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On Nowhere Dense Sets

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Abstract. We introduce two types of strongly nowhere dense sets, namely (s, v)-strongly nowhere dense set, $(s, v)^{\star}$-strongly nowhere dense set and analyze their characteristics in a bigeneralized topological space (BGTS). Further, it is also given some relations between these two types of strongly nowhere dense sets along with its various properties for $(s, v)^{\star}$-strongly nowhere dense set. Finally, the necessary and sufficient condition is found between μ-strongly nowhere dense set and $(s, v)^{\star}$-strongly nowhere dense set in a BGTS.

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1. Introduction

The concept of a generalized topological space was introduced by Császár in [4]. Let X be any non-null set. A collection μ of subsets of X is a *generalized topology* [8] in X if it contains the empty set and it closed under arbitrary union. Then the pair (X, μ) is called as a *generalized topological space* (GTS) [8]. The pair (X, μ) is called a *strong generalized topological space* (sGTS) [8] if $X \in \mu$.

If $Q \in \mu$, then Q is called a μ*-open set* and if $X - Q \in \mu$, then Q is said to be a μ*-closed set*. Let D be a subset of a GTS (X, μ). The *interior of* D [8] denoted by iD, is the union of all μ-open sets contained in D and the *closure of* D [8] denoted by cD, is the intersection of all μ-closed sets containing D when no confusion can arise. Denote $\{D \in \mu \mid D \neq \emptyset\}$ by $\tilde{\mu}$ [7] and denote $\{D \in \mu \mid x \in D\}$ by $\mu(x)$ [7].

Define a generalized topology $\mu^{\star}$ as follows; $\mu^{\star} = \{\bigcup_t (U_1^t \cap U_2^t \cap U_3^t \cap ... \cap U_{n_t}^t) \mid U_1^t, U_2^t, ..., U_{n_t}^t \in \mu\}$ [7]. Then $\mu \subset \mu^{\star}$ and $\mu^{\star}$ is closed under finite intersection [7].

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2. Preliminaries

Let (X, μ) be a GTS and $Q \subset X$. Then Q is called a *μ-nowhere dense* [6] (resp. *μ-dense* [6, 7], *μ-codense* [7]) set if $icQ = \emptyset$ (resp. $cQ = X$; $c(X - Q) = X$).

Let μ_1 and μ_2 be two generalized topologies on a non-null set X. Then (X, μ_1, μ_2) is called as a *bigeneralized topological space* (briefly, BGTS) [2].

Let (X, μ_1, μ_2) be a BGTS and $D \subset X$. Then $c_s(D)$ denote the *closure of* D and $i_s(D)$ denote the *interior of* D with respect to μ_s, respectively, for $s = 1, 2$ [2].

A subset Q of a BGTS (X, μ_1, μ_2) is called *(s, v)-closed* if $c_s(c_v(Q)) = Q$, where $s, v = 1$ or 2 ; $s \neq v$. If $X - Q$ is (s, v)-closed, then Q is called as *(s, v)-open* [2] set.

In [2], let Q be a subset of a BGTS (X, μ_1, μ_2) is called

(1) *(s, v)-g-preopen* if $Q \subseteq i_s(c_v(Q))$ where $s, v = 1$ or 2 ; $s \neq v$.

(2) *(s, v)-g-α-open* if $Q \subseteq i_s(c_v(i_s(Q)))$ where $s, v = 1$ or 2 ; $s \neq v$.

Lemma 1. [3] Let Q be a subset of a generalized topological space (X, μ). Then $y \in c(Q)$ if and only if $M \cap Q \neq \emptyset$ for any $M \in \mu(y)$.

Lemma 2. [8, Lemma 3.2] Let (X, μ) be a generalized topological space and $D, B \subset X$. If $B \in \tilde{\mu}; B \cap D = \emptyset$, then $B \cap cD = \emptyset$.

3. Nowhere dense sets

In this section, we define a set namely, $(s, v)^{\star}$-nowhere dense and give some of their properties in a BGTS.

Let Q be a subset of a generalized topological space (X, μ). Then Q is called μ-semi-open if $Q \subset c_\mu(i_\mu(Q))$ [5]. If $X - Q$ is a μ-semi-open set, then Q is called μ-semi-closed [5].

Moreover, $\sigma(\mu)$ or $\sigma(\mu(X)) = \{Q \subset X \mid Q$ is μ-semi-open set in $X\}$ [8]. Also, $i_\sigma(Q)$ denote the μ-semi-interior of $Q \subset X$ is defined by the union of all μ-semi-open subsets of (X, μ) contained in Q [8].

Let Q be a subset of a BGTS (X, μ_1, μ_2) is called *(s, v)-nowhere dense* [1] set in X if $i_s(c_v(Q)) = \emptyset$ where $s, v = 1, 2$; $s \neq v$.

Definition 1. Let (X, μ_1, μ_2) be a bigeneralized topological space and K be a non-null subset of X. Then K is called to be a *$(s, v)^{\star}$-nowhere dense* set if $i_{\sigma_v}(c_s(K)) = \emptyset$ where $s, v = 1, 2$; $s \neq v; \sigma_v = \sigma_{\mu_v}$.

Moreover, $(s, v)^{\star} - \mathcal{N}(X) = \{Q \subset X \mid Q$ is $(s, v)^{\star}$-nowhere dense set in $X\}$ where $s, v = 1, 2$; $s \neq v$.

