EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 13, No. 5, 2020, 1270-1284
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

Special Issue Dedicated to Professor Hari M. Srivastava On the Occasion of his 80th Birthday

Analysis of Electro-MHD of third grade fluid flow through porous channel

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Abstract. This paper examines the Electro-MHD flow and heat transfer of a third grade fluid passing through a porous channel. An unidirectional and one-dimensional flow is propelled with the aid of lorentz force generated due to interaction of vertically applied magnetic field along with horizontally applied electric field. The equations of momentum and energy governing the third grade fluid flow are transformed to algebraic equation from nonlinear partial differential equation by implementing fully implicit finite difference scheme and solution is obtained by damped-Newton method. Lastly, the problem is simulated using MATLAB and the influence on velocity and temperature profiles with variation of non-dimensional parameters are depicted graphically. The noteworthy findings of this study is that the increasing values of elastic parameter α and non-Newtonian parameter γ diminishes the flow velocity and results in enhancement of temperature profile. A completely contrasting effect is observed for increasing values of strength of electric and magnetic field.

2020 Mathematics Subject Classifications: 76A05, 76W05
Key Words and Phrases: Electro-MHD (EMHD), third grade fluid, porous channel, finite-difference scheme, damped-Newton method.

NOMENCLATURE

- t - time.
- A - constant having dimension LT^{-1}.
- θ - temperature.

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DOI: https://doi.org/10.29020/nybg.ejpam.v13i5.3707

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- Re - Reynolds number.
- α - viscoelastic parameter.
- γ - non-Newtonian third grade elastic parameter.
- m - Hartmann number.
- Pr - Prandtl number.
- E' - viscous dissipation parameter.
- ν - kinematic viscosity.
- ρ - density.
- μ - dynamic viscosity.
- E - strength of applied Electric field .
- B_0 - strength of applied magnetic field.
- H - Dimensionless parameter related to electric strength.
- ζ_1 - ratio of heat generated by interaction of electrical and magnetic fields due to heat conduction.
- ζ_2 - ratio of joule heating to heat conduction.
- k - thermal conductivity.
- C_p - specific heat at constant pressure.
- V - velocity of suction/injection.
- σ - electrical conductivity of the medium.
- α_1, β_3 - Material constants.

1. Introduction

Substantial progression has been marked in the mechanics of third grade fluid over the years due to its various application in engineering, geology, pharmaceutical industries and agriculture. Third grade fluids are special subclass of non-Newtonian fluids that possesses the property of shear thinning and thickening. Many useful work has been done in connection with third grade fluids. Szeri and Rajagopal [26] investigated flow of third grade fluid passing through heated parallel plates. Rajagopal [22] gave a note with regard to drag for fluids of third grade. Pakdemirli [20] discussed the boundary layer equations of steady, incompressible, two-dimensional third grade fluid using special coordinate system.

Ariel [3] described the third grade fluid flow through porous flat channel. Akinshilo [2] examined the steady flow and heat transfer of third grade fluid with porous medium and heat generation. Awais [4] employed numerical inversion of Laplace transform to study some unsteady third grade fluid flows. Nayak and Padhy [19] analysed the time-dependent fluid flow of grade three fluid passing through porous channel. Adesanya and Falade [1] studied rate of entropy generation during flow and heat transfer of hydromagnetic fluid of grade three through horizontally placed parallel plates saturated with porous materials and obtained an analytical solution for the equations governing the fluid flow using regular perturbation method. Hayat et al [11] used the homotopy analysis method to derive an analytical solution for magnetohydrodynamics flow of a grade three fluid through porous channel and concluded that Reynold's number and Hartmann number have contrasting effects on velocity.

Electro-MHD [7, 10, 23] generally focuses on effective enhancement of fluid flows by employing electric fields, magnetic fields, or their suitable combinations and this phenomena generally finds immense application in micro pumps. Many researchers have done potential work on magnetohydrodynamic micro pumps [8, 14, 29]. Experiments reveal that combined EMHD effects can possibly be utilised to enhance the flow of liquid in microchannels.

Chakraborty and Paul [5] developed a mathematical model to study the combined effect of EMHD forces to control fluid flow through parallel plate rectangular microchannel and described the important roles of several non-dimensional parameters in the flow augmentation process. Wang et al. [27] presented the EMHD flow of a third fluid flowing through a parallel microchannel and obtained the analytical and numerical solution by perturbation method and Chebyshev spectral collocation method respectively and compared the results obtained. Muhammad et al. [16] described the EMHD flow and radiative heat exchange in inelastic fluids through porous microchannel. They considered the steady pressure driven flow and heat transfer of EMHD micro pump of grade three fluid through micro parallel plates embedded in a porous medium and obtained analytical solution to the problem by Homotopy perturbation method.

