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Some Properties of Fuzzy Supra Soft Topological Spaces

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Abstract. In this paper, we introduce the notion of fuzzy supra soft topological spaces, which is a generalization to the notion of fuzzy soft topological spaces and supra soft topological spaces. Also, we consider the notion of fuzzy supra soft continuity as a generalization to fuzzy soft continuity, supported by examples and counterexamples. These examples illustrating the notions used in the paper are included. So we can see that all these concepts are independent from each other or does implies the other. Finally, as a direct application to fuzzy supra soft topological spaces, we introduce the notion of fuzzy supra soft compact (resp. fuzzy supra soft lindelöf) spaces to such spaces as a generalization to fuzzy soft compactness. Furthermore, we establish some interesting properties of this notion.

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1. Introduction

Many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets etc. can be considered as mathematical tools for dealing with uncertain data, obtained in various fields of engineering, physics, computer science, economics, social science, medical science, and of many other diverse fields. But all these theories have their own difficulties. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets, introduced by Zadeh [32] in 1965. This theory brought a paradigmatic change in mathematics. But, there exists difficulty, how to set the membership function in each particular case.

The theory of intuitionistic fuzzy sets is a more generalized concept than the theory of fuzzy sets, but this theory has the same difficulties. All the above mentioned theories are successful to some extent in dealing with problems arising due to vagueness present in the real world. But there are also cases where these theories failed to give satisfactory

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results, possibly due to inadequacy of the parametrization tool in them. As a necessary supplement to the existing mathematical tools for handling uncertainty, in 1999, Molodtsov [26] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling the problems in engineering, physics, computer science, economics, social sciences, and medical sciences. In 1968, Chang [14] introduced fuzzy topological space and in 2011, subsequently Cagman et al. [13] and Shabir et al. [31] introduced soft topological spaces and they defined basic notions of soft topological spaces. In 2011, Tanay et al. [10] introduced the notion of fuzzy soft topological spaces, which is extended in [15, 21, 28, 29]. Recently, Some researchers [11, 12, 17, 24, 27, 30] studied on the fuzzy soft compact topological spaces.

In 1984, Mashhour et al. [25] introduced supra topological space, subsequently El-sheikh et al. [16] introduced supra soft topological spaces and they defined basic notions of supra soft topological spaces, which is extended in [4, 5, 20]. In 1987, Abd El-Monsef et al. introduced fuzzy supra topological space and defined basic notions of fuzzy supra topological spaces.

Our aim of this paper, is to introduce the notion of fuzzy supra soft topological spaces, which is a generalization to the notion of fuzzy soft topological spaces [21] and supra soft topological spaces [16]. Also, we consider the notion of fuzzy supra soft continuity as a generalization to fuzzy soft continuity [8], fuzzy semi-soft continuity [19], fuzzy pre-soft continuity [1], fuzzy α-soft continuity [2], fuzzy β-soft continuity [3] and fuzzy b-soft continuity [6], supported by examples and counterexamples. We also introduce and study the concepts of fuzzy supra open (resp. closed) soft functions a generalization to fuzzy open (resp. closed) soft functions [30]. Finally, we introduce the notion of fuzzy supra soft compact (resp. fuzzy supra soft lindelöf) spaces as a generalization to such introduced in [11, 12, 17, 24, 27, 30]. We also introduce some basic definitions of fuzzy supra soft compact spaces and theorems of the concept.

2. Preliminaries

From the literature, we recall the following definitions and results for the development of fuzzy soft set theory and fuzzy soft topological spaces, which will be needed in this paper.

Definition 1. *[32] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. Here, I^X denotes the family of all fuzzy sets on X.*

Definition 2. *[26] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A), denoted by F_A, is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e*
$F_A = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 3. *[22] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X , where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$, where $\overline{0}$ is the membership function of null fuzzy set over X, which takes value 0 for all $x \in X$ i.e $\overline{0}(e) = 0 \ \forall \ x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_A$.*

Definition 4. *[29] The complement of a fuzzy soft set (f, A), denoted by $(f, A)^c$, is defined by $(f, A)^c = (f^c, A)$, $f^c_A : E \to I^X$ is a mapping given by $\mu^e_{f^c_A} = \overline{1} - \mu^e_{f_A} \ \forall \, e \in A$, where $\overline{1}(e) = 1 \ \forall \ x \in X$. Clearly $(f^c_A)^c = f_A$.*

Definition 5. *[23] A fuzzy soft set f_A over X is said to be a null fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \overline{0}$.*

Definition 6. *[23] A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A$, $f_A(e) = \overline{1}$, where $\overline{1}$ is the membership function of absolute fuzzy set over X, which takes value 1 for all for all $x \in X$. Clearly, we have $(\tilde{1}_A)^c = \tilde{0}_A$ and $(\tilde{0}_A)^c = \tilde{1}_A$.*

Definition 7. *[29] Let f_A, $g_B \in FSS(X)_E$. Then, f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \subseteq \mu^e_{g_B} \ \forall \ e \in A$, i.e. $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x) \ \forall \ x \in X$ and $\forall \ e \in A$.*

Definition 8. *[29]. The union of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C$,
$h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B} \ \forall e \in C$. Here, we write $h_C = f_A \sqcup g_B$.*

Definition 9. *[29]. The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cap B$ and for all $e \in C$,
$h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \wedge \mu^e_{g_B} \ \forall e \in C$. Here, we write $h_C = f_A \sqcap g_B$.*

Definition 10. *[10]. Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T}$ is called a fuzzy soft topology on X if*

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, *where* $\tilde{0}_E(e) = \overline{0}$ *and* $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,

(2) *The union of any members of $\mathfrak{T}$, belongs to $\mathfrak{T}$,*

(3) *The intersection of any two members of $\mathfrak{T}$, belongs to $\mathfrak{T}$.*

The triplet $(X, \mathfrak{T}, E)$ is called a fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called a fuzzy open soft in $(X, \mathfrak{T}, E)$. We denote the set of all fuzzy open soft sets by $FOS(X, \mathfrak{T}, E)$, or $FOS(X)$.

Definition 11. *[10] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, if its relative complement f^c_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X, \mathfrak{T}, E)$, or $FCS(X)$.*

Definition 12. *[28] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,*
$Fcl(f_A) = \sqcap\{h_D : h_D \text{ is fuzzy closed soft set and } f_A \sqsubseteq h_D\}$.
The fuzzy soft interior of g_B, denoted by $Fint(g_B)$ is the fuzzy soft union of all fuzzy open soft subsets of g_B.i.e.,
$Fint(g_B) = \sqcup\{h_D : h_D \text{ is fuzzy open soft set and } h_D \sqsubseteq g_B\}$.

Definition 13. *[21] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if for the element $e \in E$, $\mu^e_{f_A}(x) \neq \overline{0}$ and $\mu^{e'}_{f_A}(x) = \overline{0}$ for each $e' \in E - \{e\}$, and this fuzzy soft point is denoted by f^e_A.*

Definition 14. *[21] The fuzzy soft point f^e_A is said to be belonging to the fuzzy soft set g_B, denoted by $f^e_A \tilde{\in} g_B$, if for the element $e \in A \cap B$, $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x)$.*

Definition 15. *[21] A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called a fuzzy soft neighborhood of the fuzzy soft point f^e_A if there exists a fuzzy open soft set h_C such that $f^e_A \tilde{\in} h_C \sqsubseteq g_B$. A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called a fuzzy soft neighborhood of the fuzzy soft set k_D if there exists a fuzzy open soft set h_C such that $k_D \sqsubseteq h_C \sqsubseteq g_B$. The fuzzy soft neighborhood system of the fuzzy soft point f^e_A, denoted by $N_{\mathfrak{T}}(f^e_A)$, is the family of all its fuzzy soft neighborhoods.*

Definition 16. *[21] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h^Y_E be a fuzzy soft set over (Y, E) such that $h^Y_E : E \to I^Y$ such that $h^Y_E(e) = \mu^e_{h^Y_E}$, where*

$$\mu^e_{h^Y_E}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h^Y_E \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y, E)$is called fuzzy soft subspace topology for (Y, E) and $(Y, \mathfrak{T}_Y, E)$ is called fuzzy soft subspace of $(X, \mathfrak{T}, E)$. If $h^Y_E \in \mathfrak{T}$ (resp. $h^Y_E \in \mathfrak{T}^c$), then $(Y, \mathfrak{T}_Y, E)$ is called fuzzy open (resp. closed) soft subspace of $(X, \mathfrak{T}, E)$.

