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Fixed point results in metric-like spaces via σ-simulation functions

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Abstract. The purpose of this paper is to establish some fixed point results for (α, β)-admissible $\mathcal{Z}$-contraction mappings in complete metric-like spaces. Our results generalize and extend several known results on literature. Two illustrated examples are also presented.

2010 Mathematics Subject Classifications: 47H10, 54H25

Key Words and Phrases: Fixed point, metric-like space, simulation function, (α, β)-admissible mapping

1. Introduction and preliminaries

Fixed point theory is an essential tool to resolve many equations appeared in applied science such as Biology, Physics, Economics, Engineering and Game Theory. Banach contraction principle [12] is considered the most important tool in fixed point theory. It was extended in several directions. For more details, see [13, 17, 18, 20–25]. Going in this direction, Harandi [16] reintroduced the concept of metric-like spaces.

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Definition 1. *[16] Let X is a nonempty set. A function $\sigma : X \times X \to [0, \infty)$ is said to be a metric-like space (or a dislocated metric) on X if for any $x, w, y \in X$, the following conditions hold:*

(σ_1) $\sigma(x, y) = 0$ *implies that* $x = y$;

(σ_2) $\sigma(x, y) = \sigma(y, x)$;

(σ_3) $\sigma(x, y) \leq \sigma(x, z) + \sigma(z, y)$.

The pair (X, σ) is called a metric-like space.

It is clear that every metric space and partial metric space is a metric-like space, but the converse is not true.

Example 1. *Let $X = \{0, 1\}$ and*

$$\sigma(x, y) = \begin{cases} 2, & \text{if } x = y = 0; \\ 1, & \text{otherwise.} \end{cases}$$

Then (X, σ) is a metric-like space. It is neither a partial metric space ($\sigma(0, 0) \not\leq \sigma(0, 1)$), nor a metric space ($\sigma(0, 0) = 2 \neq 0$).

Following [16], we have the following topological concepts. Each metric-like σ on X generates a topology τ_σ on X whose base is the family of open σ-balls

$$B_\sigma(x, \epsilon) = \{y \in X :\mid \sigma(x, y) - \sigma(x, x) \mid < \epsilon\}, \; for \; all \; x \in X \; and \; \epsilon > 0.$$

Now, let (X, σ) be a metric-like space. The mapping $T : X \to X$ is said σ-continuous at $x \in X$ if for all $\varepsilon > 0$, there exists $\delta > 0$ such that $T(B_\sigma(x, \delta)) \subseteq B_\sigma(Tx, \varepsilon)$. Consequently, if $T : X \to X$ is σ-continuous, then if $\lim_{n\to\infty} x_n = x$, we have $\lim_{n\to\infty} Tx_n = Tx$. A sequence $\{x_n\}_{\infty}^{n=0}$ of elements of X is called σ-Cauchy if the limit $\lim_{n,m\to\infty} \sigma(x_n, y_m)$ exists and is a finite number. The metric-like space (X, σ) is called complete if for each σ-Cauchy sequence $\{X_n\}_{\infty}^{n}$, there is some $y \in Y$ such that

$$\lim_{n\to\infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{n,m\to\infty} \sigma(x_n, x_m).$$

A subset A of a metric-like space (X, σ) is bounded if there is a point $b \in X$ and a positive constant K such that $\sigma(a, b) \leq K$ for all $a \in A$.

Remark 1. *Let $X = \{0, 1\}$ be endowed with $\sigma(x, y) = 1$ for each $x, y \in X$. Take $x_n = 1$ for each $n \in \mathbb{N}$. Using the convergence definition, it is is easy to see that $x_n \to 0$ and $x_n \to 1$. In metric-like spaces, the limit of a convergent sequence is not necessarily unique.*

The following lemma is known and useful for the rest of paper.

Lemma 1. *[5, 16] Let (X, σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \to x$ where $x \in X$ and $\sigma(x, y) = 0$. Then for all $y \in X$, we have $\lim_{n\to\infty} \sigma(x_n, y) = \sigma(x, y)$.*

In literature, there are several (common) fixed point works in the setting of metric-like spaces. For instance, see [6, 8, 10].
On the one hand, Samet [26] presented the concept of α-admissible mappings and proved some fixed point theorems in metric spaces. Recently, Chandok [14] introduced the notion of (α, β)-admissible mappings and obtained some fixed point theorems.

