Analytical discussion and sensitivity analysis of parameters of magnetohydrodynamic free convective flow in an inclined plate

S. Thamizh Suganya[1], P. Balaganesan[1], L. Rajendran[1], Marwan Abukhaled[2,*]

[1] *Department of Mathematics, Academy of Maritime Education & Training (Deemed to be University), Tamil Nadu, India*

[2] *Department of Mathematics and Statistics, American University of Sharjah, Sharjah, UAE*

Abstract. A mathematical model of the magnetohydrodynamic free convective flow of a viscous incompressible fluid, which is based on a system of coupled steady-state nonlinear deferential equations, is discussed. A new approach of the homotopy perturbation method is employed to derive analytical expressions of the fluid velocity, fluid temperature, and species concentration. The efficiency and accuracy of the derived results are tested against highly accurate and widely used numerical methods. The obtained analytical expressions are employed to study the effects of the magnetic field, chemical reaction, and other relevant flow parameters on fluid velocity, fluid temperature, and species concentration. Sensitivity analysis of these parameters is also presented.

2020 Mathematics Subject Classifications: 34B60, 65L10, 76W05

Key Words and Phrases: Analytic solution, mathematical modelling, boundary value problem, magnetohydrodynamic, porous medium, inclined plate

1. Introduction

The basic idea of magnetohydrodynamic (MHD) oscillatory flow in a channel is that, if the heat source is connected to a heat sink by an oscillating fluid, then the convective motion implies sharp spikes in velocity leading to optimal heat transport over pure conduction [24]. Underground water and energy storage systems, plasma physics, petroleum industries, nuclear reactors, and crystal growth are just a few applications of MHD convection with heat transfer.

In recent years, the development of heat and mass transfer processes, due to the effects of external forces, has been intensively studied in science and engineering research.

Examples of these studies include the effect of heat transfer on MHD oscillatory flow in an asymmetric wavy channel [24], the MHD convective flow through a porous medium in a horizontal channel [20], the effect of viscous dissipation on the MHD boundary layer flow [5], the MHD free convection and mass transfer flow with a heat generation [14], and the chemical and radiation effects on MHD heat and mass transfer flow along a moving vertical porous plate and a long a porous medium bounded by an inclined surface [22].

The effect of magnetic fields on the MHD boundary flow layers has been examined in several research. As examples, we mention the studies on the effect of induced magnetic and convective heating on MHD stagnation point flow [13], the heat transfer of unsteady MHD convective assuming that the semi-infinite plate is moving in a transverse magnetic field [8], and the effect of an electromagnetic field on natural convection in an inclined porous medium [6].

The mathematical model of the MHD free convective heat and mass transfer flow is represented by a system of nonlinear boundary differential equations for which no exact solution exists. Despite the fact that reliable numerical methods have been implemented to find approximate solutions, the need for reliable analytical solutions is necessary to investigate the effects of parameters variation on the governing system and hence improve the efficiency and control of the heat and mass transfer processes. Of the numerical methods that have been implemented by researchers, we mention a sixth-order Runge-Kutta coupled with a shooting method [29], an implicit finite difference method of Crank-Nicolson-type [5], a numerical integration scheme over the entire range of physical parameters [19], and a DuFort-Frankel finite difference method [21].

In the last two decades, researchers have carried out several theoretical investigations to study the effect of thermal radiation on magnetohydrodynamics flow and heat transfer. However, efficient and reliable analytical methods to find accurate approximate analytical solutions for the underlined nonlinear differential equations in unsteady MHD flow are still largely outnumbered by numerical simulations despite some remarkable techniques. For example, an analytical solution was obtained in terms of two-term harmonic and nonharmonic functions to discuss the MHD free convective flow through a porous medium past a vertical plate in the presence of heat absorption [27]. A Laplace transform technique was employed by Hussein et al. in a series of articles to investigate the MHD heat and mass transfer under various assumptions. Of these articles, we mention their studies on: The combined effects of Hall current and rotation on unsteady MHD free convective heat and mass transfer flow, the effect of thermal radiation on magneto-nanofluids free convective flow in the presence of an inclined magnetic field, and the effect of magnetic field, heat absorption and chemical reaction on fluid flow (see [11, 12, 26] and the references therein). Also a Laplace transform approach was used to study the effect of hall current on MHD natural convection heat and mass transfer of Casson fluid flow past a vertical plate with ramped wall temperature [25].