Definition 17. *[8] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then, the map f_{pu} is called a fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,*

(1) *If $f_A \in FSS(X)_E$. Then, the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\, k \in p(E)$, $\forall\, y \in Y$,*

$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} [\vee_{p(e)=k}(f_A(e))](x), & x \in u^{-1}(y), \\ 0, & otherwise. \end{cases}$$

(2) *If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f^{-1}_{pu}(g_B)$, where $\forall\, e \in p^{-1}(K)$, $\forall\, x \in X$,*

$$f^{-1}_{pu}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)), & p(e) \in B, \\ 0, & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 18. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be two fuzzy soft topological spaces and $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a fuzzy soft mapping. Then, f_{pu} is called*

(1) *[8]Fuzzy soft continuous if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \ \forall \ g_B \in \mathfrak{T}_2$.*

(2) *[15]Fuzzy open soft if $f_{pu}(g_A) \in \mathfrak{T}_2 \forall \ g_A \in \mathfrak{T}_1$.*

() *[15]Fuzzy closed soft if $f_{pu}(h_B) \in \mathfrak{T^c}_2 \forall \ h_B \in \mathfrak{T^c}_1$.*

Theorem 1. *[8] Let $FSS(X)_E$ and $FSS(Y)_K$ be two families of fuzzy soft sets. For the fuzzy soft function $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$, the following statements hold,*

(a) $f_{pu}^{-1}((g_B)^c) = (f_{pu}^{-1}(g_B))^c \forall \ g_B \in FSS(Y)_K$.

(b) $f_{pu}(f_{pu}^{-1}((g_B))) \sqsubseteq g_B \forall \ g_B \in FSS(Y)_K$. *If f_{pu} is surjective, then the equality holds.*

(c) $f_A \sqsubseteq f_{pu}^{-1}(f_{pu}(f_A)) \forall \ f_A \in FSS(X)_E$. *If f_{pu} is injective, then the equality holds.*

(d) $f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. *If f_{pu} is surjective, then the equality holds.*

(e) $f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ *and* $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(f) *If $f_A \sqsubseteq g_A$, then $f_{pu}(f_A) \sqsubseteq f_{pu}(g_A)$.*

(g) *If $f_B \sqsubseteq g_B$, then $f_{pu}^{-1}(f_B) \sqsubseteq f_{pu}^{-1}(g_B) \ \forall \ f_B, g_B \in FSS(Y)_K$.*

3. Fuzzy Supra Soft Topological Spaces

Our aim of this section, is to introduce the notion of fuzzy supra soft topological spaces, which is a generalization to the notion of fuzzy soft topological spaces [21] and supra soft topological spaces [16]. We introduce and study the properties of the operations which will help researcher enhance and promote the further study on fuzzy supra soft topology.Furthermore, we list the main properties of the operations which give the deviations between these operations and that in fuzzy soft topological spaces.

Definition 19. *[29]. Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T}$ is called a fuzzy supra soft topology on X if*

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, *where* $\tilde{0}_E(e) = \overline{0}$ *and* $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,

(2) *The union of any members of $\mathfrak{T}$, belongs to $\mathfrak{T}$.*

The triplet $(X, \mathfrak{T}, E)$ is called a fuzzy supra soft topological space (fssts for short) over X. Also, each member of $\mathfrak{T}$ is called a fuzzy supra open soft in $(X, \mathfrak{T}, E)$. A fuzzy soft set f_A over X is said to be fuzzy supra closed soft set in X, if its relative complement f_A^c is a fuzzy supra open soft set. We denote the set of all fuzzy supra open (closed) soft sets by $FSOS(X)$ $(FSCS(X))$.

Remarks 1. *Every fuzzy soft topological space is a fuzzy supra soft topological space, but the converse is not true in general as will shown in the following example.*

Example 1. *Let $X = \{a, b, c\}$ be the set of three cars under consideration and $E = \{e_1(Modern\ Technology), e_2(Luxurious), e_3(Costly)\}$. Let $A, B \subseteq E$ where $A = \{e_1, e_2\}$ and $B = \{e_2, e_3\}$. Let $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_{1A}, f_{2B}, f_{3E}\}$ where f_{1A}, f_{2B}, f_{3E} are fuzzy soft sets over X representing the "attractiveness of the cars" which Mr. R, Mr. S and Mr. T are going to buy, respectively which defined as follows:*
$\mu^{e_1}_{f_{1A}} = \{a_{0.6}, b_{0.75}, c_{0.3}\}$, $\mu^{e_2}_{f_{1A}} = \{a_{0.5}, b_{0.8}, c_{0.7}\}$,
$\mu^{e_2}_{f_{2B}} = \{a_{0.4}, b_{0.6}, c_{0.3}\}$, $\mu^{e_3}_{f_{2B}} = \{a_{0.3}, b_{0.35}, c_{0.45}\}$,
$\mu^{e_1}_{f_{3E}} = \{a_{0.6}, b_{0.75}, c_{0.3}\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.5}, b_{0.8}, c_{0.7}\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.3}, b_{0.35}, c_{0.45}\}$.
Then, $\mathfrak{T}$ is a fuzzy supra soft topology on X, but not fuzzy soft topology, where $f_{1A} \sqcap f_{2B} \notin \mathfrak{T}$.

Definition 20. *Let $(X, \mathfrak{T}^*, E)$ be a fuzzy soft topological space and $(X, \mathfrak{T}, E)$ be a fssts. We say that $\mathfrak{T}$ is a sfsst associated with $\mathfrak{T}^*$ if $\mathfrak{T}^* \subset \mathfrak{T}$.*

Proposition 1. *Let $(X, \mathfrak{T}, E)$ be a fssts, then it is a parameterized collection of fuzzy supra topologies on X.i.e $\mathfrak{T}_e = \{f_A(e) : f_A \in \mathfrak{T}\}$ defines a fuzzy supra topology on X [7] for each $e \in E$. The following example supports our claim.*

Example 2. *Let $X = \{a, b, c, d\}$ be the set of four jobs under consideration and $E = \{e_1(Salary), e_2(Position)\}$. Let $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}, f_{12E}\}$, where $f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}, f_{6E}, f_{7E}, f_{8E}, f_{9E}, f_{10E}, f_{11E}, f_{12E}$ are fuzzy soft sets over X representing the "the income of the jobs" which some persons are going to work, respectively which defined as follows:*
$\mu^{e_1}_{f_{1E}} = \{a_{0.5}, b_{0.4}, c_0, d_0\}$, $\mu^{e_2}_{f_{1E}} = \{a_0, b_{0.3}, c_0, d_{0.5}\}$,
$\mu^{e_1}_{f_{2E}} = \{a_0, b_{0.4}, c_0, d_{0.5}\}$, $\mu^{e_2}_{f_{2E}} = \{a_{0.4}, b_0, c_{0.6}, d_0\}$,
$\mu^{e_1}_{f_{3E}} = \{a_{0.5}, b_0, c_{0.6}, d_0\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{4E}} = \{a_1, b_1, c_1, d_1\}$, $\mu^{e_2}_{f_{4E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{5E}} = \{a_{0.5}, b_0, c_{0.6}, d_0\}$, $\mu^{e_2}_{f_{5E}} = \{a_1, b_1, c_1, d_1\}$,
$\mu^{e_1}_{f_{6E}} = \{a_{0.5}, b_{0.4}, c_0, d_{0.5}\}$, $\mu^{e_2}_{f_{6E}} = \{a_{0.4}, b_{0.3}, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{7E}} = \{a_{0.5}, b_{0.4}, c_{0.6}, d_{0.5}\}$, $\mu^{e_2}_{f_{7E}} = \{a_{0.4}, b_{0.3}, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{8E}} = \{a_1, b_1, c_1, d_1\}$, $\mu^{e_2}_{f_{8E}} = \{a_{0.4}, b_{0.3}, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{9E}} = \{a_{0.5}, b_{0.4}, c_{0.6}, d_0\}$, $\mu^{e_2}_{f_{9E}} = \{a_1, b_1, c_1, d_1\}$,
$\mu^{e_1}_{f_{10E}} = \{a_{0.5}, b_{0.4}, c_{0.6}, d_{0.5}\}$, $\mu^{e_2}_{f_{10E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{11E}} = \{a_1, b_1, c_1, d_1\}$, $\mu^{e_2}_{f_{11E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{12E}} = \{a_{0.5}, b_0, c_{0.6}, d_{0.5}\}$, $\mu^{e_2}_{f_{12E}} = \{a_1, b_1, c_1, d_1\}$.
Then, $\mathfrak{T}$ defines a fuzzy supra soft topology on X. Then it is clear that $\mathfrak{T}_{e_1}$ and $\mathfrak{T}_{e_2}$ are fuzzy supra topologies on X, where
$\mathfrak{T}_{e_1} = \{\overline{1}, \overline{0}, f_{1E}(e_1), f_{2E}(e_1), ..., f_{12E}(e_1)\}$, *and*
$\mathfrak{T}_{e_2} = \{\overline{1}, \overline{0}, f_{1E}(e_2), f_{2E}(e_2), ..., f_{12E}(e_2)\}$.