Definition 2. *[14] Let X be a nonempty set, $f : X \to X$ and $\alpha, \beta : X \times X \to \mathbb{R}^+$. We say that f is an (α, β)-admissible mapping if $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$ imply that $\alpha(fx, fy) \geq 1$ and $\beta(fx, fy) \geq 1$ for all $x, y \in X$.*

For other results using different concepts of α-admissible mappings, see [1, 2, 7, 9, 11, 15, 27–29]. On the other hand, Khojasteh et al. [19] introduced a new class of mappings called simulation functions. They [19] proved several fixed point theorems and showed that many results in the literature are simple consequences of their obtained results.

Definition 3. *[19] A function $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ is called a simulation function if ζ satisfies the following conditions:*

(ζ_1) $\zeta(0, 0) = 0$;

(ζ_2) $\zeta(t, s) < s - t$ *for all* $t, s > 0$;

(ζ_3) *if* $\{t_n\}$ *and* $\{s_n\}$ *are sequences in* $(0, \infty)$ *such that* $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = \ell \in (0, \infty)$, *then*

$$\lim_{n\to\infty} \sup \zeta(t_n, s_n) < 0.$$

In [19], the following unique fixed point theorem is established.

Theorem 1. *[19] Let (X, d) be a metric space and $f : X \to X$ be a $\mathcal{Z}$-contraction with respect to a simulation function ζ, that is,*

$$\zeta(d(fx, fy), d(x, y)) \geq 0, \ \ forall \ \ x, y \in X.$$

Then T has a unique fixed point.

It is worth mentioning that the Banach contraction is an example of $\mathcal{Z}$-contractions by defining $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ via

$$\zeta(t, s) = \gamma s - t, \ \ \forall \ \ s, t \in [0, \infty),$$

where $\gamma \in [0, 1)$.

Argoubi et al. [4] modified Definition 3 as follows.

Definition 4. *[4] A simulation function is a function* $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ *that satisfies the following conditions:*

(i) $\zeta(t, s) < s - t$ *for all* $t, s > 0$*;*

(ii) if $\{t_n\}$ *and* $\{s_n\}$ *are sequences in* $(0, \infty)$ *such that* $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = \ell \in (0, \infty)$*, then*

$$\lim_{n\to\infty} \sup \zeta(t_n, s_n) < 0.$$

It is clear that any simulation function in the sense of Khojasteh et al. (Definition 3) is also a simulation function in the sense of Argoubi et al. (Definition 4). The converse is not true. For more details, see [4].

Example 2. *[4] Define a function* $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ *by*

$$\zeta(t, s) = \begin{cases} 1 & \text{if } (s, t) = (0, 0), \\ \lambda s - t & \text{otherwise,} \end{cases}$$

where $\lambda \in (0, 1)$*. Then* ζ *is a simulation function in the sense of Argoubi et al.*

In the following, some other examples of simulation functions in the sense of Definition 3 (see [3, 19, 31]).

(i) $\zeta(t, s) = cs - t$ for all $t, s \in [0, \infty)$ where $c \in [0, 1)$.

(ii) $\zeta(t, s) = s - \phi(s) - t$ for all $t, s \in [0, \infty)$, where $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is a lower semi-continuous function such that $\phi(t) = 0$ if and only if $t = 0$.

In this paper, we introduce the concept of (α, β)-admissible $\mathcal{Z}$-contractions with respect to ζ. We also establish the existence of fixed points for this class of mappings in metric-like spaces. Our work generalizes and extends some theorems in the literature. Two illustrated examples are given to support the obtained results.

2. Main results

First, we introduce the following.

Definition 5. *Let* (X, σ) *be a metric-like space. Given* $f : X \to X$ *and* $\alpha, \beta : X \times X \to \mathbb{R}^+$*. Such* f *is said an* (α, β)*-admissible* $\mathcal{Z}$*-contraction with respect to* ζ *if*

$$\zeta(\alpha(x, y)\beta(x, y)\sigma(fx, fy), \sigma(x, y)) \geq 0 \tag{1}$$

for all $x, y \in X$*, where* ζ *is a simulation function in the sense of Definition 3.*

Now, we introduce our main theorem.

Theorem 2. *Let (X, σ) be a complete metric-like space and let f be a self-mapping on X satisfying the following conditions:*

(i) f is (α, β)-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\beta(x_0, fx_0) \geq 1$;

(iii) f is an (α, β)-admissible $\mathcal{Z}$-contraction on (X, σ);

(iv) f is $\sigma-$continuous.

Then f has a unique fixed point $u \in X$ with $\sigma(u, u) = 0$.

