scheme (23) preserves the invariants $I_i, i = 1,2,3$. Moreover, L_2 and L_∞ are satisfactorily small.

Table 1: Errors in Conservation of Mass, Momentum and Energy and L_2 and L_∞ Errors.

Time	I_1	I_2	I_3	L_2	L_∞
0.0	1.1e-8	2.5e-13	-2.8e-6	0.0	0.0
1.0	-7.6e-7	-3.7e-10	-2.8e-6	7.7e-5	2.2e-4
2.0	-1.9e-6	-3.3e-10	-2.8e-6	1.5e-4	4.3e-4
3.0	-2.2e-6	-3.8e-10	-2.8e-6	2.3e-4	6.3e-4

To see whether the proposed numerical scheme exhibits the expected convergence rates in space, we performed some further numerical experiment for various values of h and a fixed value of Δt. The rate of convergence is calculated by

$$\text{rate of convergence} \approx \frac{\ln(E(h_2)/E(h_1))}{\ln(h_2/h_1)}$$

where $E(h)$ is the L_2 or L_∞ error. We take $\Delta t = 5 \times 10^{-4}$ to minimize the temporal errors. The results corresponding to the scheme (23) are shown in Table 2 for an decreasing space length. We present the L_2-errors and the L_∞-errors for the terminating time $t = 1$ together with the observed rates of convergence in each case. The computed convergence rates agree well with the expected rates when the second order central difference approximation is used for discretization in space direction.

Table 2: Rate of Convergence.

h	L_2	Order	L_∞	Order
0.1	0.417248	-	1.060321	-
0.02	0.023961	1.77	0.068023	1.70
0.01	0.005802	2.04	0.016483	2.04
0.005	0.001436	2.01	0.004183	1.98
0.0025	0.000358	2.00	0.001048	1.99

Experiment B

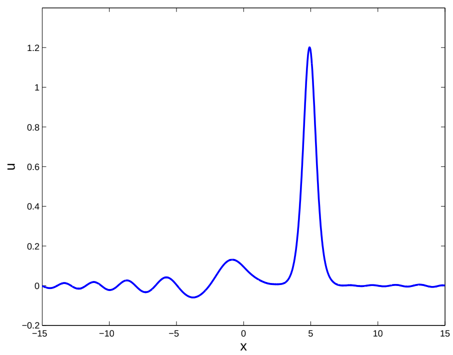
Now we consider the KdV equation with the Maxwellian initial condition

$$u(x,0) = \exp(-x^2), \qquad -15 < x < 15 \tag{26}$$

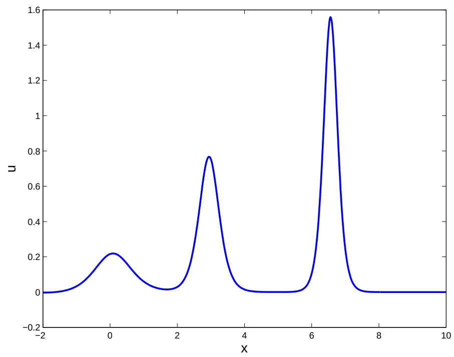
subject to the boundary conditions

$$u(-15,t) = u(15,0) = 0, \qquad t > 0. \tag{27}$$

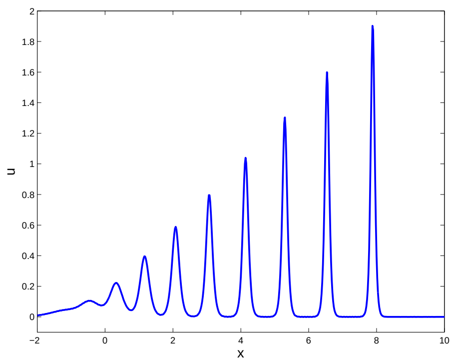
We take $\varepsilon = 1.0$ and consider for μ the set of parameters $\mu = 0.04, 0.01, 0.001, 0.0005$ and run the simulation up to time $t = 12$ with $\Delta t = 0.03$, $h = 0.02$. The results are shown in Figure 1. It is seen from the figure that Maxwellian initial condition exhibits rapidly oscillating wave packets. For example, for $\mu = 0.04$ one solitary wave with an oscillating tail is observed, while for $\mu = 0.0005$ twelve solitons are formed. This is an agreement with the earlier works [21] and [19]. The conservation of the invariants for $\mu = 0.001$ and $\mu = 0.0005$ are presented in Table 3.



