



Modeling, Simulation and Performance Analysis of a Flexible Production System

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Abstract. This paper deals with a flexible production system modeled by re-entrant queueing network; a system decomposed into two fundamental multi-productive stations and three classes, a part follows the route fixed by the system, where each one is processed first by station 1 for the first step, then by station 2 for the second step, and again by the first station for third and last step before leaving the system. We assume that there is an infinite supply of work available, so that there are always parts ready for processing step 1, and that the first station gives preemptive priority to buffer 3. Several performance measures have been used to evaluate the system performances considering two scenarios; high priority with service conservation and high priority with loss of parts. So, performances due to varying its parameters are investigated through expanded Monte Carlo simulations.

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Key Words and Phrases: Queueing models; flexible production systems; modeling; simulation; stability.

1. Introduction

Flexible production systems have emerged as one of the revolutions in the production industries in recent years. It has made it possible to produce a vast variety of parts in less time and cost.

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These systems are generally consisting in a number of machine tools, robots, material handling, automated storage and retrieval system, and computers or workstations. A typical flexible production system can fully process the members of one or more part families on a continuing basis without human intervention.

Over the last few decades, the modeling and the analysis of flexible production systems has been meticulously studied by control theorists and engineers.

The case study in the present paper mainly consists of modeling, simulation and analysis of a flexible production system.

Modeling and simulation of flexible production systems is a field of research for many people now days. However, they all share a common aim; to search for solutions to attain higher speeds and more flexibility and thus maximize/multiply manufacturing productivity.

Computer simulation is a great numeric modeling technique for the analysis of flexible manufacturing/product systems. Brucoleri *et al.* [5] suggested the simulation as a tool for defining the configuration of an flexible manufacturing system. Shnits *et al.* [29] used simulation of operating system as a decision support tool for controlling the flexible system to exploit flexibility. Tüysüz and Kahraman [30] presented an approach for modeling and analysis of time critical, dynamic and complex systems using stochastic Petri nets together with fuzzy sets.

Many other authors used Petri net approach to analyze and present the Performance evaluation of complex manufacturing systems, for simultaneously modeling and scheduling manufacturing systems for instance Liu *et al.* [22], Huang *et al.* [19], Delgado and Llano [14].

In this research work, flexible production system is modeled by a reentrant queueing system to analyze its performance measures. In addition, we compare and verify the results obtained from the simulation techniques and theoretical results given for this type of systems.

Stability and performance analysis of multi-class queueing networks is by now a well-researched field. Some preminent papers which have set the accent of this research field in the past 25 years are many, some of them are [7, 8, 18, 20]. Some notorious contributions with respect to stability analysis can be summarized in [3, 11, 15, 25, 28], additional contributions are summarized in [4, 10, 24].

Let us note that the re-entrant lines (described in Harrison [18]) are a special case of queueing systems that can be used to model complex manufacturing system such as wafer fabrication facilities. Kumar [20] defined these systems called standard re-entrant lines for convenience.

In our paper, we consider a re-entrant model consisting of two stations with infinite supply of work at the first one. Different from standard re-entrant line, In an infinite re-entrant line we assume that there are always infinitely many class 1 jobs available, which guarantees that the station serving class 1 will be always busy under non-idling service discipline. We call such a class infinite virtual queue. Infinite supply of work expresses an ability to control the arrivals and is often a reasonable way to model a processing system. In some situations there may indeed be an infinite supply of work; in production systems, the supply of parts for processing at an expensive machine may be controlled and not allowed to exhaust. We refer to this as an infinite virtual queue: it acts like an infinite queue while in fact it only contains a few jobs which are continually replenished. In standard queueing networks one can regard the input

stream as the output of a server which is fed by an infinite supply of work.

Infinite supply of work and infinite virtual queues are discussed in [1, 2, 17, 27, 31, 32]. A succinct study of these results is given in [26].

The layout of our paper is as follow; after an introduction, a mathematical model of the production system is described in Section 2. Section 3 presents some theoretical results given for such model, Section 4 is devoted to the simulation approach study. In Section 5 a general conclusion is offered.

2. Mathematical Model

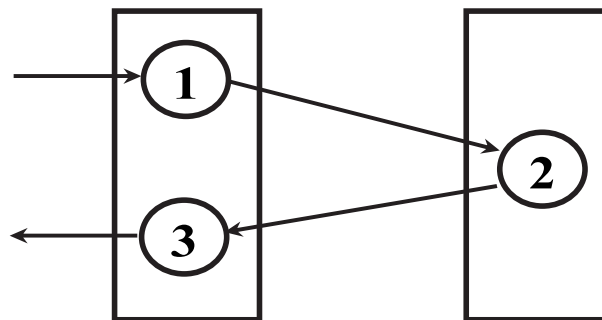


Figure 1: A Reentrant Network with Two Stations and Three Classes

The production system considered in our study is schematically shown in Figure 1. This process can be seen as a reentrant queueing network, consisting of two stations and three classes. Parts arrive according to a Poisson process with rate λ . When the part arrives at the first station, if it finds some ones before it, it waits in the queue 1, otherwise it goes directly to the service to be served with rate μ_1 , then aligns the second line requiring a second service with rate μ_2 . Finally, the part returns to the first station for the third and final service before leaving the system. Indeed, there are three types of parts, parts of class C_1 processed by the first station for the first service, parts of class C_2 processed by the second station for the second service and parts of the class C_3 processed by the first station for the third and final service.

We assume that there is an infinite supply of work available, so that there are always parts ready for processing step 1. In that case station 1 will always be busy. Each class is processed in FIFO order. Processing is non-idling, that is a station will always process a part when there is work. The present research work consider two scenarios.

The first one is as follows: station 1 gives preemptive priority to class 3; whenever there exist some parts in class 3, station 1 will work on the first of them; when class 3 empties, station 1 will instantaneously resume processing of a part in step 1. This is possible by the supposition that there is an infinite supply of work. Class 1 is called a virtual infinite queue. The queue is virtual because in practice class 1 need not contain many parts, but it needs to be controlled so it will never be empty. If during the processing of class 1 a part arrives from class

2 into class 3, station 1 will preempt its work at class 1, and immediately starts processing class 3.

In the second scenario, each part which arrives to the system, is lost if it is not at the head of the queue, it will be also lost if it is interrupted by the arriving parts from station 2 to class 3.

3. Theoretical Study

Queueing systems are excessively useful class models that have found application in many areas including communication networks, computer systems, manufacturing... Much of this success is maybe due to the constitutional tractability of a large class of queueing systems. However, the question of stability of many queueing systems and their performance evaluation are extremely exchanging issues. Stability analysis and performance evaluation of queueing systems have received a great deal of attention, this is partly due to several examples that demonstrate that the usual conditions; traffic intensity less than one at each station ($\rho_i < 1$) are not sufficient for stability, even under the well known FIFO service discipline (Dai and Vande Vate [12]). Rybko and Stolyar [28] and Lu and Kumar [23] demonstrated that in a deterministic version the usual conditions do not guarantee the stability of many queueing networks. Dai [11] provided a unified approach via fluid limit model to prove positive Harris recurrence of a network with two stations and N classes with feedback. Thereafter, Dai and Weiss [13] gave sufficient conditions for the re-entrant line to be stable (global stability) whatever the discipline of service. Chen and Zhang [9] have established sufficient conditions for the stability of multi-class queueing networks under FIFO service discipline. Weiss [31] studied a reentrant network with two stations and three classes with an infinite supply of work under LFBS service discipline (Last Buffer First served), where arrivals are not random and the service times are exponential with mean m_i for each class i .

