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# Seebeck Effect on a Magneto-Thermoelastic Long Solid Cylinder with Temperature-Dependent Thermal Conductivity

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**Abstract.** This article considers problem of generalized magneto-thermo-elasticity with dual-phase-lags in an infinitely long solid cylinder with variable thermal conductivity. Modified Ohm's law that includes effects of temperature gradient (Seebeck's phenomenon) and charge density as well as generalized Fourier's law with current density is introduced. Curved surface of cylinder is under thermal shock and placed in uniform axial magnetic field. Laplaces transform and its inversion techniques are applied to solve present problem. Different results for field quantities like temperature, displacement, flexural moment, and stress distributions are presented. In addition, the induced magnetic and electric fields are displayed in some plots. Effects of Seebeck parameter, variability of thermal conductivity parameter and applied magnetic field are also investigated.

**2010 Mathematics Subject Classifications**: 73C
**Key Words and Phrases**: Magneto-thermo-elasticity, dual-phase-lag, Seebeck effect, solid cylinder, variable thermal conductivity

## 1. Introduction

Classical thermoelasticity theories predict infinite speed of propagation for thermal field. Elastic change has no effects on temperature in classical uncoupled thermoelasticity theories. However, coupled theory of thermoelasticity eliminates paradox of uncoupled one. In addition, generalized thermoelasticity theories like Lord and Shulman [13] presented, instead of classical Fourier's law, wave-type heat conduction contains heat flux vector and its time-derivative as well as new thermal relaxation. Green and Lindsay [8] developed another generalized theory with two thermal relaxations. In addition to modification of Lord and Shulman they also modified all equations of coupled theory. Green and

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Naghdi [9] provide another theory that not accommodates dissipation of thermal energy. Recently, Zenkour [27] discussed transient thermal shock by presenting unified generalized thermoelasticity theory in context of all other theories. Tzou [24, 25] proposed a model that known as dual-phase-lag (DPL). It would be convenient to use this model to investigate heat transfer in micro-structures. Chandrasekharaiah [6] proposed a DPL model to modify classical thermoelasticity one.

Allam et al. [14, 15] used Green and Naghdi model to deduce 1-D and 2-D problems of time-dependent heat source for homogeneous perfectly conducting electro-magneto-thermo-elastic plate and infinitely long hollow cylinder. Sherief and Ezzat [23] used the Lord-Shulman theory to discuss the response of infinitely long magneto-thermo-elastic conducting annular cylinders. Santwana and Roychoudhuri [21] presented magneto-thermoelastic interactions in infinitely cylinders under periodic loading. Tianhu et al. [10] used Lord and Shulman theory to investigate electro-magneto-thermo-elastic interactions in perfectly conducting solid cylinder. Sadeq et al. [16] investigated the steady state thermoelasticity of hollow nanospheres. Abouelregal and Abo-Dahab [4] presented DPL model to investigate magneto-thermo-elastic solid with spherical cavity. Abouelregal and Abo-Dahab [5] used DPLs model to study the diffusion on electro-magneto-thermo-elastic solid cylinder. Abbas and Zenkour [2] and Zenkour and Abbas [30] used different theories to discuss electro-magneto-thermoelastic effects on bending of functionally graded cylinders.

If, instead of heating a single junction, one heats both junctions of circuit equally, the current stops flowing. Again, however, the current density will increase with an increasing temperature difference between the junctions [18]. The flow of electricity caused by heating does not necessarily depend on presence of two different metals. As shown by Thomson, a similar phenomenon is observed in a homogenous material if the latter is subjected to non-uniform temperature field. As a first approximation, it is permissible to assume a linear dependence of the current induced by heating on the temperature gradient, due to the fact that in most particle situations the gradient is small [12].

Temperature-dependent measurements of material properties should be taken into account due to progress in different fields in techno-structures [1, 3, 7, 19, 26, 28, 29, 31, 32]. All mentioned investigations are restricted to only electromagneto-thermoelasticity analysis. No efforts have been made to consider effects of induced magnetic and electric fields taking into consideration the effects of modified Ohm's and Fouriers laws and Seebeck coefficient. The present article investigates the DPL model to discuss electro-magneto-thermo-elastic interactions in an isotropic infinite conducting cylinder with temperature-dependent thermal conductivity. The cylinder is placed in primary magnetic field and its curved surface is considered traction free. The cylinder is deforming due to thermal shock and magnetic field to produce induced magnetic and electric fields. Displacement, temperature and thermal moment are numerically illustrated at different positions of the medium. Induced magnetic, electric and perturbed magnetic fields in surrounding free space are also represented. All considered variables are graphically displayed and results are comprehensively discussed.

## 2. Basic equations in magneto-thermoelasticity

Equations of electrodynamics are linearly simplified for a perfect homogeneous conducting elastic solid due to Maxwell's equations as

$$\begin{array}{c|c} \nabla \times \mathbf{h} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, & \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} = 0, & \nabla \cdot \mathbf{D} = \rho_e, \\ \mathbf{B} = \mu_0 \mathbf{H}, & \mathbf{D} = \varepsilon_0 \mathbf{E}, \end{array} \tag{1}$$

where $\mu_0$ denotes magnetic permeability and $\varepsilon_0$ denotes electric permeability, $\mathbf{J}$ denotes current density vector, $\mathbf{E}$ denotes induced electric field, $\mathbf{B}$ and $\mathbf{D}$ represent magnetic and electric induction vectors, $\rho_e$ is charge (electric current) density, $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ in which $\mathbf{H}_0$ denotes applied magnetic field and $\mathbf{h}$ denotes perturbation occurred in total magnetic field by induction.

In addition to the above field equations, the first hypothesis of Ohm's law for moving media represents one of the constitutive equations

$$\mathbf{J} = \sigma_0 \left( \mathbf{E} + \mu_0 \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right), \tag{2}$$

which turns out to be true only isothermal conditions; if a conductor (say, a metal) is heated non-uniformly, the relation becomes more complicated. Suppose, for example, that one heat a single junction of closed circuit formed of two pieces of wire, each of a different material, welded together in series. It found that a current starts flowing through the circuit, as manifested by, say, the deviation of a compass needle (Seebeck's effect).