Proof.
By (2), there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\beta(x_0, fx_0) \geq 1$. Define the sequence $\{x_n\}$ by $x_{n+1} = fx_n$ for all $n = 0, 1, 2, \cdots$. If $x_n = x_{n+1}$ for some n, then $x_n = x_{n+1} = fx_n$. So x_n is a fixed point of f, and the proof is completed. From now on, assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Since f is an (α, β)-admissible mapping, we derive

$$\alpha(x_0, fx_0) = \alpha(x_0, x_1) \geq 1 \Rightarrow \alpha(fx_0, fx_1) = \alpha(x_1, x_2) \geq 1.$$

Continuing in this process, we get

$$\alpha(x_n, x_{n+1}) \geq 1, \quad \text{for all } n \geq 0. \tag{2}$$

Similarly,

$$\beta(x_n, x_{n+1}) \geq 1, \quad \text{for all } n \geq 0. \tag{3}$$

From (1), (2) and (3), we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_n, x_{n-1})\beta(x_n, x_{n-1})\sigma(fx_n, fx_{n-1}), \sigma(x_n, x_{n-1})) \\ &= \zeta(\alpha(x_n, x_{n-1})\beta(x_n, x_{n-1})\sigma(x_{n+1}, x_n), \sigma(x_n, x_{n-1})) \\ &< \sigma(x_n, x_{n-1}) - \alpha(x_n, x_{n-1})\beta(x_n, x_{n-1})\sigma(x_{n+1}, x_n). \end{aligned} \tag{4}$$

Consequently, we derive that

$$\sigma(x_{n+1}, x_n) \leq \alpha(x_n, x_{n-1})\beta(x_n, x_{n-1})\sigma(x_{n+1}, x_n) < \sigma(x_n, x_{n-1}) \text{ for all } n \geq 0. \tag{5}$$

The sequence $\{\sigma(x_n, x_{n-1})\}$ is nondecreasing, so there exists $r \geq 0$ such that $\lim_{n\to\infty} \sigma(x_n, x_{n-1}) = r$. We prove that

$$\lim_{n\to\infty} \sigma(x_n, x_{n-1}) = 0. \tag{6}$$

Suppose that $r > 0$. By (5), we derive that

$$\lim_{n\to\infty} \alpha(x_n, x_{n-1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n-1}) = r. \tag{7}$$

Letting $s_n = \alpha(x_n, x_{n-1})\beta(x_n, x_{n-1})\sigma(x_n, x_{n+1})$ and $s_n = \sigma(x_n, x_{n-1})$ and taking (ζ_3) into account, we have

$$0 \leq \limsup_{n\to\infty} \zeta(\alpha(x_n, x_{n-1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n-1})) < 0, \tag{8}$$

which is a contradiction. Thus, $r = 0$.
Now, we will show that $\{x_n\}$ is a Cauchy sequence. Suppose on the contrary that $\{x_n\}$ is not a Cauchy sequence. Then, there exists ϵ for which we can find subsequences $\{x_{n_l}\}$ and $\{x_{m_l}\}$ of $\{x_n\}$ with $n_l > m_l > l$ such that for every l,

$$\sigma(x_{n_l}, x_{m_l}) \geq \epsilon \tag{9}$$

and n_l is the smallest number such that (9) holds. From (9), we get

$$\sigma(x_{n_l-1}, x_{m_l}) < \epsilon. \tag{10}$$

Using the triangular inequality and (10),

$$\begin{aligned} \epsilon &\leq \sigma(x_{n_l}, x_{m_l}) \\ &\leq \sigma(x_{n_l}, x_{n_l-1}) + \sigma(x_{n_l-1}, x_{m_l}) \\ &< \sigma(x_{n_l}, x_{n_l-1}) + \epsilon. \end{aligned}$$

Letting $n \to \infty$ in the above inequality and using (6), we obtain

$$\lim_{n\to\infty} \sigma(x_{n_l}, x_{m_l}) = \epsilon. \tag{11}$$

Also, from the triangular inequality, we have

$$\mid \sigma(x_{n_l+1}, x_{m_l}) - \sigma(x_{n_l}, x_{m_l}) \mid \leq \sigma(x_{n_l}, x_{n_l+1}).$$

