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Local approximation results for Stancu variant of modified Szász-Mirakjan operators

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Abstract. The aim of this paper is to obtain local approximation results for Stancu type generalization of modified Szász-Mirakjan operators. First, we calculate moments of the operators. Some direct results of the operators are investigated. The rate of convergence of the operators is evaluated. In last section of the paper, the Voronovskaya type result is obtained.
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1. Introduction

In 1950, Otto and Mirakjan [5] introduced Szász-Mirakjan operators; generalization of Bernstein operators defined by

$$M_n(f)(x) = e^{-nx} \sum_{k=0}^{\infty} \frac{(nx)^k}{k!} f\left(\frac{k}{n}\right). \tag{1}$$

In his paper D.D. Stancu [3] introduced a positive linear polynomial type operators defined by

$$B_n^{\alpha,\beta}(f)(x) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{k+\alpha}{n+\beta}\right). \tag{2}$$

where, $0 \le \alpha \le \beta$ and $0 \le x \le 1$.
Walczak [13] investigated generalization of Szász-Mirakjan operators defined by

$$S_n[f; a_n, b_n, q, x] = \sum_{k=0}^{\infty} s_{a_n,k}(x) f\left(\frac{k}{b_n+q}\right).,\tag{3}$$

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where $s_{a_n,k}(x) = e^{-a_n x}\frac{(a_n x)^k}{k!}$, for $k = 0, 1, 2, ...; q \geq 0$ is a fixed number, $(a_n)_1^\infty$ and $(b_n)_1^\infty$ are increasing and unbounded sequences such that $1 \leq a_n \leq b_n$, and $(a_n/b_n)_1^\infty$ is non-decreasing and

$$\frac{a_n}{b_n} = 1 + o\left(\frac{1}{b_n}\right)$$

Walczask [13] derived pointwise and uniform convergence of the operators (3) in exponential weight space. There are some other linear positive operators with Stancu type modification, e.g. [1], [10], [11], [12].
Recently, Gandhi and Mishra [4] introduced modification of Szász-Mirakjan operators (1) given by

$$S_n(f;x) = \sum_{k=0}^{\infty} e^{-b_n x}\frac{(b_n x)^k}{k!} f\left(\frac{k}{b_n}\right), \tag{4}$$

where $(b_n)_1^\infty$ is an increasing sequence of positive real numbers, $b_n \to \infty$ as $n \to \infty, b_1 \geq 1$. For $b_n = n$, we get the operators defined in (1).
Gandhi and Mishra discussed local and global approximation results of the operators (4) in polynomial weighted space of polynomials. Indeed, the rapid development has led to the discovery of new generalizations of approximation operators (one may refer to [2],[7], [8], [9], [15]).
In second section of this paper, we introduce our Stancu variant of modified Szász-Mirakjan operators and we evaluate moments of our operators. The uniform convergence of the operators is derived. In third section, we discuss local approximation result and rate of convergence of the operators. In the last section, we derive Voronovskaya type result for the operators.

2. Construction of the operators

Motivated by Gandhi and Mishra [4], we introduce Stancu type generalization of modified Szász-Mirakjan operators for $f \in C[0, \infty)$ as follows:

$$S_n^{\alpha,\beta}(f;x) = \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} f\left(\frac{k+\alpha}{b_n+\beta}\right), \tag{5}$$

where, $1/b_n \to 0$ as $n \to \infty$, $b_n \geq 1$. For $\alpha = \beta = 0$ we get the modified Szász-Mirakjan operators.
Now, we calculate moments of our operators (5).

Lemma 2.1. *Let $e_j(t) = t^j$ for $j = 0, 1, 2$, the followings are true:*

$$S_n^{\alpha,\beta}(1;x) = 1, \tag{6}$$

$$S_n^{\alpha,\beta}(t;x) = \frac{b_n x + \alpha}{b_n + \beta}, \tag{7}$$

$$S_n^{\alpha,\beta}(t^2;x) = \frac{b_n^2 x^2}{(b_n+\beta)^2} + \frac{(1+2\alpha)b_n}{(b_n+\beta)^2}x + \frac{\alpha^2}{(b_n+\beta)^2}. \tag{8}$$