Generally, the creation of an electromotive field be locally described Seebeck effect as [20]

$$E = -k_0 \nabla \theta, \tag{3}$$

where $k_0$ is the Seebeck coefficient (also known as thermopower or thermoelectric sensitivity) and $\theta = T - T_0$ denotes thermodynamic temperature in which $T$ represents temperature and $T_0$ is the environment one. Modified generalized Ohm's law for any solid with finite conductivity can be written as [5]

$$\mathbf{J} = \sigma_0 \left( \mathbf{E} + \mu_0 \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right) + \rho_e \frac{\partial \mathbf{u}}{\partial t} - k_0 \nabla \theta, \tag{4}$$

where $\mathbf{u}$ represents displacement vector and $\sigma_0$ denotes electric conductivity. Neglecting some unused items like inner heat sources and volume forces, and considering Lorentz force, then equations of motion become

$$\sigma_{ij} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{5}$$

where $\sigma_{ij}$ are the stress components, $\rho$ denotes mass density and $F_i$ represent Lorentz force components as

$$F_i = \mu_0 \left( \mathbf{J} \times \mathbf{H} \right)_i. \tag{6}$$

The corresponding Maxwell equations in adjoining free space of cylinder are

$$\nabla \times h^0 = \varepsilon^0 \frac{\partial E^0}{\partial t}, \quad \nabla \cdot h^0 = 0,$$
$$\nabla \times E^0 = \mu^0 \frac{\partial h^0}{\partial t}, \quad \nabla \cdot E^0 = 0, \tag{7}$$

in which $\mu^0$ and $\varepsilon^0$ represent magnetic and electric permeabilities in free space, $E^0$ denotes induced electric field, $h^0$ denotes magnetic field in free space and $\nabla$ represents gradient operator. It is assumed that relative permeability in cylinder and permeability in free space are equivalent. The remaining constitutive equations of Duhamel-Neumann are given by

$$\sigma_{ij} = \mu \left(u_{i,j} + u_{j,i}\right) + \left[\lambda \left(\nabla \cdot \mathbf{u}\right) - \gamma\theta\right] \delta_{ij}, \tag{8}$$

in which $\lambda$ and $\mu$ represent Lamé's constants, $\delta_{ij}$ denotes Kroneckers delta, $\gamma = (3\lambda + 2\mu)\,\alpha_t$, and $\alpha_t$ represents linear thermal expansion coefficient. The energy equation in context of DPL model including current density effect is expressed as [5]

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \left(K\theta_{,i}\right)_{,i} = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2}\right) \left(\rho C_e \theta + \gamma T_0 \nabla \cdot \mathbf{u}\right) + \pi_0 \nabla \cdot \mathbf{J}, \tag{9}$$

in which $K$ represents thermal conductivity, $\tau_\theta$ is PL of heat flux, $\tau_q$ represents PL of gradient of temperature, $(\tau_q > \tau_\theta > 0)$ and $C_e$ denotes specific heat at uniform strain. In above equations, $\pi_0$ represents coefficient connecting current density with heat flow density.

## 3. Problem formulation

A long, homogeneous, solid cylinder of radius $a$ with perfect conductivity is initially placed in an axial magnetic field $\mathbf{H}_0 \equiv (0, 0, H_0)$ acting directed parallel to $z$-axis as shown in Figure 1. Assuming that surface of cylinder be subjected to a thermal shock and traction free.

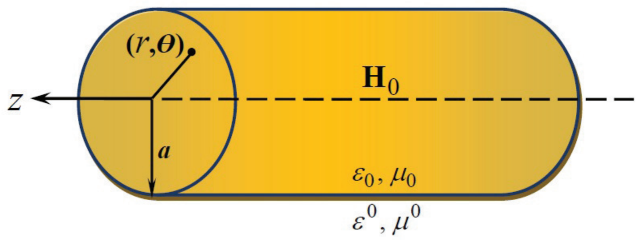


Figure 1: Geometry of a long conducting solid cylinder in an axial magnetic field vector

The initial magnetic field $\mathbf{H}_0$ is applied to the medium and generates induced magnetic $\mathbf{h}$ and electric $\mathbf{E}$ fields. Cylindrical coordinates system $(r, \varphi, z)$ is considered for the present axisymmetric problem and so all variables depend on $r$ and $t$ only. Then, $u_r = u\left(r,t\right)$ and $u_\varphi = u_z = 0$.

Components of strain tensor $e_{ij}$ are given as

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\varphi\varphi} = \frac{u}{r}, \quad e_{zz} = e_{rz} = e_{z\varphi} = e_{r\varphi} = 0. \tag{10}$$

Cubic strain dilatation $e = e_{kk}$ is thus represented as

$$e = e_{rr} + e_{\phi\phi} + e_{zz} = \frac{\partial u}{\partial r} + \frac{u}{r} = \frac{1}{r}\frac{\partial}{\partial r}(ru). \tag{11}$$

Thermoelastic stresses $\sigma_{ij}$ become

$$\begin{aligned} &\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma\theta, \quad \sigma_{\varphi\varphi} = 2\mu\frac{u}{r} + \lambda e - \gamma\theta, \\ &\sigma_{zz} = \lambda e - \gamma\theta, \quad \sigma_{r\varphi} = \sigma_{rz} = \sigma_{\varphi z} = 0. \end{aligned} \tag{12}$$

It can be easily seen that vectors $\mathbf{E}$ and $\mathbf{J}$ have their components only in $\varphi$-direction. That is

$$\mathbf{E} \equiv \left(0, E, 0\right), \quad \mathbf{J} \equiv \left(0, J, 0\right). \tag{13}$$

From Eqs. (1) and (3), we have

$$\frac{\partial h}{\partial r} = -J - \varepsilon_0\frac{\partial E}{\partial t}, \tag{14}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(Er) = -\mu_0\frac{\partial h}{\partial t}, \tag{15}$$

$$J = \sigma_0\left(E - \mu_0 H_0\frac{\partial u}{\partial t}\right) - k_0\frac{\partial \theta}{\partial r}. \tag{16}$$

Eliminating $J$ between Eqs. (14) and (16) and $E$ between Eqs. (15) and (16), leads to

$$\frac{\partial h}{\partial r} = \sigma_0\mu_0 H_0\frac{\partial u}{\partial t} - \left(\sigma_0 E + \varepsilon_0\frac{\partial E}{\partial t}\right) + k_0\frac{\partial \theta}{\partial r}, \tag{17}$$

$$\left(\nabla^2 - \sigma_0\mu_0\frac{\partial}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right)h = \sigma_0\mu_0 H_0\frac{\partial e}{\partial t} + k_0\nabla^2\theta, \tag{18}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ represents Laplace operator. Equation of motion, Eq. (4), reduces to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} + F_{rr} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{19}$$

where $F_{rr}$ represents Lorentz force which will, after applying the initial magnetic field vector, be

$$F_{rr} = \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_r = -\mu_0 H_0 \left(\frac{\partial h}{\partial r} + \varepsilon_0 \frac{\partial E}{\partial t}\right). \tag{20}$$

Thus, from Eqs. (11), (12), (19) and (20), it is found that

$$\left(\lambda + 2\mu\right) \frac{\partial e}{\partial r} - \gamma \frac{\partial \theta}{\partial r} - \mu_0 H_0 \left(\frac{\partial h}{\partial r} + \varepsilon_0 \frac{\partial E}{\partial t}\right) = \rho \frac{\partial^2 u}{\partial t^2}. \tag{21}$$

