EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 7, No. 4, 2014, 437-441
ISSN 1307-5543 – www.ejpam.com

Some Remarks on Semi Open Sets with Respect to an Ideal

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Abstract. In this article we introduce the notions of weakly semi open sets with respect to an ideal, characterize them and find some properties

2010 Mathematics Subject Classifications: 54C10

Key Words and Phrases: weakly semi open set with respect to an ideal, semi open set with respect to an ideal, semi open set

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1. Introduction

Currently many mathematicians have worked in generalized mathematical notions, see [1, 2, 4], also provided some characterizations of these notions, this is the case of Friday Ifeanyi Michael K. [1] in the article “On semi open sets with respect to an ideal” in which it generalizes the notion of semi open sets defined by Norman Levine in [3]. In this article the author define the notion of I-semi open set as follows: let X be a topological space and I an ideal on X, $A \subseteq X$ is said to be I-semi open set if there exists an open set U such that $U \setminus A \in I$ and $A \setminus cl(U) \in I$. In this article the following properties were proved:
Proposition 5. Let I be an ideal on a topological space X, where every subset of X is dense and the collection of open subsets of X satisfies the finite intersection property:

1. - If A is I-semi open and $A \subseteq B$, then B is I-semi open.

2. - If A is I- semi open, then so is $A \cup B$ for any subset B of X.

3. - If both A and B are I-semi open, then so is $A \cap B$.

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Proposition 6. Under the condition of Proposition 5, we have that A is I-semi open if and only if $cl(A)$ is I-semi open.
If we analyze with more detail, the Proposition 6, observe the following example.

Example 1. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a, c\}, \{a, b, c\}\}$ and $I = \{\emptyset\}$. Notice that the collection of nonempty subsets of X satisfies the finite intersection property and every nonempty open subset of X is dense. Now consider $A = \{b, c\}$, it is easy to see that $cl(A) = X$ is I-semi open but A is not I-semi open.*

This example shows that the Proposition 6 given in [1] is not necessarily true. Using this fact, our interest is to find some weaker condition of semi open set with respect to an ideal in order to prove that: Let $I \neq \emptyset$ be an ideal, $A \subseteq X$ satisfies the weaker condition if and only if $cl(A)$ satisfies the weaker condition.

2. Weakly Semi Open Sets with Respect to an Ideal

Let X be a topological space. Recall that $A \subset X$ is a semi open set [3], if there exists an open set U such that $U \subset A \subset cl(U)$.

Definition 1. *A subset A of X is said to be weakly semi open set with respect to an ideal I (denoted by weakly I-semi open) if $A = \emptyset$ or if $A \neq \emptyset$ there exists an open set $U \neq \emptyset$ such that $U \setminus A \in I$.*

Observe that

(i) for any ideal I any semi open set is weakly I-semi open.

(ii) For any ideal I, if $A \subseteq X$ is I-semi open then A is weakly I-semi open. Observe that if $A \in I$ not necessarily A is weakly I-semi open.

(iii) There exists weakly I-semi open sets that are neither semi open nor I-semi open.

Example 2. *Let $X = \{a, b, c\}$, with topology $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $I = \{\emptyset\}$. The set $A = \{a, b\}$ is weakly I-semi open but is neither semi open nor I-semi open.*

Now we characterize the weakly I-semi open sets.

Theorem 1. *Let $A \neq \emptyset$ a subset of X and I an ideal. A is weakly I-semi open if and only if there exists an open set U and $C \in I$ such that $(U \setminus C) \subset A$*

Proof. Suppose that $A \neq \emptyset$ is weakly I-semi open, then there exists an open set $U \neq \emptyset$ such that $U \setminus A \in I$. Take $C = U \setminus A = U \cap (X \setminus A)$. Then $U \setminus C \subset A$. Reciprocally suppose that there exists an open set U and $C \in I$ such that $(U \setminus C) \subset A$, then $(U \setminus A) \subset C$, follows that $U \setminus A \in I$. □

Definition 2. *A subset A of X is said to be weakly I-semi closed if $X \setminus A$ is weakly I-semi open.*

Theorem 2. *Let (X, τ) be a topological space, I an ideal and $A \subseteq X$. If A is weakly I-semi closed then $A \subset (K \cup B)$ for some closed subset K of X and $B \in I$.*

Proof. If A is weakly I-semi closed, then $X \setminus A$ is weakly I-semi open. If $X \setminus A = \emptyset$, then $A = X$, in consequence, the $\emptyset$ is weakly I-semi closed. If $X \setminus A \neq \emptyset$, then there exists an open set U and $B \in I$ such that $(U \setminus B) \subset (X \setminus A)$, follows that $A \subset X \setminus (U \setminus B) = X \subset (U \cap (X \setminus B)) = (X \setminus U) \cap B$. Take $K = (X \subset U)$ then $A \subset K \cup B$. □

The converse of the above Theorem is not necessarily true, as we see in the following example.