Hsiao [12] considered the MHD mixed convective heat transfer of second grade viscoelastic fluid past a wedge with porous suction/injection. Rashid et al. [24] performed corrugated walls analysis in microchannels through porous medium under the impact of Electro-MHD (EMHD) effects and investigated the analytical solutions of the velocity and volume flow rate by perturbation technique. Zhao et al [28] inspected the Electro-MHD (EMHD) flow and heat transfer characteristics of nanofluid inside a parallel plate microchannel. Parida and Padhy [21] examined the Electro-osmotic flow of a third-grade fluid past a channel having stretching walls. They used suitable similarity transformations to reduce the nonlinear partial differential equation to ordinary differential equations and obtained a numerical solution with the help of damped-Newton method.

Electric and magnetic fields have excellent ability for regulating flows and therefore finds immense application in medical diagnosis and magnetic/electromagnetic therapies and medical surgery. This phenomena is mainly used to study the blood flow, regarded as a third grade fluid through porous arteries and capillaries. This property is also implemented in micropumps. Due to these applications the present study becomes more significant. Taking into account the useful information obtained from the above mentioned studies, the principal objective of this paper is to study the combined effect of electric and magnetic field on the flow and heat transfer of third grade fluid through porous channels and a comprehensive study of impact of several non-dimensional parameters on the velocity and temperature profiles.

2. Introduction to the Problem

We have considered the Electro-MHD flow of an incompressible, viscous, unsteady third grade fluid in a porous channel with vertically applied magnetic field and horizontally applied electric field. The flow is propelled due to the lorentz force generated as a result of the combined effect of the applied forces. The lower plates coincides with $y = 0$ axis and upper plate coincides with $y = 1$ axis. A pictorial representation of the problem is shown in Figure 1.

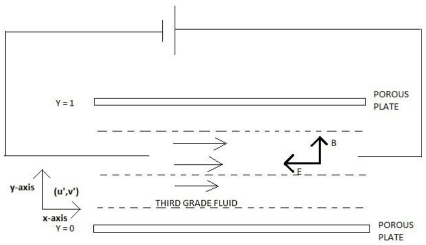

Figure 1: Geometry of the Flow Problem

3. Governing Equations

Using the stress equation of the third grade fluid [9, 25], the momentum equation is represented as

$$\rho\left(\frac{\partial u'}{\partial t'} + V\frac{\partial u'}{\partial y'}\right) = \mu\frac{\partial^2 u'}{\partial y'^2} + \alpha_1\frac{\partial^3 u'}{\partial y'^2 \partial t'} + 6\beta_3\frac{\partial^2 u'}{\partial y'^2}\frac{\partial u'}{\partial y'}^2 + \alpha_1 V\frac{\partial^3 u'}{\partial y'^3} - \sigma B_0^2 u' + \sigma B_0 E. \quad (1)$$

subjected to the following initial and boundary conditions

$$\begin{cases} t' = 0 : u' = 0, \ \forall\ y', \\ t' > 0 : u' = At^n, \ for\ y' = 0, \\ t' > 0 : u' = 0, \ for\ y' = 1. \end{cases} \quad (2)$$

where the value of n is considered to be 1. The equation of energy can be represented as :

$$\rho C_p\left(\frac{\partial\theta'}{\partial t'}+V\frac{\partial\theta'}{\partial y'}\right)=k\frac{\partial^2\theta'}{\partial y'^2}+\mu\left(\frac{\partial u'}{\partial y'}\right)^2+\alpha_1\left(\frac{\partial^2 u'}{\partial y'\partial t'}\right)\left(\frac{\partial u'}{\partial y'}\right)+\alpha_1 V\left(\frac{\partial^2 u'}{\partial y'^2}\right)\left(\frac{\partial u'}{\partial y'}\right)$$
$$+2\beta_3\left(\frac{\partial u'}{\partial y'}\right)^4+\sigma[(B_0)^2u'^2+E^2-2EB_0u']. \tag{3}$$

subjected to the subsequent initial and boundary conditions

$$\begin{cases} t'=0:\theta'=0,\ \forall\ y', \\ t'>0:\theta'=\theta_2,\ for\ y'=0, \\ t'>0:\theta'=0,\ for\ y'=1. \end{cases} \tag{4}$$