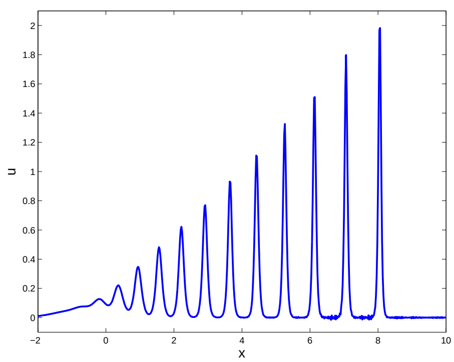
(a) $\mu = 0.04$



(b) $\mu = 0.01$



(c) $\mu = 0.001$



(d) $\mu = 0.0005$

Figure 1: KdV simulation with Maxwellian initial condition at $t = 12$ for different values of μ. (a) One soliton plus oscillating tail with $\mu = 0.04$. (b) Three solitons with $\mu = 0.01$. (c) Nine solitons with $\mu = 0.001$. (d) Twelve solitons with $\mu = 0.0005$

Table 3: Errors in Conservation of Mass, Momentum and Energy.

Time	$\mu = 0.001$			$\mu = 5 \times 10^{-4}$		
	I_1	I_2	I_3	I_1	I_2	I_3
3.0	1.7e-12	-1.3e-2	-4.3e-2	3.0e-15	-2.7e-2	-9.3e-2
6.0	1.7e-6	-1.6e-2	-5.1e-2	-8.7e-8	-3.4e-2	-1.1e-1
9.0	-9.7e-8	-1.6e-2	-5.1e-2	-2.4e-5	-3.5e-2	-1.1e-1
12.0	7.0e-6	-1.6e-2	-5.1e-2	-3.5e-5	-3.5e-2	-1.1e-1

4.2. Burgers' Simulation

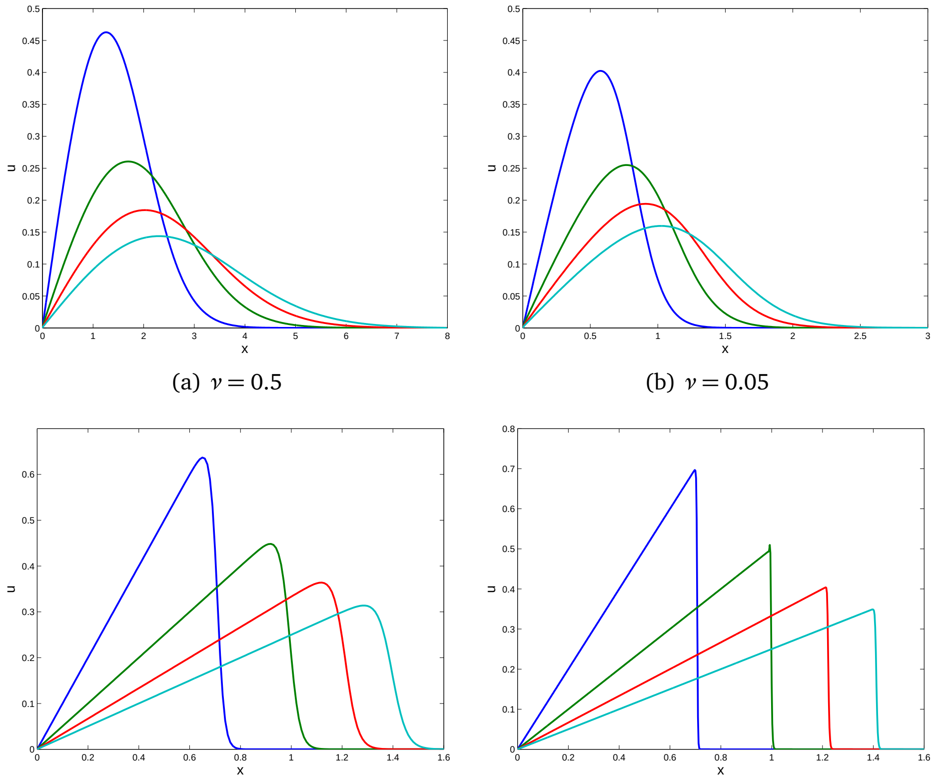
Now we set $\mu = 0$ and examine the performance of the proposed splittings (14)-(15) by using the composition (23) for the Burgers' equation. Burgers's equation has an analytic solution of the form [12]