Now we show that the converse of Proposition 1 does not hold in general by giving the following counterexample.

Example 3. *Let $X = \{a, b, c, d\}$ be the set of four houses under consideration and $E = \{e_1(Wooden), e_2(Luxurious)\}$. Define the fuzzy soft sets $f_i : E \to I^X$ on X, $1 \leq i \leq 5$, representing the "the goodness of the houses" which some persons are going to buy, respectively which defined as follows:*
$\mu^{e_1}_{f_{1E}} = \{a_{0.5}, b_{0.4}, c_0, d_0\}$, $\mu^{e_2}_{f_{1E}} = \{a_0, b_{0.3}, c_0, d_{0.5}\}$,
$\mu^{e_1}_{f_{2E}} = \{a_0, b_{0.4}, c_0, d_{0.5}\}$, $\mu^{e_2}_{f_{2E}} = \{a_{0.4}, b_0, c_{0.6}, d_0\}$,
$\mu^{e_1}_{f_{3E}} = \{a_{0.5}, b_0, c_{0.6}, d_0\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{4E}} = \{a_1, b_1, c_1, d_1\}$, $\mu^{e_2}_{f_{4E}} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$,
$\mu^{e_1}_{f_{5E}} = \{a_{0.5}, b_0, c_{0.6}, d_0\}$, $\mu^{e_2}_{f_{5E}} = \{a_1, b_1, c_1, d_1\}$.
Then,

$$\mathfrak{T}_{e_1} = \{\overline{1}, \overline{0}, f_{1E}(e_1), f_{2E}(e_1), ..., f_{12E}(e_1)\}, \text{ and}$$
$$\mathfrak{T}_{e_2} = \{\overline{1}, \overline{0}, f_{1E}(e_2), f_{2E}(e_2), ..., f_{12E}(e_2)\}.$$

are fuzzy supra topologies on X, at the time that the collection $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_{1E}, f_{2E}, f_{3E}, f_{4E}, f_{5E}\}$ is not fssts, where $f_{1E} \sqcup f_{2E}, f_{2E} \sqcup f_{3E}, f_{3E} \sqcup f_{5E}, f_{1E} \sqcup f_{3E}, f_{1E} \sqcup f_{5E}, f_{2E} \sqcup f_{5E} \notin \mathfrak{T}$.

Proposition 2. *Let $(X, \mathfrak{T}_1, E)$ and $(X, \mathfrak{T}_2, E)$ be two fssts over the same universe X, then $(X, \mathfrak{T}_1 \wedge \mathfrak{T}_2, E)$ is a fssts on X.*

Proof.

(1) Since $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}_1$ and $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}_2$, $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}_1 \wedge \mathfrak{T}_2$.

(2) Consider the collection of fuzzy soft sets $\{f_{iE} : i \in I\} \in \mathfrak{T}_1 \wedge \mathfrak{T}_2$. Then $\{f_{iE} : i \in I\} \in \mathfrak{T}_1$ and $\{f_{iE} : i \in I\} \in \mathfrak{T}_2$. Hence, $\bigsqcup_{i \in I} f_{iE} \in \mathfrak{T}_1$ and $\bigsqcup_{i \in I} f_{iE} \in \mathfrak{T}_2$. It is follows, $\bigsqcup_{i \in I} f_{iE} \in \mathfrak{T}_1 \wedge \mathfrak{T}_2$. Therefore, $\mathfrak{T}_1 \wedge \mathfrak{T}_2$ is a fssts on X.

Remarks 2. *The union of fssts need not to be a fssts in general, as will shown in the following example.*

Example 4. *Let $X = \{a, b, c\}$ be the set of four watches under consideration and $E = \{e_1(Expensive), e_2(Luxurious)\}$. Let $A, B \subseteq E$ where $A = \{e_1, e_2\}$ and $B = \{e_2, e_3\}$. Let $\mathfrak{T}_1 = \{\tilde{1}_E, \tilde{0}_E, f_{1A}, f_{2B}, f_{3E}\}$ and $\mathfrak{T}_2 = \{\tilde{1}_E, \tilde{0}_E, g_{1E}, g_{2B}, g_{3E}\}$ where $f_{1A}, f_{2B}, f_{3E}, g_{1E}, g_{2B}, g_{3E}$ are fuzzy soft sets over X, representing the "the compositions of the watches" which some persons are going to buy, respectively which defined as follows:*
$\mu^{e_1}_{f_{1A}} = \{a_{0.65}, b_{0.7}, c_{0.35}\}$, $\mu^{e_2}_{f_{1A}} = \{a_{0.5}, b_{0.3}, c_{0.9}\}$,
$\mu^{e_2}_{f_{2B}} = \{a_{0.4}, b_{0.7}, c_{0.25}\}$, $\mu^{e_3}_{f_{2B}} = \{a_{0.3}, b_{0.35}, c_{0.45}\}$,
$\mu^{e_1}_{f_{3E}} = \{a_{0.65}, b_{0.7}, c_{0.35}\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.5}, b_{0.7}, c_{0.9}\}$, $\mu^{e_2}_{f_{3E}} = \{a_{0.3}, b_{0.35}, c_{0.45}\}$,
$\mu^{e_1}_{g_{1E}} = \{a_{0.3}, b_{0.5}, c_0\}$, $\mu^{e_2}_{g_{1E}} = \{a_{0.4}, b_{0.7}, c_{0.1}\}$, $\mu^{e_3}_{g_{1E}} = \{a_{0.3}, b_{0.2}, c_{0.95}\}$,
$\mu^{e_2}_{g_{2B}} = \{a_{0.5}, b_{0.6}, c_1\}$, $\mu^{e_3}_{g_{2B}} = \{a_{0.8}, b_0, c_{0.6}\}$,
$\mu^{e_1}_{g_{3E}} = \{a_{0.3}, b_{0.5}, c_0\}$, $\mu^{e_2}_{g_{3E}} = \{a_{0.5}, b_{0.7}, c_1\}$, $\mu^{e_2}_{g_{3E}} = \{a_{0.8}, b_{0.2}, c_{0.95}\}$.
Then $\mathfrak{T}_1$ and $\mathfrak{T}_2$ defines fssts on X. But, $\mathfrak{T}_1 \vee \mathfrak{T}_2 = \{\tilde{1}_E, \tilde{0}_E, f_{1A}, f_{2B}, f_{3E}, g_{1E}, g_{2B}, g_{3E}\}$ is not fssts on X, where $f_{1A} \sqcup g_{1E} \notin \mathfrak{T}_1 \vee \mathfrak{T}_2$.

Definition 21. *A fuzzy soft set g_B in a fssts $(X, \mathfrak{T}, E)$ is called fuzzy supra soft neighborhood (briefly: fss neighborhood) of the fuzzy soft point $f_A^e \tilde{\in} X_E$ if there exists a fuzzy supra open soft set h_C such that $f_A^e \tilde{\in} h_C \sqsubseteq g_B$. The fuzzy supra soft neighborhood system of a fuzzy soft point f_A^e, denoted by $N_{\mathfrak{T}}(f_A^e)$, is the family of all its fuzzy supra soft neighborhoods.*

Example 5. *Consider the fuzzy supra soft topological space $(X, \mathfrak{T}, E)$ In Example 1. The fuzzy soft set g_A, where*
$\mu_{g_A}^{e_1} = \{a_{0.7}, b_{0.8}, c_{0.4}\}$, $\mu_{g_A}^{e_2} = \{a_{0.6}, b_1, c_{0.8}\}$
is a fss-neighborhood of the fuzzy soft point $k_A^{e_1} = \{a_{0.5}, b_{0.4}, c_{0.1}\}$.

Theorem 2. *A fuzzy soft set in a fssts is fuzzy supra soft open if and only if it is a fss-neighborhood of each of its fuzzy soft points.*

Proof. Obvious.