Introducing the non-dimensional parameters and values as follows:
$y=\frac{y'}{\sqrt{\nu T}},\ \ t=\frac{t'}{T},\ u=\frac{u'}{A},\ \theta=\frac{\theta'-\theta_1}{\theta_2-\theta_1},\ Re=\frac{V\sqrt{T}}{\sqrt{\nu}},\ \alpha=\frac{\alpha_1}{\rho\nu T}\ ,\ \gamma=\frac{6\beta_3A^2}{\rho\nu^2T}\ ,\ m^2=\frac{\sigma B_0^2T}{\rho},$
$\Pr=\frac{\nu\rho C_p}{k},\ E'=\frac{A^2}{C_p(\theta_2-\theta_1)},\ H=\frac{\sigma B_0ET}{\rho A},\ \zeta_1=\frac{2\nu A\sigma B_0ET}{k(\theta_2-\theta_1)},\ \zeta_2=\frac{\sigma E^2T\nu}{k(\theta_2-\theta_1)}.$
The above notations are employed in (1) and (3) to obtain the following dimensionless form

$$\frac{\partial u}{\partial t}+Re\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+Re\alpha\frac{\partial^3 u}{\partial y^3}+\gamma\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-m^2u+H. \tag{5}$$

subjected to the subsequent conditions

$$\begin{cases} t=0:u=0\ ,\ \forall\ y, \\ t>0:u=t^n,\ for\ y=0, \\ t>0:u=0,\ when\ y=1. \end{cases} \tag{6}$$

$$\frac{\partial\theta}{\partial t}+Re\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+E'\left(\frac{\partial u}{\partial y}\right)^2+\alpha E'\left(\frac{\partial^2 u}{\partial y\partial t}\right)\frac{\partial u}{\partial y}+\alpha ReE'\left(\frac{\partial^2 u}{\partial y^2}\right)\frac{\partial u}{\partial y}$$
$$+2\gamma E'\left(\frac{\partial u}{\partial y}\right)^4+m^2E'u^2-\zeta_1u+\zeta_2. \tag{7}$$

subjected to the boundary conditions

$$\begin{cases} t=0:\theta=0,\ \forall\ y, \\ t>0:\theta=1\ for\ y=0, \\ t>0:\theta=0,\ for\ y=1. \end{cases} \tag{8}$$

The nonlinear partial differential equation of velocity and temperature along with the boundary conditions are solved by implicit finite difference method and the numerical solution is obtained by implementing damped-Newton method as stated in [6].

4. Numerical Solution Procedure

We choose to take up the following solution strategy to solve the above (5). We implement the implicit finite difference scheme of crank-Nickolson type for discretisation in space as well as in time with a uniform mesh of space step h and time step k. The used scheme is unconditionally stable and suffices the second order convergence in time as well as in space. The derivatives at the nodes $(ih, j\triangle t),\ i = 0(1)N+1,\ j = 0(1)M-1$ are reckoned as

$$\frac{\partial u}{\partial t} \approx \frac{u_i^{j+1} - u_i^j}{\triangle t}.$$

$$\frac{\partial u}{\partial y} \approx \frac{1}{4h}((u_{i+1}^{j+1} - u_{i-1}^{j+1}) + (u_{i+1}^j - u_{i-1}^j)).$$

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{1}{2h^2}((u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}) + (u_{i+1}^j - 2u_i^j + u_{i-1}^j)).$$

$$\frac{\partial^3 u}{\partial y^2 \partial t} \approx \frac{1}{h^2 \triangle t}((u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}) - (u_{i+1}^j - 2u_i^j + u_{i-1}^j)).$$

$$\frac{\partial^3 u}{\partial y^3} \approx \frac{1}{2h^3}\left((-u_{i-2}^{j+1} + 2u_{i-1}^{j+1} - 2u_{i+1}^{j+1} + u_{i+2}^{j+1}) + (-u_{i-2}^j + 2u_{i-1}^j - 2u_{i+1}^j + u_{i+2}^j)\right),\ i \neq 1, N. \tag{9}$$

$$\text{At point } (1, j\triangle t), \frac{\partial^3 u}{\partial y^3} \approx \frac{1}{2h^3}\Big((-3u_{i-1}^{j+1} + 10u_i^{j+1} - 12u_{i+1}^{j+1} + 6u_{i+2}^{j+1} - u_{i+3}^{j+1}) + (-3u_{i-1}^j + 10u_i^j - 12u_{i+1}^j + 6u_{i+2}^j - u_{i+3}^j)\Big). \tag{10}$$

$$\text{At point } (N, j\triangle t), \frac{\partial^3 u}{\partial y^3} \approx \frac{1}{2h^3}\Big((u_{i-3}^{j+1} - 6u_{i-2}^{j+1} + 12u_{i-1}^{j+1} - 10u_i^{j+1} + 3u_{i+1}^{j+1}) + (u_{i-3}^j - 6u_{i-2}^j + 12u_{i-1}^j - 10u_i^j + 3u_{i+1}^j)\Big). \tag{11}$$

