Further results on fuzzy soft BCK/BCI-algebras

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Abstract. In this paper, we obtain further results on fuzzy soft BCK/BCI-algebras. In fact, we introduce the notion of fuzzy soft sub-BCK/BCI-algebra and investigate related properties.

1. Introduction

Soft set theory was introduced initially by Molodtsov in 1999 [12] as a mathematical tool to model uncertainty and vagueness. In [1], Ali et al. studied some operations between two soft sets. Furthermore, they improved the definition of the complement of a soft set then studied DeMorgan's type results in soft set theory. Soft set theory has been applied in different directions some are shown in [12] and other applications are shown in [5] and [11]. Based on soft set theory many researches has been done (see for example [3, 7, 9, 14–18]). Wong [20] used fuzzy set theory introduced by Zadeh [21] to extend general topology to fuzzy topology. Jun [6] studied fuzzy subalgebras of BCK/BCI-algebras based on the relations belongs to and quasi-coincidence with.

The authors in [4] introduced and studied fuzzy soft groups and fuzzy soft homomorphisms. Maji et al. [10] defined and studied fuzzy soft sets. Roy and Maji [19] considered the notion of fuzzy soft set and used it to present a theoretic approach to decision making problems and Jun et al. [8] applied the same notion to BCK/BCI-algebras. Recently, Al-Masarwah and Ahmad [2] applied the notion of m-polar fuzzy sets to BCK/BCI-algebras.

As shown above, the BCK/BCI-algebra which is introduced by Iséki was extensively investigated by several researchers and this paper gives further results on fuzzy soft BCK/BCI-algebras. We start by recalling the definition of the algebras we are studying and the basic definitions of soft sets and fuzzy soft sets and the definition of some operations related. Then we investigate further results that are not studied in [8].

2. Preliminaries

An algebra $(B; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

1. $(\forall \beta, \gamma, \delta \in B) \, (((\beta * \gamma) * (\beta * \delta)) * (\delta * \gamma) = 0)$,

2. $(\forall \beta, \gamma \in B) \, ((\beta * (\beta * \gamma)) * \gamma = 0)$,

3. $(\forall \beta \in B) \, \beta * \beta = 0)$,

4. $(\forall \beta, \gamma \in B) \, (\beta * \gamma = 0, \gamma * \beta = 0 \Rightarrow \beta = \gamma)$.

If a *BCI*-algebra B satisfies $(0 * \beta = 0)$, $(\forall \beta \in B)$ then B is called a *BCK-algebra*. Any *BCK*-algebra B satisfies the following properties:

- $(\forall \beta \in B) \, (\beta * 0 = \beta)$,

- $(\forall \beta, \gamma, \delta \in B) \, (\beta \leq \gamma \Rightarrow \beta * \delta \leq \gamma * \delta, \, \delta * \gamma \leq \delta * \beta)$,

- $(\forall \beta, \gamma, \delta \in B) \, ((\beta * \gamma) * \delta = (\beta * \delta) * \gamma)$,

- $(\forall \beta, \gamma, \delta \in B) \, ((\beta * \delta) * (\gamma * \delta) \leq \beta * \gamma)$

where $\beta \leq \gamma$ if and only if $\beta * \gamma = 0$.

Any *BCI*-algebra B satisfies the properties:

- $(\forall \beta, \gamma, \delta \in B) \, (0 * (0 * ((\beta * \delta) * (\gamma * \delta))) = (0 * \gamma) * (0 * \beta))$,

- $(\forall \beta, \gamma \in B) \, (0 * (0 * (\beta * \gamma)) = (0 * \gamma) * (0 * \beta))$.

For a nonempty subset A of a *BCK/BCI*-algebra B, if $\beta * \gamma \in A$ for all $\beta, \gamma \in A$ then A is said to be a *BCK/BCI-subalgebra* of B.

A fuzzy set ϱ in a *BCK/BCI*-algebra B which satisfies

$$(\forall \beta, \gamma \in B) \, (\varrho(\beta * \gamma) \geq \min\{\varrho(\beta), \varrho(\gamma)\}) \tag{1}$$

is said to be a *fuzzy BCK/BCI-algebra*.