The stability conditions associated with our production system are those removed from the model studied by Weiss [31]. Indeed, the purpose of his work revolves around the following question: Under what conditions on the system parameters, the Markov process reaches the steady state ?

So, we assume that there is an infinite supply of work available, we analyze this system under the LBFS policy: station 1 gives priority to parts in class 3 over parts in class 1, we assume that this priority is preemptive; whenever a part arrives in class 3, station 1 will preempt the part in class 1 and start processing the part in class 3, and it will resume work on the part in class 1 only when class 3 is empty.

Let note that in general, it is well know that if parts arrive at this system in a renewal stream, at rate λ , then under the condition $\rho_1 = \lambda(m_1 + m_3) < 1$, and $\lambda m_2 < 1$ the queues of parts waiting for each step are stable, and in fact the system is positive Harris recurrent, for any work conserving policy, readers are referred to Kumar and Kumar [21] and Dai and Weiss [13], for more details.

If however the arrival rate a is high enough to equal the bottleneck processing rate, i.e., $\max\{\rho_1, \rho_2\} = 1$, then the system is weakly stable but not stable: The departure rate from the queues is equal to λ , but as time increases, the queue length at some of the classes will converge

weakly to infinity. Consequently such a system cannot work at a rate $\max\{\rho_1, \rho_2\} = 1$, without accumulating unbounded queues.

Weiss [31], established a sufficient condition for the stability for such system. So if $\lambda\left(\frac{1}{\mu_1} + \frac{1}{\mu_3}\right) > \alpha\frac{1}{\mu_2}$ then under LBFS policy $\rho_1 = 1$, but the queues for steps 2 and 3 will be stable, and the system will be positive recurrent.

Theorem 1 ([31]). *If*

$$\lambda\left(\frac{1}{\mu_1} + \frac{1}{\mu_3}\right) > \alpha\frac{1}{\mu_2}, \quad (1)$$

then the system is recurrent positive.

Adan and Weiss [1] kept considering the case $\lambda\left(\frac{1}{\mu_1} + \frac{1}{\mu_3}\right) > \alpha\frac{1}{\mu_2}$; station 1 works all the time and the system is weakly stable, then by solving the balance equations of such system a stationary distribution network is obtained.

Guo and Zhang [17] obtained sufficient and necessary condition for the stability for such network.

Guo [16] using a fluid model approach, obtained a sufficient condition for the model considered. In addition, the author got necessary conditions for the corresponding fluid model to be weakly stable.

Proposition 1. [16] *If*

$$\rho_2 > 1, \quad (2)$$

the the fluid model is weakly stable. Thus the reentrant network is unstable.

Yoni Nazarathy and Gideon Weiss [27] considered the same model given in Weiss [31], and proposed a method for the control over a finite time horizon, the model was approximated by a fluid network and formulated a fluid optimization problem. The optimal fluid solution partitions the time horizon to intervals in which constant fluid flow rates were maintained. Then, they used a policy by which the queueing network tracks the fluid solution. To that end, they modeled the deviations between the queueing and the fluid network in each of the intervals by a multi-class queueing network with some infinite virtual queues. These deviations were kept stable by an adaptation of a maximum pressure policy.

Now, let us note that from previous results, one leads to the following consequence.

Consequence 1. *Consider the network presented in Figure 1, if*

$$\rho_1 = \lambda\left(\frac{1}{\mu_1} + \frac{1}{\mu_3}\right) < 1, \quad (3)$$

$$\rho_2 = \frac{\lambda}{\mu_2} < 1, \quad (4)$$

$$\rho_1 > \rho_2, \quad (5)$$

then the network is stable.

4. Simulation Study Approach

Performance analysis of any system is one of the most studied topics in Operations Research. In practice, we are often interested in numbers of customers in the system and their waiting time.

However, the expression of these performances depends heavily on how customers arrive to the system, the manner on which customers request services and service disciplines. To control them, we are led to the problem by modeling these systems via queueing models.

So, a queueing network is a set of interconnected queues, at which circulate one or more classes of customers. Two important parameters (generally stochastic) determine the behavior of the network (open or closed network) over the time:

- The inter-arrival times of customers.
- The service times.

The main objective of this work is to model the production line, to analyze, to evaluate and improve its performance using computer simulation techniques. Finally, conclusions are drawn from the analysis made and then recommendations are given based on those concluded points.

Therefore, it is believed that the work will add some value to the existing knowledge.

Analysis and evaluation of a production system usually uses performance indicators capable of assessing the adequacy of the model used with respect to the real system. We first start by specifying performance measures which we consider interesting to study:

- Mean number of customers in the overall system N and in each sub-system N_i , $i = \overline{1,3}$.
- Mean number of customers waiting in the overall system Q and in each sub-system Q_i , $i = \overline{1,3}$.
- Total load of the overall system ρ_C and the load of each sub-system ρ_{C_i} , $i = \overline{1,3}$.

Our main goal is to evaluate these performance measures, via Monte Carlo simulation technique for the network described in Section 2 considering two scenarios:

Scenario 1: Station 1 gives preemptive priority to C_3 , when this latter empties, station 1 will instantaneously resume processing of a part in step 1.

Scenario 2: Station 1 gives preemptive priority to C_3 , and customer of class 1 is lost if it is not at the head of the queue, it will be also lost if it is interrupted by the arriving customer from station 2 to C_3 .

To this end, we develop a simulation algorithm under MATLAB at discrete event. The latter allows us to have two types of results:

Numerical results: Performance measures for the analysis of results.

Graphical results: Representation of the stationary probabilities of the states of the overall system and subsystems, depending on its parameters. The abscissa axis represents the number of customers k in the system (respectively, k_i in the i^{th} sub-system, $i = \overline{1,3}$) and the ordinate axis represents the stationary probability π_k having k parts in the system (respectively, $\pi_{k_i}^{(i)}$ in the i^{th} sub-system), where $k = \sum_{i=1}^3 k_i$.

Subsequently, we analyze the influence of parameters of the considered systems (Scenario 1 and Scenario 2), by varying the arrival and service rates λ (resp. $\mu_i, i = \overline{1,3}$), after that, we determine some values of parameters $\rho_j, j = 1,2$ which are in terms of λ and μ_i , to check whether the stability conditions of such systems described by the inequalities (3), (4) and (5) are fulfilled. Firstly, we fix the service rate ($\mu_i, j = \overline{1,3}$) and let varying the arrival rate λ . Secondly, the procedure is taken inversely (we fix λ and let varying $\mu_i, i = \overline{1,3}$), so as to obtain the different states of the network (stable, unstable, weakly stable).