5. Solution of Velocity Equation

The above differences are implemented to the non-dimensional velocity equation to obtain the system of algebraic equations.

$$
\begin{aligned}
&u_i^{j+1} - u_i^j + \frac{Re\triangle t}{4h}\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^j - u_{i-1}^j\right) - \frac{\triangle t}{4h^2}\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1} + u_{i+1}^j - 2u_i^j + u_{i-1}^j\right)\\
&-\frac{\alpha}{h^2}\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1} - u_{i+1}^j + 2u_i^j - u_{i-1}^j\right)\\
&-\frac{\gamma\triangle t}{32h^4}\left(\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^j - u_{i-1}^j\right)^2\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1} + u_{i+1}^j - 2u_i^j + u_{i-1}^j\right)\right)\\
&-\frac{Re\alpha\triangle t}{2h^3}\left(-u_{i-2}^{j+1} + 2u_{i-1}^{j+1} - 2u_{i+1}^{j+1} + u_{i+2}^{j+1} - u_{i-2}^j + 2u_{i-1}^j - 2u_{i+1}^j + u_{i+2}^j\right)\\
&+m^2\triangle t\frac{u_i^{j+1} + u_i^j}{2} - H\triangle t = 0.\\
&for\ i = 2, 3, ..., N-1\ and\ j = 1, 2, ...M.
\end{aligned}
\tag{12}
$$

The discretised initial and boundary condition subjected to (12) is as follows

$$
\begin{cases}
u_i^0 = 0,\ i = 0(1)N+1,\\
u_0^j = (j\triangle t)^n\ and,\\
u_{N+1}^j = 0,\ j = 1(1)M.
\end{cases}
\tag{13}
$$

The above discretised velocity equation along with the subsequent conditions is solved by aforesaid numerical scheme.

A good initial guess is very important in order to achieve the desired level of accuracy. The next step is to calculate the residue matrix (R_i, where i ranges from 1 to N) and the Jacobian matrix $\left(\left(\frac{\partial R_i}{\partial u_j}\right),\ i = 1(1)N\ ,\ j = 1(1)M\right)$ as computed in [17, 18]. The system of equations formed by $Jh = -R$ is solved by Gauss-seidel method. The value of i is determined in a manner such that

$$
i = min\left(j : 0 \le j \le j_{max} \mid \parallel residue(x^k + \frac{h}{2^j}) \parallel_2 < \parallel residue(x^k) \parallel_2\right),
$$

and the next updated iteration is evaluated as $x^{k+1} = \left(x^k + \frac{h}{2^i}\right)$, which is repeated till the difference between two consecutive nodes become less that a prescribed error ϵ as in [13].

6. SOLUTION FOR ENERGY EQUATION

Discretised form of (7) is represented as

$$\theta_i^{j+1} - \theta_i^j + \frac{Re\triangle t}{4h}\left(\theta_{i+1}^{j+1} - \theta_{i-1}^{j+1} + \theta_{i+1}^{j} - \theta_{i-1}^{j}\right) - \frac{\triangle t}{2Prh^2}\left(\theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1} + \theta_{i+1}^{j} - 2\theta_i^{j} + \theta_{i-1}^{j}\right)$$
$$\frac{E'\triangle t}{4h}\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^{j} - u_{i-1}^{j}\right)^2 - \frac{\alpha E'}{8h^2\triangle t}\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^{j} - u_{i-1}^{j}\right)^2$$
$$- \frac{\alpha E' Re\triangle t}{8h^3}\left[\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1} + u_{i+1}^{j} - 2u_i^{j} + u_{i-1}^{j}\right)\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^{j} - u_{i-1}^{j}\right)\right]$$
$$- \frac{2\gamma E'\triangle t}{256h^4}\left(u_{i+1}^{j+1} - u_{i-1}^{j+1} + u_{i+1}^{j} - u_{i-1}^{j}\right)^4 - m^2\triangle t E'\left(\frac{u_i^{j+1} + u_i^j}{2}\right)^2 + \zeta_1\left(\frac{u_i^{j+1} + u_i^j}{2}\right) - \zeta_2 = 0. \tag{14}$$

subjected to the subsequent conditions

$$\begin{cases} \theta_i^0 = 0, \ i = 0(1)N+1, \\ \theta_0^j = 1 \ and, \\ \theta_{N+1}^j = 0, \ j = 1(1)M. \end{cases} \tag{15}$$

The energy equation is arranged in tridiagonal form. The initial guess for temperature is approximated according to the aforesaid conditions along with implementing the values obtained from (12) and a numerical solution is obtained by using an exponentially fitted scheme described in [15].