Example 2. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}\}$ and $\mu_2 = \{\emptyset, \{p, s\}, \{q, s\}, \{p, q, s\}\}$. Then $\sigma_1 = \{\emptyset, \{s\}, \{p, q\}, \{q, r\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$ and $\sigma_2 = \{\emptyset, \{r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$.

1. Take $E = \{s\}$. Then $i_{\sigma_2}(c_1(E)) = i_{\sigma_2}(E) = \emptyset$. Thus, E is a $(1, 2)^{\star}$-nowhere dense set in X.

2. Choose $F = \{p, r\}$. Then $i_{\sigma_1}(c_2(F)) = i_{\sigma_1}(\{p, r\}) = \emptyset$. Then F is a $(2, 1)^{\star}$-nowhere dense in X.

In a bigeneralized topological space, if $K \in (s, v)^{\star} - \mathcal{N}(X)$ and $L \subset K$, then $L \in (s, v)^{\star} - \mathcal{N}(X)$ where $s, v = 1, 2$ and $s \neq v$. Also, every $(s, v)^{\star}$-nowhere dense set where $s, v = 1, 2$ and $s \neq v$, is a μ_v-codense set for $v = 1, 2$ in X.

Moreover, any $(s, v)^{\star}$-nowhere dense set is a (v, s)-nowhere dense set in a bigeneralized topological space (X, μ_1, μ_2) where $s, v = 1, 2$ and $s \neq v$, since $\mu \subset \sigma$ [3].

Example 3. Consider the BGTS (X, μ_1, μ_2) where $X = \{p, q, r, s\}$ and μ_1, μ_2 are defined in Example 2.
Take $P = \{s\}$. Then P is $(1, 2)^{\star}$-nowhere dense set, by Example 2. Now $i_2(c_1(P)) = i_2(P) = \emptyset$. Therefore, P is $(2, 1)$-nowhere dense set in X.
Choose $D = \{p, r\}$. In Example 2, D is $(2, 1)^{\star}$-nowhere dense set in X. Here $i_1(c_2(D)) = i_1(D) = \emptyset$. Thus, D is $(1, 2)$-nowhere dense set in X.

Theorem 4. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the followings are true.*
(a) If (X, μ_1) is a sGTS and $Q \subset X$ is a $(1, 2)$-nowhere dense set, then $Q \in (2, 1)^{\star} - \mathcal{N}(X)$.
(b) If (X, μ_2) is a sGTS and $J \subset X$ is a $(2, 1)$-nowhere dense set, then $J \in (1, 2)^{\star} - \mathcal{N}(X)$.

Proof. (a). Assume that, (X, μ_1) is a sGTS and Q is a $(1, 2)$-nowhere dense set. Then $i_1(c_2(Q)) = \emptyset$. Suppose $i_{\sigma_1}(c_2(Q)) \neq \emptyset$. Then there exist $G \in \tilde{\sigma}_1$ such that $G \subset c_2(Q)$. Since $G \in \tilde{\sigma}_1$ we have $G \subset c_1(i_1(G))$ which implies $c_1(i_1(G)) \neq \emptyset$ which turn implies that $i_1(G) \neq \emptyset$, by assumption. Thus, $i_1(G) \in \tilde{\mu}_1$ and $i_1(G) \subset c_2(Q)$. Then $i_1(c_2(Q)) \neq \emptyset$ which is not possible. Therefore, $i_{\sigma_1}(c_2(Q)) = \emptyset$.
(b). Follows from the similar arguments in (a).

In Theorem 4, the condition "μ_1 is a sGT" is necessary as shown by the below Example 5. The condition "μ_2 is a sGT" in Theorem 4 is necessary as shown by Example 6.

Example 5. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s, t\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{p, s\}, \{p, q, s\}\}$; $\mu_2 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}\}$. Here μ_1 is not a sGT. Then $\sigma_1 = \{\emptyset, \{r\}, \{t\}, \{r, t\}, \{p, q\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, q, t\}, \{p, r, s\}, \{p, s, t\}, \{p, q, r, s\}, \{p, q, r, t\}, \{p, q, s, t\}, \{p, r, s, t\}, X\}$.
Take $D = \{r\}$. Then $i_1(c_2(D)) = i_1(\{r, s, t\}) = \emptyset$. Thus, D is a $(1, 2)$-nowhere dense set in X. But $i_{\sigma_1}(c_2(D)) = i_{\sigma_1}(\{r, s, t\}) = \{r, t\} \neq \emptyset$. Thus, $D \notin (2, 1)^{\star} - \mathcal{N}(X)$.

Example 6. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s, t\}$; $\mu_1 = \{\emptyset, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, \{p, q, r, s\}\}$ and $\mu_2 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}\}$. Here μ_2 is not a sGT. Then $\sigma_2 = \{\emptyset, \{s\}, \{t\}, \{s, t\}, \{p, q\}, \{q, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, t\}, \{q, r, s\}, \{q, r, t\}, \{p, q, r, s\}, \{p, q, r, t\}, \{p, q, s, t\}, \{q, r, s, t\}, X\}$.
Choose $D = \{s\}$. Then $i_2(c_1(D)) = i_2(\{s, t\}) = \emptyset$. Thus, D is a $(2, 1)$-nowhere dense set in X. But $i_{\sigma_2}(c_1(D)) = i_{\sigma_2}(\{s, t\}) = \{s\} \neq \emptyset$. Thus, $D \notin (1, 2)^{\star} - \mathcal{N}(X)$.