Other methods that are prone to deliver reliable analytical or semi-analytical solutions include, but not limited to, the variational iteration method [1, 17], the homotopy analysis method [16], and a Greens function based method [2–4, 15]. In this article, analytical expressions of the velocity, temperature, and concentration distributions are
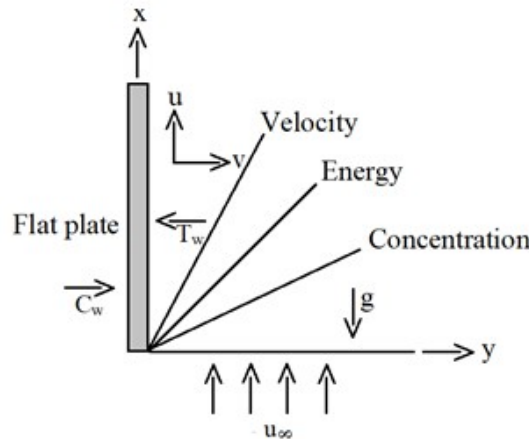
Figure 1: Physical model and coordinate system.

derived using a modified simple approach of the homotopy perturbation method. A sensitivity analysis is presented to explain the effect of parameter variation on the velocity profile.

2. Mathematical formulation of the problem

Consider a steady laminar, two-dimensional free convective flow of chemically reacting and viscous incompressible and electrically conducting along an inclined non-conducting plate kept at uniform temperature T'_w. It is assumed that the x'-axis is along the plate, the y'-axis is normal to it, and the flux is uniformly in the y' direction. Initially, the fluid and the plate are assumed to have the same temperature, and for $t' > 0$, the plate temperature is raised to T'_w, and the concentration level close to the plate is also raised to C'_w. The physical model and the coordinate system are illustrated in Figure 1.

The nonlinear equations of momentum, energy, and diffusion for steady state are, respectively, given by:

$$\nu\frac{d^2u'}{dy'^2} + g\beta(T'_w - T'_\infty)\cos\alpha + g\beta'(C'_w - C'_\infty)\cos\alpha - \frac{\sigma B_0{}^2}{\rho}u' - \frac{v'}{K'}u' = 0, \quad (1)$$

$$\kappa\frac{d^2T'}{dy'^2} + \mu\Big(\frac{du'}{dy'}\Big)^2 = 0, \quad (2)$$

$$D\frac{d^2C'}{dy'^2} - K'_r(C'_w - C'_\infty) = 0. \quad (3)$$

The corresponding initial and boundary conditions are:

$$u' = 0, \ T' = T'_w, \ C' = C'_w \quad \text{at} \ y' = 0, \quad (4)$$

$$u' \to 0, \ T' \to T'_\infty, \ C' \to C'_\infty \quad \text{as} \ y' \to \infty, \tag{5}$$

where $\nu = \frac{\mu}{\rho}$ (μ is the viscosity and ρ is the constant density of the fluid), g is the acceleration due to gravity, β is the coefficient of thermal expansion, β' is the coefficient of concentration expansion, α is the inclination angle from the vertical direction, σ is the electrical conductivity, B_0 is the magnetic induction and K' is the permeability of the porous medium, κ is the thermal conductivity, D is the molecular, and K'_r is the chemical reaction constant. Introduce the following dimensionless variables:

$$u = u'\Big(\nu g(T'_w - T'_\infty)\Big)^{1/3}, \qquad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \qquad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \qquad y = y'\Big(\frac{g\beta(T'_w - T'_\infty)}{\nu^2}\Big)^{1/3},$$

$$N = \frac{\beta^*(C'_w - C'_\infty)}{\beta(T' - T'_\infty)}, \qquad u_0 = \Big(\nu g\beta(T'_w - T'_\infty)\Big)^3, \qquad Pr = \frac{\mu C_p}{\kappa}, \qquad Ec = \frac{u_0^2}{C_p(T'_w - T'_\infty)},$$

$$K = \frac{K'\nu^2}{\nu_0^2}, \qquad M = \frac{\nu\sigma B_0^2}{u_0^2\rho}, \qquad Sc = \frac{\nu}{D}, \tag{6}$$

where N, M, K, Ec, Pr, Sc and Kr denote the buoyancy ratio parameter, magnetic parameter (Hartmann number), permeability parameter, Eckert number, Prandtl number, Schmidt number, and chemical reaction parameter, respectively. Substituting the variables in Eq. (6) into Eqs. (1)-(3) lead to the dimensionless steady state nonlinear equations of momentum, energy and diffusion

$$\frac{d^2u}{dy^2} - \Big(M + \frac{1}{k}\Big)u + \theta\cos\alpha + NC\cos\alpha = 0, \tag{7}$$

$$\frac{1}{PrEc}\frac{d^2\theta}{dy^2} + \Big(\frac{du}{dy}\Big)^2 = 0, \tag{8}$$

$$\frac{d^2C}{dy^2} - Sc\,Kr\,C = 0, \tag{9}$$

with the new dimensionless boundary conditions:

$$u = 0, \ \theta = 1, \ C = 1 \quad \text{at} \ y = 0, \tag{10}$$

$$u = 0, \ \theta = 0, \ C = 0 \quad \text{as} \ y \to \infty, \tag{11}$$

where u, θ, and C are dimensionless velocity, temperature, and concentration of fluid, respectively.