It is better to apply the operator $\left(\frac{1}{r}\right)\left(\frac{\partial}{\partial r}\right)(r)$ to both sides of Eq. (21) to get

$$\left(\lambda + 2\mu\right) \nabla^2 e - \gamma \nabla^2 \theta - \mu_0 H_0 \nabla^2 h + \mu_0^2 H_0 \varepsilon_0 \frac{\partial^2 h}{\partial t^2} = \rho \frac{\partial^2 e}{\partial t^2}. \tag{22}$$

Maxwell electromagnetic stress tensor in cylinder is given due to induced fields in the form [7]

$$M_{ij} = \mu_0 \left(H_i h_j + H_j h_i - \delta_{ij} H_k h_k\right), \tag{23}$$

which are reduced to two components $M_{rr}$ and $M_{rr}^0$. They read

$$M_{rr} = -\mu_0 H_0 h, \quad M_{rr}^0 = -\mu_0 H_0 h^0. \tag{24}$$

From Eqs. (6), the induced field components in adjoining free space will be

$$\mathbf{E}^0 \equiv \left(0, E^0, 0\right), \quad \mathbf{h}^0 \equiv \left(0, 0, h^0\right). \tag{25}$$

Using the relation $\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$, then Eq. (7) and (25) give

$$\nabla^2 h^0 - \mu_0 \varepsilon_0 \frac{\partial^2 h^0}{\partial t^2} = 0. \tag{26}$$

The thermal conductivity $K$ is temperature-dependent in the linear form [19]

$$K = K(\theta) = K_0 + \beta_0 \theta, \tag{27}$$

where $K_0$ denotes thermal conductivity at $T_0$ and $\beta_0 = K_0 \beta_1$ represents slope of thermal conductivity-temperature curve in which $\beta_1$ is constant.

Now, we introduce the mapping [17]

$$\psi = \frac{1}{K_0} \int_0^\theta K(\theta) \mathrm{d}\theta, \tag{28}$$

in which $\psi$ is new function of heat conduction. Substituting Eq. (27) into Eq. (28) gives [31]

$$\psi = \theta \left(1 + \frac{1}{2} \beta_1 \theta\right). \tag{29}$$

Differentiating Eq. (28) with respect to $r$, one obtains

$$K_0\psi_{,r} = K(\theta)\theta_{,r}. \tag{30}$$

Also, differentiating Eq. (30) once again with respect to $r$, one gets

$$K_0\psi_{,rr} = \left[K(\theta)\theta_{,r}\right]_{,r}. \tag{31}$$

In addition, the first derivative of mapping with respect to $t$ gives

$$K_0\dot{\psi}_{,rr} = K(\theta)\dot{\theta}. \tag{32}$$

Using Eqs. (31) and (32), Eq. (9) becomes

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2\psi = \left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\frac{1}{k}\frac{\partial\psi}{\partial t} + \frac{\gamma T_0}{K_0}\frac{\partial e}{\partial t}\right), \tag{33}$$

where $\rho C_e = K/k$ and $k$ is the diffusivity.

For linearity, and remember that $\theta = T - T_0$ with $|\theta/T_0| << 1$, then Eqs. (22), (17) and (18) take the forms

$$(\lambda+2\mu)\nabla^2 e - \gamma\nabla^2\psi - \mu_0 H_0\nabla^2 h + \mu_0^2 H_0\varepsilon_0\frac{\partial^2 h}{\partial t^2} = \rho\frac{\partial^2 e}{\partial t^2}. \tag{34}$$

$$\frac{\partial h}{\partial r} = \sigma_0\mu_0 H_0\frac{\partial u}{\partial t} - \sigma_0 E - \varepsilon_0\frac{\partial E}{\partial t} + k_0\frac{\partial\psi}{\partial r}, \tag{35}$$

$$\left(\nabla^2 - \sigma_0\mu_0\frac{\partial}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right)h = \sigma_0\mu_0 H_0\frac{\partial e}{\partial t} + k_0\nabla^2\psi. \tag{36}$$

The constitutive relations given in Eqs. (12), for linearity will be in the forms

$$\begin{aligned}\sigma_{rr} &= 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma\psi,\\ \sigma_{\varphi\varphi} &= 2\mu\frac{u}{r} + \lambda e - \gamma\psi,\\ \sigma_{zz} &= \lambda e - \gamma\psi.\end{aligned} \tag{37}$$

## 4. Solution of the problem

For convenience, one can dimensionless all quantities as follows [24]

$$\begin{aligned}&u' = gc_1\eta u, \quad r' = c_1\eta r, \quad a' = c_1\eta a, \quad t' = c_1^2\eta t, \quad \tau_q' = c_1^2\eta\tau_q,\\ &M_{rr}' = \frac{gM_{rr}}{\mu}, \quad E' = \frac{\eta g E}{\sigma_0\mu_0^2 H_0 c_1}, \quad \psi' = \frac{\psi}{T_0}, \quad J' = \frac{\eta g J}{\sigma_0^2\mu_0^2 H_0 c_1},\\ &\tau_\theta' = c_1^2\eta\tau_\theta, \quad h' = \frac{\eta g h}{\sigma_0\mu_0 H_0}, \quad e' = ge, \quad K_1' = T_0 K_1,\\ &\sigma_{ij}' = \frac{g\sigma_{ij}}{\mu}, \quad c_1^2 = \frac{\lambda+2\mu}{\rho}, \quad \eta = \frac{\rho C_e}{K}, \quad g = \frac{\gamma}{\rho C_e}.\end{aligned} \tag{38}$$

In terms of dimensionless quantities, Eqs. (33)-(37) and Eq. (30) (suppressing primes for simplicity in the notation), are reduced to

$$\frac{\partial h}{\partial r} = \frac{\partial u}{\partial t} - v_1 E - V^2 \frac{\partial E}{\partial t} + S \frac{\partial \psi}{\partial r}, \tag{39}$$

$$\left( \nabla^2 - v_1 \frac{\partial}{\partial t} - V^2 \frac{\partial^2}{\partial t^2} \right) h = \frac{\partial e}{\partial t} + S \nabla^2 \psi, \tag{40}$$

$$\nabla^2 e - \varepsilon \nabla^2 \psi - v_1 g_2 \nabla^2 h + v_1 g_2 V^2 \frac{\partial h}{\partial t} = \frac{\partial^2 e}{\partial t^2}, \tag{41}$$

$$\left( 1 + \tau_\theta \frac{\partial}{\partial t} \right) \nabla^2 \psi = \frac{\partial}{\partial t} \left( 1 + \tau_q \frac{\partial}{\partial t} \right) (\psi + e), \tag{42}$$