Proof. For $i = 0$, the result is obvious.
For $i = 1$,

$$\begin{aligned}
S_n^{\alpha,\beta}(t;x) &= \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} \left(\frac{k+\alpha}{b_n+\beta} \right) \\
&= \frac{1}{b_n+\beta} \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} k + \frac{\alpha}{b_n+\beta} S_n(1;x) \\
&= \frac{b_n x + \alpha}{b_n + \beta}.
\end{aligned}$$

For $i = 2$,

$$\begin{aligned}
S_n^{\alpha,\beta}(t^2;x) &= \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} \left(\frac{k+\alpha}{b_n+\beta} \right)^2 \\
&= \frac{1}{(b_n+\beta)^2} \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} k^2 + \frac{2\alpha}{(b_n+\beta)^2} \sum_{k=0}^{\infty} \frac{(b_n x)^k}{k!} e^{-b_n x} k + \frac{\alpha^2}{(b_n+\beta)^2} \\
&= \frac{b_n^2 x^2}{(b_n+\beta)^2} + \frac{(1+2b_n\alpha)}{(b_n+\beta)^2} b_n x + \frac{\alpha^2}{(b_n+\beta)^2}.
\end{aligned}$$

Hence, lemma is proved.

Lemma 2.2. *The central moments* $\Phi_m^{\alpha,\beta}(x) = S_n^{\alpha,\beta}((t-x)^m;x)$ *for* $m = 1, 2$ *are as follows:*

$$\Phi_1^{\alpha,\beta}(x) = \frac{b_n x + \alpha}{b_n + \beta} - x, \tag{9}$$

$$\Phi_2^{\alpha,\beta}(x) = \left(\frac{b_n}{b_n+\beta} - 1 \right)^2 x^2 + \left(\frac{(1+2\alpha)b_n}{(b_n+\beta)^2} - \frac{2\alpha}{b_n+\beta} \right) x + \frac{\alpha^2}{(b_n+\beta)^2}. \tag{10}$$

Proof. Using lemma:(2.1) we get the result.

Now, we obtain the uniform convergence of the operators $S_n^{\alpha,\beta}$ to f, $f \in C_\xi[0,\infty)$,

$$C_\xi[0,\infty) = \{ f \in C[0,\infty) : |f(x)| \le M(1+t)^\xi \}$$

for $M > 0, \xi > 0$.

Theorem 2.3. $S_n^{\alpha,\beta}(f;x)$ *converges uniformly to* $f(x)$ *for* $0 \le x \le a$, $f \in C_\xi[0,\infty), \xi \ge 2, a > 0$.

Proof. Using Korovkin theorem, it is sufficient to show that

$$\lim_{n\to\infty} \| S_n^{\alpha,\beta}(t^j;x) - x^j \|_{C_\xi[0,\infty)} = 0$$

for $j = 0, 1, 2$.
The result is trivial for the case $j = 0$ using (6). For $j = 1$, the result can be obtained using (7), as follows:

$$\begin{aligned}\lim_{n\to\infty} \|S_n^{\alpha,\beta}(t;x) - x\|_{C_\xi[0,\infty)} &= \lim_{n\to\infty} \left\| \frac{b_n x + \alpha}{b_n + \beta} - x \right\|_{C_\xi[0,\infty)} \\ &= 0.\end{aligned}$$

Finally, for $j = 2$, using (8), we get

$$\begin{aligned}\lim_{n\to\infty} \|S_n^{\alpha,\beta}(t^2;x) - x^2\|_{C_\xi[0,\infty)} &= \lim_{n\to\infty} \left\| \frac{b_n^2 x^2}{(b_n+\beta)^2} + \frac{(1+2\alpha)b_n}{(b_n+\beta)^2}x + \frac{\alpha^2}{(b_n+\beta)^2} - x^2 \right\|_{C_\xi[0,\infty)} \\ &= \lim_{n\to\infty} \left\| \frac{b_n^2 x^2}{(b_n+\beta)^2} - x^2 \right\|_{C_\xi[0,\infty)} \\ &= 0.\end{aligned}$$