5. First scenario: Results and Discussion

In this part, we suppose that when the third class is empty $n_3 = 0$, parts are processed out of class 1 and n_2 increases until the second station completes the processing of a part out of it class at which time the state is $(n_2 - 1, 1)$, and station 1 switches to class C_3 at that time customers of class C_1 are interrupted, the interrupted customer joins the QRIC queue (queue of capacity 1 reserved for interrupted customer).

While class C_3 is not empty, parts arrive at class C_3 from class C_2 at rate μ_2 and depart out of class C_3 at rate μ_3 , so class C_3 behaves like an $M/M/1$ queue, except that the total number of arrivals into class C_3 cannot exceed μ_2 . Mathematical model associated with this network is shown in Figure 2.

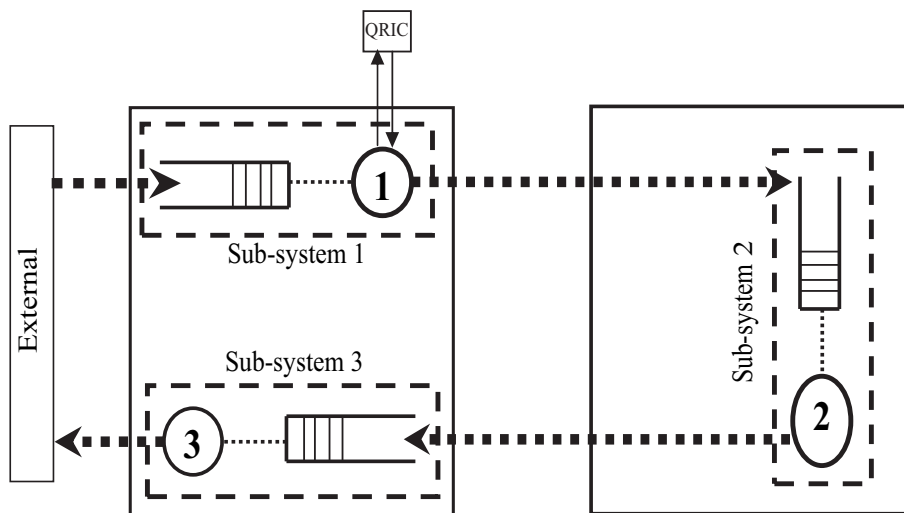


Figure 2: The Mathematical Model (Scenario 1)

5.1. Variation of the Parameter λ

Let $Tmax = 10000$ time units be the simulation duration and $MC = 100$ the number of replications. For $\mu_i = 1/3$, ($i = \overline{1,3}$), we vary λ ($\lambda = [0.10, 0.13, 0.16, 0.17]$). The mean number of customers in the sub-systems, in the overall system and the in the queues as the loads are mainly summarized in the Tables 1, 2 and 3.

Table 1: Variation of N and N_i in Terms of λ

λ	N_1	N_2	N_3	N
0.10	0.95	0.46	0.36	1.78
0.13	2.45	0.73	0.53	3.71
0.16	14.36	1.13	0.73	16.24
0.17	37.34	1.24	0.78	39.37

Table 2: Variation of Q and Q_i in Terms of λ

λ	Q_1	Q_2	Q_3	Q
0.10	0.47	0.16	0.06	0.70
0.13	1.75	0.34	0.14	2.24
0.16	13.43	0.66	0.26	14.36
0.17	36.37	0.75	0.29	37.41

Table 3: Variation of ρ_C and ρ_{C_i} in Terms of λ

λ	ρ_{C_1}	ρ_{C_2}	ρ_{C_3}	ρ_C
0.10	47.50	30.13	29.70	73.53
0.13	69.53	38.94	39.16	88.35
0.16	92.72	47.42	47.63	97.89
0.17	97.83	49.20	49.22	99.44

5.2. Discussion of Results

From Tables 1, 2 and 3, we remark that:

- ✓ The mean number of customers in the system and in the queue of each sub-system increases with respect to λ . The number of customers increases significantly in the sub-system 1 compared to subsystems 2 and 3, which generates a big number of customers in the queue 1 and the overall system. The overall system load increases in terms of λ (loads of subsystems 2 and 3 are nearly equal in all cases).
- ✓ For $\lambda \leq 0.13$, the three subsystems and the overall system have an average load. A decrease in the mean number of customers in each sub-system and in the queue is observed,

which indicates that there is a smooth transit from a sub-system to another, which means that customers are not blocked at any sub-system. In addition, the load of the overall system is $\rho_C \leq 90\%$ (the system is stable, because of the stability conditions given in (1) are verified).

- ✓ For $\lambda \geq 0.16$, the system load is considerable ($\rho_C > 97\%$) which is generated by a high load at the first sub-system ($\rho_{C_1} \geq 92\%$). Moreover, when λ increases (from 0.16 to 0.17), the network moves from stability to instability because of the the first station. While the loads of the two subsystems 2 and 3 are average.
- ✓ A significant difference between the mean number of customers in the 1st sub-system and subsystems 2 and 3, because of blocked customers of class C_1 , caused by the high priority given to the class C_3 and the service rate which is below the arrival rate.

Figures 3 and 4 represent the stationary probabilities of the states of the network for $\lambda = 0.1$ and $\lambda = 0.17$; we conclude that:

- ✓ More λ increases, more the probability that the system is empty decreases (π_0). Unlike the stability case (Figure 3), the probability π_0 is almost negligible in the instability case (Figure 4 for the case $\lambda = 0.17$).
- ✓ The probability that it would have a very large number of customers in the first sub-system, π_{k_1} , is considerable (Figure 4), the station 1 is saturated. Consequently, the global system becomes saturated, this is due to the fact that the first condition of stability is not verified ($\rho_1 = 1.02$).

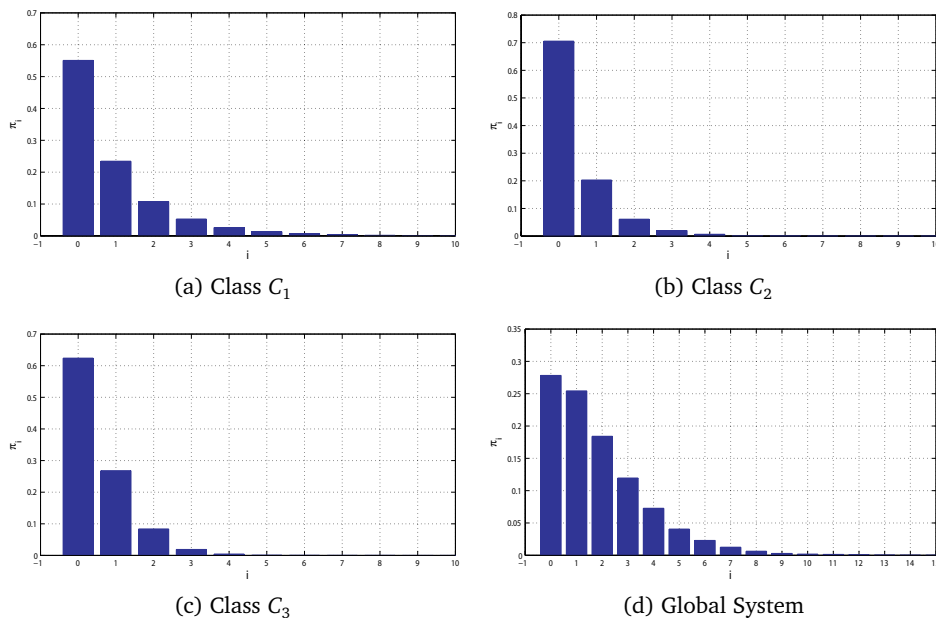


Figure 3: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.10$ and $\mu_i = 1/3$