$$\nabla^2 h^0 - V^2 \frac{\partial^2 h^0}{\partial t^2} = 0, \tag{43}$$

$$\frac{1}{r} \frac{\partial \left( E^0 r \right)}{\partial r} = - \frac{\partial h^0}{\partial t}, \tag{44}$$

$$\begin{aligned} \sigma_{rr} &= \beta^2 e - 2\frac{u}{r} - b_1 \psi, \\ \sigma_{\varphi\varphi} &= \beta^2 e - 2\frac{\partial u}{\partial r} - b_1 \psi, \\ \sigma_{zz} &= \left( \beta^2 - 2 \right) e - b_1 \psi, \end{aligned} \tag{45}$$

$$M_{rr} = -b_2 h. \tag{46}$$

where

$$\begin{aligned} &v_1 = \frac{\sigma_0 \mu_0}{\eta}, \quad V^2 = \frac{c_1^2}{c^2}, \quad c^2 = \frac{1}{\mu_0 \varepsilon_0}, \\ &S = \frac{k_0 \eta g T_0}{\sigma_0 \mu_0 H_0}, \quad g_2 = \frac{\mu_0 H_0^2}{\rho c_1^2}, \quad \varepsilon_0 = \frac{\gamma g T_0}{\rho c_1^2}, \\ &\beta^2 = \frac{\lambda + 2\mu}{\mu}, \quad b_1 = \frac{\gamma T_0}{\mu}, \quad b_2 = \beta^2 v_1 g_2. \end{aligned} \tag{47}$$

## 5. Laplace transforms domain solutions

The problem may be solved after applying its initial and boundary conditions. Here, the initial conditions are assumed to be homogeneous as

$$u(r,0) = \left.\frac{\partial u(r,t)}{\partial t}\right|_{t=0} = 0, \quad \psi(r,0) = \left.\frac{\partial \psi(r,t)}{\partial t}\right|_{t=0} = 0,$$
$$E(r,0) = \left.\frac{\partial E(r,t)}{\partial t}\right|_{t=0} = 0, \quad h(r,0) = \left.\frac{\partial h(r,t)}{\partial t}\right|_{t=0} = 0. \tag{48}$$

The above conditions are supplemented by adding boundary conditions. Firstly, it is assumed that the transverse components of **E** are continuous across the cylindrical surface, i. e.,

$$E(r,t) = E^0(r,t), \quad r = a, \ \ t > 0, \tag{49}$$

in which $E^0$ denotes electric field intensity component in free surrounding cylinder in the direction of $\varphi$. Also, the same components of **h** are also continuous across the cylindrical surface, i.e.,

$$h(r,t) = h^0(r,t), \quad r = a, \ \ t > 0, \tag{50}$$

in which $h^0$ denotes induced magnetic field intensity component in free surrounding cylinder in $z$-direction. Finally, the cylindrical surface is considered traction free,

$$\sigma_{rr}(a,t) = 0, \tag{51}$$

and lies under a thermal shock,

$$\theta(a,t) = \theta_0 H(t), \tag{52}$$

in which $H(t)$ denotes unit-step function.

Now, it is known that Laplace transform is defined by

$$\bar{f}(r,s) = \int_0^\infty \mathrm{e}^{-st} f(r,t)\mathrm{d}t, \tag{53}$$

and it will be applied into Eqs. (39)-(44) under initial conditions given in Eqs. (48), leads to

$$\frac{\partial \bar{h}}{\partial r} = s\bar{u} - \left(v_1 + V^2 s\right)\bar{E} + S\frac{\partial \bar{\psi}}{\partial r}, \tag{54}$$

$$\left(\nabla^2 - v_1 s - V^2 s^2\right)\bar{h} = s\bar{e} + S\nabla^2\bar{\psi}, \tag{55}$$

$$\nabla^2\bar{e} - \varepsilon\nabla^2\bar{\psi} - v_1 g_2 \nabla^2\bar{h} + s v_1 g_2 V^2 \bar{h} = s^2\bar{e}, \tag{56}$$

$$(1+\tau_\theta s)\,\nabla^2\bar{\psi} = s\,(1+\tau_q s)\,(\bar{\psi}+\bar{e}), \tag{57}$$

$$\nabla^2\bar{h}^0 - V^2 s^2 \bar{h}^0 = 0, \tag{58}$$

$$\frac{1}{r}\frac{\partial(\bar{E}^0 r)}{\partial r} = -s\bar{h}^0. \tag{59}$$

Eliminating $\bar{e}$ and $\bar{h}$ from Eqs. (55) and (56), the following sixth order differential equation for $\bar{\psi}$ is formed as follows

$$\left(\nabla^6 - A\nabla^4 + B\nabla^2 - C\right)\bar{\psi} = 0, \tag{60}$$

in which $A$, $B$ and $C$ are some coefficients given as

$$\begin{aligned} A &= q^2 + s^2 + g_2 q v_1\left(K_1 + \frac{s}{q}\right) + s\left(sV^2 + v_1\right) + q\varepsilon, \\ B &= qs^2 + s^2\left(S + \frac{s}{q}\right)V^2 + g_2 q s v_1 + s\left(sV^2 + v_1\right)\left(q^2 + s^2 + q\varepsilon\right), \\ C &= g_2 q s^3 V^2 v_1 + q s^3\left(sV^2 + v_1\right), \quad q = \frac{s + \tau_q s^2}{1 + \tau_\theta s}. \end{aligned} \tag{61}$$

In a similar manner, we can show that $\bar{e}$ and $\bar{h}$ satisfy the equation

$$\left(\nabla^6 - A\nabla^4 + B\nabla^2 - C\right)\{\bar{e}, \bar{h}\} = 0, \tag{62}$$

Introducing $m_i$ $(i = 1, 2, 3)$ into Eq. (60), we find

$$\left(\nabla^2 - m_1^2\right)\left(\nabla^2 - m_2^2\right)\left(\nabla^2 - m_3^2\right)\bar{\psi} = 0, \tag{63}$$

where $m_i^2$ are the roots of the characteristic equation

$$m^6 - Am^4 + Bm^2 - C = 0. \tag{64}$$

These roots are given by [10]

$$\begin{aligned} m_1^2 &= \frac{1}{3}\left[2p\sin(q) + A\right], \\ m_2^2 &= -\frac{p}{3}\left[\sqrt{3}\cos(q) + \sin(q)\right] + \frac{A}{3}, \\ m_3^2 &= -\frac{p}{3}\left[\sqrt{3}\cos(q) - \sin(q)\right] + \frac{A}{3}, \end{aligned} \tag{65}$$

in which

$$q = \frac{1}{3}\sin^{-1}(R), \quad p = \sqrt{A^2 - 3B}, \quad R = -\frac{2a^3 - 9AB + 27C}{2p^3}. \tag{66}$$