3. Direct result

In this section, we give some local results for the operators. Let $C_B[0,\infty)$ be the space of all real valued continuous bounded functions defined on $[0,\infty)$. The norm on the space $C_B[0,\infty)$ is the supremum norm $\|f\| = \sup\limits_{x\in[0,\infty)} |f(x)|$. Further, Peetre's K-functional is defined by

$$K_2(f,\delta) = \inf_{g\in W^2}\{\|f-g\| + \delta\|g''\|\},$$

here $W^2 = \{g \in C_B[0,\infty) : g', g'' \in C_B[0,\infty)\}$, By [14] there exists a positive constant $C > 0$ such that $K_2(f,\delta) \le C\omega_2(f,\delta^{1/2}), \delta > 0$, where

$$\omega_2(f,\delta^{1/2}) = \sup_{0<h<\delta^{1/2}}, x \in [0,\infty)|f(x+2h) - 2f(x+h) + f(x)|$$

is the second order modulus of continuity of function $f \in C_B[0,\infty)$. Also. for $f \in C_B[0,\infty)$ the usual modulus of continuity is given by

$$\omega(f,\delta^{1/2}) = \sup_{0<h<\delta^{1/2}, x\in[0,\infty)} |f(x+h) - f(x)|.$$

Theorem 3.1. *Let $f \in C_B[0,\infty)$, then for all $n \in \mathbb{N}$, there exists an absolute constant $C > 0$ such that*

$$|S_n^{\alpha,\beta}(f;x) - f(x)| \le C\omega_2(f,\delta_n(x)) + \omega(f,\alpha_n(x)), \tag{11}$$

where

$$\delta_n(x) = [S_n^{\alpha,\beta}((t-x)^2;x) + (S_n^{\alpha,\beta}((t-x);x))^2]^{1/2}$$

and

$$\alpha_n(x) = \left|\frac{b_n x+\alpha}{b_n+\beta} - x\right|.$$

Proof. For $0 \le x < \infty$, we consider the auxiliary operators $\hat{S}_n^{\alpha,\beta}(f;x)$ defined by

$$\hat{S}_n^{\alpha,\beta}(f;x) = S_n^{\alpha,\beta}(f;x) + f(x) - f(\frac{b_n x+\alpha}{b_n+\beta}).$$

Using above operators and (9), we get

$$\begin{aligned}\hat{S}_n^{\alpha,\beta}(t-x;x) &= S_n^{\alpha,\beta}(t-x;x) - (\frac{b_n x+\alpha}{b_n+\beta} - x)\\ &= 0.\end{aligned}$$

Now, $0 \le x < \infty$ and $g \in W^2$. Applying Taylor's formula, we get

$$g(t) = g(x) + (t-x)g'(x) + \int_x^t (t-u)g''(u)du.$$

Applying $\hat{S}_n^{\alpha,\beta}$ on the both sides of the above equation, we obtain

$$\begin{aligned}\hat{S}_n^{\alpha,\beta}((g;x)-g(x)) &= \hat{S}_n^{\alpha,\beta}((t-x)g'(x);x) + \hat{S}_n^{\alpha,\beta}\left(\int_x^t (t-u)g''(u)du;x\right)\\ &= g'(x)\hat{S}_n^{\alpha,\beta}((t-x);x) + S_n^{\alpha,\beta}\left(\int_x^t (t-u)g''(u)du;x\right)\\ &- \int_x^{\frac{b_n x+\alpha}{b_n+\beta}} \left(\frac{b_n x+\alpha}{b_n+\beta} - u\right)g''(u)du\\ &= S_n^{\alpha,\beta}\left(\int_x^t (t-u)g''(u)du;x\right) - \int_x^{\frac{b_n x+\alpha}{b_n+\beta}} \left(\frac{b_n x+\alpha}{b_n+\beta} - u\right)g''(u)du.\end{aligned}$$

Also,

$$\left|\int_x^t (t-u)g''(u)du\right| \le \int_x^t |t-u||g''(u)|du \le \|g''\| \int_x^t |t-u|du \le (t-x)^2\|g''\|.$$

and

$$\left|\int_x^{\frac{b_n x+\alpha}{b_n+\beta}} \left(\frac{b_n x+\alpha}{b_n+\beta} - u\right)g''(u)du\right| \le \left(\frac{b_n x+\alpha}{b_n+\beta} - x\right)^2 \|g''\|.$$