7. Results and Discussion

The unsteady EMHD flow characteristic of fluid of grade three passing between two infinite long porous plates is investigated through several graphs depicted in Fig. 2 to 23.

Fig. 2 represents the behaviour of velocity and temperature profiles with increasing value of elastic parameter α. For $\alpha = 0$ and $\gamma = 0$ the fluid behaves as a Newtonian fluid and the velocity graph takes a convex shape implying maximum flow velocity due to minimum elasticity. As we slightly increase the value of α with $\gamma = 0$, it coincides with second grade fluid and leads to decreasing momentum boundary layer thickness with decreasing velocity profile where as an increasing behaviour in the temperature profile is noticed as shown in Fig. 3.
When the non-Newtonian parameter γ is slightly increased to 0.01, the momentum boundary layer appears to increase with decreasing velocity profile and the temperature profile remains unaltered as depicted in Fig. 4 and 5.
Fig. 6 represents the impact on velocity profile with increasing values of α when $\gamma = 2$ and other parameters kept fixed. A sudden rise is noticed near the walls of the plate initially with gradual decrease in velocity where as the temperature increases by 20 times as compared to small values of elastic and non-Newtonian parameters α and γ respectively as

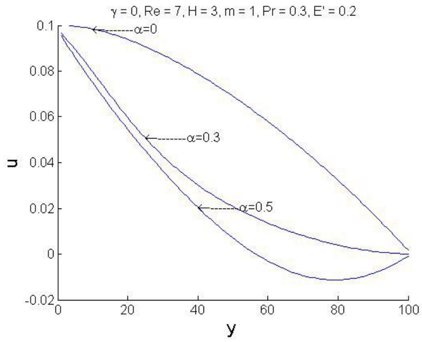

Figure 2: Influence on velocity with variations in α.

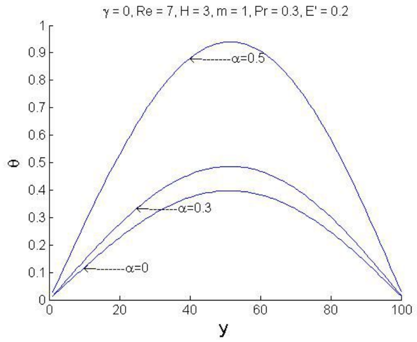

Figure 3: Influence on temperature with variations in α.

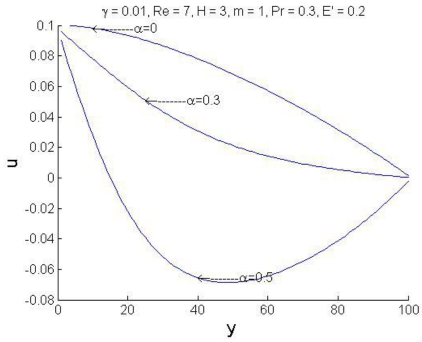

Figure 4: Influence on velocity with variations in α.

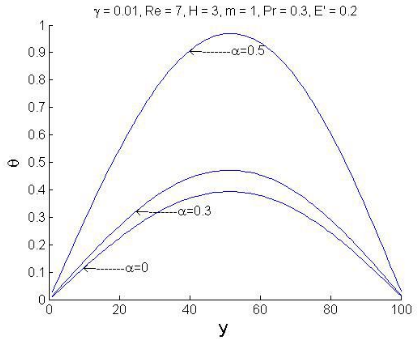

Figure 5: Influence on temperature with variations in α.

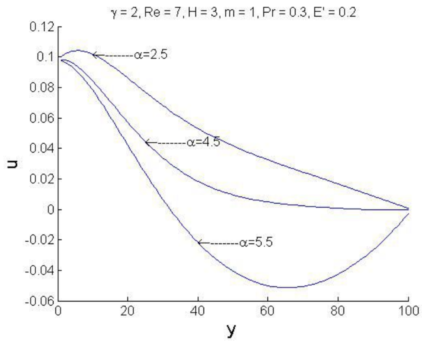

Figure 6: Influence on velocity with variations in α.