Theorem 3. *The fuzzy supra soft neighborhood system $N_{\mathfrak{T}}(f_A^e)$ in a fssts $(X, \mathfrak{T}, E)$ has the the following properties:*

(1) *For all fuzzy soft points f_A^e, $N_{\mathfrak{T}}(f_A^e) \neq \tilde{0}_E$,*

(2) *If $k_B \in N_{\mathfrak{T}}(f_A^e)$,then $f_A^e \tilde{\in} k_B$,*

(3) *If $k_D \sqsubseteq g_B$ and $k_D \in N_{\mathfrak{T}}(f_A^e)$, then $k_d \in N_{\mathfrak{T}}(f_A^e)$,*

(4) *If $e \in E$ is the support of the fuzzy supra soft set f_A, then $N_{\mathfrak{T}}(f_A^e) = \cap\{N_{\mathfrak{T}}(f_A^{e'}) : 0 < f_A(e') < f_A(e)\}$.*

(5) *If $k_B \in N_{\mathfrak{T}}(f_A^e)$, then there exists $h_C \in N_{\mathfrak{T}}(f_A^e)$ such that $h_C \sqsubseteq K_B$ and $h_C \in N_{\mathfrak{T}}(h_C^e)$.*

Proof.

(1) Since $\tilde{1}_E$ is a fss-neighborhood of each of its fuzzy soft points f_A^e from Theorem 2, $\tilde{1}_E \in N_{\mathfrak{T}}(f_A^e)$. Then, $N_{\mathfrak{T}}(f_A^e) \neq \tilde{0}_E$

(2) Let $k_B \in N_{\mathfrak{T}}(f_A^e)$. Then, there exists a fuzzy supra open soft set h_C such that $f_A^e \tilde{\in} h_C \sqsubseteq k_B$. Hence, $f_A^e \tilde{\in} k_B$.

(3) Let $k_D \in N_{\mathfrak{T}}(f_A^e)$. Then, there exists a fuzzy supra open soft set h_C such that $f_A^e \tilde{\in} h_C \sqsubseteq k_D$. Since $k_D \sqsubseteq g_B$, $f_A^e \tilde{\in} h_C \sqsubseteq k_D \sqsubseteq g_B$. Therefore, $k_D \in N_{\mathfrak{T}}(f_A^e)$.

(4) Obvious.

(5) Let $k_B \in N_{\mathfrak{T}}(f_A^e)$. Then, there exists a fuzzy supra open soft set s_F such that $f_A^e \tilde{\in} s_F \sqsubseteq k_B$. But, s_F is a fss-neighborhood of each of its fuzzy soft points from Theorem 2. Hence, $s_F \in N_{\mathfrak{T}}(s_F^e)$.

Theorem 4. *Let the collection $N(g_B^e$ be a fuzzy supra soft neighborhood system of the fuzzy soft point g_B^e with the properties (1)-(5) in Theorem 3. Then, there exists a unique fssts $\mathfrak{T}$ on X for which $N(g_B^e)$ coincides with the family of fss-neighborhood $N_{\mathfrak{T}}(g_B^e)$ of g_B^e with respect to $\mathfrak{T}$.*

Proof. Consider the collection $\mathfrak{T} = \{f_A : f_A \in N(g_B^e)\ and\ g_B^e \tilde{\in} f_A\}$. We want to prove that $\mathfrak{T}$ is a fssts on X.

(1) Since $\tilde{0}_E$ contains no fuzzy soft point, $\tilde{0}_E \in \mathfrak{T}$. Also, from condition (1), $N_{\mathfrak{T}}(f_A^e) \neq \tilde{0}_E$, so there exist some neighborhood of every fuzzy soft point in X, which is a superset of each of the fss-neighborhood s. Hence, $\tilde{1}_E \in \mathfrak{T}$ from condition (3).

(2) Let $\{(f_A)_j : j \in J\} \subseteq \mathfrak{T}$. Then, $(f_A)_j \in N(g_B^e)$, $j \in J$. By condition (3), $\sqcup_{j\in J}(f_A)_j \in N(g_B^e)$. Therefore, $\mathfrak{T}$ is a fssts on X.

Now, we prove that $N_{\mathfrak{T}}(g_B^e) = N(g_B^e)$. Let $h_A \in N(g_B^e)$, then there exists $k_C \in N_{\mathfrak{T}}(g_B^e)$ such that $k_C \sqsubseteq K_B$ and $k_C \in N(k_C^e)$ from condition (5). It is follows, $g_B^e \tilde{\in} k_C$ from condition (2). Hence, $k_C \in \mathfrak{T}$ from Theorem 2. This means, h_A is $\mathfrak{T}$-fss-neighborhood of g_B^e. Thus,

$$N_{\mathfrak{T}}(g_B^e) \subseteq N(g_B^e). \tag{1}$$

Conversely, Let $n_Y \in N_{\mathfrak{T}}(g_B^e)$, then there exists a fuzzy supra open soft set m_Z such that $g_B^e \tilde{\in} m_Z \sqsubseteq n_Y$. It is follows, $m_Z \in N(g_B^e)$ from Theorem 2. But, $m_Z \sqsubseteq n_Y$, then $n_Y \in N(g_B^e)$ from condition (3). Hence,

$$N(g_B^e) \subseteq N_{\mathfrak{T}}(g_B^e). \tag{2}$$

from (3.1) and (3.2) we have $N_{\mathfrak{T}}(g_B^e) = N(g_B^e)$.

Definition 22. *Let $(X, \mathfrak{T}, E)$ be a fuzzy supra soft topological space over and $g_B \in SS(X)_E$. Then the fuzzy supra soft interior of g_B, denoted by $Fint^s(g_B)$ is defined as*

$$Fint^s(g_B) = \sqcup\{h_A : h_A\ is\ fuzzy\ supra\ open\ soft\ set\ and\ h_A \sqsubseteq g_B\}. \tag{3}$$

Also, the fuzzy supra soft closure of g_B, denoted by $Fcl^s(g_B)$ is defined as

$$Fcl^s(g_B) = \sqcap\{h_A : h_A\ is\ fuzzy\ supra\ closed\ soft\ set\ and\ g_B \sqsubseteq h_A\}. \tag{4}$$

Definition 23. *Let $(X, \mathfrak{T}, E)$ be a fssts over X and $g_B \in SS(X)_E$. Then, $f_A^e \in SS(X)_E$ is called fuzzy supra limit soft point of f_A if $(g_B - f_A^e) \sqcap h_C \neq \tilde{0}_E\ \forall\ h_C \in FSOS(X)$. The set of all fuzzy supra limit soft points of g_B is called the fuzzy supra soft derived of g_B and denoted by $d_f^s(g_B)$.*

Theorem 5. *Let $(X, \mathfrak{T}, E)$ be a supra soft topological space and $f_A, g_B \in SS(X)_E$. Then*

(1) $Fint^s(\tilde{1}_E) = \tilde{1}_E$ *and* $Fint^s(\tilde{0}_E) = \tilde{0}_E$.

(2) $Fcl^s(\tilde{1}_E) = \tilde{1}_E$ *and* $Fcl^s(\tilde{0}_E) = \tilde{0}_E$.

(3) *f_A is fuzzy supra open soft if and only if $Fint^s(f_A) = f_A$.*

(4) *$Fint^s(Fint^s(f_A)) = Fint^s(f_A)$ and $Fint^s(f_A) \sqsubseteq f_A$.*

(5) *f_A is fuzzy supra closed soft set if and only if $Fcl^s(f_A) = f_A$.*

(6) *$Fcl^s(Fcl^s(f_A)) = Fcl^s(f_A)$ and $f_A \sqsubseteq Fcl^s(f_A)$.*

(7) *If $f_A \sqsubseteq g_B$, then $Fint^s(f_A) \sqsubseteq Fint^s(g_B)$ and $Fcl^s(f_A) \sqsubseteq Fcl^s(g_B)$.*

(8) *$Fcl^s(f_A) = [Fint^s(f_A^c)]^c$.*

Proof. Obvious.

Theorem 6. *Let $(X, \mathfrak{T}, E)$ be a fssts on X and $f_A \in SS(X)_E$. Then:*

(1) *$Fint^s(f_A^c) = \tilde{1}_E - [Fcl^s(f_A)]$.*

(2) *$Fcl^s(f_A^c) = \tilde{1}_E - [Fint^s(f_A)]$.*

Proof.