$$u(x,t) = \frac{x/t}{1 + (t/t_0)\exp(x^2/4\nu t)} \tag{28}$$

where $t_0 = \exp(1/8\nu)$. Initial condition is obtained by evaluating the Eq. (28) at $t = 1$. The homogenous boundary conditions $u(x_L, t) = u(x_R, t) = 0$, $(t \geq 1)$ are used. We set $\varepsilon = 1.0$, $1 \leq t \leq 5$, $\Delta t = 0.02$ and $h = 0.01$ for all simulations. On the interval $0 \leq x \leq 8$ we used $h = 0.01$ $\nu = 0.5, 0.05, 5 \times 10^{-3}$ and on the interval $0 \leq x \leq 2$ we used $h = 0.0025$ for $\mu = 5 \times 10^{-4}$. The Figure 2 shows the development of the solution for different values of ν for the KdVB equations (14) and (15) with $\alpha = \beta = 0.5$ by using (23). In the Figure 2, the top curve shows the solution at $t = 1$ while the bottom curve shows the solution at $t = 4.0$. It can be seen that the amplitudes of the solutions decrease in time. Moreover, decreasing the viscosity parameter results in shock waves. All the results reported for different values of ν are in good agreement with the earlier work of [21] and [19]. The error norms are recorded in Table 4.

Table 4: Errors Norms for Burgers' Type Solution.

Time	$\nu = 0.5$		$\nu = 0.05$		$\nu = 0.005$		$\nu = 5 \times 10^{-4}$	
	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	8.2e-2	6.0e-2	2.9e-2	4.1e-2	9.6e-3	4.0e-2	3.9e-3	6.2e-2
3.0	9.8e-2	6.5e-2	4.0e-2	5.0e-2	1.4e-2	5.3e-2	4.3e-3	5.2e-2
4.0	9.8e-2	6.2e-2	4.5e-2	5.1e-2	1.6e-2	5.8e-2	5.1e-3	5.6e-2
5.0	9.8e-2	5.8e-2	4.8e-2	5.1e-2	1.8e-2	6.0e-2	5.6e-3	5.8e-2



(a) $\nu = 0.5$ (b) $\nu = 0.05$

(c) $\nu = 5 \times 10^{-3}$ (d) $\nu = 5 \times 10^{-4}$

Figure 2: Burgers's Type Solutions at $t = 1, 2, 3, 4$ (from top to bottom).

4.3. The KdV–Burger Type Solutions

Now we consider the new splitting (14) and (15) with $\alpha = 0.2$ and $\beta = 0.8$. We recall that for $\alpha = \beta = 0$ the equation (14) reduces to Burgers' equation and the equation (15) reduces to the KdV equation. Therefore the composition (23) corresponds to Burger-KdV-Burger composition for $\alpha = \beta = 0$ which will be abbreviated by BKB simulation. However for $\alpha = \beta = 1$ the equation (14) reduces to the KdV equation and the equation (15) reduces to the Burgers' equations. Therefore the composition (23) corresponds to KdV-Burger-KdV compositon for $\alpha = \beta = 1$ which will be abbreviated by KBK simulation. For $0 < \alpha, \beta < 1$, both $\widetilde{S1}$ and $\widetilde{S2}$ are KdVB equation; therefore the composition (23) will be abbreviated by KdVB simulation for which we choose $\alpha = 0.2$ and $\beta = 0.8$.