On taking limit as $l \to \infty$ on both sides of above inequality and using (6) and (11), we get

$$\lim_{l\to\infty} \sigma(x_{n_l+1}, x_{m_l}) = \epsilon. \tag{12}$$

Similarly, it is easy to show that

$$\lim_{l\to\infty} \sigma(x_{n_l+1}, x_{m_l+1}) = \epsilon. \tag{13}$$

Moreover, since f is an (α, β)-admissible mapping, we have

$$\alpha(x_{n_l}, x_{m_l}) \geq 1 \text{ and} \beta(x_{n_l}, x_{m_l}) \geq 1. \tag{14}$$

By the fact f is an (α, β)-admissible $\mathcal{Z}$-contraction with respect to ζ , together with (11), (14) and (ζ_3), we get

$$0 \leq \limsup_{l\to\infty} \zeta(\alpha(x_{n_l}, x_{m_l})\beta(x_{n_l}, x_{m_l})\sigma(x_{n_l+1}, x_{m_l+1}), \sigma(x_{n_l}, x_{m_l})) < 0,$$

which is a contradiction. Hence $\{x_n\}$ is a Cauchy sequence. Owing to the fact that (X, σ) is a complete metric-like space, there exists some $u \in X$ such that

$$\lim_{n\to\infty} \sigma(x_n, u) = \sigma(u, u) = \lim_{n\to\infty} \sigma(x_n, x_m) = 0, \tag{15}$$

which implies that $\sigma(u,u)=0$. Moreover, the continuity of f implies that

$$\lim_{n\to\infty}\sigma(x_{n+1},fu)=\sigma(fx_n,fu)=\sigma(fu,fu).$$

By Lemma 1 and (15), we obtain

$$\lim_{n\to\infty}\sigma(x_{n+1},fu)=\sigma(u,fu). \tag{16}$$

Combining (15) and (16), we have $\sigma(fu,fu)=\sigma(u,fu)$, that is, $fu=u$.. To prove the uniqueness of the fixed point, suppose that there exists $w\in X$ such that $fw=w$ and $w\neq u$. Then

$$0\leq\zeta(\alpha(u,w)\beta(u,w)\sigma(fu,fw),\sigma(u,w))<\sigma(u,w)-\alpha(u,w)\beta(u,w)\sigma(fu,fw)\leq 0,$$

which is a contradiction, so $u=w$.

Theorem 2 remains true if we drop the continuity hypothesis by the following property:

(H): If $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1})\geq 1$ and $\beta(x_n,x_{n+1})\geq 1$ for all n, then there exists a subsequence $\{x_{n_l}\}$ of $\{x_n\}$ such that $\alpha(x_{n_l},x_{n_l+1})\geq 1$ and $\beta(x_{n_l},x_{n_l+1})\geq 1$ for all $l\in\mathbb{N}$ and $\alpha(x,fx)\geq 1$ and $\beta(x,fx)\geq 1$.

Theorem 3. *Let (X,σ) be a complete metric-like space and let f be a self-mapping on X satisfying the following conditions:*

(i) f is (α,β)-admissible;

(ii) there exists $x_0\in X$ such that $\alpha(x_0,fx_0)\geq 1$ and $\beta(x_0,fx_0)\geq 1$;

(iii) f is an (α,β)-admissible $\mathcal{Z}$-contraction on (X,σ);

(iv) (H) holds.

Then f has a unique fixed point $u\in X$ with $\sigma(u,u)=0$.

Proof. Following the proof of Theorem 2, we construct a sequence $\{x_n\}$ in X defined by $x_{n+1}=fx_n$, which converges to some $u\in X$. From definition (2) and (H), there exists a subsequence $\{x_{n_l}\}$ of $\{x_n\}$ such that $\alpha(x_{n_l},x_{n_l})\geq 1$ and $\beta(x_{n_l},x_{n_l})\geq 1$ for all $l\in\mathbb{N}$. Thus applying (1) for all l, we have

$$\begin{aligned}0 &\leq \zeta(\alpha(x_{n_l},u)\beta(x_{n_l},u)\sigma(fx_n,fu),\sigma(x_{n_l},u))\\ &= \zeta(\alpha(x_{n_l},u)\beta(x_{n_l},u)\sigma(x_{n+1},fu),\sigma(x_{n_l},u))\\ &< \sigma(x_{n_l},u)-\alpha(x_{n_l},u)\beta(x_{n_l},u)\sigma(x_{n_l+1},fu) \end{aligned}\tag{17}$$

which is equivalent to

$$\sigma(x_{n_l}+1,fu)=\sigma(fx_{n_l},fu)\leq\alpha(x_{n_l},u)\beta(x_{n_l},u)\sigma(fx_n,fu)\leq\sigma(x_{n_l},u). \tag{18}$$

Letting $l\to\infty$ in the above equality, we have $\sigma(u,fu)=0$. Using similar arguments as above, we can show that u is a fixed point of f. The uniqueness of the fixed point of f is obtained by similar arguments as those given in the proof of Theorem 2.

