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$\mathcal{N}$-soft p-ideals of BCI-algebras

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Abstract. In this paper, using the notions of soft sets and $\mathcal{N}$-structures, the notion of $\mathcal{N}$-soft p-ideals in BCI-algebras is introduced, and related properties are investigated. Furthermore, relations between $\mathcal{N}$-soft ideals and $\mathcal{N}$-soft p-ideals are discussed. Finally, conditions for an $\mathcal{N}$-soft ideal to be an $\mathcal{N}$-soft p-ideal are established.

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1. Introduction

Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own limitations which are pointed out in [15]. Maji et al. [13] and Molodtsov [15] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [15] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. He pointed out several directions for the applications of soft sets. Later on, Maji et al. [13] described the application of soft set theory to a decision making problem. Maji et al. [12] also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [21]. Roy et al. [20] presented some results on an application of fuzzy soft sets in decision making problem.

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Aygünoğlu et al. [2] introduced the notion of fuzzy soft group and studied its properties. Ali et al. [3] discussed new operations in soft set theory. Jun [6] applied the notion of soft set to BCK/BCI-algebras, and Jun et al. [8] considered applications of soft set theory in the ideals of d-algebras. Also, Muhiuddin et al. studied the soft set theory on various aspects (see for e.g., [1], [16], [17]), [18], [19]).

A (crisp) set A in a universe X can be defined in the form of its characteristic function $\mu_A : X \to \{0, 1\}$ yielding the value 1 for elements belonging to the set A and the value 0 for elements excluded from the set A. So far most of the generalization of the crisp set have been conducted on the unit interval $[0, 1]$ and they are consistent with the asymmetry observation. In other words, the generalization of the crisp set to fuzzy sets relied on spreading positive information that fit the crisp point $\{1\}$ into the interval $[0, 1]$. Because no negative meaning of information is suggested, so one should be interested to deal with negative information and to supply a mathematical tool for the same. Considering this fact, Jun et al. [9] introduced a new function which is called negative-valued function, and constructed $\mathcal{N}$-structures. They applied $\mathcal{N}$-structures to BCK/BCI-algebras, and discussed $\mathcal{N}$-subalgebras and $\mathcal{N}$-ideals in BCK/BCI-algebras. Jun et al. [10] considered closed ideals in BCH-algebras based on $\mathcal{N}$-structures. Jun et al. [11] introduced the notion of $\mathcal{N}$-soft sets which are a soft set based on $\mathcal{N}$-structures, and then they applied it to both a decision making problem and a BCK/BCI-algebra. Jun et al [7] introduced the notion of (closed) $\mathcal{N}$-ideal over a BCI-algebra based on soft sets and $\mathcal{N}$-structures, and investigated related properties. They established relations between $\mathcal{N}$-BCI-algebras and $\mathcal{N}$-ideals. They also provided characterizations of a (closed) $\mathcal{N}$-ideal over a BCI-algebra, and considered conditions for an $\mathcal{N}$-ideal to be an $\mathcal{N}$-BCI-algebra.

In this paper, we apply the soft sets and $\mathcal{N}$-structures to p-ideals in BCI-algebras. We introduce the notion of $\mathcal{N}$-soft p-ideals in BCI-algebras, and investigate related properties. We provide relations between $\mathcal{N}$-soft ideals and $\mathcal{N}$-soft p-ideals, and establish conditions for an $\mathcal{N}$-soft ideal to be an $\mathcal{N}$-soft p-ideal.

2. Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)$ $(((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)$ $((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)$ $(x * x = 0)$,

(IV) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

Define a binary relation $\leq$ on X by letting $x * y = 0$ if and only if $x \leq y$. Then $(X, \leq)$ is a partially ordered set. A BCI-algebra X satisfying $0 \leq x$ for all $x \in X$, is called BCK-algebra.

Theorem 1. *Let X be a BCI-algebra. Then following hold*

(a1) $(\forall x \in X)\ ((x * 0 = x))$,

(a2) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$,

(a3) $(\forall x \in X)\ (0 * (0 * (0 * x)) = 0 * x)$,

(a4) $(\forall x, y, z \in X)\ (0 * (0 * ((x * z) * (y * z))) = (0 * y) * (0 * x))$,

(a5) $(\forall x, y \in X)\ (0 * (0 * (x * y)) = (0 * y) * (0 * x))$

A non-empty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset A of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies:

(c1) $0 \in A$,

(c2) $(\forall x, y \in X)\,(x * y \in A,\ y \in A \Rightarrow x \in A)$.