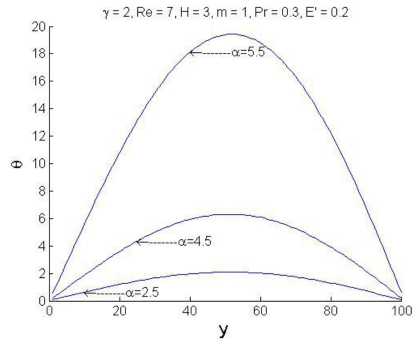

Figure 7: Influence on temperature with variations in α.

shown in Fig. 7. Hence, elasticity hinders the fluid flow and the elastic stresses developed in the fluid consequently enhances the temperature.

As the value of γ increases, shear thickening of fluid increases which results in fall of fluid velocity. The decreasing behaviour of velocity becomes more pronounced in the middle portion of the channel and it can be concluded that an inverse relationship exists between the non-Newtonian parameter and velocity profile for elastic parameter values 0, 0.9, 3 as shown in Fig. 8, 10 and 12. Temperature can be seen as a decreasing function of γ when $\alpha = 0$ but it completely reverses its behaviour because as we go on increasing the values of α, the entropy generation increases thus increasing the temperature profiles as represented in Fig. 9, 11 and 13. It should be mentioned here that keeping other parameters fixed,

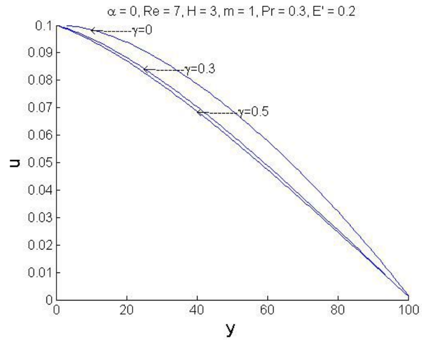

Figure 8: Influence on velocity with variations in γ.

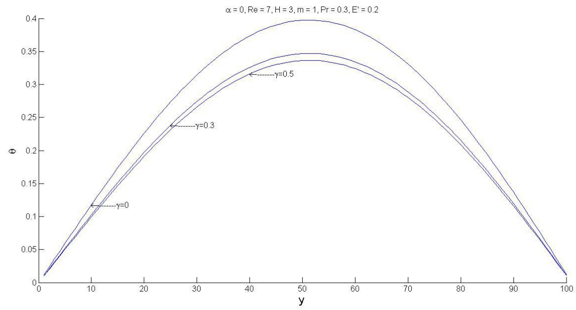

Figure 9: Influence on temperature with variations in γ.

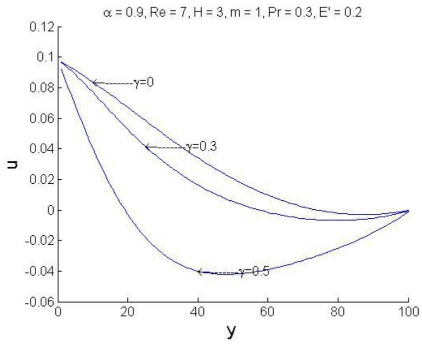

Figure 10: Influence on velocity with variations in γ.

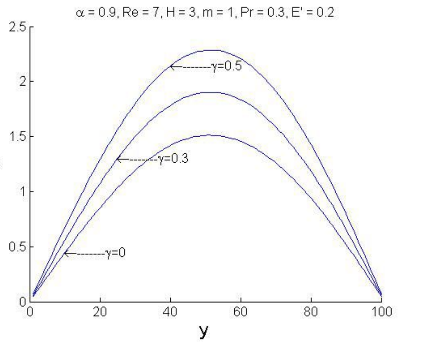

Figure 11: Influence on temperature with variations in γ.

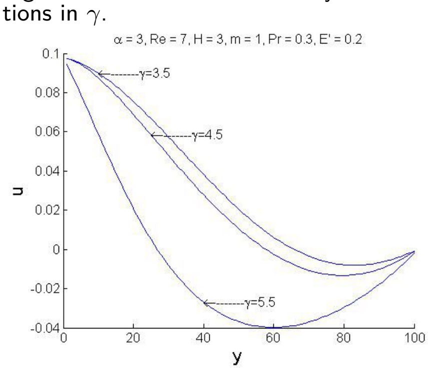

Figure 12: Influence on velocity with variations in γ.