Therefore, we can conclude that

$$|\hat{S}_n^{\alpha,\beta}((g;x)-g(x))| = \left|S_n^{\alpha,\beta}\left(\int_x^t (t-u)g''(u)du;x\right)\right|$$

$$
\begin{aligned}
&+\left|\int_x^{\frac{b_nx+\alpha}{b_n+\beta}}\left(\frac{b_nx+\alpha}{b_n+\beta}-u\right)g''(u)du\right|\\
&\le \|g''\|S_n^{\alpha,\beta}((t-x)^2;x)+\left(\frac{b_nx+\alpha}{b_n+\beta}-x\right)^2\|g''\|\\
&=\delta_n^2\|g''\|.
\end{aligned}
$$

Also, we get

$$|\hat{S}_n^{\alpha,\beta}((g;x)| \le |S_n^{\alpha,\beta}(f;x)|+2\|f\| \le 3\|f\|.$$

Therefore,

$$
\begin{aligned}
|S_n^{\alpha,\beta}(f;x)-f(x)| &\le |\hat{S}_n^{\alpha,\beta}((f-g);x)-(f-g)(x)|+|\hat{S}_n^{\alpha,\beta}((g;x)-g(x))|\\
&+\left|f(x)-f(\frac{b_nx+\alpha}{b_n+\beta})\right|\\
&\le 4\|f-g\|+\|g''\|\delta_n^2(x)+\omega\left(f:\left|\frac{b_nx+\alpha}{b_n+\beta}-x\right|\right).
\end{aligned}
$$

Hence, taking the infimum on the right hand side over all $g\in W^2$, we obtain

$$|S_n^{\alpha,\beta}(f;x)-f(x)| \le 4K_2(f,\delta_n^2(x))+\omega(f,\alpha_n(x)).$$

By using property of K-functional, we have

$$|S_n^{\alpha,\beta}(f;x)-f(x)| \le C\omega_2(f,\delta_n(x))+\omega(f,\alpha_n(x)).$$

Hence the result is obtained.

Now, we consider the following class of functions:
$H_{x^2}[0,\infty)=\{f:[0,\infty)\to\mathbb{R}:|f(x)|\le M_f(1+x^2)$ here M_f is constant depending on the function $f\}$,
$C_{x^2}[0,\infty)=\{f\in H_{x^2}[0,\infty):f$ is continuous$\}$,
$C^*_{x^2}[0,\infty)=\{f\in C_{x^2}[0,\infty):\lim_{|x|\to\infty}\frac{f(x)}{(1+x^2)}$ is finite$\}$.
The norm on the space $C^*_{x^2}[0,\infty)*$ is defined by $\|f\|_{x^2}=\sup_{x\in[0,\infty)}|\frac{f(x)}{1+x^2}|$.
We denote the modulus of continuity of f on closed interval $[0,a], a>0$ by:

$$\omega_a(f;\delta)=\sup_{|t-x|\le\delta,x,t\in[0,a]}|f(t)-f(x)|.$$

Theorem 3.2. *For $f\in C_{x^2}[0,\infty);\omega_a(f;\delta)$ be its modulus of continuity on the interval $[0,a+1]\subset[0,\infty), a>0$, we have*

$$\|S_n^{\alpha,\beta}(f;x)-f(x)\| \le 6M_f(1+a^2)\lambda_n+2\omega_{a+1}(f;\sqrt{\lambda_n}),$$

here

$$\lambda_n=\left(1-\frac{b_n^2}{(b_n+\beta)^2}\right)a^2+\left(\frac{b_n-2\alpha\beta}{b_n+\beta}\right)\frac{a}{b_n+\beta}+\frac{\alpha^2}{(b_n+\beta)^2}.$$

Proof. For $0 \le x \le a$ and $t \ge 0$, we have [6]

$$|f(t) - f(x)| \le 6M_f(1 + a^2)(t - x)^2 + \omega_{a+1}(f; \delta_n)\left(\frac{|t - x|}{\delta_n} + 1\right).$$