Theorem 7. *Let (X, μ_1, μ_2) be a BGTS and $E \subset X$. Then the followings are true.*
(a) If (X, μ_2) is a sGTS and if $c_1(E)$ does not contain a non-null μ_2-open set, then $E \in$

$(1,2)^{\star} - \mathcal{N}(X)$.
(b) If (X, μ_1) is a sGTS and if $c_2(E)$ does not contain a non-null μ_1-open set, then $E \in (2,1)^{\star} - \mathcal{N}(X)$.

Proof. (a). Assume that, (X, μ_2) is a sGTS. Suppose $i_{\sigma_2}(c_1(E)) \neq \emptyset$. Then there is a non-null σ_2-open set M such that $M \subset c_1(E)$. Since M is a non-null σ_2-open set we have $M \subset c_2(i_2(M))$. This implies that $c_2(i_2(M)) \neq \emptyset$ which implies $i_2(M) \neq \emptyset$, by assumption. Thus, $c_1(E)$ contain a non-null μ_2-open set which is not possible. Therefore, $E \in (1,2)^{\star} - \mathcal{N}(X)$.
(b). By similar arguments in (a), we get the proof.

Theorem 8. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If $\mu_s \subset \mu_v$ and $Q \in (s,v)^{\star} - \mathcal{N}(X)$, then Q is a μ_v-nowhere dense set in X where $s, v = 1, 2$; $s \neq v$.*

Proof. Take $s = 1$ and $v = 2$. Suppose $\mu_1 \subset \mu_2$ and $Q \in (1,2)^{\star} - \mathcal{N}(X)$. Then $i_{\sigma_2}(c_1(Q)) = \emptyset$. This implies that $i_{\mu_2}(c_{\mu_1}(Q)) = \emptyset$ which implies $i_{\mu_2}(c_{\mu_2}(Q)) = \emptyset$, by hypothesis. Hence Q is a μ_2-nowhere dense set in X.
Similarly, we can prove the result for $s = 2$ and $v = 1$.

In Theorem 8, the conditions "$\mu_1 \subset \mu_2$" and "$\mu_2 \subset \mu_1$" are can not be dropped as shown by the below Example 9.

Example 9. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$ and $\mu_2 = \{\emptyset, \{p, s\}, \{q, s\}, \{p, q, s\}\}$. Here $\mu_1 \nsubseteq \mu_2$. Now $\sigma_2 = \{\emptyset, \{r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$. Take $Q = \{p, q\}$. Then $i_{\sigma_2}(c_1(Q)) = i_{\sigma_2}(\{p, q\}) = \emptyset$. Thus, Q is a $(1,2)^{\star}$-nowhere dense set in X. Here $i_2(c_2(Q)) = i_2(X) = \{p, q, s\} \neq \emptyset$. Thus, Q is not a μ_2-nowhere dense set in X.
(b) Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}; \mu_1 = \{\emptyset, \{p, r\}, \{q, r\}, \{p, q, r\}\}$ and $\mu_2 = \{\emptyset, \{p, q\}, \{q, s\}, \{p, q, s\}\}$. Here $\mu_2 \nsubseteq \mu_1$. Now $\sigma_1 = \{\emptyset, \{s\}, \{p, r\}, \{q, r\}, \{p, q, r\}, \{p, r, s\}, \{q, r, s\}, X\}$. Choose $H = \{r\}$. Then $i_{\sigma_1}(c_2(H)) = i_{\sigma_1}(\{r\}) = \emptyset$. Thus, H is a $(2,1)^{\star}$-nowhere dense set in X. But $i_1(c_1(H)) = i_1(X) = \{p, q, r\} \neq \emptyset$. Thus, H is not a μ_1-nowhere dense set in X.

Theorem 10. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If $\mu_v \subset \mu_s$ and if μ_v is a strong generalized topology, then any μ_v-nowhere dense set in X is a $(s,v)^{\star}$-nowhere dense set in X where $s, v = 1, 2$ and $s \neq v$.*

Proof. Take $s = 1$ and $v = 2$. Assume that, $\mu_2 \subset \mu_1$ and Q is a μ_2-nowhere dense set in X. Then $i_{\mu_2}(c_{\mu_2}(Q)) = \emptyset$. Suppose $i_{\sigma_2}(c_1(Q)) \neq \emptyset$. Then there exists $M \in \tilde{\mu}_{\sigma_2}$ such that $M \subset c_1(Q)$. Since $M \in \tilde{\mu}_{\sigma_2}$ we have $c_2(i_2(M)) \neq \emptyset$. Then by hypothesis, $i_2(M) \neq \emptyset$ and so $i_2(c_1(Q)) \neq \emptyset$. By hypothesis, $i_2(c_2(Q)) \neq \emptyset$, which is not possible. Therefore, $i_{\sigma_2}(c_1(Q)) = \emptyset$. Hence $Q \in (1,2)^{\star} - \mathcal{N}(X)$.
Similarly, we can prove the result for $s = 2$ and $v = 1$.

The following Example 11 shows that the hypothesis of Theorem 10 can not be dropped.