Let ϱ be a fuzzy set in a set B defined by:

$$\varrho(\gamma) := \begin{cases} k \in (0, 1] & \text{if } \gamma = \beta, \\ 0 & \text{if } \gamma \neq \beta. \end{cases}$$

Then ϱ is called a *fuzzy point* with support β and value k and is denoted by β_k.

For an initial universe set U, let $P(U)$ denotes the power set of U and for a set of parameters E, let $M \subset E$. Molodtsov [12] defined the soft set as follows.

Definition 1 ([12])**.** *A soft set over U, is a pair (μ, M) where μ is a mapping given by*

$$\mu : M \to P(U).$$

Clearly, a soft set is not a set. Several examples have been considered by Molodtsov in [12].

Definition 2 ([10]). *Let E be a set of parameters and $M \subseteq E$. A fuzzy soft set over an initial universe set U is a pair $(\tilde{\mu}, M)$ where $\tilde{\mu}$ is a mapping from M to the set of all fuzzy sets in U.*

In general, for every $m \in M$, $\tilde{\mu}[m]$ is a fuzzy set in U and it is called *fuzzy value set* of parameter m.

Definition 3 ([10]). *The "union" of two fuzzy soft sets $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ over a common universe U, is the fuzzy soft set $\left(\tilde{\xi}, Q\right)$ satisfying the following conditions:*

(i) $Q = M \cup N$,

(ii) *for all $q \in Q$,*

$$\tilde{\xi}[q] = \begin{cases} \tilde{\mu}[q] & \text{if } q \in M \setminus N, \\ \tilde{\eta}[q] & \text{if } q \in N \setminus M, \\ \tilde{\mu}[q] \cup \tilde{\eta}[q] & \text{if } q \in M \cap N. \end{cases}$$

We write $(\tilde{\mu}, M) \, \tilde{\cup} \, (\tilde{\eta}, N) = \left(\tilde{\xi}, Q\right)$.

Definition 4 ([10]). *For two fuzzy soft sets $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ over a common universe U, the $(\tilde{\mu}, M)$ "AND" $(\tilde{\eta}, N)$ denoted by $(\tilde{\mu}, M) \, \tilde{\wedge} \, (\tilde{\eta}, N)$ is defined by*

$$(\tilde{\mu}, M) \, \tilde{\wedge} \, (\tilde{\eta}, N) = \left(\tilde{\xi}, M \times N\right),$$

where $\tilde{\xi}[m, n] = \tilde{\mu}[m] \cap \tilde{\eta}[n]$ for all $(m, n) \in M \times N$.

Definition 5 ([1]). *The "extended intersection" of two soft sets $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ over a common universe U, is the soft set $(\tilde{\xi}, Q)$ satisfying the following conditions:*

(i) $Q = M \cup N$,

(ii) *for all $q \in Q$,*

$$\tilde{\xi}[q] = \begin{cases} \tilde{\mu}[q] & \text{if } q \in M \setminus N, \\ \tilde{\eta}[q] & \text{if } q \in N \setminus M, \\ \tilde{\mu}[q] \cap \tilde{\eta}[q] & \text{if } q \in M \cap N. \end{cases}$$

We write $(\tilde{\mu}, M) \, \tilde{\cap}_{\mathrm{e}} \, (\tilde{\eta}, N) = (\tilde{\xi}, Q)$.

Definition 6 ([1]). *The "restricted intersection" of two soft sets $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ over a common universe U where $M \cap N \neq \emptyset$ is denoted by $(\tilde{\mu}, M) \, \tilde{\cap}_{\mathrm{r}} \, (\tilde{\eta}, N)$ and is defined as $(\tilde{\mu}, M) \, \tilde{\cap}_{\mathrm{r}} \, (\tilde{\eta}, N) = (\tilde{\xi}, Q)$, where $Q = M \cap N$ and for all $q \in Q$, $\tilde{\xi}[q] = \tilde{\mu}[q] \cap \tilde{\eta}[q]$.*

3. Fuzzy soft BCK/BCI-algebras

In what follows, B is a BCK/BCI-algebra and E is a set of parameters.