3. Derivation of analytical expression

Combining classical perturbation with homotopy theory, He [9, 10] developed the homotopy perturbation method (HPM), where the requirement of small parameters is waved. Over the past two decades, HPM has been employed by many researchers to obtain approximate analytical solutions for many nonlinear engineering dynamical systems

[7, 18]. In this section, a modified homotopy perturbation method [23, 28] is employed to obtain an analytical solution of the MHD free convective flow past an inclined plate under a steady state condition. The basic idea of the HPM is described in Appendix A.

Letting $A = M + \frac{1}{k}$ in Eqs. (7)-(8) gives

$$\frac{d^2u}{dy^2} - Au + \theta \cos \alpha + NC \cos \alpha = 0, \tag{12}$$

$$\frac{1}{Pr\,Ec}\frac{d^2\theta}{dy^2} + \left(\frac{du}{dy}\right)^2 = 0, \tag{13}$$

with boundary conditions

$$u = 0, \ \theta = 1, \quad \text{at } y = 0, \tag{14}$$

$$u = 0, \ \theta = 0, \quad \text{as } y \to \infty, \tag{15}$$

The exact solution of Eq. (9) is readily obtained

$$C = e^{-y\sqrt{Sc\,Kr}}. \tag{16}$$

Applying the homotopy as described in Eq. (A4) to Eqs. (12) and (13) gives

$$(1-p)\left(\frac{d^2u}{dy^2} - Au\right) + p\left(\frac{d^2u}{dy^2} - Au + \theta \cos \alpha + NC \cos \alpha\right) = 0, \tag{17}$$

$$(1-p)\left(\frac{1}{Pr\,Ec}\frac{d^2\theta}{dy^2} - \theta\right) + p\left(\frac{1}{Pr\,Ec}\frac{d^2\theta}{dy^2} - \left(\frac{du}{dy}\right)^2 - \theta + \theta\right) = 0, \tag{18}$$

for which the approximate series solutions are, respectively, given by

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \cdots, \tag{19}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3 + \cdots. \tag{20}$$

Direct substitution of Eqs. (19) and (20) into Eqs. (17) and (18) leads to

$$(1-p)\left(\frac{d^2(u_0 + pu_1 + p^2u_2 + \cdots)}{dy^2} - A(u_0 + pu_1 + p^2u_2 + \cdots)\right)$$
$$+ p\left(\frac{u_0 + pu_1 + p^2u_2 + \cdots}{dy^2} - A(u_0 + pu_1 + p^2u_2 + \cdots)\right. \tag{21}$$
$$\left. + (\theta_0 + p\theta_1 + p^2\theta_2 + \cdots)\cos \alpha + NC \cos \alpha\right) = 0,$$

$$(1-p)\left(\frac{1}{Pr\,Ec}\frac{d^2(\theta_0 + p\theta_1 + p^2\theta_2 + \cdots)}{dy^2} - A(\theta_0 + p\theta_1 + p^2\theta_2 + \cdots)\right)$$
$$+ p\left(\frac{1}{Pr\,Ec}\frac{d^2(\theta_0 + p\theta_1 + p^2\theta_2 + \cdots)}{dy^2} - \left(\frac{d(u_0 + pu_1 + p^2u_2 + \cdots)}{dy}\right)^2\right. \tag{22}$$
$$\left. - (\theta_0 + p\theta_1 + p^2\theta_2 + \cdots) + (\theta_0 + p\theta_1 + p^2\theta_2 + \cdots)\right) = 0.$$

Rearranging Eq. (21) according to the powers of p, gives the following set of equations:

$$p^0 : \quad \frac{d^2 u_0}{dy^2} - A u_0 = 0, \tag{23}$$

$$p^1 : \quad \frac{d^2 u_1}{dy^2} - A u_1 + \theta_0 \cos\alpha + NC \cos\alpha = 0, \tag{24}$$

$$\vdots$$

and from Eq. (22), we obtain the system

$$p^0 : \quad \frac{1}{Pr\,Ec} \frac{d^2 \theta_0}{dy^2} - \theta_0 = 0, \tag{25}$$

$$p^1 : \quad \frac{1}{Pr\,Ec} \frac{d^2 \theta_1}{dy^2} - \theta_1 + \left(\frac{du_0}{dy}\right)^2 + \theta_0 = 0, \tag{26}$$

$$\vdots$$

The sets of corresponding boundary conditions are, respectively

$$u_0 = -1, \quad \theta_0 = 1 \quad \text{at} \quad y = 0, \tag{27}$$
$$u_0 = 0, \quad \theta_0 = 0 \quad \text{as} \quad y \to \infty, \tag{28}$$

and

$$u_1 = 1, \quad \theta_1 = 0 \quad \text{at} \quad y = 0, \tag{29}$$
$$u_1 = 0, \quad \theta_1 = 0 \quad \text{as} \quad y \to \infty. \tag{30}$$