3. Consequences

In this section, we apply Theorem 2 to obtain different results known in literature. The first one is of Banach type.

Corollary 1. *Let (X, σ) be a complete metric-like space and let f be a self-mapping on X satisfying the following conditions:*

(i) f is (α, β)-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\beta(x_0, fx_0) \geq 1$;

(iii)

$$\alpha(x, y)\beta(x, y)\sigma(fx, fy) \leq \lambda\sigma(x, y),$$

for all $x, y \in X$ and $\lambda \in [0, 1)$;

(iv) f is σ-continuous.

Then f has a unique fixed point $u \in X$ with $\sigma(u, u) = 0$.

Proof. Following the lines of Theorem 2, by taking as a σ-simulation function,

$$\zeta(t, s) = \lambda s - t.$$

Corollary 2. *Let (X, σ) be a complete metric-like space and let f be a self-mapping on X satisfying the following conditions:*

(i) f is (α, β)-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$ and $\beta(x_0, fx_0) \geq 1$;

(iii) there exists a lower semi-continuous function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ with $\varphi^{-1} = \{0\}$ such that

$$\alpha(x, y)\beta(x, y)\sigma(fx, fy) \leq \sigma(x, y) - \varphi(\sigma(x, y))$$

for all $x, y \in X$;

(iv) f is σ-continuous.

Then f has a unique fixed point $u \in X$ with $\sigma(u, u) = 0$.

Proof. It suffices to take

$$\zeta(t, s) = s - \varphi(s) - t.$$

If we consider in Theorem 2, $\alpha(x, y) = \beta(x, y) = 1$ for all $x, y \in X$, we have

Corollary 3. *Let (X, σ) be a complete metric-like space and let f be a self-mapping on X. Suppose that there exists a σ-simulation function ζ such that*

$$\zeta(\sigma(fx, fy), \sigma(x, y)) \geq 0 \tag{19}$$

for all $x, y \in X$. Then f has a unique fixed point $u \in X$ with $\sigma(u, u) = 0$.

We present the following illustrated examples.

Example 3. *Let $X = [0, \infty), \sigma(x, y) = (x + y)$ for all $x, y \in X$ and $f : X \to X$ be defined by*

$$fx = \begin{cases} \frac{1}{4}x & \text{if } 0 \leq x \leq 1 \\ 4x & \text{otherwise.} \end{cases}$$

Consider

$$\zeta(s, t) = cs - t,$$

where $0 \leq \frac{1}{4} < c < 1$. Define $\alpha, \beta : X \times X \to \mathbb{R}_+$ as

$$\alpha(x, y) = \begin{cases} \frac{4}{3} & \text{if } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$\beta(x, y) = \begin{cases} \frac{3}{2} & \text{if } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

We shall prove that Corollary 1 can be applied. Clearly, (X, σ) is a complete metric-like space. Let $x, y \in X$ such that $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$. Since $x, y \in [0, 1]$ and so $fx \in [0, 1], fy \in [0, 1]$ and $\alpha(fx, fy) = 1$ and $\beta(fx, fy) = 1$. Hence f is (α, β)-admissible. Condition (2) is satisfied with $x_0 = 1$. Condition (4) is satisfied with $x_n = f^n x_1 = \frac{1}{n}$.

If $0 \leq x \leq 1$, then $\alpha(x, y) = \frac{4}{3}$ and $\beta(x, y) = \frac{3}{2}$. We have

$$\begin{aligned} \zeta(\alpha(x, y)\beta(x, y)\sigma(fx, fy), \sigma(x, y)) &= c\sigma(x, y) - \alpha(x, y)\beta(x, y)\sigma(fx, fy) \\ &= \frac{3}{4}(x + y) - 2\frac{1}{4}(x + y) \\ &= (\frac{3}{4} - \frac{1}{2})(x + y) \\ &= \frac{1}{4}(x + y) \\ &\geq 0. \end{aligned}$$

If $0 \leq x \leq 1$ and $y > 1$, then $\zeta(\alpha(x, y)\beta(x, y)\sigma(fx, fy), \sigma(x, y)) \geq 0$ since $\alpha(x, y) = \beta(x, y) = 0$. Consequently, all assumptions of Corollary 1 are satisfied and hence f has a unique fixed point, which is $u = 0$.