Applying above inequality and Cauchy-Schwarz inequality, we have

$$\begin{aligned}
\|S_n^{\alpha,\beta}(f(t); x) - f(x)\|_{C[0,a]} &\le S_n^{\alpha,\beta}(|f(t) - f(x)|; x) \\
&\le 6M_f(1 + a^2)S_n^{\alpha,\beta}((t - x)^2; x) \\
&+ \omega_{a+1}(f; \delta_n)\left(1 + \frac{1}{\delta_n^2}S_n^{\alpha,\beta}((t - x)^2; x\right)^{1/2}.
\end{aligned}$$

For $0 \le x \le a$, using lemma (2.2),

$$\begin{aligned}
S_n^{\alpha,\beta}((t - x)^2; x) &= \left(\frac{b_n}{b_n + \beta} - 1\right)^2 x^2 + \left(\frac{(1 + 2\alpha)b_n}{(b_n + \beta)^2} - \frac{2\alpha}{b_n + \beta}\right)x + \frac{\alpha^2}{(b_n + \beta)^2} \\
&\le \left(\frac{b_n}{b_n + \beta} - 1\right)^2 a^2 + \left(\frac{(1 + 2\alpha)b_n}{(b_n + \beta)^2} - \frac{2\alpha}{b_n + \beta}\right)a + \frac{\alpha^2}{(b_n + \beta)^2} \\
&\le \left(1 - \frac{b_n^2}{(b_n + \beta)^2}\right)a^2 + \left(\frac{b_n - 2\alpha\beta}{b_n + \beta}\right)\frac{a}{b_n + \beta} + \frac{\alpha^2}{(b_n + \beta)^2} \\
&= \lambda_n.
\end{aligned}$$

Taking $\delta_n = \sqrt{\lambda_n}$, we will get the theorem.

4. Voronovskaya type result

Theorem 4.1. *For $f \in C_\xi[0, \infty)$ such that $f', f'' \in C_\xi[0, \infty)$, we have*

$$\lim_{n\to\infty} b_n[S_n^{\alpha,\beta}(f; x) - f(x)] = (\alpha - \beta x)f'(x) + \frac{x}{2}f''(x). \tag{12}$$

where $0 \le x \le a,\ a > 0$.

Proof. From Taylor's formula, we have

$$f(t) = f(x) + (t - x)f'(x) + \tfrac{1}{2}(t - x)^2 f''(x) + (t - x)^2 r(t, x),$$

here, $r(t, x)$ is reminder term and $\lim_{t\to x} r(t, x) = 0$.
Therefore,

$$\begin{aligned}
b_n[S_n^{\alpha,\beta}(f; x) - f(x)] &= b_n f'(x)S_n^{\alpha,\beta}((t - x); x) \\
&+ b_n\frac{f''(x)}{2}S_n^{\alpha,\beta}((t - x)^2; x) \\
&+ b_n S_n^{\alpha,\beta}(r(t, x)(t - x)^2; x).
\end{aligned}$$

By the Cauchy-Schwarz inequality, we get

$$S_n^{\alpha,\beta}(r(t,x)(t-x)^2;x) \le \sqrt{S_n^{\alpha,\beta}(r^2(t,x);x)}\sqrt{S_n^{\alpha,\beta}(t-x)^4;x)}.$$

As $r(t,x) \in C_\xi[0,\infty)$, therefore by Theorem (2.3) and from the fact that $\lim_{t\to x} r(t,x) = 0$, we obtain

$$\lim_{n\to\infty} S_n^{\alpha,\beta}(r^2(t,x);x) = r^2(x,x) = 0.$$

Therefore,

$$\begin{aligned}\lim_{n\to\infty} b_n[S_n^{\alpha,\beta}(f;x) - f(x)] &= \lim_{n\to\infty} b_n f'(x) S_n^{\alpha,\beta}((t-x);x) \\ &\quad + \lim_{n\to\infty} b_n \frac{f''(x)}{2} S_n^{\alpha,\beta}((t-x)^2;x).\end{aligned}$$