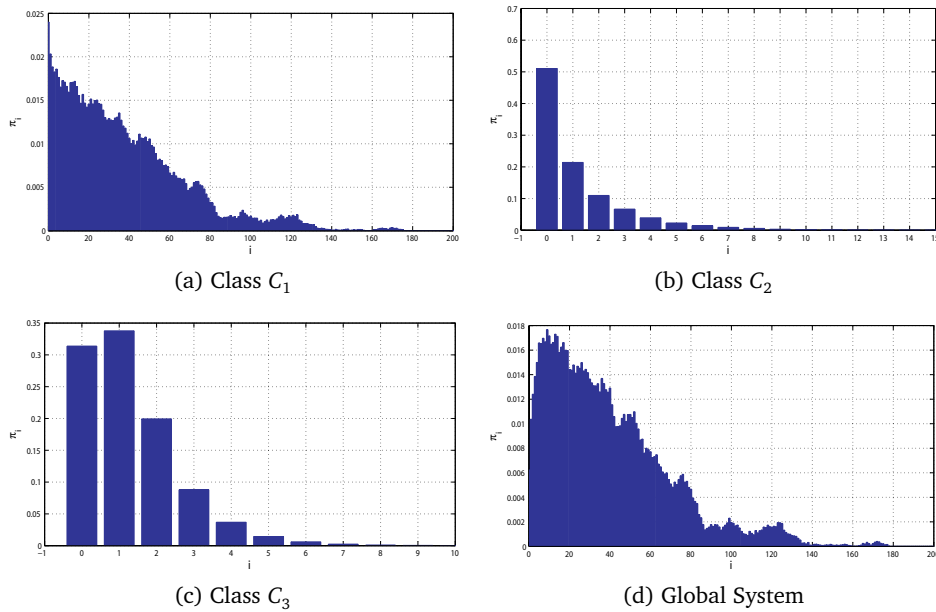


Figure 4: Stationary probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.17$ and $\mu_i = 1/3$

Table 4 summarizes the results carried out previously. Essentially, we can say that the network is unstable if at least one of the necessary conditions ($\rho_1 < 1$ and $\rho_2 < 1$) is unverified. Furthermore, the stability is not achieved if the following three conditions is violated: $\rho_1 < 1$, $\rho_2 < 1$ and $\rho_1 > \rho_2$.

Table 4: Summary, when λ Varies

λ	ρ_1	ρ_2	Ascertainment
0.10	0.60	0.30	Stable
0.13	0.78	0.39	
0.16	0.96	0.48	
0.17	1.02	0.51	unstable

5.3. Variation of Service Rates μ_1 and μ_3

Let varying the service rates μ_1 and μ_3 , and fixe the arrival rate $\lambda = 0.10$ and $\mu_2 = 0.15$. $Tmax = 10000$ time units the period simulation, $MC = 100$ the number of replications. Results provided by our simulator, are summarized in Tables 5, 6 and 7.

Table 5: Variation of N and N_i in Terms of (μ_1, μ_3)

μ_1	μ_3	N_1	N_2	N_3	N
0.75	0.10	61.58	6.25	4.33	72.16
0.70	0.15	3.29	3.44	1.50	8.24
0.65	0.20	1.25	2.40	0.84	4.49
0.60	0.25	0.80	2.24	0.59	3.64
0.55	0.30	0.63	2.18	0.46	3.28
0.50	0.35	0.57	2.09	0.37	3.04
0.45	0.40	0.55	2.10	0.32	2.98
0.44	0.41	0.55	1.97	0.30	2.84
0.40	0.45	0.59	2.07	0.28	2.94
0.35	0.50	0.63	1.97	0.24	2.85
0.30	0.55	0.76	1.96	0.21	2.95
0.25	0.60	1.00	2.00	0.19	3.20
0.20	0.65	1.53	2.05	0.18	3.76
0.15	0.70	3.69	2.00	0.16	5.86

Table 6: Variation of Q and Q_i in Terms of (μ_1, μ_3)

μ_1	μ_3	Q_1	Q_2	Q_3	Q
0.75	0.10	60.60	5.66	3.46	69.74
0.70	0.15	2.61	2.78	0.84	6.25
0.65	0.20	0.77	1.74	0.34	2.86
0.60	0.25	0.40	1.57	0.20	2.18
0.55	0.30	0.28	1.51	0.13	1.92
0.50	0.35	0.22	1.42	0.09	1.75
0.45	0.40	0.21	1.42	0.07	1.71
0.44	0.41	0.21	1.31	0.06	1.59
0.40	0.45	0.22	1.40	0.05	1.69
0.35	0.50	0.25	1.31	0.04	1.61
0.30	0.55	0.33	1.30	0.03	1.68
0.25	0.60	0.50	1.33	0.02	1.87
0.20	0.65	0.93	1.38	0.02	2.34
0.15	0.70	2.91	1.34	0.02	4.27

Table 7: Variation of ρ_C and ρ_{C_i} in Terms of (μ_1, μ_3)

μ_1	μ_3	ρ_{C_1}	ρ_{C_2}	ρ_{C_3}	ρ_C
0.75	0.10	97.52	58.11	86.90	99.82
0.70	0.15	67.31	66.25	66.04	96.88
0.65	0.20	48.17	65.77	49.44	91.74
0.60	0.25	39.60	66.69	39.77	88.53
0.55	0.30	35.64	66.85	33.08	86.24
0.50	0.35	34.27	66.47	28.44	84.55
0.45	0.40	34.57	67.34	25.05	84.44
0.44	0.41	34.44	65.90	24.28	83.44
0.40	0.45	36.22	66.76	22.42	83.71
0.35	0.50	38.19	66.28	19.81	83.53
0.30	0.55	42.90	66.18	18.15	84.32
0.25	0.60	49.74	66.69	16.66	86.27
0.20	0.65	60.41	66.83	15.42	89.07
0.15	0.70	78.45	66.46	14.26	94.08

5.4. Discussion of Results

According to the numerical results stored in Tables 5, 6 and 7, we observe that:

- ✓ For $\mu_1 = 0.75$ and $\mu_3 = 0.10$, the mean number of customers in the first sub-system is very large compared to subsystems 2 and 3. Therefore, the network is unstable, because of ($\rho_1 > 1$).
- ✓ For μ_1 and μ_3 which vary respectively from 0.45 to 0.75 and from 0.10 to 0.40, the mean number of customers in the sub-system 1 decreases. As against, for μ_1 varying from 0.15 to 0.40 and μ_3 from 0.45 to 0.70, the mean number of customers in the sub-system 1 increases. This situation is due because of high priority given to C_3 over C_1 .
- ✓ When the service rate of class C_3 is less than service rate of class C_1 , the interrupted customers of class 1 resume their services within a short time, which justifies the decrease in the number of customers in the 1st sub-system, and the converse is true.
- ✓ For $\mu_1 = 0.70$ and $\mu_3 = 0.15$, the mean number of customers in each subsystems and the loads are distributed equitably. In addition, the arriving customers are served at the end of the simulation time, then the system is stable (all stability conditions are satisfied).
- ✓ For μ_1 varying from 0.65 to 0.20 and μ_3 varying from 0.20 to 0.65, the mean number of customers is high in the 2nd sub-system and in the queue 2, compared to sub-systems 1 and 3. The load in the second station 2 is average ($\rho_{C_2} > 65\%$), the transition to the station 1 is slow and the load in the sub-system 3 is decreasing. Therefore, the system is weakly stable, this is due to the fact that condition (5) is not verified, $\rho_1 < \rho_2$.