Substituting Eq. (16) into Eq. (24) and solving Eqs. (23)-(24) with boundary conditions (BC) (27)-(28) lead to

$$u_0 = -e^{-y\sqrt{A}}, \tag{31}$$

$$u_1 = e^{-y\sqrt{A}} + \frac{\cos\alpha}{Ec\,Pr - A}\left(e^{-y\sqrt{A}} - e^{-y\sqrt{Ec\,Pr}}\right) + \frac{N\cos\alpha}{Sc\,Kr - A}\left(e^{-y\sqrt{A}} - e^{-y\sqrt{Sc\,Kr}}\right). \tag{32}$$

The sum of u_0 and u_1 gives the following two-term HPM semi-analytic formula for the velocity:

$$u(y) = \frac{\cos\alpha}{Ec\,Pr - A}\left(e^{-y\sqrt{A}} - e^{-y\sqrt{Ec\,Pr}}\right) + \frac{N\cos\alpha}{Sc\,Kr - A}\left(e^{-y\sqrt{A}} - e^{-y\sqrt{Sc\,Kr}}\right). \tag{33}$$

Similarly, solving the system (25)-(26) with BCs (29)-(30) leads to

$$\theta_0 = e^{-y\sqrt{Ec\,Pr}}, \tag{34}$$

$$\theta_1 = \frac{AEc\,Pr}{4A - Ec\,Pr}\left(e^{-y\sqrt{Ec\,Pr}} - e^{-2y\sqrt{A}}\right) + \frac{y\sqrt{Ec\,Pr}}{2}e^{-y\sqrt{Ec\,Pr}}. \tag{35}$$

The sum of θ_0 and θ_1 gives the following two-term HPM semi-analytic formula for the temperature:

$$\theta(y) = \frac{A\,Ec\,Pr}{4A - Ec\,Pr}\left(e^{-y\sqrt{Ec\,Pr}} - e^{-2y\sqrt{A}}\right) + \left(\frac{y\sqrt{Ec\,Pr}}{2} + 1\right)e^{-y\sqrt{Ec\,Pr}}. \tag{36}$$

4. Analytical expressions for the skin friction and Nusselt and Sherwood numbers

As a direct conclusion of Eqs. (33), (36) and (16), analytic expressions for the local Skin-friction coefficient (C_f), the Nusselt number (N_u), and the Sherwood number (S_h), which are essential material parameters to analyze the rate of fluid velocity and temperature near to the plate [11], can be derived.

Skin friction From Eq. (16), an analytical expression of the dimensionless skin friction is given by

$$C_f = \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{\cos\alpha}{Ec\,Pr - A}\left(\sqrt{Ec\,Pr} - \sqrt{A}\right) + \frac{N\cos\alpha}{Sc\,Kr - A}\left(\sqrt{Sc\,Kr} - \sqrt{A}\right). \tag{37}$$

Nusselt number From Eq. (36), an analytical expression of the dimensionless rate of heat transfer (Nusselt number) is given by

$$N_u = \frac{\partial\theta}{\partial y}\Big|_{y=0} = \frac{A\,Ec\,Pr}{4A - Ec\,Pr}\left(2\sqrt{A} - \sqrt{Ec\,Pr}\right) - \frac{Ec\,Pr}{2}. \tag{38}$$

Sherwood number From Eq. (16), an analytical expression of the dimensionless rate of mass transfer (Sherwood number) takes the form

$$S_h = \frac{\partial C}{\partial y}\Big|_{y=0} = -\sqrt{Sc\,Kr}. \tag{39}$$

4.1. Results and discussion

In this section, we present numerical simulations to test the accuracy and reliability of the proposed method. The analytical expressions obtained in this paper will be compared to the highly accurate numerical solutions obtained by the MATLAB routine bvp4c, which is a finite-difference code that implements the three-stage Lobatto IIIA formula.

The analytical and numerical solutions were plotted on the same coordinates for a wide range of possible values of the underlined problem parameters. Figures 2,3, 5 and 8-11 reveal that the derived analytical expressions for the velocity, temperature, and concentration are in strong agreement with numerical solutions.

Velocity profiles for different values of buoyancy ratio parameter (N) are shown in Figure 2, where it is observed that an increase in N leads to an increase in velocity. The

effect of the magnetic parameter (M) on the velocity is exactly opposite to that of N as illustrated in Figure 3. That is, an increase in M leads to a decrease in the velocity or, in other words, any decrease in the fluid angle on the inclined plate leads to an increase in the flow of the velocity profile.