Experiment A

Now we consider the KdVB equation (1) with $\epsilon = 1.0$. We pick the initial and boundary conditions from the exact solution (9)

$$u(x,0) = -\frac{3\nu^2}{25\varepsilon\mu}\left(-\operatorname{sech}^2(kx) + 2\tanh(kx) + 2\right). \tag{29}$$

We present the numerical results over the spatial interval $0 \leq x \leq 100$ and temporal interval $0 \leq t \leq 100$ with $\Delta x = 0.2$ and $\Delta t = 0.1$.

Table 5 represents the absolute errors define by

$$\text{Absolute Error } = |u(x_i,t) - U(x_i,t)|, \quad i = 0, 25, 50, 75, 100$$

for various space values at time $t = 100$. From the table we see that the three kinds of splitting gives approximately the same remarkable accuracy. These results are in good agrement with the earlier work of [16].

Table 5: Absolute Errors at $t = 100$

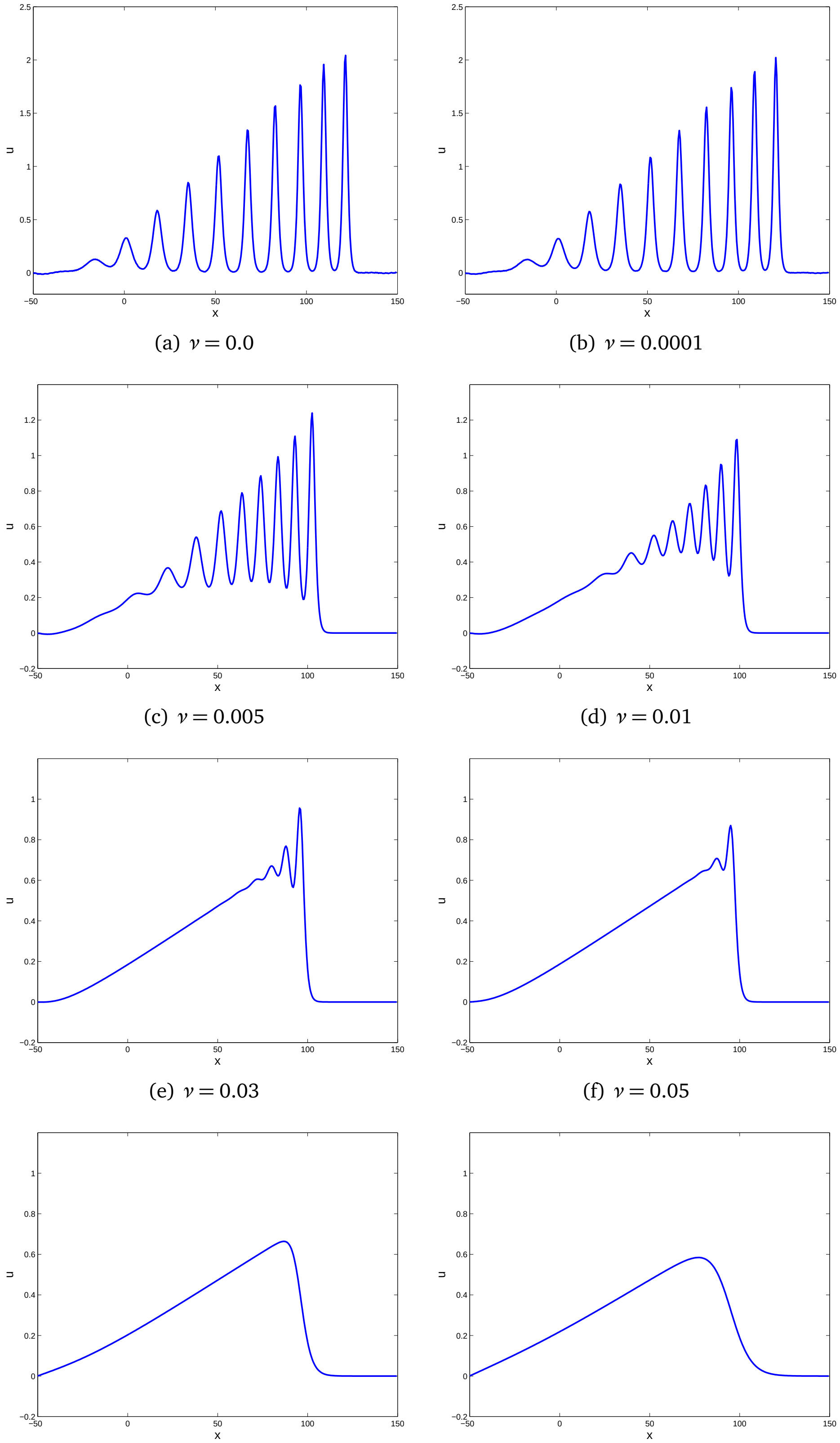
	x	KdVB $\alpha = 0.1,\ \beta = 0.8$	KBK $\alpha = \beta = 0$	BKB $\alpha = \beta = 1$
$\nu = \mu = 0.1$	0.0	1.21×10^{-6}	1.22×10^{-6}	1.21×10^{-6}
	25.0	6.63×10^{-8}	6.63×10^{-8}	6.63×10^{-8}
	50.0	4.60×10^{-10}	4.60×10^{-10}	4.60×10^{-10}
	75.0	2.90×10^{-12}	2.89×10^{-12}	3.00×10^{-12}
	100.0	9.40×10^{-10}	9.33×10^{-10}	9.37×10^{-10}
$\nu = \mu = 0.01$	0.0	1.46×10^{-8}	1.47×10^{-8}	1.46×10^{-8}
	25.0	1.02×10^{-9}	1.02×10^{-9}	1.02×10^{-9}
	50.0	7.08×10^{-12}	7.08×10^{-12}	7.10×10^{-12}
	75.0	4.77×10^{-14}	7.77×10^{-14}	4.77×10^{-14}
	100.0	5.45×10^{-15}	4.83×10^{-15}	5.36×10^{-15}