✓ For $\mu_1 = 0.15$ and $\mu_3 = 0.70$, the system reaches the steady state as all stability conditions are fulfilled.

Details of some of results given in Tables 5, 6 and 7 are illustrated in Figures 5 and 6. When the service rate of class C_3 is higher than the class C_1 , the probability of having 0 customer in the third sub-system is very high (equal almost to 1), the network is stable because all the conditions are fulfilled

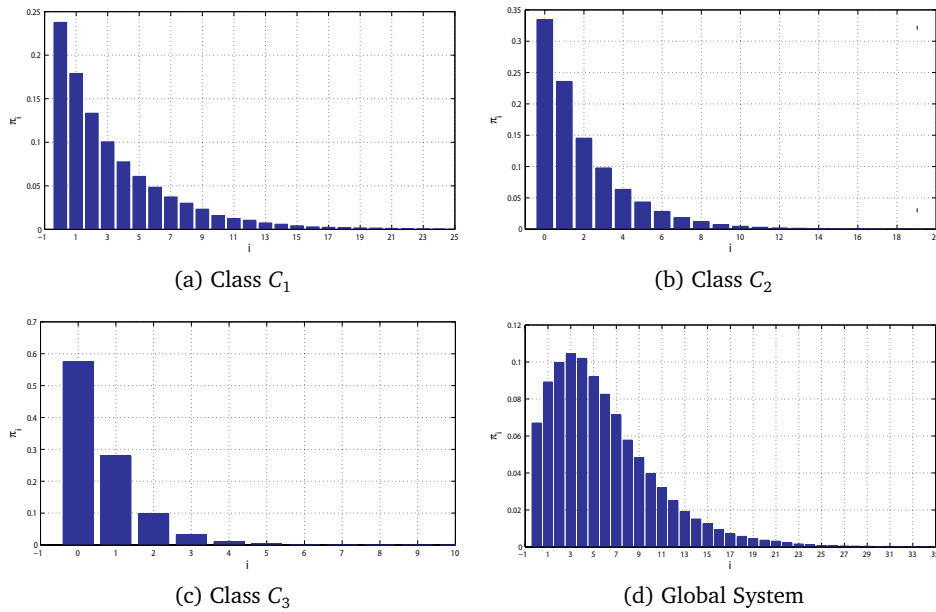


Figure 5: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.10$, $\mu_1 = 0.15$ and $\mu_3 = 0.70$

Figures 5 and 6, look the same. In Figure 6, the probability for having 0 customers in the third class decreases considerably, compared to that given in 5, because the service rate has declined (from 0.7 to 0.2). So the network converges to the steady state, but is not stable ((5) not verified).

Table 8 summarizes the different situations depending on the network parameters. Indeed, it allows us to see that the state of the network is sensitive to the variation of its parameters. Indeed, it moves from a stability state when all the conditions are verified, to instability one as one of the usual conditions is violated ($\rho_1 > 1$ or $\rho_2 > 1$), through an intermediate state (weakly stable network), if $\rho_1 < \rho_2$, which confirm the theoretical results.

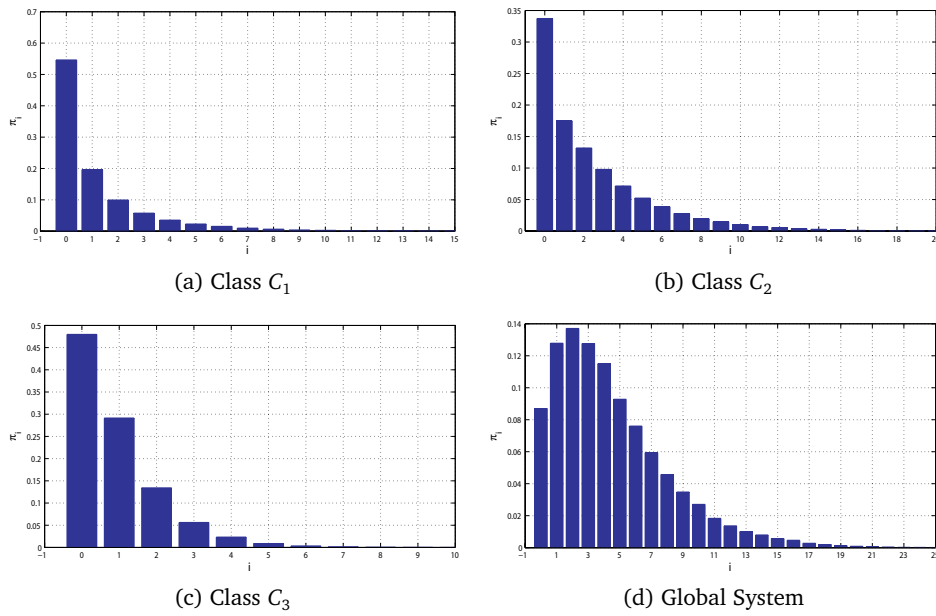


Figure 6: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.10$, $\mu_1 = 0.65$ and $\mu_3 = 0.20$

Table 8: Summary, when μ_1 and μ_3 vary

μ_1	μ_3	ρ_1	Ascertainment
0.75	0.10	1.33	unstable
0.70	0.15	0.80	weakly Stable
0.65	0.20	0.65	Stable
0.60	0.25	0.57	
0.55	0.30	0.51	
0.50	0.35	0.48	
0.45	0.40	0.46	
0.44	0.41	0.47	
0.40	0.45	0.47	
0.35	0.50	0.48	
0.30	0.55	0.51	
0.25	0.60	0.57	
0.20	0.65	0.65	weakly Stable
0.15	0.70	0.80	

5.5. Conclusion of the First Scenario

As conclusion of the first scenario, we can say that the state of the production system is strongly linked to the variation of its parameters.