Definition 7 ([8]). *For a fuzzy soft set $(\tilde{\mu}, M)$ over B where M is a subset of E, we say that $(\tilde{\mu}, M)$ is a fuzzy soft BCK/BCI-algebra based on a parameter m over B if there exists $m \in M$ such that $\tilde{\mu}[m]$ is a fuzzy BCK/BCI-algebra in B. If $(\tilde{\mu}, M)$ is a fuzzy soft BCK/BCI-algebra based on a parameter m over B for all $m \in M$, we say that $(\tilde{\mu}, M)$ is a fuzzy soft BCK/BCI-algebra over B.*

Definition 8. *Let $(\tilde{\mu}, M)$ be a fuzzy soft BCK/BCI-algebra over B. Then*

(1) *$(\tilde{\mu}, M)$ is said to be θ-identity, where $\theta \in (0, 1]$, if it satisfies:*

$$(\forall m \in M)(\forall \beta \in B) \left(\tilde{\mu}[m](\beta) = \left\{ \begin{array}{ll} \theta & \text{if } \beta = 0, \\ 0 & \text{otherwise} \end{array} \right. \right).$$

(2) *$(\tilde{\mu}, M)$ is said to be θ-absolute, where $\theta \in (0, 1]$, if $\tilde{\mu}[m](\beta) = \theta$ for all $\beta \in B$ and $m \in M$.*

Example 1. *Consider a BCI-algebra $B = \{0, 1, 2, \beta, \gamma\}$ with the following Cayley table:*

$*$	0	1	2	β	γ
0	0	0	0	β	β
1	1	0	1	γ	β
2	2	2	0	β	β
β	β	β	β	0	0
γ	γ	β	γ	1	0

(1) Let $M = \{m_1, m_2, m_3, m_4\}$ be a set of parameters and define a fuzzy soft set $(\tilde{\mu}, M)$ as follows:

$\tilde{\mu}$	0	1	2	β	γ
m_1	0.03	0	0	0	0
m_2	0.03	0	0	0	0
m_3	0.03	0	0	0	0
m_4	0.03	0	0	0	0

Then $(\tilde{\mu}, M)$ is a 0.03-identity fuzzy soft BCI-algebra over B.

(2) Let $N = \{n_1, n_2, n_3\}$ be a set of parameters and define a fuzzy soft set $(\tilde{\eta}, N)$ as follows:

$\tilde{\eta}$	0	1	2	β	γ
n_1	0.4	0.4	0.4	0.4	0.4
n_2	0.4	0.4	0.4	0.4	0.4
n_3	0.4	0.4	0.4	0.4	0.4

Then $(\tilde{\eta}, N)$ is a 0.4-absolute fuzzy soft BCI-algebra over B.

Theorem 1. *Let $\pi : B \to C$ be a homomorphism of BCK/BCI-algebras. If a fuzzy soft BCK/BCI-algebra $(\tilde{\mu}, M)$ over B satisfies:*

$$(\forall m \in M)(\forall \beta \in B) \left(\tilde{\mu}[m](\beta) = \left\{ \begin{array}{ll} \theta & \text{if } \beta \in \mathrm{Ker}\pi, \\ 0 & \text{otherwise,} \end{array} \right. \right) \tag{2}$$

then $(\pi(\tilde{\mu}), M)$ is a θ-identity fuzzy soft BCK/BCI-algebra over C.

Proof. Let $m \in M$ and $\gamma \in C$. If $\gamma = 0_C$ (the zero element of C), then $0_B \in \mathrm{Ker}\pi$ where 0_B is the zero element of B and so

$$\pi(\tilde{\mu})[m](\gamma) = \pi(\tilde{\mu}[m])(0_C) = \sup_{\beta \in \pi^{-1}(0_C)} \tilde{\mu}[m](\beta) = \sup_{\beta \in \mathrm{Ker}\pi} \tilde{\mu}[m](\beta) = \theta.$$

If $\gamma \neq 0_C$, then $\pi(\tilde{\mu})[m](\gamma) = 0$. Therefore, $(\pi(\tilde{\mu}), M)$ is a θ-identity fuzzy soft BCK/BCI-algebra over C.