Now,

$$\begin{aligned}\lim_{n\to\infty} b_n S_n^{\alpha,\beta}((t-x);x) &= lim_{n\to\infty} b_n \left(\frac{b_n x + \alpha}{b_n+\beta} - x\right) \\ &= \lim_{n\to\infty} b_n \left(\frac{b_n}{b_n+\beta} - 1\right)x + \lim_{n\to\infty} b_n \left(\frac{\alpha}{b_n+\beta}\right) \\ &= \alpha - \beta x,\end{aligned}$$

and

$$\begin{aligned}\lim_{n\to\infty} b_n S_n^{\alpha,\beta}((t-x)^2;x) &= \lim_{n\to\infty} b_n \Bigg(\left(\frac{b_n}{b_n+\beta} - 1\right)^2 x^2 \\ &\quad + \left(\frac{(1+2\alpha)b_n}{(b_n+\beta)^2} - \frac{2\alpha}{b_n+\beta}\right)x + \frac{\alpha^2}{(b_n+\beta)^2}\Bigg) \\ &= x.\end{aligned}$$

Hence, from above equations, we have

$$\lim_{n\to\infty} b_n[S_n^{\alpha,\beta}(f;x) - f(x)] = (\alpha - \beta x) f'(x) + \frac{x}{2} f''(x).$$

Hence the theorem is proved.

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References

[1] A. Kumar, L.N. Mishra, Approximation by modified Jain-Baskakov-Stancu operators, Tbilisi Mathematical Journal, 10(2): 185-199, 2017.

[2] A.R. Gairola, Deepmala, L.N. Mishra, On the $q-$derivatives of a certain linear positive operators, Iranian Journal of Science & Technology, Transactions A: Science, 2017, doi: 10.1007/s40995-017-0227-8.

[3] D.D. Stancu, Approximation of functions by means of a new generalized Bernstein operator, Calcolo 20: 211-229, 1983.

[4] R.B. Gandhi, Deepmala, V.N. Mishra, Local and global results for modified Sz*á*szMirakjan operators. Mathematical Methods in the Applied Sciences, 40(7):2491-2504, 2017.

[5] O. Sz*á*sz, Generalization of S. Bernstein's polynomials to the infinite interval, J. Res. Nat. Bur. Standards Sect. B. 45:239-245, 1950.

[6] V. Gupta, A. Aral, Convergence of the q-analogue of Sz*á*sz-Beta operators, Applied Mathematics and Computation, 216(2):374-380, 2010.

[7] V.N. Mishra, K. Khatri, L.N. Mishra, Deepmala; Inverse result in simultaneous approximation by Baskakov-Durrmeyer-Stancu operators, Journal of Inequalities and Applications, 2013, 2013:586.

[8] V.N. Mishra, L.N. Mishra, Trigonometric Approximation of Signals (Functions) in $L_p(p \geq 1)$ norm, International Journal of Contemporary Mathematical Sciences, 7(19):909-918, 2012.

[9] V.N. Mishra, K. Khatri, L.N. Mishra; Statistical approximation by Kantorovich type Discrete $q-$Beta operators, Advances in Difference Equations, 2013, 2013:345.

[10] V.N. Mishra, P. Sharma, L.N. Mishra; On statistical approximation properties of $q-$Baskakov-Szász-Stancu operators, Journal of Egyptian Mathematical Society, 24(3):396-401, 2016.

[11] V.N. Mishra, K. Khatri, L.N. Mishra; On Simultaneous Approximation for Baskakov-Durrmeyer-Stancu type operators, Journal of Ultra Scientist of Physical Sciences, 24(3):567-577, 2012.

[12] V.N. Mishra, H.H. Khan, K. Khatri, L.N. Mishra; Hypergeometric Representation for Baskakov-Durrmeyer-Stancu Type Operators, Bulletin of Mathematical Analysis and Applications, 5(3):18-26, 2013.

[13] Z. Walczak, On modified Sz*á*sz-Mirakjan operators, Novi Sad Journal of Mathematics 33(1):93-107, 2003.

[14] R.A. DeVore and G.G. Lorentz, Constructive Approximations, vo. 303, Springer, Berlin, 1993.

[15] A.R. Gairola, Deepmala, L.N. Mishra, Rate of Approximation by Finite Iterates of q-Durrmeyer Operators, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 86(2):229-234, 2016.