- ✓ While increasing the arrival rate λ from 0.1 to 0.16 ($\rho_1 < 1$), the number of customers in the system increases gradually, especially in the station 1, but the system is stable as all the conditions are satisfied. Beyond $\lambda = 0.16$, the sub-system 1 is saturated ($\rho_1 > 1$), which causes the instability of the global network.
- ✓ We can see a change in performance while fixing μ_2 and varying μ_1 and μ_3 in interval $[0.15, 0.7]$. The state of the network is greatly affected by the fluctuation service rate of the first station. Indeed, when $\lambda = 0.01$ and $\mu_2 = 0.15$, we get $\rho_2 < 1$. These performance measures are the best when $\mu_3 < \mu_1$, this is due to the priority given to customers of class C_3 . But for some values of μ_1 and μ_3 , for which the third condition is not verified, the system becomes weakly stable.

Finally, numerical results obtained by the simulation approach and the theoretical results shown by Weiss [31] correspond perfectly.

6. Scenario two: Results and Discussion

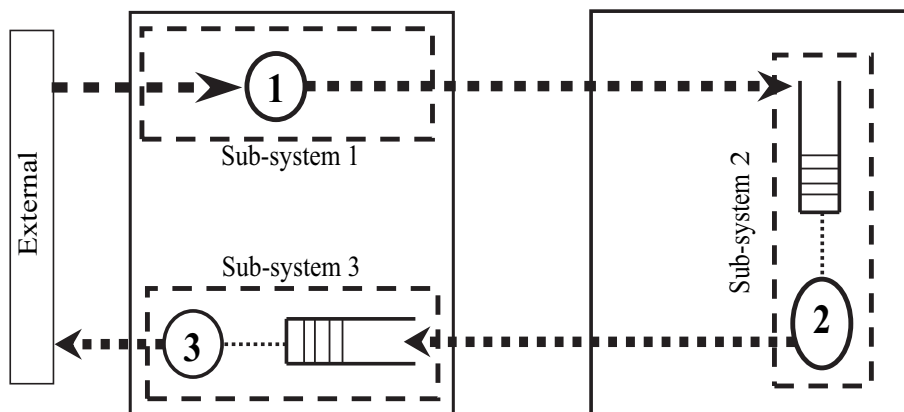


Figure 7: The mathematical model (Scenario 2)

In this part, we study the stability and performance measures of the network described in Figure 2, considering the case of Loss customers (the interrupted customer leaves definitively the system), see Figure 7. Thus, when a customer arrives and finds the first station occupied by customer of class C_3 , it leaves the system (lost customer) in order to include the assumption of perfect control of customers of class C_1 . Note that our scenario corresponds to the one given in [2], except that in the latter, author has considered that processing of a part from class 1 only starts when class 3 is empty. However, once started machine 1 will complete the processing of step 1 of the part even if parts arrive in class 3 (after being processed by machine 2). As a result, if class 2 contains n_2 parts and class 3 is empty, and processing of a part from class 1 starts, then at the end of the processing of step 1 of this part, there may be between 0 and n_2 parts in class 3.

So, in our paper, we have to evaluate the characteristics of the our system with loss parts, then we compare the results. We follow the same study that it was given for the first scenario.

6.1. Variation of the Parameter λ

In this experience, we fix the the service rate $\mu_i = 1/3$ ($i = \overline{1,3}$), and let varying λ from 0.01 to 0.17 with a pitch of 0.03. We keep the same input parameters as in the first experience (Scenario 1), let $Tmax = 10000$ time units be the simulation duration and $MC = 100$ the number of replications. The results obtained for this situation are stored in Tables 9, 10 and 11.

Table 9: Variation of N and N_i in Terms of λ

λ	N_1	N_2	N_3	N
0.01	0.02	0.02	0.02	0.08
0.04	0.10	0.10	0.10	0.31
0.07	0.18	0.17	0.16	0.52
0.10	0.24	0.24	0.21	0.70
0.13	0.30	0.30	0.25	0.86
0.16	0.37	0.34	0.29	1.01
0.17	0.39	0.37	0.30	1.07

Table 10: Variation of Q and Q_i in Terms of λ

λ	Q_1	Q_2	Q_3	Q
0.01	0.0009	0.0008	0.0005	0.002
0.04	0.011	0.011	0.005	0.03
0.07	0.03	0.03	0.01	0.07
0.10	0.05	0.05	0.02	0.13
0.13	0.08	0.07	0.03	0.20
0.16	0.12	0.10	0.04	0.27
0.17	0.13	0.11	0.05	0.29

Table 11: Variation of ρ_C and ρ_{C_i} in Terms of λ

λ	ρ_1	ρ_3	ρ_{C_1}	ρ_{C_2}	ρ_{C_3}	ρ_C
0.01	0.06	0.03	2.87	2.83	2.79	8.38
0.04	0.24	0.12	9.67	9.66	9.74	27.55
0.07	0.42	0.21	15.01	14.87	14.99	41.12
0.10	0.60	0.30	18.94	18.83	18.77	50.26
0.13	0.78	0.39	22.08	22.36	22.11	57.68
0.16	0.96	0.48	24.91	24.76	24.90	63.43
0.17	1.02	0.51	25.70	25.73	25.73	65.05

6.2. Discussion of Results

From the results given in Tables 9, 10 and 11, we conclude that:

- ✓ The number of customers in the queue and in the system increases gradually in terms of λ . Moreover, based on the important number of rejected customers of class C_1 , caused by the high priority given to the class C_3 , the number of customers increases slightly.
- ✓ For all values of λ , all arriving customers to the network are served. This is due to the fact that the mean number of customers in the system and the loads are almost the same in the three subsystems. In addition, the overall load never reaches a high level, even when the stability condition, given by the inequality (3) is not verified (ρ_C varies from 8.38%, ($\lambda = 0.01$), to 65.05%, ($\lambda = 0.17$)).
- ✓ The mean number of customers in the system and in the queue is small, compared to Scenario 1, for fixed values of λ ($\lambda = 0.10$, $\lambda = 0.13$, $\lambda = 0.16$ and $\lambda = 0.17$). This is due to the fact that all interrupted customers are lost.

Figures 8 and 9 illustrate graphically the details of some results given in Tables 9, 10 and 11. Indeed, increasing λ from 0.01 to 0.17 generates a high probability that the network is empty (Probability of having 0 customers at the end of the simulation period in the overall system is higher than 0.50). Moreover, the stationary probability of having 0 customers in each subsystem i ($i = \overline{1,3}$) is considerable ($\pi_0^{(i)} > 0.80$, $i = \overline{1,3}$). However, the network is still stable, even when the stability condition given by inequality (3) is not verified (for instance: $\rho_1 > 1$ for $\lambda = 0.17$).

A comparison between Figures 3 and 9, shows the affect of the perfect control on the change of state of the network for the same input parameters.