(1) Since $Fcl^s(f_A) = \sqcap\{h_D : h_D \in FSCS(X),\ f_A \sqsubseteq h_D\}$, $\tilde{1}_E - Fcl^s(f_A) = \sqcup\{h_D^c : h_D^c \in FSOS(X),\ h_D^c \sqsubseteq f_A^c\} = Fint^s(f_A^c)$.

(2) By a similar way.

In the next theorem, we list the main properties of the operations which give the deviations between these operations and that in fuzzy soft topological spaces.

Theorem 7. *Let $(X, \mathfrak{T}, E)$ be a supra soft topological space and $f_A, g_B \in SS(X)_E$. Then*

(1) *$Fcl^s(f_A) \sqcup Fcl^s(g_B) \sqsubseteq Fcl^s(f_A \sqcup g_B)$.*

(2) *$d_f^s(f_A) \sqcup d_f^s(g_B) \sqsubseteq d_f^s(f_A \sqcup g_B)$.*

(3) *$Fint^s(f_A \sqcap g_B) \sqsubseteq Fint^s(f_A) \sqcap Fint^s(g_B)$.*

Proof. Immediate.

Remarks 3. *The equality of each part in Theorem 7 is not true in general as will shown in the following examples.*

Example 6. **(1)** *Consider the fssts in Example 2 and consider the two fuzzy soft sets g_E and h_E over X defined by;*
$\mu_{g_E}^{e_1} = \{a_{0.5}, b_{0.4}, c_0, d_0\}$, $\mu_{g_E}^{e_2} = \{a_0, b_{0.3}, c_0, d_{0.5}\}$ *and*
$\mu_{h_E}^{e_1} = \{a_{0.5}, b_{0.6}, c_1, d_1\}$, $\mu_{h_E}^{e_2} = \{a_1, b_{0.7}, c_1, d_{0.5}\}$.
It is follows, $Fcl^s(g_E \sqcup h_E) = \tilde{1}_E$ and $Fcl^s(g_E) \sqcup Fcl^s(h_E) = k_E$, where K_E is a fuzzy soft set over X defined by:
$\mu_{k_E}^{e_1} = \{a_{0.5}, b_{0.6}, c_1, d_1\}$, $\mu_{k_E}^{e_2} = \{a_1, b_{0.7}, c_1, d_{0.5}\}$. *Therefore, $Fcl^s(g_E \sqcup h_E) \not\sqsubseteq Fcl^s(g_E) \sqcup Fcl^s(h_E)$.*

(2) *In (1),* $d^s_f(g_E \sqcup h_E) \not\sqsubseteq d^s_f(g_E) \sqcup d^s_f(h_E)$

(3) *Consider the fssts in Example 2 and consider the two fuzzy soft sets* m_E *and* n_E *over* X *defined by;*
$\mu^{e_1}_{m_E} = \{a_0, b_{0.4}, c_0, d_{0.5}\}$, $\mu^{e2}_{m_E} = \{a_{0.4}, b_0, c_{0.6}, d_0\}$,
$\mu^{e_1}_{n_E} = \{a_{0.5}, b_0, c_{0.6}, d_0\}$, $\mu^{e2}_{n_E} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$.
It is follows, $Fint^s(m_E \sqcap n_E) = \tilde{0}_E$ *and* $Fint^s(m_E) \sqcap Fint^s(n_E) = l_E$*, where* l_E *is a fuzzy soft set over* X *defined by:*
$\mu^{e_1}_{l_E} = \{a_0, b_0, c_0, d_{0.5}\}$, $\mu^{e2}_{l_E} = \{a_{0.4}, b_0, c_{0.6}, d_{0.5}\}$.
Therefore, $Fint^s(m_E) \sqcap Fint^s(n_E) \not\sqsubseteq Fint^s(m_E \sqcap n_E)$.

Definition 24. *Let* $(X, \mathfrak{T}, E)$ *be a fssts and* $Y \subseteq X$*. Let* y_E *be a fuzzy soft set over* (Y, E) *defined by:*
$y_E : E \to I^Y$ *such that* $y_E(e) = \mu^e_{y_E}$*, where*

$$\mu^e_{y_E}(x) = \begin{cases} 1, & x \in Y, \\ 0, & x \notin Y. \end{cases}$$

Then, the fssts $\mathfrak{T}_{y_E} = \{y_E \sqcap g_B : g_B \in \mathfrak{T}\}$ *is called the fuzzy supra soft subspace topology for* y_E *and* $(Y, \mathfrak{T}_{y_E}, E)$ *is called fuzzy supra soft subspace of* $(X, \mathfrak{T}, E)$.

Example 7. *Consider the fssts in Example 1, let* $Y = \{a, b\} \subseteq X$*. We consider the fuzzy soft set* y_E *over* (Y, E) *defined as follows:*
$\mu^{e_1}_{y_E} = \{a_1, b_1, c_0\}$, $\mu^{e_2}_{y_E} = \{a_1, b_1, c_0\}$, $\mu^{e_3}_{y_E} = \{a_1, b_1, c_0\}$.
Then $\mathfrak{T}_{y_E} = \{y_E \sqcap z_E : z_E \in \mathfrak{T}\}$ *where*
$y_E \sqcap \tilde{0}_E = \tilde{0}_E$, $y_E \sqcap \tilde{1}_E = y_E$, $y_E \sqcap f_{1A} = h_A$*, where*
$\mu^{e_1}_{h_A} = \{a_{0.6}, b_{0.75}, c_0\}$, $\mu^{e_2}_{h_A} = \{a_{0.5}, b_{0.8}, c_0\}$,

$y_E \sqcap f_{2B} = h_B$*, where*
$\mu^{e_2}_{h_B} = \{a_{0.4}, b_{0.6}, c_0\}$, $\mu^{e_2}_{h_B} = \{a_{0.3}, b_{0.35}, c_0\}$,

$y_E \sqcap f_{3E} = h_E$*, where*
$\mu^{e_1}_{h_E} = \{a_{0.6}, b_{0.75}, c_0\}$, $\mu^{e_2}_{h_E} = \{a_{0.5}, b_{0.8}, c_0\}$, $\mu^{e_3}_{h_E} = \{a_{0.3}, b_{0.35}, c_0\}$.
Thus, the collection $\mathfrak{T}_{y_E} = \{y_E \sqcap z_E : z_E \in \mathfrak{T}\}$ *is a fuzzy supra soft subspace of* $\mathfrak{T}$.

Proposition 3. *Let* $(Y, \mathfrak{T}_{y_E}, E)$ *be a fuzzy supra soft subspace of a fssts* $(X, \mathfrak{T}, E)$ *and* $g_B \in FSS(X)_E$*. Then,* g_B *is* $\mathfrak{T}_{y_E}$*-fuzzy supra closed soft if and only if* $g_B = y_E \sqcap k_c$ *for some* $\mathfrak{T}$*-fuzzy supra closed soft set* k_C.

Proof. Immediate.

Proposition 4. *Let* $(Y, \mathfrak{T}_{y_E}, E)$ *be a fuzzy supra soft subspace of a fssts* $(X, \mathfrak{T}, E)$ *and* $g_B \in FSS(X)_E$*. Then,* $\mathfrak{T}_{y_E}$*-fuzzy supra soft closure of* g_B*, denoted by* $Fcl^s_{\mathfrak{T}_{y_E}}$*, where* $Fcl^s_{\mathfrak{T}_{y_E}}(g_B) = y_E \sqcap Fcl^s(g_B)$.

Proof. Immediate.

4. Fuzzy Supra Soft Continuous Mappings

In this section, we consider the notion of fuzzy supra soft continuity as a generalization to fuzzy soft continuity [8], fuzzy semi-soft continuity [19], fuzzy pre-soft continuity [1], fuzzy α-soft continuity [2], fuzzy β-soft continuity [3] and fuzzy b-soft continuity [6], supported by examples and counterexamples. We also introduce and study the concepts of fuzzy supra open (resp. closed) soft functions a generalization to fuzzy open (resp. closed) soft functions [30].