Theorem 2. *Let $\pi : B \to C$ be a homomorphism of BCK/BCI-algebras. If $(\tilde{\mu}, M)$ is a θ-absolute fuzzy soft BCK/BCI-algebra over B, then $(\pi(\tilde{\mu}), M)$ is a θ-absolute fuzzy soft BCK/BCI-algebra over C.*

Proof. Direct.

Definition 9. *Let $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ be two fuzzy soft BCK/BCI-algebras over B. We say that $(\tilde{\mu}, M)$ is a fuzzy soft sub-BCK/BCI-algebra of $(\tilde{\eta}, N)$ if*

(1) *$M \subseteq N$,*

(2) *$\tilde{\mu}[m]$ is a fuzzy sub-BCK/BCI-algebra of $\tilde{\eta}[m]$ for all $m \in M$, that is, $\tilde{\mu}[m]$ is a fuzzy BCK/BCI-algebra satisfying the condition:*

$$(\forall \beta \in B) \left(\tilde{\mu}[m](\beta) \leq \tilde{\eta}[m](\beta) \right).$$

Example 2. *Consider a BCK-algebra $B = \{0, 1, 2, 3, 4\}$ with the following Cayley table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	4	4	0

Let $N = \{n_1, n_2, n_3, n_4, n_5\}$ be a set of parameters and let $(\tilde{\eta}, N)$ be a fuzzy soft set over B given as follows:

$\tilde{\eta}$	0	1	2	3	4
n_1	0.9	0.7	0.5	0.4	0.2
n_2	0.8	0.7	0.6	0.4	0.3
n_3	0.9	0.8	0.6	0.5	0.3
n_4	0.7	0.6	0.4	0.3	0.1
n_5	0.9	0.6	0.5	0.6	0.5

Then, $(\tilde{\eta}, N)$ is a fuzzy soft BCK-algebra over B. Now let $M = \{n_2, n_5\}$ be a subset of N. Define a soft set $(\tilde{\mu}, M)$ over B as follows:

$\tilde{\mu}$	0	1	2	3	4
n_2	0.78	0.67	0.56	0.34	0.23
n_5	0.89	0.56	0.45	0.56	0.45

Then, $(\tilde{\mu}, M)$ is a fuzzy soft sub-BCK-algebra of $(\tilde{\eta}, N)$.

The following theorem is obvious.

Theorem 3. *Let $(\tilde{\mu}, M)$ and $(\tilde{\eta}, M)$ be fuzzy soft BCK/BCI-algebras over B. If $\tilde{\mu}[m] \subseteq \tilde{\eta}[m]$ for all $m \in M$, then $(\tilde{\mu}, M)$ is a fuzzy soft sub-BCK/BCI-algebra of $(\tilde{\eta}, M)$.*

Lemma 1 ([8]). *If $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ are fuzzy soft BCK/BCI-algebras over B, then the extended intersection of $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ is a fuzzy soft BCK/BCI-algebra over B.*

Theorem 4. *Let $\left(\tilde{\xi}, Q\right)$ be a fuzzy soft BCK/BCI-algebra over B. If $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ are fuzzy soft sub-BCK/BCI-algebras of $\left(\tilde{\xi}, Q\right)$, then so is the extended intersection of $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$.*

Proof. It is proved by Definition 9 and Lemma 1.

Lemma 2 ([8]). *Let $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ be fuzzy soft BCK/BCI-algebras over B. If M and N are disjoint, then the union $(\tilde{\mu}, M) \,\tilde{\cup}\, (\tilde{\eta}, N)$ is a fuzzy soft BCK/BCI-algebra over B.*

Theorem 5. *Let $\left(\tilde{\xi}, Q\right)$ be a fuzzy soft BCK/BCI-algebra over B. If $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ are fuzzy soft sub-BCK/BCI-algebras of $\left(\tilde{\xi}, Q\right)$, then so is the union of $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ whenever M and N are disjoint.*

Proof. It is proved by Definition 9 and Lemma 2.