The solution of $\bar{\psi}$ will be

$$\bar{\psi} = \sum_{i=1}^{3} \left( A_i I_0(m_i r) + B_i K_0(m_i r) \right), \tag{67}$$

where $A_i$ and $B_i$ $(i = 1, 2, 3)$ represent parameters depending on $s$ and $I_0(\cdot)$, $K_0(\cdot)$ represent Modified Bessels function of first and second kinds of order zero, respectively. Since the cylinder is solid, then solution should be continuous everywhere. So, $B_i$ should be vanished and the solution for dimensionless $\bar{\psi}$ in Laplace domain is

$$\bar{\psi} = \sum_{i=1}^{3} A_i(s) I_0(m_i r). \tag{68}$$

In a similar manner

$$\{\bar{e}, \bar{h}\} = \sum_{i=1}^{3} \{A_i^{'}(s), A_i^{''}(s)\} I_0(m_i r). \tag{69}$$

The compatibility between Eqs. (55) and (57) and Eqs. (69), gives

$$A_i^{'} = \frac{m_i^2 - q_1}{q_1} A_i = \Gamma_i A_i, \quad A_i^{''} = \frac{s(m_i^2 - q_1) + q_1 S m_i^2}{q_1(m_i^2 - v_1 s - V^2 s^2)} A_i = \Omega_i A_i. \tag{70}$$

The displacement in Laplace domain comes as follows

$$\bar{u} = \sum_{i=1}^{3} \frac{1}{m_i} A_i^{'}(s) I_1(m_i r), \tag{71}$$

which is derived by using the well-known relation of Bessel function,

$$\int z I_0(z) \mathrm{d}z = z I_1(z). \tag{72}$$

If we differentiate Eq. (71) and using the formula

$$\frac{\mathrm{d}}{\mathrm{d}z} \left[ I_n(z) \right] = I_{n-1}(z) - \frac{n}{z} I_n(z), \tag{73}$$

we obtain

$$\frac{\partial \bar{u}}{\partial r} = \sum_{i=1}^{3} A_i^{'}(s) \left( I_0(m_i r) - \frac{1}{m_i r} I_1(m_i r) \right). \tag{74}$$

Substituting from Eqs. (68), (69) and (71) into Eq. (54), leads to

$$\bar{E} = \sum_{i=1}^{3} C_i(s) I_1(m_i r), \tag{75}$$

where

$$C_i = \frac{s\Gamma_i + SA_i - \Omega_i}{m_i(v_1 + V^2 s)} A_i = \phi_i A_i. \tag{76}$$

Now, one can substitute solutions for $\bar{u}$ and $\bar{\psi}$ in Eqs. (45)-(48) to obtain stress and moment components as

$$\bar{\sigma}_{rr} = \sum_{i=1}^{3} \left[ (\beta^2\Gamma_i - b_1) I_0(m_i r) - \frac{2\Gamma_i}{m_i r} I_1(m_i r) \right] A_i(s), \tag{77}$$

$$\bar{\sigma}_{\varphi\varphi} = \sum_{i=1}^{3} \left[ (\beta^2\Gamma_i - b_1 - 2\Gamma_i) I_0(m_i r) + \frac{2\Gamma_i}{m_i r} I_1(m_i r) \right] A_i(s), \tag{78}$$

$$\bar{\sigma}_{zz} = \sum_{i=1}^{3} \left[ \Gamma_i(\beta^2 - 2) - b_1 \right] I_0(m_i r) A_i(s), \tag{79}$$

$$\bar{M}_{rr} = -b_2 \sum_{i=1}^{3} \Omega_i I_0(m_i r) A_i(s). \tag{80}$$

Also, the induced fields in free space $\bar{h}^0$ and $\bar{E}^0$, which are bounded at infinity, maybe deduced from Eqs. (60) and (61) in the form

$$\bar{h}^0 = K_0(Vsr) A_4(s), \tag{81}$$

$$\bar{E}^0 = \frac{1}{V} K_1(Vsr) A_4(s), \tag{82}$$

where $A_4(s)$ is a parameter depending on $s$ only. The boundary conditions given in Eqs. (48) may be written, after using Laplace transform, in the form

$$\bar{E}(a, s) = \bar{E}^0(a, s), \tag{83}$$

$$\bar{h}(a, s) = \bar{h}^0(a, s), \tag{84}$$

$$\bar{\sigma}_{rr}(a, s) = 0, \tag{85}$$

$$\bar{\psi}(R, s) = \theta_0 \left( \frac{1}{s} + \frac{K_1}{2s} \right) = \bar{G}(s). \tag{86}$$

After applying the above boundary conditions we obtain four equations in unknown parameters $A_j$ as

$$\sum_{i=1}^{3} C_i(s) I_1(m_i a) - A_4(s) K_0(V s a) = 0, \tag{87}$$

$$\sum_{i=1}^{3} \phi_i A_i(s) I_0(m_i a) - A_4(s) K_0(V s a) = 0, \tag{88}$$

$$\sum_{i=1}^{3} \left[ (\beta^2 \Gamma_i - b_1) I_0(m_i a) - \frac{2\Gamma_i}{m_i a} I_1(m_i a) \right] A_i(s) = 0, \tag{89}$$

$$\sum_{i=1}^{3} A_i(s) I_0(m_i a) = \bar{G}(s). \tag{90}$$

It is easy to find the constants $A_j$ due to the above equations and then the solution in Laplace transform domain will be completed. Moreover, temperature increment $\bar{\theta}$ can be deduced in terms of $\bar{\psi}$ by using Eq. (29) as

$$\bar{\theta}(r, s) = \frac{-1 + \sqrt{1 + 2K_1 \bar{\psi}}}{K_1}. \tag{91}$$

## 6. Numerical results and discussions

Laplace inversions for all obtained variables in Laplace transform domain should be used to get the final solutions in physical domain. Numerical inversion method based on Fouriers series expansion [11] obvious that inversion $g(r, t)$ of Laplace transform $\bar{g}(s)$ maybe approximated as

$$g(t) = \frac{\mathrm{e}^{ct}}{t_1} \left[ \frac{g(c)}{2} + \mathrm{Re} \left\{ \sum_{k=1}^{N} \mathrm{e}^{\mathrm{i}k\pi t/t_1} g\left( c + \frac{\mathrm{i}k\pi t}{t_1} \right) \right\} \right], \quad 0 \le t \le 2t_1, \tag{92}$$

in which $N$ presents number of terms in truncated infinite Fourier series. It must be chosen such that

$$\mathrm{e}^{ct} \mathrm{Re} \left\{ \mathrm{e}^{\mathrm{i}N\pi t/t_1} g\left( c + \frac{\mathrm{i}N\pi t}{t_1} \right) \right\} \le \varepsilon_1, \tag{93}$$

where $\varepsilon_1$ represents persecuted small positive number, $c$ represents positive free parameter that must be greater than real parts of all singularities of $\bar{g}(s)$ [28].