Experiment B

In the last experiment we compute the numerical solution of Eq.(1) by using the scheme (23) with the initial condition

$$u(x,0) = 0.5\left[1 - \tanh\frac{|x| - 25}{5}\right]$$

and boundary conditions $u(-50,t) = u(150,t) = 0$. We choose $\varepsilon = 0.2$, $\mu = 0.1$, $\Delta t = 0.4$, $h = 0.5$ and study the effect of increasing the viscosity and hence the dispersion term on the solution. For this we run the scheme (23) for different values of ν. The results for shown in Fig. 3. From the figure when we see that increase the viscosity ν, the solution of the KdVB equation (1) behaves like the solution of the Burger's equation (4) as in the work of Zaki [21].



(a) $\nu = 0.0$ (b) $\nu = 0.0001$

(c) $\nu = 0.005$ (d) $\nu = 0.01$

(e) $\nu = 0.03$ (f) $\nu = 0.05$

(g) $\nu = 0.2$ (h) $\nu = 0.4$

Figure 3: KdV–Burgers's Solutions at $t = 800$ with $\varepsilon = 0.2$, $\mu = 0.1$.

5. Conclusion

Since most of the nonlinear PDEs that contain dissipation and/or dispersion does not have exact solution, developing some new techniques for the numerical solution of these types of nonlinear PDEs are essential to understand solution behavior. In this paper, we considered a numerical solution of the KdVB equation based on the splitting. The KdVB equation can be splitted as KdV equation and the Burgers's equation or vice a versa. However, since the KdVB equation is a nonlinear diffusive dispersive equation, the numerical solution of this equation should represent all qualitative behavior of both the KdV equation and the Burger's equation. To capture this feature a new kind of splitting, which is different from the splitting in the literature, is introduced for KdVB equation. For this we introduce a real parameters α and β for the KdVB equation and then split the equation into two parts where both splitted equations are KdVB equation. The real parameters enable us to play with the splitted equations in the sense that each splitted equation can be made arbitrarily close to the KdV equation or the Burger's equation. From the above tables and figures we conclude that the numerical results based on the new splitting are feasible and valid. We see that the numerical results are not sensitive for the added parameters namely playing with the real parameters α and β does not chance the numerical results. The splitting method we introduce in this paper is less restrictive and comprises the numerical solutions that exist in the literature. This splitting approach can be extended to solve other types of equations such as nonlinear Klein-Gordon-Maxwell equation and the Klein-Gordon-Schrödinger equation. It is an open question that whether playing with such an added parameters chance the numerical results or not.