Lemma 3 ([8]). *If $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ are two fuzzy soft BCK/BCI-algebras over B, then $(\tilde{\mu}, M) \,\tilde{\wedge}\, (\tilde{\eta}, N)$ is a fuzzy soft BCK/BCI-algebra over B.*

Theorem 6. *Let $\left(\tilde{\xi}, Q\right)$ be a fuzzy soft BCK/BCI-algebra over B. If $(\tilde{\mu}, M)$ and $(\tilde{\eta}, N)$ are fuzzy soft sub-BCK/BCI-algebras of $\left(\tilde{\xi}, Q\right)$, then $(\tilde{\mu}, M) \,\tilde{\wedge}\, (\tilde{\eta}, N)$ is a fuzzy soft sub-BCK/BCI-algebra of $\left(\tilde{\xi}, Q\right)$.*

Proof. It is proved by Definition 9 and Lemma 3.

Let $(\tilde{\mu}, M)$ be a fuzzy soft set over B and let $k \in [0,1]$. For a parameter m in M, consider the following sets:

$$(\tilde{\mu}, M)_m^{\geq k} := \{\beta \in B \mid \tilde{\mu}[m](\beta) \geq k\},$$

and

$$(\tilde{\mu}, M)_{\bar{k}}^{\geq} := \{\beta \in B \mid \tilde{\mu}[m](\beta) \geq k \text{ for all } m \in M\}.$$

Obviously, $(\tilde{\mu}, M)_{\bar{k}}^{\geq} = \bigcap\limits_{m \in M} (\tilde{\mu}, M)_m^{\geq k}$.

Theorem 7. *For a fuzzy soft set $(\tilde{\mu}, M)$ over B, the following two statements are equivalent:*

(1) $(\tilde{\mu}, M)$ *is a fuzzy soft BCK/BCI-algebra over B based on a parameter $m \in M$.*

(2) $(\tilde{\mu}, M)_m^{\geq k}$ *is a subalgebra of B for all $k \in [0,1]$ with $(\tilde{\mu}, M)_m^{\geq k} \neq \emptyset$.*

Proof. $(1) \Rightarrow (2)$. Assume that $(\tilde{\mu}, M)$ is a fuzzy soft BCK/BCI-algebra over B based on a parameter $m \in M$. Let $k \in [0,1]$ such that $(\tilde{\mu}, M)_m^{\geq k} \neq \emptyset$. Let $\beta, \gamma \in (\tilde{\mu}, M)_m^{\geq k}$. Then, $\tilde{\mu}[m](\beta) \geq k$ and $\tilde{\mu}[m](\gamma) \geq k$. Thus, $\tilde{\mu}[m](\beta * \gamma) \geq \min\{\tilde{\mu}[m](\beta), \tilde{\mu}[m](\gamma)\} \geq k$ which implies that $\beta * \gamma \in (\tilde{\mu}, M)_m^{\geq k}$. Therefore, $(\tilde{\mu}, M)_m^{\geq k}$ is a subalgebra of B.

$(2) \Rightarrow (1)$. Suppose that the second assertion is valid and that $(\tilde{\mu}, M)$ is not a fuzzy soft BCK/BCI-algebra over B based on a parameter $m \in M$. Then,

$$\tilde{\mu}[m](\beta * \gamma) < k_0 \leq \min\{\tilde{\mu}[m](\beta), \tilde{\mu}[m](\gamma)\}$$

for some $\beta, \gamma \in B$ and $k_0 \in (0,1]$. It follows that $\beta, \gamma \in (\tilde{\mu}, M)_m^{\geq k_0}$ but $\beta * \gamma \notin (\tilde{\mu}, M)_m^{\geq k_0}$, which is a contradiction. Hence, $(\tilde{\mu}, M)$ is a fuzzy soft BCK/BCI-algebra over B based on a parameter $m \in M$.

Corollary 1. *A fuzzy soft set $(\tilde{\mu}, M)$ over B is a fuzzy soft BCK/BCI-algebra over B if and only if $(\tilde{\mu}, M)_{\bar{k}}^{\geq}$ is a subalgebra of B for all $k \in [0,1]$ with $(\tilde{\mu}, M)_{\bar{k}}^{\geq} \neq \emptyset$.*

4. Conclusion

The main goal of the present paper is to obtain further results on fuzzy soft BCK/BCI-algebras. In fact, the notion of fuzzy soft sub-BCK/BCI-algebra is introduced and related properties are investigated.

In our future study, we intend to apply the notions of the present paper to different algebras such as BL-algebras, MTL-algebras, $R0$-algebras, MV-algebras, EQ-algebras and lattice implication algebras etc.