Now, numerical results are carried out for Cooper, which has the following material constants [22].

$$\begin{aligned} &K = 368 \text{ Wm}^{-1}\text{ K}^{-1}, \quad \alpha_t = 1.78 \times 10^{-5}\text{ K}^{-1}, \quad C_e = 383.1 \text{ JKg}^{-1}\text{ K}^{-1}, \\ &\rho = 8954 \text{ Kgm}^{-3}, \quad \lambda = 7.76 \times 10^{10}\text{ Nm}^{-2}, \quad \mu = 3.86 \times 10^{10}\text{ Nm}^{-2}, \\ &\eta = 8886.73 \text{ sm}^{-2}, \quad T_0 = 293 \text{ K}, \quad \varepsilon = 0.0168, \quad g = 1.61, \quad \sigma_0 = 5.7 \times 10^{7}, \\ &\varepsilon_0 = 10^{-9}/36\pi \text{ Fm}^{-1}, \quad \mu_0 = 4\pi \times 10^{-7}\text{ Hm}^{-1}, \quad H_0 = 10^{7}/4\pi \text{ Am}^{-1}. \end{aligned} \tag{94}$$

Numerical results for physical quantities for dimensionless time $t = 0.1$ can be conducted using this data. Distributions of dimensionless temperature $\theta$, radial displacement $u$, stress component $\sigma_{rr}$, Maxwell's stress $M_{rr}$, induced magnetic field $h$ and induced electric field component $E$, induced field components in adjoining $h^0$ and $E^0$ distributions were evaluated and are presented graphically. These distributions are shown in Figs. 2-17. Comparisons among different cases may also be made with the help of these figures. From the figures, it is deduced that the field quantities depend not only on state and space variables $t$ and $r$, but also depend on variability thermal conductivity parameter, the modified Fourier's and Ohm's laws.

Numerical computations were carried out for two cases as illustrated in the following:

Case I investigates dimensionless temperature, displacement, stresses Maxwell's stress, induced magnetic field induced electric field component, induced field components in the adjoining with different values of variability thermal conductivity parameter $K_1$ when Seebeck parameter $S$, PL of heat flux $\tau_q$ and PL of temperature gradient $\tau_\theta$ remain constants.

Case II illustrates dimensionless field quantities with different values of Seebeck parameter $S$ when variability thermal conductivity parameter $K_1$, $\tau_q$ and $\tau_\theta$ remain constants.

In case I, three different values of variability thermal conductivity parameter $K_1$ are considered to discuss effect of temperature on thermal conductivity. We take $K_1 = -0.25, -0.5$ for variable thermal conductivity and $K_1 = 0$ when thermal conductivity is temperature-independent. Seebeck parameter and other parameters are fixed to be $S = 0.5$, $\tau_q = 0.2$ and $\tau_\theta = 0.1$. It is obvious from Figs. 2-9 that $K_1$ has significant effects on all field quantities.

Figure 2 describes variation of temperature $\theta$ versus radial distance $r$. It is seen that $\theta$ is not vanished only in a bounded region of space at a given instant. However, outside this region, $\theta$ is vanished and this means that region has not felt thermal disturbance yet. Figure 3 plots displacement $u$ against $r$. The displacement gets its maximum magnitudes at the boundary where thermal shock is applied. It is seen that medium adjacent to cylindrical surface undergoes expansion deformation due to thermal shock while others compressive deformation.

In Fig. 4, radial stress $\sigma_{rr}$ at cylindrical surface is always zero since the surface is traction free. The medium close to cylindrical surface suffers from tensile stress. Also, it is clear that tensile stress region becomes larger while compressed becomes smaller with passage of time. Moreover, Fig. 4 shows that at some instant non-zero region of stress is finite which indirectly proves wave effect of heat.

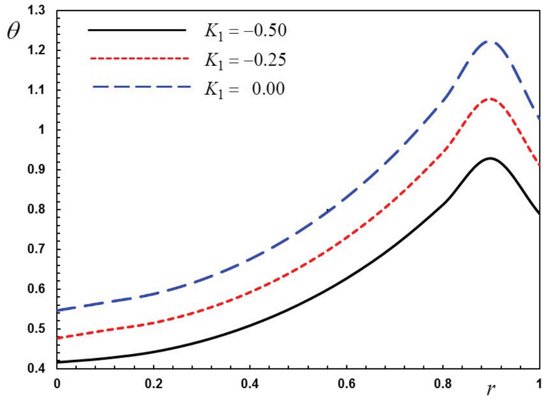


Figure 2: Distribution of $\theta$ along the radial direction of the solid cylinder for different values of $K_1$

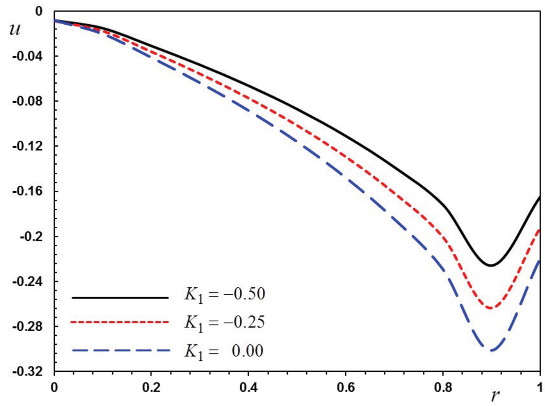


Figure 3: Distribution of $u$ along the radial direction of the solid cylinder for different values of $K_1$

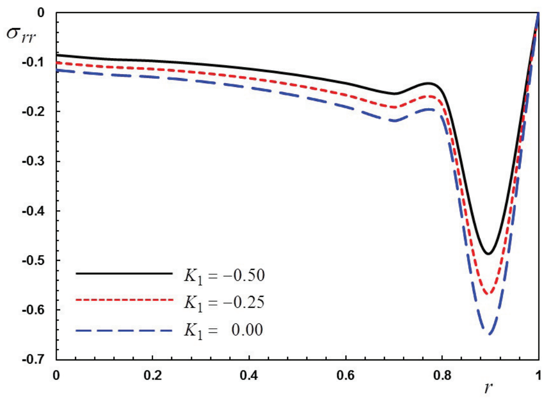


Figure 4: Distribution of $\sigma_{rr}$ along the radial direction of the solid cylinder for different values of $K_1$

Figures 5 and 6 illustrate the induced magnetic field and electric field distribution effects. It is well known that electromagnetic medium is placed in initial magnetic field, and it deforms due to thermal shock.