We refer the reader to the books [5, 14] for further information regarding BCK/BCI-algebras.

For any family $\{a_i \mid i \in \Lambda\}$ of real numbers, we define

$$\bigvee\{a_i \mid i \in \Lambda\} := \begin{cases} \max\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \sup\{a_i \mid i \in \Lambda\} & \text{otherwise.} \end{cases}$$

$$\bigwedge\{a_i \mid i \in \Lambda\} := \begin{cases} \min\{a_i \mid i \in \Lambda\} & \text{if } \Lambda \text{ is finite,} \\ \inf\{a_i \mid i \in \Lambda\} & \text{otherwise.} \end{cases}$$

Denote by $F(X, [-1, 0])$ the collection of functions from a set X to $[-1, 0]$. We say that an element of $F(X, [-1, 0])$ is a *negative-valued function* from X to $[-1, 0]$ (briefly, $\mathcal{N}$-*function* on X). By an $\mathcal{N}$-*structure* we mean an ordered pair (X, η) of X and an $\mathcal{N}$-function η on X.

Definition 1 ([9])**.** *By a subalgebra of a BCK/BCI-algebra X based on $\mathcal{N}$-function η (briefly, $\mathcal{N}$-subalgebra of X), we mean an $\mathcal{N}$-structure (X, η) in which η satisfies the following assertion:*

$$(\forall x, y \in X)\left(\eta(x * y) \leq \bigvee \{\eta(x), \eta(y)\}\right). \tag{1}$$

Definition 2 ([9]). *By an ideal of a BCK/BCI-algebra X based on $\mathcal{N}$-function η (briefly, $\mathcal{N}$-ideal of X), we mean an $\mathcal{N}$-structure (X, η) in which η satisfies the following assertion:*

$$(\forall x, y \in X)\left(\eta(0) \leq \eta(x) \leq \bigvee \{\eta(x * y), \eta(y)\}\right). \tag{2}$$

3. p-ideals based on $\mathcal{N}$-soft sets

In what follows let E denote a set of attributes unless otherwise specified. We will use the terminology "soft machine" which means that it produces a BCI-algebra, that is, consider a soft machine "$\uplus(-,-)$" for which $\uplus(x, y) = z$ means that if we input a couple (x, y) of informations to $\uplus(-,-)$ then we get a new information z.

Definition 3 ([11]). *Let X be an initial universe set. By an $\mathcal{N}$-soft set over X we mean a pair (η, A) where $A \subset E$ and η is a mapping from A to $F(X, [-1, 0])$, i.e., for each $a \in A$, $\eta(a) := \eta_a$ is an $\mathcal{N}$-function on X.*

Denote by $\mathcal{N}(X, E)$ the collection of all $\mathcal{N}$-soft sets over X with attributes from E and we call it an $\mathcal{N}$-soft class.

Definition 4 ([11]). *Let (η, A) be an $\mathcal{N}$-soft set over a BCK/BCI-algebra X where A is a subset of E. If there exists an attribute $u \in A$ for which the $\mathcal{N}$-structure (X, η_u) is an $\mathcal{N}$-subalgebra of X, then we say that (η, A) is an $\mathcal{N}$-soft BCK/BCI-algebra related to the attribute u (briefly, $\mathcal{N}_u$-soft BCK/BCI-algebra). If (η, A) is an $\mathcal{N}_u$-soft BCK/BCI-algebra for all $u \in A$, we say that (η, A) is an $\mathcal{N}$-soft BCK/BCI-algebra.*

Definition 5 ([7]). *Let (η, A) be an $\mathcal{N}$-soft set over a BCK/BCI-algebra X where A is a subset of E. If there exists an attribute $u \in A$ for which the $\mathcal{N}$-structure (X, η_u) is an $\mathcal{N}$-ideal of X, then we say that (η, A) is an $\mathcal{N}$-soft ideal of X related to the attribute u (briefly, $\mathcal{N}_u$-soft ideal). If (η, A) is an $\mathcal{N}_u$-soft ideal of X for all $u \in A$, we say that (η, A) is an $\mathcal{N}$-soft ideal over X.*