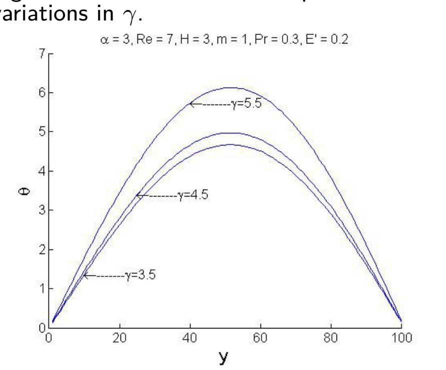

Figure 13: Influence on temperature with variations in γ.

when we increase α values, the flow field obtained is unsystematic.

The interaction of electric and magnetic forces gives rise to lorentz force. This force with magnitude $\sigma B_0 u'$ in axial direction is an inhibiting factor that opposes the fluid flow and with magnitude $\sigma B_0 E$ facilitates the fluid flow resulting increasing velocity of fluid as shown in Fig. 14 and 16 respectively.

Because of increase in resistive forces of the fluid the collision among the fluid molecules and between the plate and the fluid molecules increases as a result the overall thermal energy of the system is enhanced as depicted in Fig. 15.

The increased velocity enhances advection transport of heat energy from fluid molecules

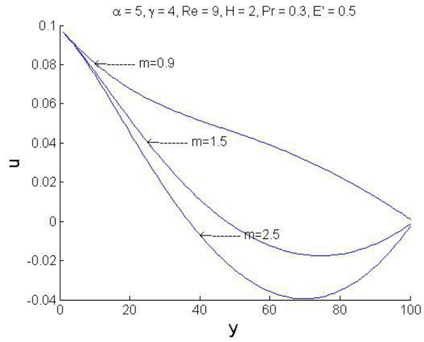

Figure 14: Influence on velocity with variations in m.

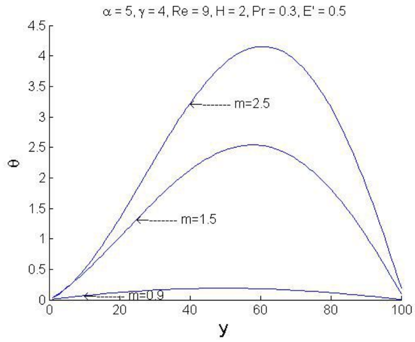

Figure 15: Influence on temperature with variations in m.

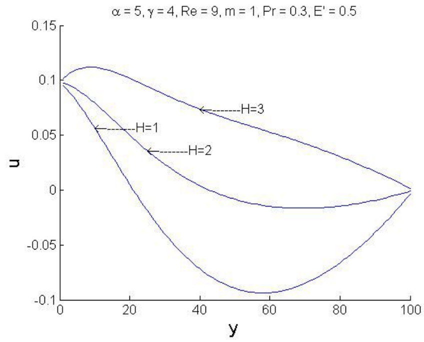

Figure 16: Influence on velocity with variations in H.

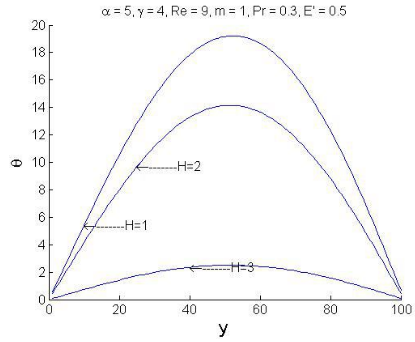

Figure 17: Influence on temperature with variations in H.

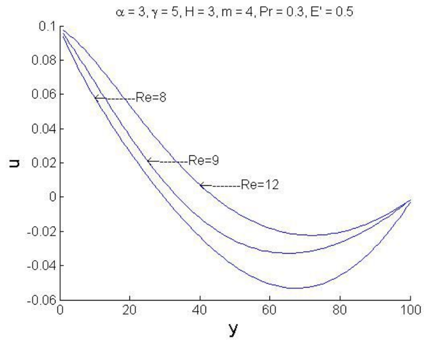

Figure 18: Influence on velocity with variations in Re.

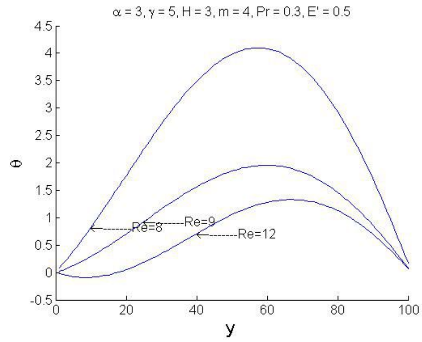

Figure 19: Influence on temperature with variations in Re.

to ambient surroundings through the porous plates resulting in fall in thermal energy of the system as depicted in Fig. 17.