Eckert number (Ec), which characterizes the influence of self-heating of a fluid also affects the velocity profile. From Figure 4, we notice that increasing Ec implies a decrease in velocity. A Similar effect on the velocity profile is caused by Schmidt number (Sc), which is the ratio of the kinetic viscosity to the molecular diffusion coefficient. The inverse proportionality relation between Sc and velocity is presented in Figure 5.

Prandtl number is a dimensionless quantity that puts the viscosity of a fluid in correlation with the thermal conductivity. Figure 6 shows how an increase in Pr results in a decrease in the velocity profile. The chemical reaction parameter (Kr) has a similar affect on the velocity. That is, an increase in Kr leads to a decrease in the velocity, as illustrated in Figure 7. In fact, Schmidt, Prandtl, and Eckert numbers are all inversely proportional to velocity, as seen in Figures 4-7.

The exponential analytical expression of the temperature (Eq. (30)) justifies its inverse proportionality relation with Eckert and Prandtl numbers as well as the magnetic parameter, as depicted in Figures 810. Figure 11 shows that the increase in temperature that resulted from increasing the permeability parameter is insignificant.

The effects of problem parameters are summarized in the sensitivity analysis chart depicted in Figure 12. In this figure, where the rate of change of momentum u was computed (using the experimental values $N = 30, \alpha = 0.8, M = 1, k = 1, Ec = 1, Pr = 1, Sc = 0.6$ and $Kr = 2$), it is shown that α has the most impact (72.3%) on the rate of change of velocity followed by Schmidt number with 13.6% impact on the rate of change of velocity. For the same experimental values, the sensitivity analysis of problem parameters reveals that Ec and Pr have the most impact (45% each) on the rate of change of temperature distribution. As for the concentration, the exact solution of equation (9) explains the reason why Schmidt number Sc and the permeability parameter Kr have an identical impact on the concentration profile (50% each).

5. Conclusion

In this paper, a free convective and mass transfer magnetic field of a viscous incompressible fluid in an inclined plate was presented. A modified version of the homotopy perturbation method was employed to derive analytical expressions for the fluid velocity, temperature, and concentration of species. These analytical expressions were used to study the effects of the system parameters on temperature and velocity profiles. Analytical expressions for the Skin-friction and Nusselt and Sherwood numbers were also derived. The accuracy of the analytical solutions was confirmed by noticing a strong agreement with MATLAB-generated numerical simulations.
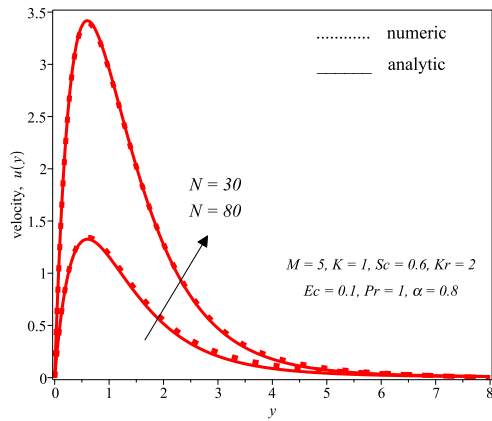
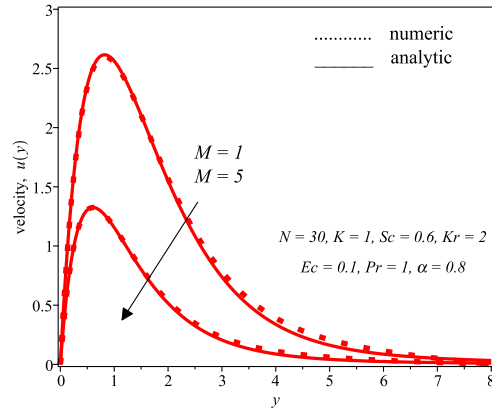Figure 2: Effect of buoyancy ratio (N) on velocity profile.

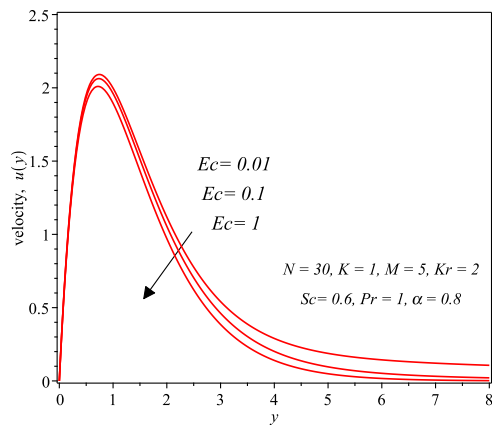Figure 3: Effect of the magnetic parameter (M) on velocity profile.