Definition 6. *By a p-ideal of a BCI-algebra X based on $\mathcal{N}$-function ψ (briefly, p$\mathcal{N}$-ideal of X), we mean an $\mathcal{N}$-structure (X, ψ) in which ψ satisfies the following assertions:*

(i) $(\forall x \in X)\,(\psi(0) \leq \psi(x))$,

(ii) $(\forall x, y, z \in X)\,(\psi(x) \leq \bigvee \{\psi((x * z) * (y * z)), \psi(y)\})$.

Definition 7. *Let (η, A) be an $\mathcal{N}$-soft set over a BCI-algebra X where A is a subset of E. If there exists an attribute $u \in A$ for which the $\mathcal{N}$-structure (X, η_u) is a p$\mathcal{N}$-ideal of X, then we say that (η, A) is an $\mathcal{N}$-soft p-ideal of X related to the attribute u (briefly, $\mathcal{N}_u$-soft p-ideal). If (η, A) is an $\mathcal{N}_u$-soft p-ideal of X for all $u \in A$, we say that (η, A) is an $\mathcal{N}$-soft p-ideal over X.*

Example 1. *Let U be a initial universe set consists of 'white', 'reddish', 'green' and 'yellow'. The soft machine "$\uplus(-,-)$" is equipped as follows:*

Table 1: Tabular representation of (η, A)

(η, A)	white	reddish	green	yellow
beautiful	-0.8	-0.7	-0.3	-0.3
fine	-0.6	-0.5	-0.4	-0.4
smart	-0.7	-0.5	-0.1	-0.1

$$\uplus(x,y) = y \quad \textit{if } x = \text{white} \textit{ and } y \in U,$$
$$\uplus(x,y) = \begin{cases} \text{reddish} & \text{if } (x,y) = (\text{reddish}, \text{white}), \\ \text{white} & \text{if } (x,y) = (\text{reddish}, \text{reddish}), \\ \text{yellow} & \text{if } (x,y) = (\text{reddish}, \text{green}), \\ \text{green} & \text{if } (x,y) = (\text{reddish}, \text{yellow}), \end{cases}$$
$$\uplus(x,y) = \begin{cases} \text{green} & \text{if } (x,y) = (\text{green}, \text{white}), \\ \text{yellow} & \text{if } (x,y) = (\text{green}, \text{redish}), \\ \text{white} & \text{if } (x,y) = (\text{green}, \text{green}), \\ \text{reddish} & \text{if } (x,y) = (\text{green}, \text{yellow}), \end{cases}$$
$$\uplus(x,y) = \begin{cases} \text{yellow} & \text{if } (x,y) = (\text{yellow}, \text{white}), \\ \text{green} & \text{if } (x,y) = (\text{yellow}, \text{redish}), \\ \text{reddish} & \text{if } (x,y) = (\text{yellow}, \text{green}), \\ \text{white} & \text{if } (x,y) = (\text{yellow}, \text{yellow}). \end{cases}$$

Then the soft machine "$\uplus(-,-)$" makes U into a BCI-algebra. Consider a set of attributes:

$$A := \{\text{beautiful}, \text{fine}, \text{smart}\},$$

and let (η, A) be an $\mathcal{N}$-soft sets over U with the tabular representations which is given by Table 1. Then the $\mathcal{N}$-structures $(U, \eta_{\text{beautiful}})$, (U, η_{fine}) and (U, η_{smart}) are p$\mathcal{N}$-ideals of U. Hence (η, A) is an $\mathcal{N}$-soft p-ideal over U.