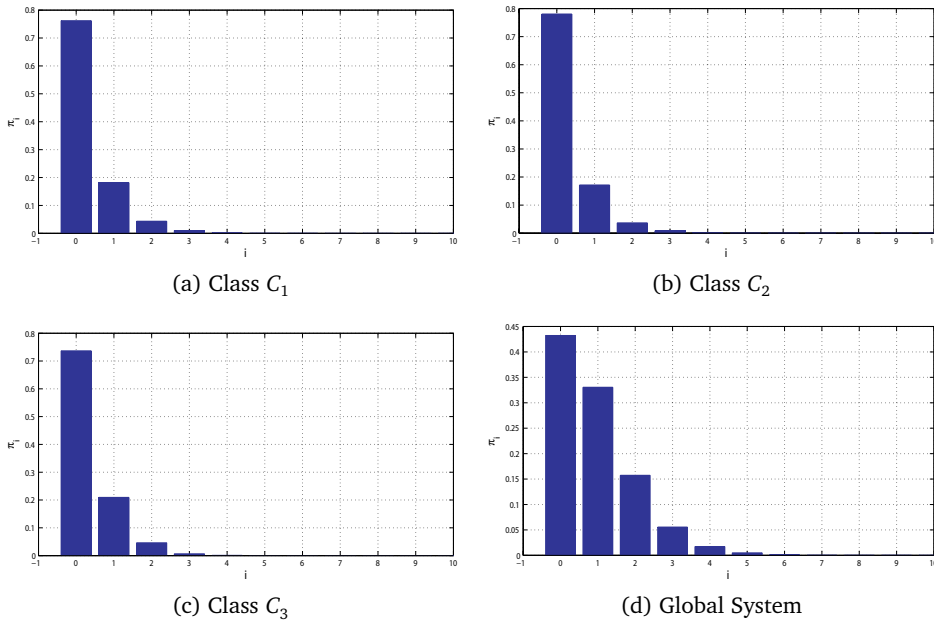


Figure 8: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.01$ and $\mu_i = 1/3$

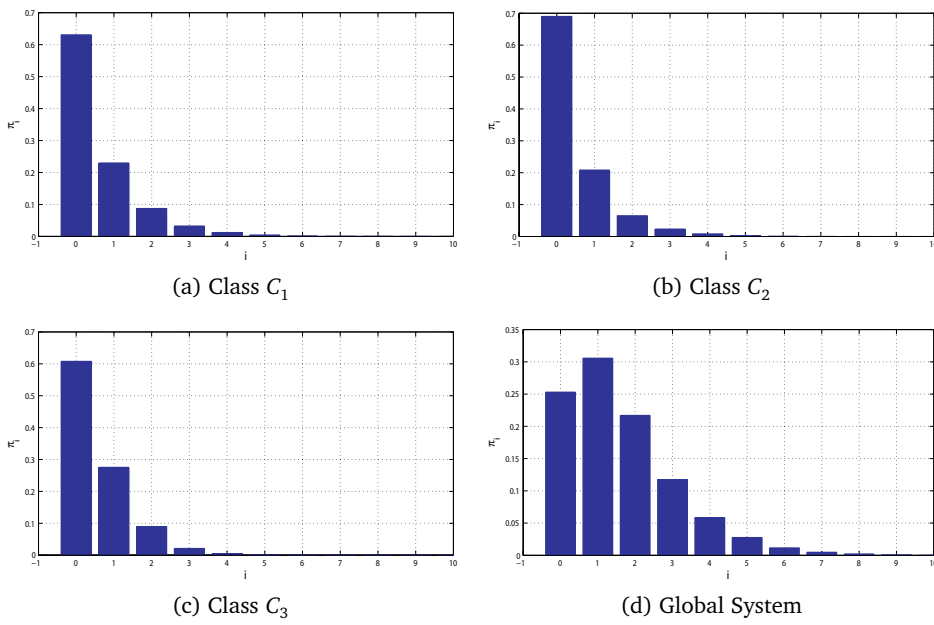


Figure 9: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k for $\lambda = 0.17$ and $\mu_i = 1/3$

6.3. Variation of Service Rates

In this part, some comparisons with results of the first scenario will be done, to this end we keep the same input parameters ($\lambda = 0.1$, $\mu_2 = 0.15$ and let varying μ_1 and μ_3 in the interval $[0.1, 0.75]$). The characteristics obtained are summarized in tables 12, 13 and 14.

Table 12: Variation of N and N_i in Terms of (μ_1, μ_3)

μ_1	μ_3	N_1	N_2	N_3	N
0.75	0.10	0.07	0.56	0.67	1.31
0.70	0.15	0.09	0.69	0.51	1.29
0.65	0.20	0.11	0.77	0.39	1.28
0.60	0.25	0.12	0.83	0.32	1.28
0.55	0.30	0.14	0.84	0.27	1.26
0.50	0.35	0.16	0.88	0.23	1.28
0.45	0.40	0.18	0.89	0.20	1.28
0.44	0.41	0.19	0.88	0.19	1.27
0.40	0.45	0.21	0.88	0.17	1.27
0.35	0.50	0.25	0.86	0.15	1.26
0.30	0.55	0.30	0.81	0.13	1.25
0.25	0.60	0.36	0.76	0.11	1.26
0.20	0.65	0.46	0.70	0.10	1.26
0.15	0.70	0.62	0.59	0.08	1.30

Table 13: Variation of Q and Q_i in Terms of (μ_1, μ_3)

μ_1	μ_3	Q_1	Q_2	Q_3	Q
0.75	0.10	0.0088	0.24	0.20	0.45
0.70	0.15	0.01	0.31	0.13	0.46
0.65	0.20	0.01	0.36	0.09	0.47
0.60	0.25	0.01	0.39	0.06	0.48
0.55	0.30	0.02	0.40	0.04	0.47
0.50	0.35	0.02	0.43	0.03	0.49
0.45	0.40	0.03	0.43	0.03	0.49
0.44	0.41	0.03	0.42	0.02	0.49
0.40	0.45	0.04	0.42	0.02	0.49
0.35	0.50	0.05	0.41	0.01	0.48
0.30	0.55	0.07	0.37	0.01	0.46
0.25	0.60	0.10	0.33	0.01	0.45
0.20	0.65	0.15	0.29	0.0088	0.46
0.15	0.70	0.25	0.22	0.0061	0.48

Table 14: Variation of ρ_C and ρ_{C_i} in Terms of (μ_1, μ_3)

μ_1	μ_3	ρ_1	C_1	C_2	C_3	C
0.75	0.10	1.33	06.35	31.62	47.47	73.32
0.70	0.15	0.80	08.06	37.40	37.54	69.47
0.65	0.20	0.65	09.53	41.12	30.76	67.42
0.60	0.25	0.57	10.79	43.32	26.11	66.15
0.55	0.30	0.51	12.25	44.40	22.33	65.05
0.50	0.35	0.48	13.62	45.39	19.52	64.56
0.45	0.40	0.47	15.36	45.86	17.24	64.27
0.44	0.41	0.47	15.77	45.94	16.88	64.36
0.40	0.45	0.47	17.17	45.84	15.29	64.00
0.35	0.50	0.48	19.43	45.29	13.64	63.79
0.30	0.55	0.51	22.43	44.50	12.06	63.90
0.25	0.60	0.57	25.94	43.20	10.83	64.15
0.20	0.65	0.65	30.49	40.64	09.42	64.41
0.15	0.70	0.80	37.26	36.83	07.96	65.24

6.4. Discussion of Results

According to the numerical results given in Tables 12, 13 and 14, we conclude that:

- ✓ When μ_1 decreases the number of customers in the first sub-system increases. And when μ_3 increases the number of customers in the third sub-system decreases, this is explained by the fact that when the processing time of customer class C_3 is small, so fewer customers are rejected. Therefore, the number of customers in the first sub-system increases and vice versa.
- ✓ While varying μ_1 from 0.75 to 0.10 and μ_3 from 0.10 to 0.75, in this case the load of the global system is still average. It strongly depends on the smallest service rate of station station 1 ($\min(\mu_1, \mu_3)$).
- ✓ The network can move from the stability to the weak stability but never reaches the instability.