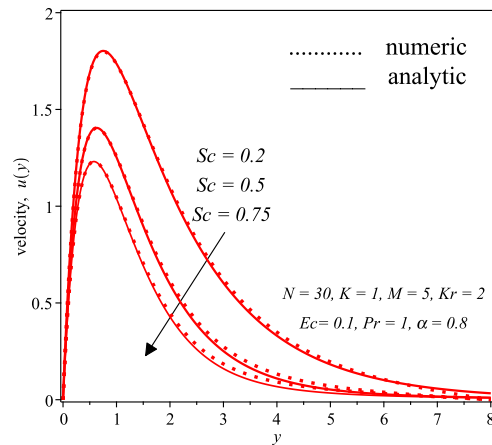Figure 4: Effect of Eckert number (Ec) on velocity profile.

Figure 5: Effect of Schmidt number (Sc) on velocity profile.

Appendix A. The homotopy perturbation method

Consider the nonlinear differential equation

$$A(u) - f(r) = 0, \;\; r \in \Omega, \tag{A1}$$

with the boundary condition

$$B\left(u, \frac{du}{dr}\right) = 0, \;\; r \in \Gamma, \tag{A2}$$

where $A, B, f(r)$ and Γ are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain Ω , respectively. Expressing $A(u)$ as the sum of linear (L) and nonlinear (N) parts, Eq. (A1) becomes
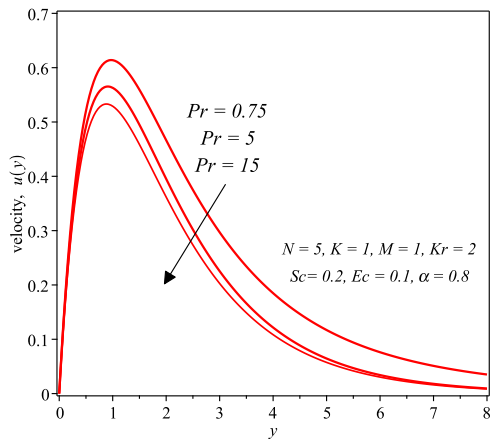
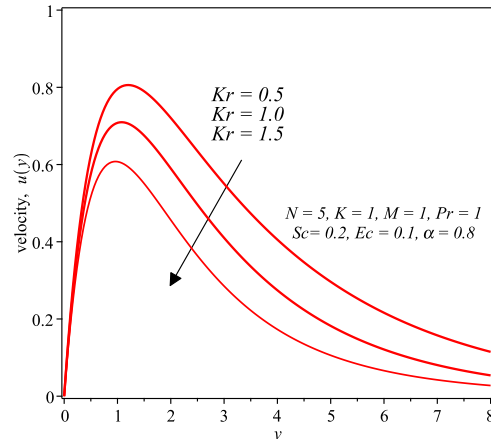Figure 6: Effect of Prandtl number (Pr) on velocity profile.

Figure 7: Effect of chemical reaction parameter (Kr) on velocity profile.
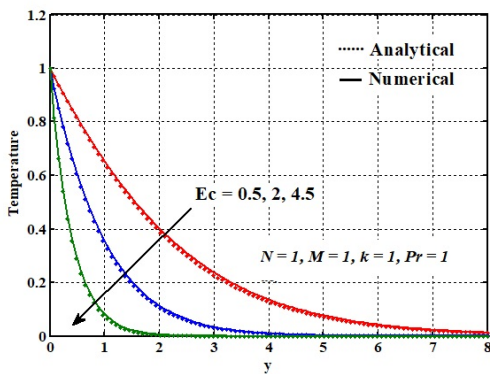

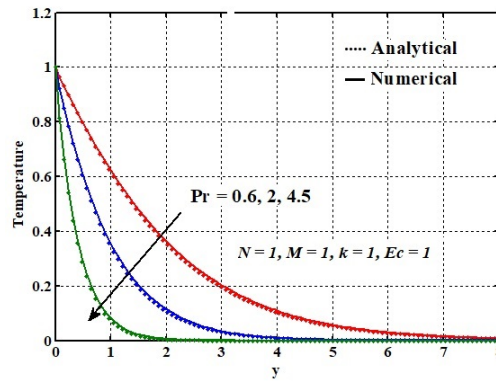Figure 8: Effect of Eckert number Ec on temperature profile.