Proposition 1. *For any attribute $u \in A$, every $\mathcal{N}_u$-soft p-ideal (η, A) over a BCI-algebra X satisfies the following inequality:*

$$(\forall x \in X)\,(\eta_u(x) \leq \eta_u\,(0 * (0 * x)))\,. \tag{3}$$

Proof. Using Definition 6(ii), we have

$$\eta_u(x) \leq \bigvee \{\eta_u((x * z) * (y * z)), \eta_u(y)\} \tag{4}$$

for all $x, y, z \in X$. If we substitute x for z, and 0 for y in (4), then

$$\begin{aligned} \eta_u(x) &\leq \bigvee \{\eta_u((x * x) * (0 * x)), \eta_u(0)\} \\ &= \bigvee \{\eta_u(0 * (0 * x)), \eta_u(0)\} \\ &= \eta_u(0 * (0 * x)) \end{aligned}$$

by using (III) and Definition 6(i). This completes the proof.

Corollary 1. *Every $\mathcal{N}$-soft p-ideal (η, A) over a BCI-algebra X satisfies the following inequality:*

$$(\forall x \in X)\,(\eta(x) \leq \eta(0 * (0 * x)))\,.$$

Theorem 2. *For any attribute $u \in A$, every $\mathcal{N}_u$-soft p-ideal over a BCI-algebra X is an $\mathcal{N}_u$-soft ideal over X.*

Proof. Let (η, A) be an $\mathcal{N}_u$-soft p-ideal over a BCI-algebra X. Since $x * 0 = x$ for all $x \in X$, it follows from Definition 6(ii) and (a1) that

$$\eta_u(x) \leq \bigvee \{\eta_u((x * 0) * (y * 0)), \eta_u(y)\} = \bigvee \{\eta_u(x * y), \eta_u(y)\}$$

for all $x, y \in X$. Therefore (η, A) is an $\mathcal{N}_u$-soft ideal over X.

Corollary 2. *Every $\mathcal{N}$-soft p-ideal over a BCI-algebra X is an $\mathcal{N}$-soft ideal over X.*

The converse of Theorem 2 is not true as seen in the following example.

Example 2. *Let U be an initial universe set consists of 'white', 'blackish', 'reddish', 'green' and 'yellow'. The soft machine "$\uplus(-,-)$" is equipped as follows:*

$$\uplus(x,y) = x \quad \text{if } x \in U \text{ and } y = \text{blackish},$$

$$\uplus(x,y) = \begin{cases} \text{blackish} & \text{if } (x,y) \in \{(\text{blackish}, \text{reddish}), (\text{reddish}, \text{reddish})\}, \\ x & \text{if } (x,y) \in \{(\text{green}, \text{reddish}), (\text{yellow}, \text{reddish}), (\text{white}, \text{reddish})\}, \end{cases}$$

$$\uplus(x,y) = \begin{cases} \text{white} & \text{if } (x,y) \in \{(\text{blackish}, \text{green}), (\text{reddish}, \text{green})\}, \\ \text{blackish} & \text{if } (x,y) = (\text{green}, \text{green}), \\ \text{green} & \text{if } (x,y) = (\text{yellow}, \text{green}), \\ \text{yellow} & \text{if } (x,y) = (\text{white}, \text{green}), \end{cases}$$

$$\uplus(x,y) = \begin{cases} \text{yellow} & \text{if } (x,y) \in \{(\text{blackish}, \text{yellow}), (\text{reddish}, \text{yellow})\}, \\ \text{white} & \text{if } (x,y) = (\text{green}, \text{yellow}), \\ \text{blackish} & \text{if } (x,y) = (\text{yellow}, \text{yellow}), \\ \text{green} & \text{if } (x,y) = (\text{white}, \text{yellow}), \end{cases}$$

$$\uplus(x,y) = \begin{cases} \text{green} & \text{if } (x,y) \in \{(\text{blackish}, \text{white}), (\text{reddish}, \text{white})\}, \\ \text{yellow} & \text{if } (x,y) = (\text{green}, \text{white}), \\ \text{white} & \text{if } (x,y) = (\text{yellow}, \text{white}), \\ \text{blackish} & \text{if } (x,y) = (\text{white}, \text{white}). \end{cases}$$