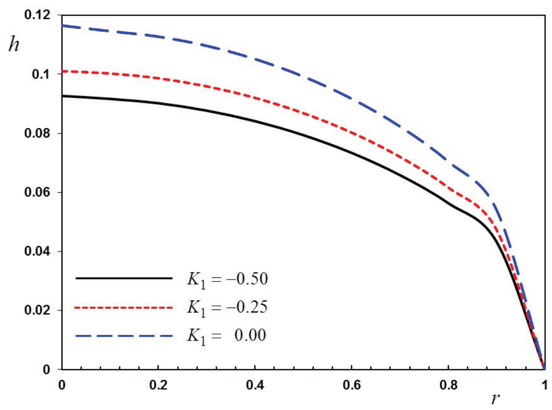


Figure 5: Distribution of $h$ along the radial direction of the solid cylinder for different values of $K_1$

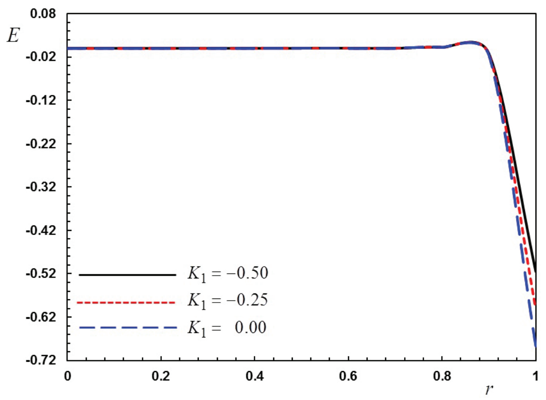


Figure 6: Distribution of $E$ along the radial direction of the solid cylinder for different values of $K_1$

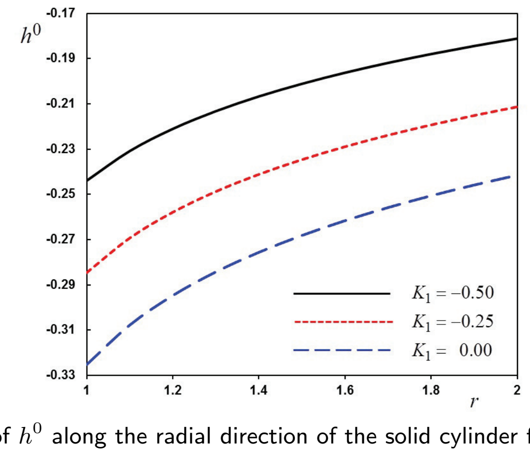


Figure 7: Distribution of $h^0$ along the radial direction of the solid cylinder for different values of $K_1$

Figures 7 and 8 give the variations of induced field components in the adjoining $h^0$ and $E^0$ distributions against distance for various values of variability thermal conductivity coefficient. It is observed from the gures that these distributions are increase with distance

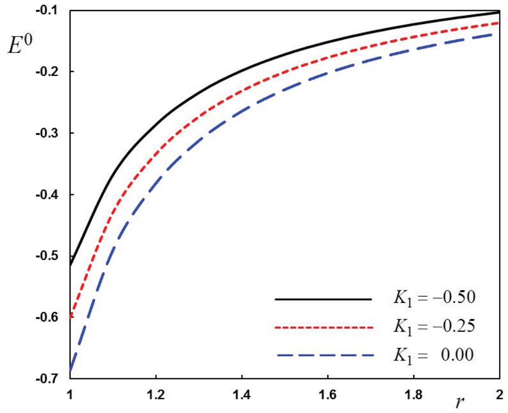


Figure 8: Distribution of $E^0$ along the radial direction of the solid cylinder for different values of $K_1$

and takes negative values.

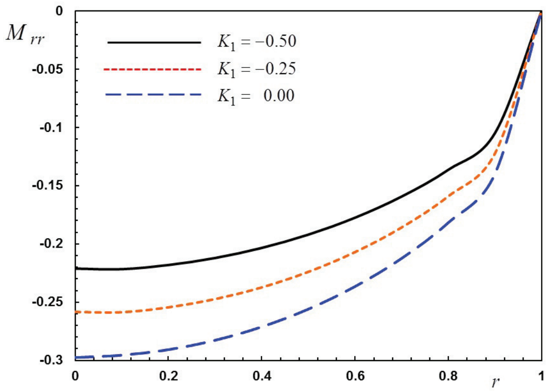


Figure 9: Distribution of $M_{rr}$ along the radial direction of the solid cylinder for different values of $K_1$

Figure 9 depicts the variation of Maxwell's stress $M_{rr}$ along the radial distance $r$ of the cylinder. This figure shows the difference between generalized theory of thermoelasticity with constant thermal conductivity and those of variable thermal conductivity. It is noted that as parameter $K_1$ increases effective region is decreased.

It is obvious that change of thermal conductivity has very small effect on induced electric field component $E$. Its effect on the other functions considered is more noticeable. The increases in conductivity tends to increase absolute value of temperature $\theta$, radial stress $\sigma_{rr}$, induced electric field component $E$, induced field components in the adjoining $h^0$ and $E^0$ and Maxwell's stress $M_{rr}$.

In case II, the distributions of temperature $\theta$, radial displacement $u$, stress component $\sigma_{rr}$, Maxwell's stress $M_{rr}$, induced magnetic $h$ and electric $E$ fields, and induced field components in adjoining $h^0$ and $E^0$ distributions are evaluated and presented graphically in Figures 10-17. Two different values of Seebeck parameter $S$ were considered ($S = 0.5, 0.25$). Additional results for field quantities are given without Seebeck effect ($S = 0$).

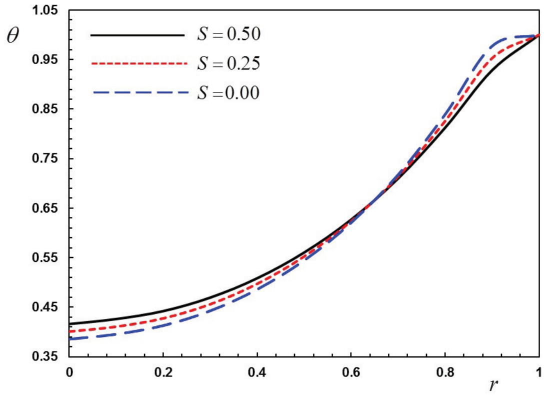


Figure 10: Distribution of $\theta$ along the radial direction of the solid cylinder for different values of $K_1$