Figure 9: The effect of Prandtl number on (Pr) temperature profile.

$$L(u) + N(u) - f(r) = 0. \tag{A3}$$

The homotopy technique begins by defining $v(r, p) : \Omega \times [0, 1] \rightarrow R$, such that

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{A4}$$

where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of Eq. (A1) that satisfies boundary conditions (A2). Evidently, Eq.(A4) implies that

$$
\begin{aligned}
H(v, 0) &= L(v) - L(u_0) = 0, \tag{A5} \\
H(v, 1) &= A(v) - f(r) = 0. \tag{A6}
\end{aligned}
$$

As p changes from 0 to 1, $v(r, p)$ changes from u_0 to u_r, a process known as a homotopy. The solution of Eq. (A4) may be expressed in terms of a power series in the form:
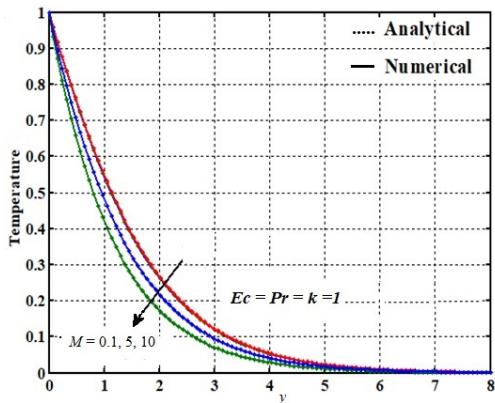
$$v = v_0 + p v_1 + p^2 v_2 + \cdots . \tag{A7}$$

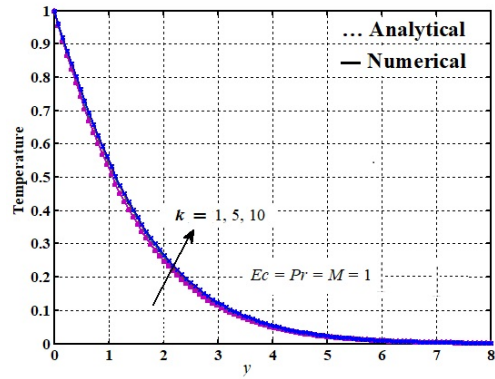Figure 10: The effect of Prandtl number (Pr) concentration profile.

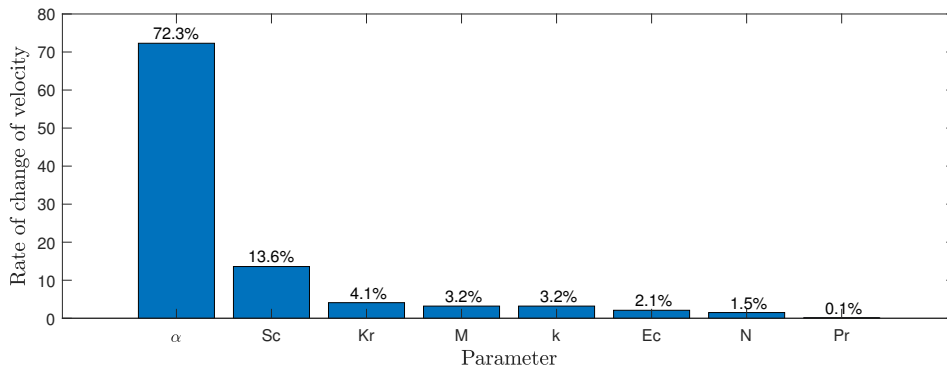Figure 11: The effect of Schmidt number on (Sc) concentration profile.

Figure 12: Sensitive analysis of parameters on velocity profile

With $p = 1$, an approximate solution to Eq. (A4) is given by:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots. \tag{A8}$$

Acknowledgements

References

[1] M. Abukhaled. Variational iteration method for nonlinear singular two-point boundary value problems arising in human physiology. *Journal of Mathematics*, 2013:1–4, 2013.

[2] M. Abukhaled. Green's function iterative approach for solving strongly nonlinear oscillators. *Journal of Computational and Nonlinear Dynamics*, 12(5):051021, 2017.

[3] M. Abukhaled and S. Khuri. An Efficient Semi-Analytical Solution of a One-Dimensional Curvature Equation that Describes the Human Corneal Shape. *Mathematical and Computational Applications*, 24(1):8, 2019.

[4] M. Abukhaled and S.A. Khuri. A semi-analytical solution of amperometric enzymatic reactions based on Green's functions and fixed point iterative schemes. *Journal of Electroanalytical Chemistry*, 792:66–71, 2017.

[5] R.N. Barik and G.C. Dash. Thermal radiation effect on an unsteady magnetohydrodynamic flow past inclined porous heated plate in the presence of chemical reaction and viscous dissipation. *Applied Mathematics and Computation*, 226:423–434, 2014.

[6] W. Bian, P Vasseur, E Bilgen, and F Meng. Effect of an electromagnetic field on natural convection in an inclined porous layer. 17(1):36–44, 1996.

[7] J. Biazar, F. Badpeima, and F. Azimi. Application of the homotopy perturbation method to ZakharovKuznetsov equations. *Computers & Mathematics with Applications*, 58(11-12):2391–2394, 2009.

[8] Ali J. Chamkha. Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. *International Journal of Engineering Science*, 42(2):217–230, 2004.

[9] J.H. He. Homotopy perturbation technique. *Comput. Methods Appl. Mech. Engrg.*, 178:257–262, 1999.