Acknowledgements

References

[1] M. I. Ali, F. Feng, X, Liu, W. K. Min and M. Shabir, On some new operations in soft set theory. *Comput. Math. Appl.*, **57** (2009) 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009

[2] A. Al-Masarwah and A.G. Ahmad, m-polar fuzzy ideals of BCK/BCI-algebras. *J. King Saud Univ.-Sci.*, **31** (4) (2019) 1220-1226.

[3] A. Al-roqi, G. Muhiuddin and S. Aldhafeeri, Normal Unisoft Filters in R0-algberas. *Cogent Mathematics*, **1** (4) (2017) 1-9.

[4] A. Aygünoğlu and H. Aygün, Introduction to fuzzy soft groups. *Comput. Math. Appl.*, **58** (2009) 1279-1286.

[5] D. Chen, E. C.C. Tsang, D. S. Yeung, X. Wang, The parameterization reduction of soft set and its applications. *Computers and Mathematics with Applications*, **49** (2005) 757-763.

[6] Y. B. Jun, On (α, β)-fuzzy subalgebras of BCK/BCI-algebras. *Bull. Korean Math. Soc.*, **42** (2005) 703-711. https://doi.org/10.4134/BKMS.2005.42.4.703

[7] Y. B. Jun, S. S. Ahn and K. J. Lee, Intersection-soft filters in R_0-algberas. *Discrete Dynamics Nature and Society*, **2013**, Article ID 950897, 7 pages. https://doi.org/10.1155/2013/950897

[8] Y. B. Jun, K. J. Lee and C. H. Park, Fuzzy soft set theory applied to BCK/BCI-algebras. *Comput. Math. Appl.*, **59** (2010) 3180-3192. https://doi.org/10.1016/j.camwa.2010.03.004

[9] Y. B. Jun, G. Muhiuddin, M. A. Ozturk and E. H. Roh, Cubic soft ideals in BCK/BCI-algebras. *Journal of Computational Analysis and Applications*, **22** (5) (2017) 929-940.

[10] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets. *J. Fuzzy Math.*, **9**(3) (2001) 589-602.

[11] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in a decision making problem. *Comput. Math. Appl.*, **44** (2002) 1077-1083. https://doi.org/10.1016/S0898-1221(02)00216-X

[12] D. Molodtsov, Soft set theory - First results. *Comput. Math. Appl.*, **37** (1999) 19-31.

[13] G. Muhiuddin and A. M. Al-roqi, Cubic soft sets with applications in BCK/BCI-algebras. *Annals of Fuzzy Mathematics and Informatics*, **8** (2) (2014) 291-304.

[14] G. Muhiuddin and A. M. Al-roqi, Unisoft Filters in R0-algebras, *Journal of Computational Analysis and Applications*, **19** (1) (2015) 133-143.

[15] G. Muhiuddin, A. M. Al-roqi and S. Aldhafeeri, Filter theory in MTL-algebras based on Uni-soft property. *Bulletin of the Iranian Mathematical Society*, **43** (7) (2017) 2293-2306.

[16] G. Muhiuddin and M. Balamurugan, Hesitant intuitionistic fuzzy soft b-ideals of BCK-algebras, *Annals of Communications in Mathematics*, **3** (1) (2020) 26–34

[17] G. Muhiuddin, F. Feng and Y. B. Jun, Subalgebras of BCK/BCI-Algebras Based on Cubic Soft Sets. *The Scientific World Journal*, **2014**, Article ID 458638 (2014) 9 pages.

[18] E. H. Roh and Y. B. Jun, Positive implicative ideals of BCK-algebras based on intersectional soft sets. *J. Appl. Math.*, **2013**, Article ID 853907, 9 pages. http://dx.doi.org/10.1155/2013/853907

[19] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems. *J. Comput. Appl. Math.*, **203** (2007) 412-418. https://doi.org/10.1016/j.cam.2006.04.008

[20] C.K. Wong, Fuzzy Topology. *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, Elsevier, (1975) 171-190.

[21] L. A. Zadeh, Fuzzy sets. *Inform. Control*, **8** (1965) 338-353.