We also notice that (19) is not satisfied. In fact, for $x = 1, y = 2$, we get

$$\sigma(f1, f2) = (\frac{33}{4})^2 > 3 = \sigma(x, y).$$

Example 4. *Consider* $X = \{0, 1, 3\}$ *and define* $\sigma : X \times X \to \mathbb{R}^+$ *as follows:*
$\sigma(0,0) = 0, \sigma(1,0) = \sigma(0,1) = \frac{1}{10}, \sigma(0,3) = \sigma(3,0) = \frac{1}{2}, \sigma(1,3) = \sigma(3,1) = \frac{2}{3}, \sigma(1,1) = \frac{1}{2}, \sigma(3,3) = \frac{7}{2}$.
Note that $\sigma(3,3) \neq 0$, *so* (X,σ) *is not a metric and* $\sigma(3,3) > \sigma(0,3)$, *so* (X,σ) *is not a partial metric. Clearly,* (X,σ) *is metric-like space. Let* $f : X \to X$ *be defined by* $f0 = f1 = 0$ *and* $f3 = 1$. *Take* $\alpha, \beta : X \times X \to \mathbb{R}^+$ *given as*

$$\alpha(x,y) = \begin{cases} \frac{5}{2}, & \text{if } x \in \{0,1,3\}, \\ 0, & \text{otherwise,} \end{cases}$$

$$\beta(x,y) = \begin{cases} 1, & \text{if } x \in \{0,1,3\}, \\ 0, & \text{otherwise.} \end{cases}$$

Take $\zeta : X \times X \to \mathbb{R}^+$ *by* $\zeta(t,s) = \frac{1}{2}s - t$. *Let* $x, y \in X$ *be such that* $\alpha(x,y) \geq 1$ *and* $\beta(x,y) \geq 1$, *then* $\alpha(fx,fy) \geq 1$ *and* $\beta(fx,fy) \geq 1$, *that is,* f *is* (α,β)*-admissible. Now, we consider the following cases:*

(i) Case 1: $x = 0$ *and* $y = 0$. *We have*

$$\zeta(\alpha(0,0)\beta(0,0)\sigma(f0,f0),\sigma(0,0)) = \zeta(\frac{5}{2}.1.0,0) = \zeta(0,0) = 0.$$

(ii) Case 2: $x = 0$ *and* $y = 1$. *Here,*

$$\zeta(\alpha(0,1)\beta(0,1)\sigma(f0,f1),\sigma(0,1)) = \zeta(\frac{5}{2}.1.0,1) = \zeta(0,\frac{1}{10}) = \frac{1}{20} > 0.$$

(iii) Case 3: $x = 0$ *and* $y = 3$. *We have*

$$\zeta(\alpha(0,3)\beta(0,3)\sigma(f0,f3),\sigma(0,3)) = \zeta(\frac{5}{2}.\frac{1}{10},\frac{1}{2}) = \zeta(\frac{1}{4},\frac{1}{2}) = 0.$$

(iv) Case 4: $x = 1$ *and* $y = 1$. *Here,*

$$\zeta(\alpha(1,1)\beta(1,1)\sigma(f1,f1),\sigma(1,1)) = \zeta(\frac{5}{2}.1.0,\frac{1}{2}) = \zeta(0,\frac{1}{2}) = \frac{1}{4} > 0.$$

(v) Case 5: $x = 1$ *and* $y = 3$. *We have*

$$\zeta(\alpha(1,3)\beta(1,3)\sigma(f1,f3),\sigma(1,3)) = \zeta(\frac{5}{2}.1.\frac{1}{10},\frac{2}{3}) = \zeta(\frac{1}{4},\frac{2}{3}) = \frac{1}{12} > 0.$$

(vi) Case 6: $x = 3$ *and* $y = 3$. *Here,*

$$\zeta(\alpha(3,3)\beta(3,3)\sigma(f3,f3),\sigma(3,3)) = \zeta(\frac{5}{2}.1.\frac{1}{2},\frac{7}{2}) = \zeta(\frac{5}{4},\frac{7}{2}) = \frac{1}{2} > 0.$$

Thus, f *is an* (α,β)*-admissible* $\mathcal{Z}$*-contraction with respect to* ζ. *Hence all conditions of Theorem 2 are satisfied and* f *has a unique fixed point, which is,* $u = 0$.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the manuscript.

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