Reynolds number is defined as the ratio between inertial force and viscous force. Increase Reynolds number decreases fluid viscosity and the inertial forces becomes more dominant than the viscous forces facilitating fluid flow through the porous channels. It is recognisable from Fig. 19 that temperature is a decreasing function of Reynolds number.

The effect of Prandtl number mirrors to that of Reynolds number. The decrease in thermal boundary layer thickness decreases the average temperature within the boundary layer resulting in overall fall in temperature profile as represented in Fig. 20.

Increase in the value of E' encourages entropy generation rate as a result of excessive heat

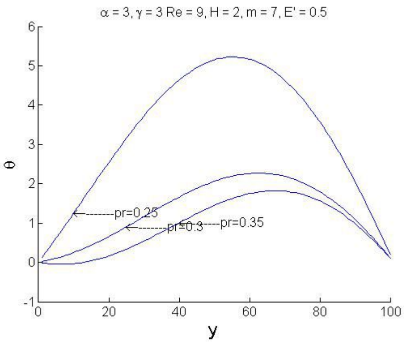

Figure 20: Influence on velocity with variations in Pr.

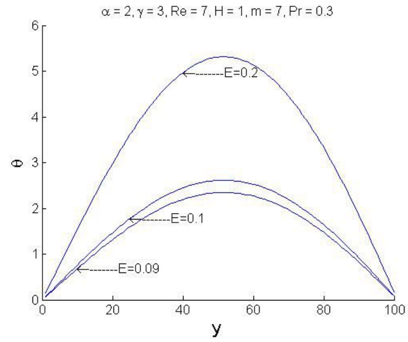

Figure 21: Influence on temperature with variations in E'.

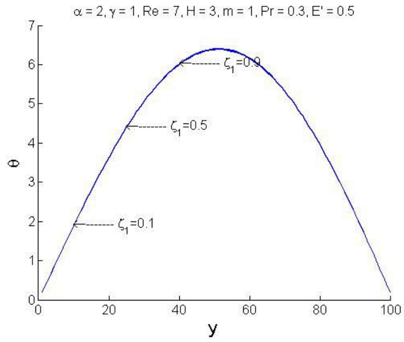

Figure 22: Influence on velocity with variations in ζ_1.

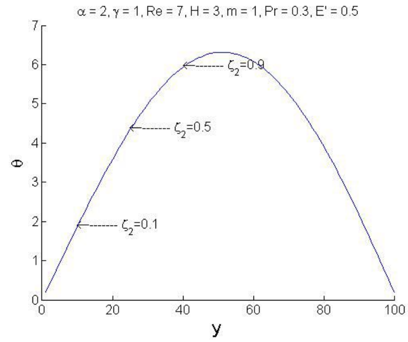

Figure 23: Influence on temperature with variations in ζ_2.

production within channel due to increased viscous heating. Hence increase in E' enhances the temperature at any point in the fluid as shown in Fig. 21.
It is apparent from Fig. 22 and 23 that ζ_1 and ζ_2 has negligible impact on the temperature field of third grade fluid as the variations are not prominent and only a marginal effect is observed.

8. Conclusion

In this study the momentum and energy equations, transformed to system of algebraic equations, is solved using fully implicit finite difference technique and numerical solution is obtained by damped-Newton method. The significance of aforesaid method is for every change in boundary conditions we don't have to repeat the whole complex calculations, some necessary modifications can give satisfactory results for both small and large parametric values. The damping criteria implemented in the problem ensures decrease in residual error in every iteration and better results are obtained as compared to Newton's method.
The subsequent conclusions can be drawn from the above depicted graphs.
For positive values of elastic parameter α and non-Newtonian parameter γ, the free flow velocity is restricted and temperature of the overall system increases. It can also be concluded that α has a stronger impact than the non-Newtonian parameter γ.

The imposed electric and magnetic field have completely opposite impacts on velocity and temperature profiles. Increased value of Hartmann number strength increases resistance to fluid flow and frequency of collisions within the system whereas increase in values of non dimensional parameter H facilitates fluid flow and encourages heat exchange to the surroundings. The remarkable impact of increasing values of Pr and Re in reducing the overall thermal energy of the system and E' in enhancing it is clearly verified from the graphical representations.

Acknowledgements

The authors would like to keep the record of the 80th birthday of Prof. H. M. Srivastava for his tremendous contribution to many significant developments in Mathematical research. Also, the authors express their heartfelt thanks to the editors and anonymous referees for their most valuable comments and constructive suggestions which led to the improvement of the earlier version of the manuscript.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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