Definition 25. *Let $(X, \mathfrak{T}^*{}_1, E)$, $(Y, \mathfrak{T}^*{}_2, K)$ be two fuzzy soft topological spaces, $\mathfrak{T}_1$ be an associated fssts with $\mathfrak{T}^*{}_1$ and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a soft function. Then, f_{pu} is called fuzzy supra soft continuous (fss-continuous, in short) if $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \ \forall\ g_B \in \mathfrak{T}^*{}_2$.*

Theorem 8. *Let $(X, \mathfrak{T}^*{}_1, E)$, $(Y, \mathfrak{T}^*{}_2, K)$ be two fuzzy soft topological spaces, $\mathfrak{T}_1$, $\mathfrak{T}_2$ be associated fssts with $\mathfrak{T}^*{}_1$, $\mathfrak{T}^*{}_2$, respectively. Let $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a soft function. Then, the following are equivalent:*

(1) *f_{pu} is fss-continuous.*

(2) $f_{pu}^{-1}(h_B) \in \mathfrak{T}^{\mathfrak{c}}{}_1 \ \forall\ h_B \in \mathfrak{T}^{*\mathfrak{c}}{}_2$.

(3) $f_{pu}(Fcl^s(g_A)) \sqsubseteq Fcl^s(f_{pu}(g_A)) \ \forall\ g_A \in FSS(X)_E$.

(4) $Fcl^s(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl^s(h_B)) \ \forall\ h_B \in FSS(Y)_K$.

(5) $f_{pu}^{-1}(Fint^s(h_B)) \sqsubseteq Fint^s(f_{pu}^{-1}(h_B)) \ \forall\ h_B \in FSS(Y)_K$.

Proof.

(1) $\Rightarrow$ (2) Let $h_B \in \mathfrak{T}^{*\mathfrak{c}}{}_2$. Then, $h_B^c \in \mathfrak{T}^*{}_2$ and $f_{pu}^{-1}(h_B^c) \in \mathfrak{T}_1$ from Definition 25. Since $f_{pu}^{-1}(h_B^c) = (f_{pu}^{-1}(h_B))^c$ from Theorem 1, $f_{pu}^{-1}(h_B) \in \mathfrak{T}^{\mathfrak{c}}{}_1$.

(2) $\Rightarrow$ (3) Let $g_A \in FSS(X)_E$. Since $g_A \sqsubseteq f_{pu}^{-1}(f_{pu}(g_A)) \sqsubseteq f_{pu}^{-1}(Fcl^s(f_{pu}(g_A))) \in \mathfrak{T}^{\mathfrak{c}}{}_1$ from (2) and Theorem 1. Then, $g_A \sqsubseteq Fcl^s(g_A) \sqsubseteq f_{pu}^{-1}(Fcl^s(f_{pu}(g_A)))$. Hence, $f_{pu}(Fcl^s(g_A)) \sqsubseteq f_{pu}(f_{pu}^{-1}(Fcl^s(f_{pu}(g_A)))) \sqsubseteq Fcl^s(f_{pu}(g_A)))$ from Theorem 1. Thus, $f_{pu}(Fcl^s(g_A)) \sqsubseteq Fcl^s(f_{pu}(g_A))$.

(3) $\Rightarrow$ (4) Let $h_B \in FSS(Y)_K$ and $g_A = f_{pu}^{-1}(h_B)$. Applying (3), $f_{pu}(Fcl^s f_{pu}^{-1}(h_B)) \sqsubseteq Fcl^s(f_{pu}(f_{pu}^{-1}(h_B)))$. Hence, $Fcl^s(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(f_{pu}(Fcl^s(f_{pu}^{-1}(h_B)))) \sqsubseteq f_{pu}^{-1}(Fcl^s(f_{pu} (f_{pu}^{-1}(h_B)))) \sqsubseteq f_{pu}^{-1}(Fcl^s(h_B))$ from Theorem 1. Thus, $Fcl^s(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl^s(h_B))$.

(4) $\Rightarrow$ (2) Let $h_B \in \mathfrak{T}^{*\mathfrak{c}}{}_2$. Then, $Fcl^s(h_B) = h_B$ and $Fcl^s(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl^s(h_B)) = f_{pu}^{-1}(h_B)$ from (4). But, we have $f_{pu}^{-1}(h_B) \sqsubseteq Fcl^s(f_{pu}^{-1}(h_B))$. It is follows, $f_{pu}^{-1}(h_B) = Fcl^s(f_{pu}^{-1}(h_B))$, and consequently $f_{pu}^{-1}(h_B) \in \mathfrak{T}^{\mathfrak{c}}{}_1$.

(1) $\Rightarrow$ (5) Let $h_B \in FSS(Y)_K$. Since $Fint^s(h_B) \in \mathfrak{T}^*{}_2$, $f_{pu}^{-1}(Fint^s(h_B)) \in \mathfrak{T}_1$ from (1). Hence, $f_{pu}^{-1}(Fint^s(h_B)) = Fint^s(f_{pu}^{-1} Fint^s(h_B)) \sqsubseteq Fint^s(f_{pu}^{-1}(h_B))$. Thus, $f_{pu}^{-1}(Fint^s(h_B)) \sqsubseteq Fint^s(f_{pu}^{-1}(h_B))$.

(5) $\Rightarrow$ (1) Let $h_B \in \mathfrak{T}^*{}_2$. Then $Fint^s(h_B) = h_B$ and $f_{pu}^{-1}(Fint^s(h_B)) = f_{pu}^{-1}((h_B)) \sqsubseteq Fint^s(f_{pu}^{-1}(h_B))$ from (5). But, we have $Fint^s(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(h_B)$. It is follows, $Fint^s(f_{pu}^{-1}(h_B)) = f_{pu}^{-1}(h_B) \in \mathfrak{T}_1$. Hence, f_{pu} is a fss-continuous.

Theorem 9. *Every fuzzy soft continuous function [8] is a fss-continuous.*

Proof. Immediate from Definition 18 (1) and Definition 25.

Remarks 4. *The converse of Theorem 9 is not true in general, as will shown in the following example.*

Example 8. *Let* $X = \{a, b, c\}$, $Y = \{l, m, n\}$, $E = \{e_1, e_2, e_3\}$ *and* $K = \{k_1, k_2, k_3\}$. *Define* $u : X \to Y$ *and* $p : A \to B$ *as follows:*
$u(a) = \{n\}$, $u(b) = \{m\}$, $u(c) = \{m\}$ *and* $p(e_1) = \{k_1\}$, $p(e_2) = \{k_1\}$, $p(e_3) = \{k_3\}$.
Let $(X, \mathfrak{T}^*{}_1, E)$ *be a fuzzy soft topological space over* X, *where* $\mathfrak{T}^*{}_1 = \{\tilde{1}_E, \tilde{0}_E, f_E\}$, *where* f_E *is a fuzzy soft set over* X *defined as follows:*
$\mu_{f_E}^{e_1} = \{a_{0.1}, b_{0.5}, c_{0.7}\}$, $\mu_{f_E}^{e_2} = \{a_{0.2}, b_{0.7}, c_{0.5}\}$, $\mu_{f_E}^{e_3} = \{a_{0.7}, b_{0.4}, c_{0.3}\}$.
Consider the associated fssts $\mathfrak{T}_1$ *with* $\mathfrak{T}^*{}_1$, *where* $\mathfrak{T}_1 = \{\tilde{1}_E, \tilde{0}_E, f_E, h_E, k_E\}$, *where* f_E, h_E, k_E *are fuzzy soft sets over* X *defined as follows:*
$\mu_{h_E}^{e_1} = \{a_{0.2}, b_{0.6}, c_{0.6}\}$, $\mu_{h_E}^{e_2} = \{a_{0.2}, b_{0.6}, c_{0.6}\}$, $\mu_{h_E}^{e_3} = \{a_{0.9}, b_{0.1}, c_{0.1}\}$,
$\mu_{k_E}^{e_1} = \{a_{0.2}, b_{0.6}, c_{0.7}\}$, $\mu_{k_E}^{e_2} = \{a_{0.2}, b_{0.7}, c_{0.5}\}$, $\mu_{k_E}^{e_3} = \{a_{0.9}, b_{0.4}, c_{0.3}\}$.
Let $(X, \mathfrak{T}^*{}_2, K)$ *be a fuzzy soft topological space over* Y, *where* $\mathfrak{T}^*{}_2 = \{\tilde{1}_K, \tilde{0}_K, g_K\}$, *where* g_K *is a fuzzy soft set over* Y *defined by:*
$\mu_{g_K}^{k_1} = \{a_{0.5}, b_{0.6}, c_{0.2}\}$, $\mu_{g_K}^{k_2} = \{a_{0.8}, b_{0.3}, c_{0.4}\}$, $\mu_{g_K}^{k_3} = \{a_{0.7}, b_{0.1}, c_{0.9}\}$.
Let $f_{pu} : (X, \mathfrak{T}^*{}_1, E) \to (Y, \mathfrak{T}^*{}_2, K)$ *be a soft function. Next, for* $p(e_i) \in K$, $i = 1, 2, 3$ *we calculate* $f_{pu}^{-1}(g_K)$ *as follows:*
$f_{pu}^{-1}(g_K)(e_1)(a) = g_K(p(e_1))(u(a)) = g_K(k_1)(n) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(n) = 0.2$,
$f_{pu}^{-1}(g_K)(e_1)(b) = g_K(p(e_1))(u(b)) = g_K(k_1)(m) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(m) = 0.6$,
$f_{pu}^{-1}(g_K)(e_1)(c) = g_K(p(e_1))(u(c)) = g_K(k_1)(m) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(m) = 0.6$,