Example 11. (a). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, s\}, \{q, s\}, \{p, q, s\}\}$ and $\mu_2 = \{\emptyset, \{p, r\}, \{p, s\}, \{q, r\}, \{p, r, s\}, \{p, q, r\}, X\}$. Here $\mu_2 \nsubseteq \mu_1$ but μ_2 is a sGT. Now $\sigma_2 = \{\emptyset, \{p, r\}, \{p, s\}, \{q, r\}, \{p, r, s\}, \{p, q, r\}, X\}$. Take $H = \{s\}$. Then $i_2(c_2(H)) = i_2(H) = \emptyset$ and so H is μ_2-nowhere dense set in X. But $i_{\sigma_2}(c_1(H)) = i_{\sigma_2}(X) = X \neq \emptyset$. Thus, H is not a $(1, 2)^\star$-nowhere dense set in X.
(b). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$ and $\mu_2 = \{\emptyset, \{p, s\}, \{q, s\}, \{p, q, s\}\}$. Here $\mu_2 \subset \mu_1$ but μ_2 is not a sGT. Now $\sigma_2 = \{\emptyset, \{r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$. Choose $P = \{r\}$. Then $i_2(c_2(P)) = i_2(P) = \emptyset$ so that P is a μ_2-nowhere dense set in X. But $i_{\sigma_2}(c_1(P)) = i_{\sigma_2}(P) = P \neq \emptyset$. Thus, P is not a $(1, 2)^\star$-nowhere dense set in X.
(c). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}, X\}$ and $\mu_2 = \{\emptyset, \{p, r\}, \{q, r\}, \{p, q, r\}\}$. Clearly, $\mu_1 \nsubseteq \mu_2$ but μ_1 is a sGT. Here $\sigma_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$. Take $Q = \{r, s\}$. Then $i_1(c_1(Q)) = i_1(Q) = \emptyset$ so that Q is a μ_1-nowhere dense set in X. But $i_{\sigma_1}(c_2(Q)) = i_{\sigma_1}(X) = X \neq \emptyset$. Hence Q is not a $(2, 1)^\star$-nowhere dense set in X.
(d). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{q, r\}, \{q, s\}, \{q, r, s\}\}$ and $\mu_2 = \{\emptyset, \{p, q\}, \{q, r\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$. Here $\mu_1 \subset \mu_2$ but μ_1 is not a sGT. Now $\sigma_1 = \{\emptyset, \{p\}, \{q, r\}, \{q, s\}, \{q, r, s\}, \{p, q, r\}, \{p, q, s\}, X\}$. Let $K = \{p, r\}$. Then $i_1(c_1(K)) = i_1(K) = \emptyset$ so that K is a μ_1-nowhere dense set in X. But $i_{\sigma_1}(c_2(K)) = i_{\sigma_1}(K) = \{p\} \neq \emptyset$. Hence K is not a $(2, 1)^\star$-nowhere dense set in X.

4. (s, v)-strongly nowhere dense sets

In this section, we define a set namely, (s, v)-strongly nowhere dense set and give some of its properties in a BGTS (X, μ_1, μ_2).

Let Q be a subset of a GTS (X, μ). Then Q is called *μ-strongly nowhere dense* [7] set if for every $K \in \tilde{\mu}$, there is $P \in \tilde{\mu}$ such that $P \subset K$ and $P \cap Q = \emptyset$.

A generalized topology μ on X is said to satisfy the $\mathcal{I}$-property [9] whenever $W_1, W_2, \ldots, W_n \in \mu$ with $W_1 \cap W_2 \cap \cdots \cap W_n \neq \emptyset, i_\mu(W_1 \cap W_2 \cap \cdots \cap W_n) \neq \emptyset$.

A GTS (X, μ) is called as a hyperconnected space [6] if $c_\mu(Q) = X$ for each $Q \in \tilde{\mu}$.

Definition 12. Let B be a non-null subset of a bigeneralized topological space (X, μ_1, μ_2). Then B is said to be *(s, v)-strongly nowhere dense* if for every $P \in \tilde{\mu}_v$ there is $Q \in \tilde{\mu}_s$ such that $Q \subset P$ and $Q \cap B = \emptyset$ where $s, v = 1, 2$; $s \neq v$.

Moreover, $(s, v) - \mathfrak{S}(X) = \{Q \subset X \mid Q$ is a (s, v)-strongly nowhere dense set in $X\}$ where $s, v = 1, 2$; $s \neq v$.

In a bigeneralized topological space, if $P \in (s, v) - \mathfrak{S}(X)$ and $Q \subset P$, then $Q \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$. Moreover, every non-null μ_v-open set is need not be a (s, v)-strongly nowhere dense set in X where $s, v = 1, 2$; $s \neq v$.

Example 13. (a). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}\}$ and $\mu_2 = \{\emptyset, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$. Let $P = \{s\}$. Then $P \in (1, 2) - \mathfrak{S}(X)$.

(b). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$; $\mu_1 = \{\emptyset, \{p, q\}, \{p, r\}, \{p, q, r\}\}$ and $\mu_2 = \{\emptyset, \{p\}, \{q, r\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}$. Let $J = \{q, r\}$. Then $J \in (2, 1) - \mathfrak{S}(X)$.

Proposition 14. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $D \subset X$. Then $D \in (s, v) - \mathfrak{S}(X)$ if and only if $c_s(D) \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Example 15 shows that the collection $(s, v) - \mathfrak{S}(X)$ is need not be closed under finite union in a BGTS (X, μ_1, μ_2) where $s, v = 1, 2$ and $s \neq v$.