[10] J.H. He. Application of homotopy perturbation method to nonlinear wave equations. *Chaos, Solitons & Fractals*, 26(3):695–700, 2005.

[11] S. M. Hussain, J. Jain, G. S. Seth, and M. M. Rashidi. Effect of thermal radiation on magneto-nanofluids free convective flow over an accelerated moving ramped temperature plate. *Scientia Iranica*, 25(3):1243–1257, 2017.

[12] S.M. Hussain, J. Jain, G.S. Seth, and M.M. Rashidi. Free convective heat transfer with hall effects, heat absorption and chemical reaction over an accelerated moving plate in a rotating system. *Journal of Magnetism and Magnetic Materials*, 422:112–123, 2017.

[13] W. Ibrahim. The effect of induced magnetic field and convective boundary condition on MHD stagnation point flow and heat transfer of upper-convected Maxwell fluid in the presence of nanoparticle past a stretching sheet. *Propulsion and Power Research*, 5(2):164–175, 2016.

[14] S. Islam, S. Ara, and P. Dey. MHD free convection and mass transfer flow with heat generation through an inclined plate. *Annals of Pure and Applied Mathematics*, 3(2):129–141, 2013.

[15] S. A. Khuri and M. Abukhaled. Efficient numerical treatment of a conductive-radiative fin with temperature-dependent thermal conductivity and surface emissivity. *International Journal for Computational Methods in Engineering Science and Mechanics*, pages 1–10, 2020.

[16] S. Liao. *Homotopy analysis method in nonlinear differential equations.* Higher Education Press ; Springer, Beijing : Heidelberg ; New York, N.Y, 2012.

[17] Junfeng Lu. Variational iteration method for solving a nonlinear system of second-order boundary value problems. *Computers & Mathematics with Applications*, 54(7-8):1133–1138, 2007.

[18] A. Meena and L. Rajendran. Mathematical modeling of amperometric and potentiometric biosensors and system of non-linear equations Homotopy perturbation approach. *Journal of Electroanalytical Chemistry*, 644(1):50–59, 2010.

[19] H. Mondal, D. Pal, S. Chatterjee, and P. Sibanda. Thermophoresis and Soret-Dufour on MHD mixed convection mass transfer over an inclined plate with non-uniform heat source/sink and chemical reaction. *Ain Shams Engineering Journal*, 9(4):2111–2121, 2018.

[20] K. Raju, T. Sudhakar Reddy, M. Raju, P. Satya Narayana, and S. Venkataramana. MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating. *Ain Shams Engineering Journal*, 5(2):543–551, 2014.

[21] R. Ramana and J. Kumar. Chemical reaction effects on MHD free convective flow past an inclined plate. *Asian J. Current Engineering and Maths*, 3:66–71, 2014.

[22] G.V. Ramana Reddy, N. Bhaskar Reddy, and R.S.R. Gorla. Radiation and chemical reaction effects on MHD flow along a moving vertical porous plate. *International Journal of Applied Mechanics and Engineering*, 21(1):157–168, 2016.

[23] R. Saravanakumar, P. Pirabaharan, M. Abukhaled, and L. Rajendran. Theoretical analysis of voltammetry at a rotating disk electrode in the absence of supporting electrolyte. *The Journal of Physical Chemistry B*, 124(3):443–450, 2020.

[24] P.V. Satya Narayana, B. Venkateswarlu, and B. Devika. Chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel. *Ain Shams Engineering Journal*, 7(4):1079–1088, 2016.

[25] G.S. Seth, A. Bhattacharyya, and R. Tripathi. Effect of hall current on MHD natural convection heat and mass transfer flow of rotating fluid past a vertical plate with ramped wall temperature. *Frontiers in Heat and Mass Transfer*, 9, 2017.

[26] G.S. Seth, S.M. Hussain, and S. Sarkar. Hydromagnetic natural convection flow with heat and mass transfer of a chemically reacting and heat absorbing fluid past an accelerated moving vertical plate with ramped temperature and ramped surface

concentration through a porous medium. *Journal of the Egyptian Mathematical Society*, 23(1):197–207, 2015.

[27] G. Sreedhar and B. Rama Bhupal Reddy. Chemical reaction effect on unsteady MHD flow past an infinite vertical porous plate in the presence of heat absorption. *IJARET*, 10(1), 2019.

[28] R. Swaminathan, K. Venugopal, M. Rasi, M. Abukhaled, and L. Rajendran. Analytical expressions for the concentration and current in the reduction of hydrogen peroxide at a metal-dispersed conducting polymer film. *Qumica Nova*, 2019.

[29] Md. Nasir Uddin, M. Alim, and M. Chowdhury. Effects of mass transfer on MHD mixed convective flow along inclined porous plate. *Procedia Engineering*, 90:491–496, 2014.