Then the soft machine "$\uplus(-,-)$" makes U into a BCI-algebra. Consider a set of attributes:

$$A := \{\text{beautiful}, \text{fine}, \text{moderate}\},$$

and let (η, A) be an $\mathcal{N}$-soft sets over U with the tabular representation which is given by Table 2. Then (η, A) is an $\mathcal{N}_{\text{fine}}$-soft ideal over U, but it is not an $\mathcal{N}_{\text{fine}}$-soft p-ideal over U since

$$\begin{aligned} \eta_{\text{fine}}(\text{reddish}) &= -0.5 > -0.7 = \eta_{\text{fine}}(\text{blackish}) \\ &= \bigvee \{\eta_{\text{fine}}(\uplus(\uplus(\text{reddish}, \text{green}), \uplus(\text{blackish}, \text{green}))), \eta_{\text{fine}}(\text{blackish})\}\,. \end{aligned}$$

Table 2: Tabular representation of (η, A)

(η, A)	blackish	reddish	green	yellow	white
beautiful	-0.9	-0.2	-0.4	-0.6	-0.1
fine	-0.7	-0.5	-0.2	-0.3	-0.4
moderate	-0.8	-0.4	-0.3	-0.2	-0.5

Proposition 2. *For any attribute $u \in A$, every $\mathcal{N}_u$-soft p-ideal (η, A) over a BCI-algebra X satisfies the following inequality:*

$$(\forall x, y, z \in X)\left(\eta_u(x * y) \geq \eta_u\left((x * z) * (y * z)\right)\right). \tag{5}$$

Proof. Let $u \in A$. If (η, A) is an $\mathcal{N}_u$-soft p-ideal over a BCI-algebra X, then it is an $\mathcal{N}_u$-soft ideal over X by Theorem 2. Note that $((x * z) * (y * z)) * (x * y) = 0$ for all $x, y, z \in X$. Thus we have

$$\begin{aligned}\eta_u\left((x * z) * (y * z)\right) &\leq \bigvee\left\{\eta_u\left(((x * z) * (y * z)) * (x * y)\right), \eta_u(x * y)\right\} \\ &= \bigvee\left\{\eta_u(0), \eta_u(x * y)\right\} = \eta_u(x * y)\end{aligned}$$

for all $x, y, z \in X$.

We provide conditions for an $\mathcal{N}$-soft ideal to be an $\mathcal{N}$-soft p-ideal.

Theorem 3. *For any attribute $u \in A$, let (η, A) be an $\mathcal{N}_u$-soft ideal over a BCI-algebra X that satisfies:*

$$(\forall x, y, z \in X)\left(\eta_u\left(x * y\right) \leq \eta_u\left((x * z) * (y * z)\right)\right). \tag{6}$$

Then (η, A) is an $\mathcal{N}_u$-soft p-ideal over X.

Proof. If an $\mathcal{N}_u$-soft ideal (η, A) over a BCI-algebra X satisfies (6), then

$$\eta_u(x) \leq \bigvee\left\{\eta_u(x * y), \eta_u(y)\right\} \leq \bigvee\left\{\eta_u\left((x * z) * (y * z)\right), \eta_u(y)\right\}$$

for all $x, y, z \in X$. Therefore (η, A) is an $\mathcal{N}_u$-soft p-ideal over X.

Lemma 1 ([7])**.** *For any attribute $u \in A$, every $\mathcal{N}_u$-soft ideal (η, A) over a BCI-algebra X satisfies the following inequality:*

$$(\forall x \in X)\left(\eta_u(0 * (0 * x)) \leq \eta_u(x)\right). \tag{7}$$

Theorem 4. *For any attribute $u \in A$, let (η, A) be an $\mathcal{N}_u$-soft ideal over a BCI-algebra X that satisfies:*

$$(\forall x \in X)\left(\eta_u(x) \leq \eta_u\left(0 * (0 * x)\right)\right). \tag{8}$$

Then (η, A) is an $\mathcal{N}_u$-soft p-ideal over X.

Proof. By using Lemma 1, (a4), (a5) and (8), we have

$$\begin{aligned}\eta_u\left((x*z)*(y*z)\right) &\geq \eta_u\left(0*(0*((x*z)*(y*z)))\right)\\ &= \eta_u\left((0*y)*(0*x)\right)\\ &= \eta_u\left(0*(0*(x*y))\right)\\ &\geq \eta_u(x*y)\end{aligned}$$

for all $x, y, z \in X$. It follows from Theorem 3 that (η, A) is an $\mathcal{N}_u$-soft p-ideal over X.

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