$f_{pu}^{-1}(g_K)(e_2)(a) = g_K(p(e_2))(u(a)) = g_K(k_1)(n) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(n) = 0.2$,
$f_{pu}^{-1}(g_K)(e_2)(b) = g_K(p(e_2))(u(b)) = g_K(k_1)(m) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(m) = 0.6$,
$f_{pu}^{-1}(g_K)(e_2)(c) = g_K(p(e_2))(u(c)) = g_K(k_1)(m) = (\{a_{0.5}, b_{0.6}, c_{0.2}\})(m) = 0.6$,

$f_{pu}^{-1}(g_K)(e_3)(a) = g_K(p(e_3))(u(a)) = g_K(k_3)(n) = (\{a_{0.9}, b_{0.1}, c_{0.1})(n) = 0.9$,
$f_{pu}^{-1}(g_K)(e_3)(b) = g_K(p(e_3))(u(b)) = g_K(k_3)(m) = (\{a_{0.9}, b_{0.1}, c_{0.1})(m) = 0.1$,
$f_{pu}^{-1}(g_K)(e_3)(c) = g_K(p(e_3))(u(c)) = g_K(k_3)(m) = (\{a_{0.9}, b_{0.1}, c_{0.1})(m) = 0.1$.
Hence, $f_{pu}^{-1}(g_K) = \{(e_1, \{a_{0.2}, b_{0.6}, c_{0.6}\}), (e_2, \{a_{0.2}, b_{0.6}, c_{0.6}\}), (e_3, \{a_{0.9}, b_{0.1}, c_{0.1})\}$. *It is follows,* $f_{pu}^{-1}(g_K) \in \mathfrak{T}_1$ *and* $f_{pu}^{-1}(g_K) \notin \mathfrak{T}^*{}_1$. *Therefore,* f_{pu} *is a fss-continuous but not fuzzy soft continuous.*

Definition 26. *Let* $(X, \mathfrak{T}^*{}_1, E)$, $(Y, \mathfrak{T}^*{}_2, K)$ *be two fuzzy soft topological spaces,* $\mathfrak{T}_2$ *be an associated fssts with* $\mathfrak{T}^*{}_2$ *and* $f_{pu} : FSS(X)_E \to FSS(Y)_K$ *be a soft function. Then,*

(1) f_{pu} *is called fuzzy supra open soft if* $f_{pu}(g_E) \in \mathfrak{T}_2 \; \forall \; g_E \in \mathfrak{T}^*{}_1$.

(2) *f_{pu} is called fuzzy supra closed soft if $f_{pu}(g_E) \in \mathfrak{T}^{\mathfrak{c}}{}_2\ \forall\ g_E \in \mathfrak{T}^{*\mathfrak{c}}{}_1$.*

Theorem 10. *Every fuzzy open (resp. closed) soft function [15] is a fuzzy supra open (resp. closed) soft.*

Proof. Immediate from Definition 25 (2), (3) and Definition 26.

Remarks 5. *The converse of Theorem 10 is not true in general, as will shown in the following example.*

Example 9. *In Example 8, let $(X, \mathfrak{T}^*{}_1, E)$ be a fuzzy soft topological space over X where, $\mathfrak{T}^*{}_1 = \{\tilde{1}_E, \tilde{0}_E, z_E\}$, where z_E is a fuzzy soft set over X defined as follows:
$\mu^{e_1}_{z_E} = \{a_{0.5}, b_0, c_{0.8}\}$, $\mu^{e_2}_{z_E} = \{a_{0.1}, b_{0.9}, c_{0.5}\}$, $\mu^{e_3}_{z_E} = \{a_{0.4}, b_{0.3}, c_{0.6}\}$.
Let $(X, \mathfrak{T}^*{}_2, K)$ be a fuzzy soft topological space over Y where,
$\mathfrak{T}^*{}_2 = \{\tilde{1}_K, \tilde{0}_K, g_K\}$, where g_K is a fuzzy soft set over Y defined by:
$\mu^{k_1}_{g_K} = \{a_{0.3}, b_{0.2}, c_{0.7}\}$, $\mu^{k_2}_{g_K} = \{a_1, b_{0.3}, c_{0.5}\}$, $\mu^{k_3}_{g_K} = \{a_{0.2}, b_{0.9}, c_{0.1}\}$.
Consider the associated fssts $\mathfrak{T}_2$ with $\mathfrak{T}^*{}_2$, where $\mathfrak{T}_2 = \{\tilde{1}_K, \tilde{0}_K, g_K, h_K, i_K\}$, where g_K, h_K, i_K are fuzzy soft sets over X defined as follows:
$\mu^{k_1}_{h_K} = \{a_0, b_{0.9}, c_{0.5}\}$, $\mu^{k_2}_{h_K} = \{a_0, b_{0.6}, c_{0.4}\}$, $\mu^{k_3}_{h_K} = \{a_0, b_0, c_0\}$,
$\mu^{k_1}_{i_K} = \{a_{0.3}, b_{0.9}, c_{0.7}\}$, $\mu^{k_2}_{i_K} = \{a_1, b_{0.6}, c_{0.5}\}$, $\mu^{k_3}_{i_K} = \{a_{0.2}, b_{0.9}, c_{0.1}\}$.
Let $f_{pu} : (X, \mathfrak{T}^*{}_1, E) \to (Y, \mathfrak{T}^*{}_2, K)$ be a soft function. Hence, $f_{pu}(z_E) = \{(k_1, \{a_0, b_{0.9}, c_{0.5}\}), (k_2, \{a_0, b_{0.6}, c_{0.4}\}), (k_3, \{a_0, b_0, c_0\})\}$. It is follows, $f_{pu}(z_E) \in \mathfrak{T}_2$ and $f_{pu}(z_E) \notin \mathfrak{T}^*{}_2$. Therefore, f_{pu} is a fuzzy supra open (resp. closed) soft function but not fuzzy open (resp. closed) soft.*

Theorem 11. *Let $(X, \mathfrak{T}^*{}_1, E)$, $(Y, \mathfrak{T}^*{}_2, K)$ be two fuzzy soft topological spaces, $\mathfrak{T}_2$ be an associated fssts with $\mathfrak{T}^*{}_2$ and $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a soft function. Then,*

(1) *f_{pu} is a fuzzy supra open soft mapping if and only if $f_{pu}(Fint^s(g_E)) \sqsubseteq Fint^s(f_{pu}(g_E))\ \forall\ g_E \in FSS(X)_E$.*

(2) *f_{pu} is a fuzzy supra closed soft mapping if and only if $Fcl^s(f_{pu}(g_E)) \sqsubseteq f_{pu}(Fcl^s(g_E))\ \forall\ g_E \in FSS(X)_E$.*

Proof.

(1) ($\Rightarrow$:) Let $g_E \in FSS(X)_E$. Since $Fint^s(g_E) \sqsubseteq f_A$, $f_{pu}(Fint^s(g_E)) \sqsubseteq f_{pu}(g_E)$ from Theorem 1. Since f_{pu} is fuzzy supra open soft mapping, $Fint^s(f_{pu}(Fint^s(g_E))) = f_{pu}(Fint^s(g_E)) \sqsubseteq Fint^s(f_{pu}(f_A))$.
(:$\Leftarrow$) Let $g_E \in \mathfrak{T}^*{}_1$. Then, $Fint^s(g_E) = g_E$. By hypothesis, $f_{pu}(Fint^s(g_E)) \sqsubseteq Fint^s(f_{pu}(g_E))$. Hence, $f_{pu}(g_E) \sqsubseteq Fint^s(f_{pu}(g_E)) \sqsubseteq f_{pu}(g_E)$. It is follows, $f_{pu}(g_E) = Fint^s(f_{pu}(g_E))$. Thus, $f_{pu}(g_E) \in \mathfrak{T}_2$. This completes the proof.