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References

[1] A Aydin and B Karasözen. Operator Splitting of the KdV-Burgers Type Equation with Fast and Slow Dynamics. In A Kilicman C Ozel, editor, *International Conference On Mathematical Science.*, pages 562–566, Bolu, 2010. AIP Conference Proceedings(1309).

[2] MT Darvishia, F Khanib, and S Kheybaria. A Numerical Solution of the KdV-Burgers-Equation by Spectral Collocation Method and Darvishi-s Preconditionings. *International Journal of Contemporary Mathematical Sciences*, 2:1085–1095, 2007.

[3] H Demiray. Nonlinear waves in a thick walled viscoelastic tube filled with an inviscid fluid. *International Journal of Engineering Science*, 36:359–362, 1998.

[4] H Demiray. A note on the exact travelling wave solution to the KdV–Burgers equation. *Wave Motion*, 38:367–369, 2003.

[5] H Demiray. A travelling wave solution of the KdV–Burgers equation. *Applied Mathematics and Computation*, 154:665–670, 2004.

[6] Z Feng. The first–integral method to study the Burgers-Korteweg–de Vries equation. *Journal of Physics A: Mathematical and General*, 35:343–349, 2002.

[7] Z Feng. On travelling wave solutions of the burgers-korteweg-de vries equation. *Nonlinearity*, 20:343–356, 2007.

[8] E Hairere, C Lubich, and G Wanner. *Geometric Numerical Integration:Structure–Preserving Algorithms for Ordinary Differential Equations.*, volume 31. Springer, 2002.

[9] S Haq, S Islam, and M uddin. A mesh-free method for the numerical solution of the kdv-burger equation. *Applied Mathematical Modelling*, 33:3442–3449, 2009.

[10] MA Helal and MS Mehanna. A comparison between two different methods for solving KdV-Burgers equation. *Chaos Solitons and Fractals*, 28:320–326, 2006.

[11] H Holden, KH Karlsen, KA Lie, and NH Risebro. *Splitting Methods for Partial Differential Equations with Rough Solutions: Analysis and Matlab Programs.* EMS Series of Lectures in Mathematics, European Mathematical Society Publishing House, Zürich, 2010.

[12] E Hopf. The partial differential equation $u_t + uu_x = \mu u_{xx}$. *Communications on Pure and Applied Mathematics*, 3:201–230, 1950.

[13] H Jonson. Nonlinear equation incorporating damping and dispersion. *Journal of Fluid Mechanics*, 42:49–60, 1970.

[14] W Kahan and R Li. Unconventional schemes for a class of ordinary differential equations with applications to the Korteweg-de Vries Equation. *Journal of Computational Physics*, 134:316–331, 1997.

[15] RI McLachlan and RGW Quispel. Splitting methods. *Acta Numerica*, 11:341–434, 2002.

[16] AA Soliman. A numerical simulation and explicit solutions of KdV–Burgers' and Lax's seventh–order KdV equations. *Chaos Solitons and Fractals*, 29:294–302, 2006.

[17] CH Su and CS Gardner. Derivation of the Korteweg de–Vries and Burgers' equation. *Journal of Mathematical Physics*, 10:536–539, 1969.

[18] GB Whitham. *Linear and Nonlinear Waves.* Wiley, New York, 1974.

[19] Y Xu and CW Shu. Local discontinuous Galerkin methods for three classes of nonlinear wave equations. *Journal of Computational Mathematics*, 22:250–274, 2004.

[20] H Yoshida. Construction of higher-order symplectic integrators. *Physics Letters A*, 150:262–269, 1990.

[21] SI Zaki. A quintic B–spline finite elements scheme for the KdVB equation. *Computer Methods in Applied Mechanics and Engineering*, 188:121–134, 2000.