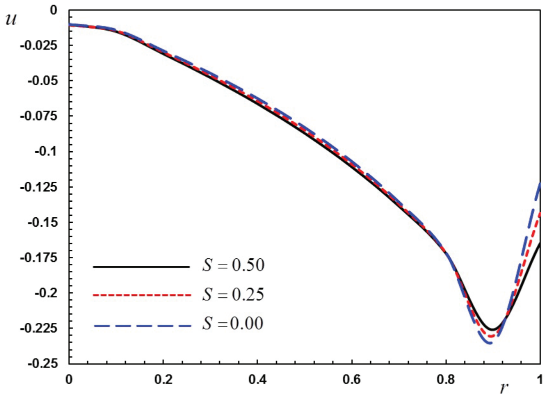


Figure 11: Distribution of $u$ along the radial direction of the solid cylinder for different values of $S$

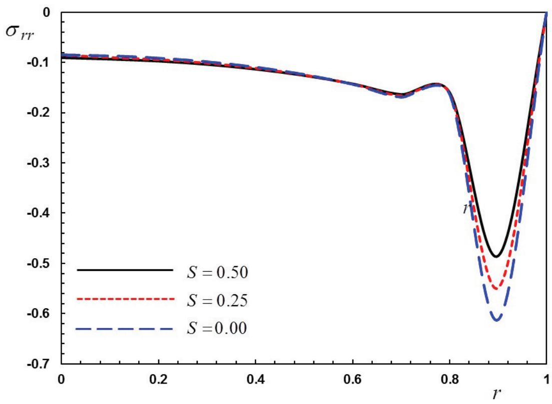


Figure 12: Distribution of $\sigma_{rr}$ along the radial direction of the solid cylinder for different values of $S$

In this case variability thermal conductivity parameter is fixed to be $K_1 = -0.5$. It is noticed that Seebeck parameter $S$ is more pronounced and has significant effects on all considered functions.

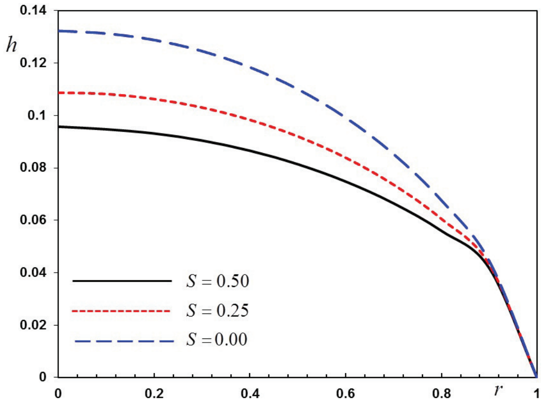


Figure 13: Distribution of $h$ along the radial direction of the solid cylinder for different values of $S$

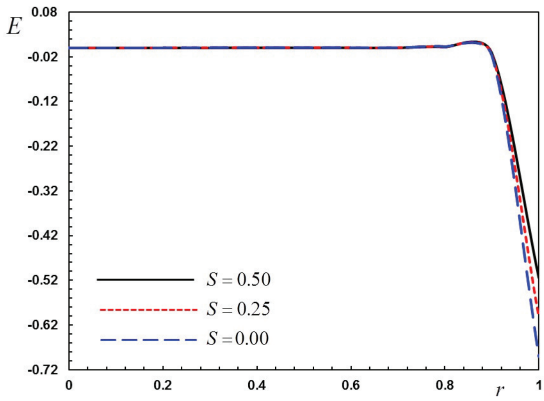


Figure 14: Distribution of $E$ along the radial direction of the solid cylinder for different values of $S$

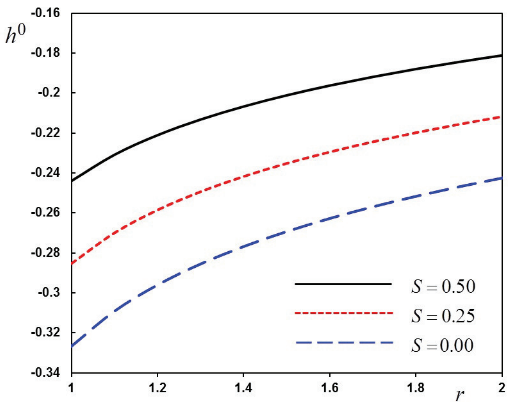


Figure 15: Distribution of $h^0$ along the radial direction of the solid cylinder for different values of $S$

## 7. Conclusions

In this article we present new model of equations of generalized electro-magneto-thermoelasticity for thermally, isotropic and electrically conducting infinitely solid cylinder

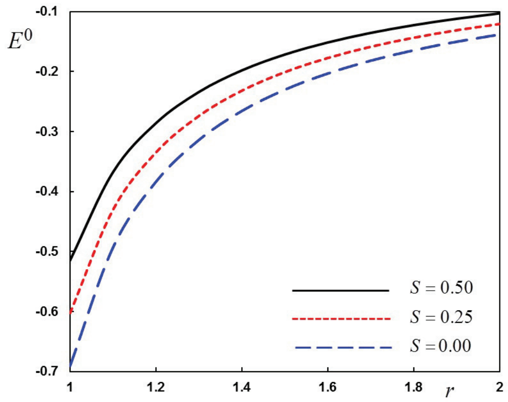


Figure 16: Distribution of $E^0$ along the radial direction of the solid cylinder for different values of $S$

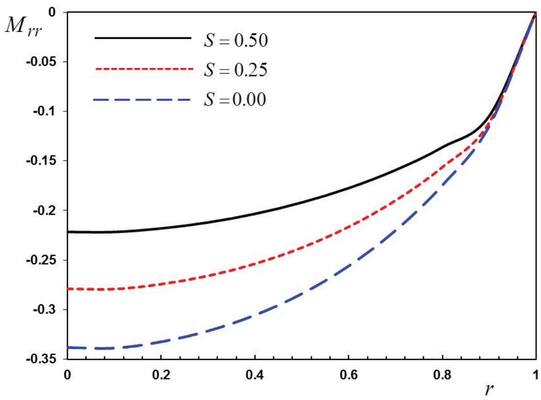


Figure 17: Distribution of $M_{rr}$ along the radial direction of the solid cylinder for different values of $S$

whose surface is subjected to thermal shock. The influences of phase-lags, magnetic field, thermal shock, Seebeck's coefficient and variable thermal conductivity are considered under variable thermal conductivity and magnetic field.

The problem has been solved by means of Laplace transform and numerical Laplace inversion. According to above, it is concluded that variability thermal conductivity parameter has significant effects on speed of wave propagation of the studied fields. The temperature-dependent thermal conductivity has significant effect on thermal and mechanical interactions. Seebeck parameter has significant influence on all distributions. Other theories of coupled thermoelasticity, generalized thermoelasticity with one relaxation time can be obtained as special cases of the present model. In generalized magneto-thermoelasticity theory with phase-lags heat propagates as wave with finite velocity instead of infinite velocity in medium. The numerical results presented here should prove useful to researchers in science and technology as well as to the development of solid-state mechanisms.

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