Example 15. (a). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s, t\}$; $\mu_1 = \{\emptyset, \{s\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{p, q, r, s\}\}$ and $\mu_2 = \{\emptyset, \{p, q, r\}, \{p, r, s\}, \{p, q, r, s\}\}$. Take $K = \{q, s\}, L = \{r, t\}$. Then $K, L \in (1, 2) - \mathfrak{S}(X)$. Now $K \cup L = \{q, r, s, t\}$. But $K \cup L \notin (1, 2) - \mathfrak{S}(X)$. Because, Here, for every $G \in \tilde{\mu}_2$ there is no $J \in \tilde{\mu}_1$ such that $J \subset G$ and $J \cap (K \cup L) = \emptyset$.
(b). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s, t\}$; $\mu_1 = \{\emptyset, \{p, q, s\}, \{p, r, s\}, \{p, q, r, s\}\}$ and $\mu_2 = \{\emptyset, \{r\}, \{p, q\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{p, q, r, s\}\}$. Take $L = \{q, r\}, M = \{s, t\}$. Then $L, M \in (2, 1) - \mathfrak{S}(X)$. Now $L \cup M = \{q, r, s, t\}$. But $L \cup M \notin (2, 1) - \mathfrak{S}(X)$. Here, for every $H \in \tilde{\mu}_1$ there is no $K \in \tilde{\mu}_2$ such that $K \subset H$ and $K \cap (L \cup M) = \emptyset$.

Theorem 16. *Let (X, μ_1, μ_2) be a BGTS where $\mu_2 = \mu_1^\star$. Then the family $(s, v) - \mathfrak{S}(X)$ is closed under finite union where $s, v = 1, 2$; $s \neq v$.*

Theorem 17. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If (X, μ_s) is hyperconnected and μ_s satisfy the $\mathcal{I}$-property, then $A_1 \cup A_2 \in (s, v) - \mathfrak{S}(X)$ whenever $A_1, A_2 \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Proof. *Take $s = 1$ and $v = 2$. Assume that, (X, μ_1) is hyperconnected and μ_1 satisfy the $\mathcal{I}$-property. Suppose that, A_1 and A_2 are $(1, 2)$-strongly nowhere dense sets in X. Take $D = A_1 \cup A_2$. Let $G \in \tilde{\mu}_2$. Then there exists $H_i \in \tilde{\mu}_1$ such that $H_i \subset G$ and $H_i \cap A_i = \emptyset$ for $i = 1, 2$. By our assumption, $i_{\mu_1}(H_1 \cap H_2) \neq \emptyset$. Take $J = i_{\mu_1}(H_1 \cap H_2)$. Then $J \in \tilde{\mu}_1$. Thus, there is $J \in \tilde{\mu}_1$ such that $J \subset G$ and $J \cap D = \emptyset$. Hence $D \in (1, 2) - \mathfrak{S}(X)$. Similarly, we can prove the result for $s = 2$ and $v = 1$.*

Corollary 18. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If (X, μ_s) is a hyperconnected space and μ_s satisfy the $\mathcal{I}$-property, then the family $(s, v) - \mathfrak{S}(X)$ is closed under finite union where $s, v = 1, 2$; $s \neq v$.*

The following Example 19 shows that (s, v)-strongly nowhere dense and (s, v)-nowhere dense sets are not comparable in a BGTS.

Example 19. (a). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = [0, 3]$; $\mu_1 = \{\emptyset, [0, 1), \{\frac{3}{2}\}, [1, 2], [0, 1) \cup \{\frac{3}{2}\}, [0, 2]\}$ and $\mu_2 = \{\emptyset, [0, \frac{3}{2}), [1, 3], [0, 3]\}$. Let $G = (2, 3]$. Then $G \in (1, 2) - \mathfrak{S}(X)$. But G is not a $(1, 2)$-nowhere dense set in X.
(b). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = [0, 3]$; $\mu_1 = \{\emptyset, [0, 2), (1, 3], [0, 3]\}$ and $\mu_2 = \{\emptyset, [0, 1), (1, 2), \{2\}, [0, 1) \cup \{2\}, (1, 2], [0, 1) \cup (1, 2), [0, 1) \cup$

$(1, 2]\}$. Let $H = (2, 3]$. Then $H \in (2, 1) - \mathfrak{S}(X)$. But H is not a $(2, 1)$-nowhere dense set in X.

(c). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = [0, 3]$; $\mu_1 = \{\emptyset, [0, 2), (1, 3], [0, 3]\}$ and $\mu_2 = \{\emptyset, [0, 1), [1, 2), [0, 2)\}$. Let $K = [2, 3]$. Then K is a (s, v)-nowhere dense set in X where $s, v = 1, 2$ and $s \neq v$. But $K \notin (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Theorem 20. *Let (X, μ_1, μ_2) be a BGTS and $Q \subset X$. If $Q \in (s, v) - \mathfrak{S}(X)$, then Q is a (v, s)-nowhere dense set in X where $s, v = 1, 2$; $s \neq v$.*

Proof. Suppose $Q \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$. Assume that, $i_v(c_s(Q)) \neq \emptyset$ where $s, v = 1, 2$; $s \neq v$. Then there is a set $J \in \tilde{\mu}_v$ such that $J \subset c_s(Q)$ where $s, v = 1, 2$ and $s \neq v$ which implies that $Q \notin (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$ which is not possible. Therefore, Q is a (v, s)-nowhere dense set in X where $s, v = 1, 2; s \neq v$.