For fixed values of λ ($\lambda = 0.10$) and μ_2 ($\mu_2 = 0.15$), Figure 10 (respectively, Figure 11) shows the stationary probability distributions $\pi_{k_i}^{(i)}$ and π_k for the case $\mu_1 = 0.75$ and $\mu_3 = 0.10$ (respectively, for the case $\mu_1 = 0.15$ and $\mu_3 = 0.70$).

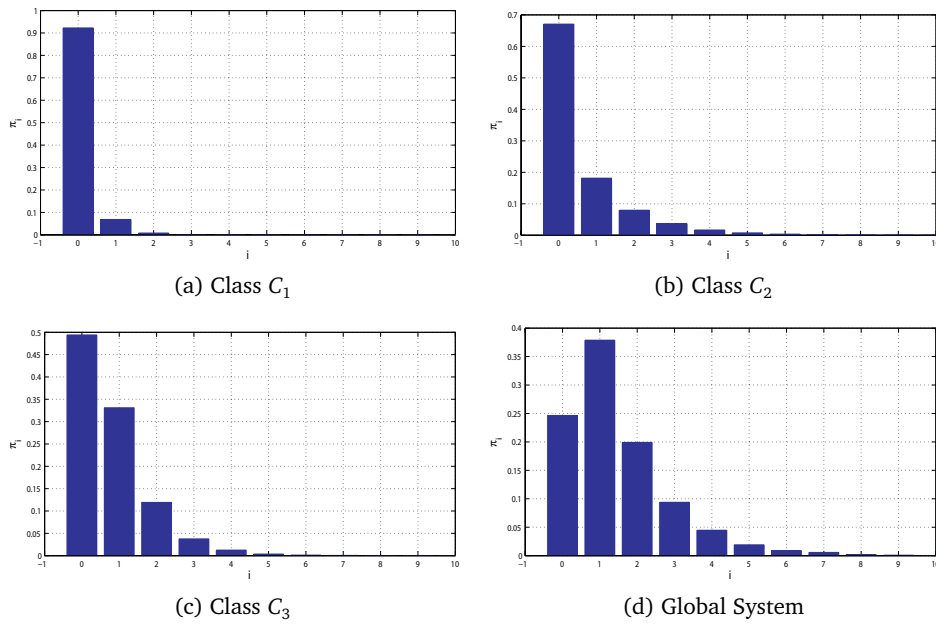


Figure 10: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k

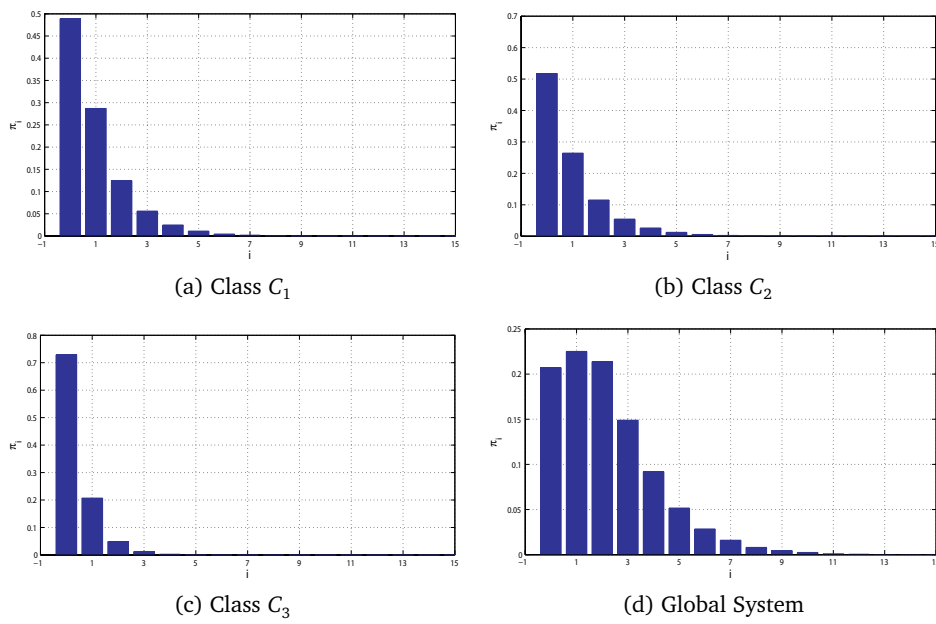


Figure 11: Stationary Probabilities $\pi_{k_i}^{(i)}$ and π_k

6.5. Conclusion of the Second Scenario

In this scenario, we conclude that:

- ✓ The arrival rate of customers, large enough, does not affect the stability of the network. Indeed, it never saturates, even if the usual stability condition (assumed necessary) is not verified ($\rho_1 < 1$). And this is due to the fact that the arriving customers which find the station 1 occupied and those interrupted by class C_3 are lost.
- ✓ The condition given by inequality (3) has no reason to be satisfied in this case, since we are assuming that all arriving customers are controlled so that the network does not saturate.
- ✓ The condition given by inequality (5) is too strict.
- ✓ There is no correlation between the stability conditions from the simulation and those obtained by Weiss [31] concerning the case of controlled arrivals.

7. General Conclusion

Simulation technique can significantly help to analyze and optimize the capacity utilization by identifying the problems of existing production facilities.

The presented case study shows the significance of simulation modeling for the production systems improvement. A flexible production system modeled by a reentrant queueing model consisting of two stations and three classes is studied. Simulation modeling and analysis using Monte Carlo simulation approach helped us to understand the behavior and reserves of the system and to suggest and verify the impact of changes on the system performance. The simulation model proposed is applicable also for solving other complex operational problems.

Two scenarios for the model were considered; Scenario 1; Whenever there exist some parts in class 3; station 1 will work on the first of them. When class 3 empties, station 1 will instantaneously resume processing of a part in step 1. The results obtained for this scenario correspond perfectly with the theoretical results obtained by Weiss [31]. While for the second scenario; a customer of class 1 is interrupted and/or when it arrives and finds the first station occupied by a customer of class C_3 , it leaves definitively the system and is considered as lost customer.

Contrariwise, the simulation results do not coincide with the assumptions given by the author, the network is still stable even if the stability conditions are not fulfilled. This allows us to think that, this is due because the author [31] has neglected a key result in the theory of queues, given by Theorem of Burke [6]. Indeed, Weiss [31] has assumed first, that arriving customers at the first station are controlled (not Poisson). On the other hand, he considered that the second station behaves as an $M/M/1$ queue, which contradicts the Theorem of Burke [6]. Therefore, the stability conditions ($\rho_1 < 1$, $\rho_2 < 1$ and $\rho_1 > \rho_2$) are suitable only when arrivals follow a Poisson process.

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