(2) By a similar way.

Definition 27. *Let $(X, \mathfrak{T}^*_1, E)$, $(Y, \mathfrak{T}^*_2, K)$ be two fuzzy soft topological spaces, $\mathfrak{T}_1$, $\mathfrak{T}_2$ be associated fssts with $\mathfrak{T}^*_1$, $\mathfrak{T}^*_2$, respectively. Let $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a soft function. Then, f_{pu} is fuzzy supra soft homeomorphism if it is bijection, fss-continuous and f_{pu}^{-1} is fss-continuous.*

Theorem 12. *Let $(X, \mathfrak{T}^*_1, E)$, $(Y, \mathfrak{T}^*_2, K)$ be two fuzzy soft topological spaces, $\mathfrak{T}_2$ be an associated fssts with $\mathfrak{T}^*_2$ and $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ be a bijective soft function. Then, the following are equivalent:*

(1) *f_{pu} is a fuzzy supra soft homeomorphism.*

(2) *f_{pu} is a fss-continuous and fuzzy supra closed soft mapping.*

(3) *f_{pu} is a fss-continuous and fuzzy supra open soft mapping.*

Proof. Immediate.

5. Fuzzy Supra Soft Compact Spaces

In this section, we introduce the notion of fuzzy supra soft compact (resp. fuzzy supra soft lindelöf) spaces as a generalization to such introduced in [11, 12, 17, 24, 27, 30]. We also introduce some basic definitions of fuzzy supra soft compact spaces and theorems of the concept.

Definition 28. *Let $(X, \mathfrak{T}, E)$ be a fssts. A family $\Psi = \{u_{iE} : i \in \Lambda\}$ of fuzzy soft sets is said to be a fuzzy supra open soft cover, if each member of Ψ is fuzzy supra open soft set.*

Definition 29. *A fuzzy soft subset f_A of the space $(X, \mathfrak{T}, E)$ is said to be fuzzy supra soft compact (resp. fuzzy supra soft lindelöf), if every fuzzy supra open soft cover $\{u_{iE} : i \in \Lambda\}$ of f_A has a finite (resp. countable) subfamily Λ_o of Λ such that*

$$f_A \sqsubseteq \tilde{\sqcup}_{i \in \Lambda_o} u_{iE}.$$

The space $(X, \mathfrak{T}, E)$ is said to be a fuzzy supra soft compact if $\tilde{1}_E$ is a fuzzy supra soft compact as a fuzzy soft subset.

Example 10. *Let $(X, \mathfrak{T}_1, E)$ and $(X, \mathfrak{T}_2, E)$ be two fssts such that $\mathfrak{T}_1 \subseteq \mathfrak{T}_2$. If $\mathfrak{T}_2$ is a fuzzy supra soft compact, then also $\mathfrak{T}_1$.*

Proposition 5. *If X is finite (resp. countable), then $(X, \mathfrak{T}, E)$ is fuzzy supra soft compact (resp. fuzzy supra soft lindelöf) for any fuzzy supra soft topology $\mathfrak{T}$ on X.*

Proof. It is easy to be obtained.

Theorem 13. *Every fuzzy supra closed soft subspace of a fuzzy supra soft compact space is a fuzzy supra soft compact.*

Proof. Let z_A be a fuzzy supra closed soft subspace of fuzzy supra soft compact space $(X, \mathfrak{T}, E)$ and $\{u_{iE} : i\in\Lambda\}$ be a fuzzy supra open soft cover of z_A. Hence, $\{u_{iE} : i\in\Lambda\} \sqcup z_A^c$ is a fuzzy supra open soft cover of $\tilde{1}_E$, and for $\tilde{1}_E$ is fuzzy supra soft compact, there exists a finite subcover $\{u_{iE} : i\in\Lambda_o\} \sqcup z_A^c)$ for $\tilde{1}_E$. Now, $[\{u_{iE} : i\in\Lambda_o\} \sqcup (z_A^c)] - (z_A^c)$ is a finite subcover of $\{u_{iE} : i\in\Lambda\}$ for z_A. So, z_A is a fuzzy supra soft compact.

Theorem 14. *Every supra closed soft subspace of fuzzy supra soft lindelöf space is a fuzzy supra soft lindelöf.*

Proof. It is similar to the proof of Theorem 13.

Theorem 15. *Every fuzzy supra soft subspace of fssts $(X, \mathfrak{T}, E)$ is a fuzzy supra soft compact if and only if every fuzzy supra open soft subspace of $\tilde{1}_E$ is a fuzzy supra soft compact.*

Proof. Let $(Y, \mathfrak{T}_{y_E}, E)$ be a fuzzy supra open soft subspace of a fssts $(X, \mathfrak{T}, E)$ and $\{u_{\alpha E} : \alpha\in\Lambda\}$ be a fuzzy supra open soft cover of $(Y, \mathfrak{T}_{y_E}, E)$. Assume that $v_C = \sqcup_{\alpha\in\Lambda} u_{\alpha E}$. Hence v_C is a fuzzy supra open soft subspace of $\tilde{1}_E$. By hypothesis, v_C is a fuzzy supra soft compact. So, $\{u_{\alpha E} : \alpha \in \Lambda_o, \Lambda_o \ is \ finite\}$ is a finite subcover of v_C. It is follows, $v_C \sqsubseteq \sqcup_{\alpha\in\Lambda_o} u_{\alpha E}$. Thus, $y_E \sqsubseteq v_C \sqsubseteq \sqcup_{\alpha\in\Lambda_o} u_{\alpha E}$. Therefore, $(Y, \mathfrak{T}_{y_E}, E)$ is a fuzzy supra soft compact.
For the necessity, it is clear.

Theorem 16. *Every fuzzy supra soft subspace of fssts $(X, \mathfrak{T}, E)$ is a fuzzy supra soft lindelöf if and only if every fuzzy supra open soft subspace of $\tilde{1}_E$ is a fuzzy supra soft lindelöf.*

Proof. It similar to the proof of Theorem 15.

Theorem 17. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be two fssts and let f_{pu}:$(X, \mathfrak{T}_1, E)\to(Y, \mathfrak{T}_2, K)$ be a surjective and fss-continuous function. If $(X, \mathfrak{T}_1, E)$ is fuzzy supra soft compact, then $(Y, \mathfrak{T}_2, K)$ is also fuzzy supra soft compact.*

Proof. Let $\{u_{iK} : i\in\Lambda\}$ be a fuzzy supra open soft cover of $\tilde{1}_K$. Since f_{pu} is a fss-continuous, $\{f_{pu}^{-1}(u_{iK}) : i\in\Lambda\}$ is a fuzzy supra open soft cover of $\tilde{1}_E$, and for $\tilde{1}_E$ is a fuzzy supra soft compact, there exists a finite subfamily Λ_o of Λ such that $\{f_{pu}^{-1}(u_{iK}) : i\in\Lambda_o\}$ also forms a fuzzy supra open soft cover of $\tilde{1}_E$. Since f_{pu} is surjective, $\{f_{pu}(f_{pu}^{-1}(u_{iK}))$ $: i\in\Lambda_o\} = \{u_{iK} : i\in\Lambda_o\}$ forms a finite fuzzy supra open soft cover of $\tilde{1}_K$. This competes the proof.

Theorem 18. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be two fssts and let f_{pu}:$(X, \mathfrak{T}_1, E)\to(Y, \mathfrak{T}_2, K)$ be a surjective and fss-continuous function. If $(X, \mathfrak{T}_1, E)$ is fuzzy supra soft lindelöf, then $(Y, \mathfrak{T}_2, K)$ is also fuzzy supra soft lindelöf.*

Proof. It is similar to the proof of Theorem 17.

6. Conclusion

In this paper, we introduce and study the notion of fuzzy supra soft topological spaces. We introduce the some new concepts in fuzzy supra soft topological spaces. Since the authors introduced topological structures on fuzzy soft sets [10, 18], so some of the fuzzy soft topological properties is generalized here to fuzzy supra soft topological spaces which are basic for further research on fuzzy supra soft topological spaces. In future, we will introduce and generalize more fuzzy soft topological properties in previous studies to such spaces and the future research will be undertaken in this direction.

Conflict of Interest

We declare that, there is no conflict of interest regarding the publication of this manuscript.

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