Definition 21. Let Q be a non-null subset of a BGTS (X, μ_1, μ_2). Then for $s, v = 1, 2$ and $s \neq v$,

(a) Q is called (s, v)-*meager* if $Q = \bigcup_{m \in \mathbb{N}} D_m$ where each D_m is a (s, v)-nowhere dense set in X.

(b) Q is called (s, v)-*residual* if $X - Q$ is a (s, v)-meager set in X.

(c) Q is of (s, v)-*second category* set if Q is not a (s, v)-meager set in X.

Definition 22. Let B be a non-null subset of a BGTS (X, μ_1, μ_2). Then for $s, v = 1, 2$ and $s \neq v$,

(a) B is said to be a (s, v)-*s-meager* set if $B = \bigcup_{m \in \mathbb{N}} B_m$ for each $B_m \in (s, v) - \mathfrak{S}(X)$.

(b) B is called as a (s, v)-*s-residual* set if $X - B$ is a (s, v)-s-meager set in X.

(c) B is of (s, v)-*s-second category* set if B is not a (s, v)-s-meager set in X.

Corollary 23. *Let (X, μ_1, μ_2) be a BGTS and $D \subset X$. For $s, v = 1, 2$ and $s \neq v$, the followings are true.*

(a) If D is (s, v)-s-meager, then it is a (v, s)-meager set.

(b) If D is (s, v)-s-residual, then it is a (v, s)-residual set.

(c) If D is of (s, v)-second category set, then it is of (v, s)-s-second category set.

Corollary 24. *Let (X, μ_1, μ_2) be a BGTS. Then the followings are true.*

(a) If μ_2 is a strong generalized topology, then $(1, 2) - \mathfrak{S}(X) \subset (1, 2)^\star - \mathcal{N}(X)$.

(b) If μ_1 is a strong generalized topology, then $(2, 1) - \mathfrak{S}(X) \subset (2, 1)^\star - \mathcal{N}(X)$.

Proof. (a). Assume that, μ_2 is a strong generalized topology. Let $Q \in (1, 2) - \mathfrak{S}(X)$. By Theorem 20, Q is a $(2, 1)$-nowhere dense set in X. By our assumption and Theorem 4 (b), Q is a $(1, 2)^\star$-nowhere dense set in X.

(b). Suppose that, μ_1 is a strong generalized topology. Let $D \in (2, 1) - \mathfrak{S}(X)$. By Theorem 20 and Theorem 4 (a), $D \in (2, 1)^\star - \mathcal{N}(X)$.

Definition 25. Let (X, μ_1, μ_2) satisfy the condition;

$$\text{if } B_1 \in \tilde{\mu}_s, B_2 \in \tilde{\mu}_v \text{ and } B_1 \cap B_2 \neq \emptyset, \text{ then } i_s(B_1 \cap B_2) \neq \emptyset$$

where $s, v = 1, 2$ and $s \neq v$. Then the BGTS (X, μ_1, μ_2) is said to satisfy the *$\mathcal{I}_S$-property.*

Theorem 26. *Let (X, μ_1, μ_2) be a BGTS which has the $\mathcal{I}_S$-property and $K, L, Q \subset X$. Then*
(a) If $K \in (s, v) - \mathfrak{S}(X)$ and $L \in (v, s) - \mathfrak{S}(X)$, then $K \cup L \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$.
(b) If L is a (v, s)-nowhere dense set, then $L \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$.
(c) If $Q \in (s, v)^{\star} - \mathcal{N}(X)$, then $Q \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$; $s \neq v$.

Proof. (a). Let $G \in \tilde{\mu}_v$ for $v = 1, 2$. Then there is a set $J \in \tilde{\mu}_s$ such that $J \subset G$ and $J \cap K = \emptyset$ for $s = 1, 2$. By hypothesis, there is a set $M_1 \in \tilde{\mu}_v$ such that $M_1 \subset J$ and $M_1 \cap L = \emptyset$ for $v = 1, 2$. Take $P = J \cap M_1$. Then $P \subset G$ and $i_s(P) \neq \emptyset$, by hypothesis for $s = 1, 2$. Also, $i_s(P) \cap (K \cup L) = \emptyset$ for $s = 1, 2$. Thus, there is $i_s(P) \in \tilde{\mu}_s$ such that $i_s(P) \subset G$ and $i_s(P) \cap (K \cup L) = \emptyset$ where $s, v = 1, 2$; $s \neq v$. Therefore, $K \cup L \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.
(b). Suppose L is a (v, s)-nowhere dense set where $s, v = 1, 2$ and $s \neq v$. Then $X - c_s(L)$ is μ_v-dense and also μ_s-open set where $s, v = 1, 2$ and $s \neq v$. Let $V \in \tilde{\mu}_v$ for $v = 1, 2$. Then $V \cap (X - c_s(L)) \neq \emptyset$ for $s = 1, 2$. By hypothesis, $i_s(V \cap (X - c_s(L)) \neq \emptyset$ for $s = 1, 2$. Take $P = i_s(V \cap (X - c_s(L))$ for $s = 1, 2$. Then $P \subset V$ and $P \cap L = \emptyset$. Therefore, L is a (s, v)-strongly nowhere dense set in X where $s, v = 1, 2$; $s \neq v$.
(c). It follows from (b) and the fact that every $(s, v)^{\star}$-nowhere dense set is a (v, s)-nowhere dense set where $s, v = 1, 2$; $s \neq v$.