Example 3. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Take $I = \{\emptyset\}$ and $A = \{a, c\}$. If $K = X$ and $B = \emptyset$, $A \subset K \cup B$ but A is not weakly I-semi closed, because $X \setminus A$ is not weakly I-semi open.*

Theorem 3. *The arbitrary union of weakly I-semi open sets is weakly I-semi open.*

Proof. Let $\{A_\alpha\}_{\alpha \in J}$ be a collection of weakly I- semi open sets, then for each A_α with $\alpha \in J$, there exists U_α, $\alpha \in J$ such that $U_\alpha \setminus A_\alpha \in I$. Now if we take a fixed α' in J then $U'_\alpha \setminus \bigcup_{\alpha \in J} A_\alpha \subset U'_\alpha \setminus A'_\alpha \in I$. In consequence, $\bigcup_{\alpha \in J} A_\alpha$ is weakly I-semi open. □

The intersection of weakly I-semi open sets is not necessarily weakly I-semi open as we can see in the following example.

Example 4. *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\emptyset\}$. Consider $A = \{a, b\}$ and $B = \{b, c\}$. It is easy to see that A and B are weakly I-semi open sets but $A \cap B = \{b\}$ is not weakly I-semi open.*

Remark 1. *We denote by $SO_I(X, \tau)$ as the family of all weakly I-semi open sets in the topological space X, then $SO_I(X, \tau)$ is a minimal structure that satisfies the Maki condition [4].*

From Definition 1, we obtain that, if $\emptyset \neq A \subset B$ and A is weakly I-semi open, then B is also weakly I-semi open in consequence we have the following corollary.

Corollary 1. *If A is weakly I-semi open, then so is $A \cup B$, for any subset B of X, in particular $cl(A)$ is weakly I-semi open.*

The converse of the above Corollary is not necessarily true as we see in the following example.

Example 5. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}\}$ and I any ideal such that $\{b\} \notin I$. Take $A = \{a\}$, $cl(A) = \{a, b\}$ is weakly I-semi open but A is not weakly I- semi open.*

The following theorem give to us a sufficient condition in order to obtain that $SO_I(X, \tau) = P(X)$.

Theorem 4. *Let (X, τ) be a topological space and I an ideal such that there exist an unitary set that belongs to the topology and the ideal, then $SO_I(X, \tau) = P(X)$.*

Proof. Suppose that the unitary set $\{a\} \in I$. Let $\{b\}$ any unitary set in X, then $\{b\} \in SO_I(X,\tau)$, because $\{a\} \setminus \{b\} \in I$. Now using Theorem 3, we obtain that any subset A of X belongs to $SO_I(X,\tau)$. □

At this point we want to determinate under what conditions, if $A \subseteq X$ is $cl(A)$ weakly I-semi open set then A is weakly I-semi open.

Observe the following facts:

(i) If $cl(A) = X$ then A is not necessarily weakly I-semi open.

(ii) If there exists $A \subset X$, such that $cl(A)$ is a clopen set then A is not necessarily weakly I-semi open.

Example 6. *Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}\}$.*

(i) If we take $I = \{\emptyset\}$ and $A = \{b, d\}$, $cl(A) = X$ is weakly I-semi open but A is not.

(ii) If we take $I = \{\emptyset, \{c\}\}$ and $A = \{a\}$, $cl(A) = \{a, b\}$ is weakly I-semi open but A is not.

Theorem 5. *Let X be a topological space and I an ideal such that the collection of open sets satisfies the finite intersection property, if A and B are weakly I-semi open, then so is $A \cap B$.*

Proof. Since A and B are weakly I-semi open sets, there exist open sets U, V such that $U \setminus A \in I$ and $V \setminus B \in I$, therefore, $(U \cap V) \setminus (A \cap B) = (U \setminus A) \cap V \cup U \cap (V \setminus B) \in I$.

Remark 2. *The following theorem characterizes the subsets $A \subseteq X$ such that the $cl(A)$ is weakly I- semi open under some conditions of the ideal and the collections of open sets of X.*

Theorem 6. *Let X be a topological space, $I \neq \emptyset$ an ideal on X and the collection of open subsets of X satisfies the finite intersection property. If $A \subset X$ such that $cl(A) \neq X$. $cl(A)$ is weakly I-semi open if and only if A is weakly I-semi open.*

Proof. If A is weakly I-semi open, then $cl(A)$ is weakly I-semi open by Corollary 1. Conversely, suppose that $cl(A)$ is weakly I-semi open, then $cl(A) = \emptyset$ or $cl(A) \neq \emptyset$. If $cl(A) = \emptyset$, then $A \in SO_I(X,\tau)$. If $cl(A) \neq \emptyset$, there exists an open set $U \neq \emptyset$ such that $U \setminus cl(A) \in I$, take the open set $V = U \setminus cl(A)$. Using the hypothesis $V \neq \emptyset$ and $V \in I$. Observe that $V \setminus A = (U \setminus cl(A)) \setminus A = U \setminus cl(A) \in I$. In consequence, A is weakly I-semi open. □

Remark 3. *Observe that if in the Theorem 6:*

(i) $I \neq \emptyset$ and $cl(A) \neq X$ are omitted, then the result may be false, (see Example 1).

(ii) If we change $cl(A) \neq X$ by $cl(A) = X$, the result may be false. If in the Example 1, $I = \{\emptyset, \{c\}\}$ and $A = \{a, d\}$, then $cl(A)$ is weakly I-semi open but A is not weakly I-semi open.

(iii) The case $I = \emptyset$ and $cl(A) \neq X$ never happens.

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