Theorem 27. *Let (X, μ_1, μ_2) be a BGTS. If $\mu_s \subset \mu_v$ and $Q \in (s, v) - \mathfrak{S}(X)$, then Q is a μ_s-strongly nowhere dense set in X where $s, v = 1, 2$ and $s \neq v$.*

Definition 28. Let (X, μ_1, μ_2) satisfy the condition;

$$\text{if } B_1 \in \tilde{\mu}_s, B_2 \in \tilde{\mu}_v \text{ and } B_1 \cap B_2 \neq \emptyset, \text{ then } i_v(B_1 \cap B_2) \neq \emptyset$$

where $s, v = 1, 2$ and $s \neq v$. Then the BGTS (X, μ_1, μ_2) is said to satisfy the *$\mathcal{I}_V$-property.*

Theorem 29. *Let (X, μ_1, μ_2) be a BGTS which has the $\mathcal{I}_V$-property. If $D \in (s, v) - \mathfrak{S}(X)$, then D is a μ_v-strongly nowhere dense set in X where $s, v = 1, 2$ and $s \neq v$.*

Proof. Take $s = 1$ and $v = 2$. Assume that, the bigeneralized topological space (X, μ_1, μ_2) satisfy the $\mathcal{I}_V$-property. Let $D \in (1, 2) - \mathfrak{S}(X)$ and $G \in \tilde{\mu}_2$. Then there is a set $J \in \tilde{\mu}_1$ such that $J \subset G$ and $J \cap D = \emptyset$. Here $G \in \tilde{\mu}_2, J \in \tilde{\mu}_1$ and $J \cap G \neq \emptyset$. By our assumption, $i_{\mu_2}(G \cap J) \neq \emptyset$. Take $K = i_{\mu_2}(G \cap J)$. Then $K \in \tilde{\mu}_2$. Thus, there is $K \in \tilde{\mu}_2$ such that $K \subset G$ and $K \cap D = \emptyset$. Therefore, D is a μ_2-strongly nowhere dense set in X. Similarly, we can prove that the result is true for the case $s = 2$ and $v = 1$.

Theorem 30. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If $\mu_v \subset \mu_s$ where $s, v = 1, 2$ and $s \neq v$, then the following hold.*
(a) If Q is a μ_v-strongly nowhere dense set, then $Q \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

(b) If J is a μ_s-strongly nowhere dense set, then $J \in (s,v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Proof. Assume that, $\mu_v \subset \mu_s$ where $s, v = 1, 2$ and $s \neq v$.
(a). Suppose that, Q is a μ_v-strongly nowhere dense set where $v = 1, 2$. Take $s = 1$ and $v = 2$. Then Q is a μ_2-strongly nowhere dense set and $\mu_2 \subset \mu_1$. Let $G \in \tilde{\mu}_2$. Then there is $H \in \tilde{\mu}_2$ such that $H \subset G$ and $H \cap Q = \emptyset$. By hypothesis, $H \in \tilde{\mu}_1$. Thus, there is a set $H \in \tilde{\mu}_1$ such that $H \subset G$ and $H \cap Q = \emptyset$. Therefore, $Q \in (1,2) - \mathfrak{S}(X)$.
Similarly, we can prove that the result is true for the case $s = 2$ and $v = 1$.
(b). Let J be a μ_s-strongly nowhere dense set for $s = 1, 2$. Choose $s = 1$ and $v = 2$. Then J is a μ_1-strongly nowhere dense set and $\mu_2 \subset \mu_1$. Let $H \in \tilde{\mu}_2$. Then $H \in \tilde{\mu}_1$ and so there is a set $K \in \tilde{\mu}_1$ such that $K \subset H$ and $K \cap J = \emptyset$. Thus, there is a set $K \in \tilde{\mu}_1$ such that $K \subset H$ and $K \cap J = \emptyset$. Hence $J \in (1,2) - \mathfrak{S}(X)$.
By Similar arguments, we can prove that the result is true for the case $s = 2$ and $v = 1$.

5. (s, v)*-strongly nowhere dense sets

In this section, we introduce $(s,v)^\star$-strongly nowhere dense set and analzye its nature in a BGTS (X, μ_1, μ_2).

Definition 31. Let (X, μ_1, μ_2) be a BGTS and B be a non-null subset of X. Then B is called $(s,v)^\star$-*strongly nowhere dense* if for every $K \in \tilde{\mu}_s$ there is $M \in \tilde{\sigma}_v$ such that $M \subset K$ and $M \cap B = \emptyset$ where $s, v = 1, 2$; $s \neq v$.

Moreover, $(s,v)^\star - \mathfrak{S}(X) = \{Q \subset X \mid Q$ is a $(s,v)^\star$-strongly nowhere dense set in $X\}$ where $s, v = 1, 2$; $s \neq v$.

Moreover, every non-null μ_s-open set is need not be an element of $(s,v)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Definition 32. Let D be a non-null subset of a BGTS (X, μ_1, μ_2). Then for $s, v = 1, 2$ and $s \neq v$,
(a) D is said to be a $(s,v)^\star$-*s-meager* set if $D = \bigcup_{m \in \mathbb{N}} D_m$, for each $D_m \in (s,v)^\star - \mathfrak{S}(X)$.
(b) D is called $(s,v)^\star$-*s-residual* if $X - D$ is a $(s,v)^\star$-s-meager set in X.
(c) D is of a $(s,v)^\star$-*s-second category* set if D is not a $(s,v)^\star$-s-meager set in X.

In a bigeneralized topological space, if $P \in (s,v)^\star - \mathfrak{S}(X)$ and $Q \subset P$, then $Q \in (s,v)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Moreover, $(s,v) - \mathfrak{S}(X) \subset (v,s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Theorem 33. *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then the following hold.*
(a) If μ_2 is a strong generalized topology, then $(1,2)^\star - \mathfrak{S}(X) \subset (2,1) - \mathfrak{S}(X)$.
(b) If μ_1 is a strong generalized topology, then $(2,1)^\star - \mathfrak{S}(X) \subset (1,2) - \mathfrak{S}(X)$.

Proof. (a). Suppose μ_2 is a strong generalized topology and $Q \in (1,2)^\star - \mathfrak{S}(X)$. Let $G \in \tilde{\mu}_1$. Then there is a set $P \in \tilde{\sigma}_2$ such that $P \subset G$ and $P \cap Q = \emptyset$. Since $P \in \tilde{\sigma}_2$ we have $i_2(P) \in \tilde{\mu}_2$, by assumption. Take $J = i_2(P)$. Then $J \in \tilde{\mu}_2$ and $J \subset G$. Also, $J \cap Q = \emptyset$.

Thus, there is a set $J \in \tilde{\mu}_2$ such that $J \subset G$ and $J \cap Q = \emptyset$. Therefore, $Q \in (2,1) - \mathfrak{S}(X)$. By Similar arguments, we get the proof for (b).

Moreover, the family $(s,v)^\star - \mathfrak{S}(X)$ is need not be closed under finite union where $s, v = 1, 2$ and $s \neq v$ as shown by Example 34.

Example 34. (a). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}; \mu_1 = \{\emptyset, \{p, q\}, \{q, r\}, \{p, q, r\}\}; \mu_2 = \{\emptyset, \{p\}, \{q\}, \{p, q\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$. Then $\sigma_2 = \{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, \{p, s\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$. Take $P = \{q, s\}$ and $Q = \{r, s\}$. Then P and Q are $(1,2)^\star$-strongly nowhere dense sets in X. But $P \cup Q = \{q, r, s\} \notin (1,2)^\star - \mathfrak{S}(X)$.
(b). Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}; \mu_1 = \{\emptyset, \{r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}; \mu_2 = \{\emptyset, \{q, r\}, \{r, s\}, \{q, r, s\}, \{p, q, s\}, X\}$. Then $\sigma_1 = \{\emptyset, \{r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$. Take $K = \{p\}$ and $L = \{q\}$. Then K and L are $(2,1)^\star$-strongly nowhere dense sets in X. But $K \cup L = \{p, q\} \notin (2,1)^\star - \mathfrak{S}(X)$.

The following Example 35 shows that
a. $P \cup Q \notin (s,v)^\star - \mathfrak{S}(X)$ even if $P \in (s,v)^\star - \mathfrak{S}(X)$ and $Q \in (v,s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.
b. $P \cup Q \notin (v,s)^\star - \mathfrak{S}(X)$ even if $P \in (s,v)^\star - \mathfrak{S}(X)$ and $Q \in (v,s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Example 35. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}; \mu_1 = \{\emptyset, \{p\}, \{r\}, \{p, r\}, \{p, q\}, \{q, r\}, \{p, q, r\}\}$ and $\mu_2 = \{\emptyset, \{p\}, \{p, q\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$. Then $\sigma_1 = \{\emptyset, \{p\}, \{r\}, \{s\}, \{p, r\}, \{p, q\}, \{p, s\}, \{q, r\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$ and $\sigma_2 = \{\emptyset, \{p\}, \{r\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$.
Take $P = \{q, s\}$ and $Q = \{q, r\}$. Then $P \in (1,2)^\star - \mathfrak{S}(X)$ and $Q \in (2,1)^\star - \mathfrak{S}(X)$. Here $P \cup Q = \{q, r, s\}$. But $P \cup Q \notin (1,2)^\star - \mathfrak{S}(X)$. Also, $P \cup Q \notin (2,1)^\star - \mathfrak{S}(X)$.

Theorem 36. *Let (X, μ_1, μ_2) be a BGTS. If μ_1 and μ_2 are strong generalized topologies, then the followings are true.*
(a) $(1,2)^\star - \mathfrak{S}(X) \subset (2,1)^\star - \mathcal{N}(X)$.
(b) $(2,1)^\star - \mathfrak{S}(X) \subset (1,2)^\star - \mathcal{N}(X)$.

Proof. It is enough to prove (a) only. Let $E \in (1,2)^\star - \mathfrak{S}(X)$. Suppose $i_{\sigma_1}(c_2(E)) \neq \emptyset$. Then there exist $G \in \tilde{\sigma}_1$ such that $G \subset c_2(E)$. Since $G \in \tilde{\sigma}_1$ we have $i_1(G) \neq \emptyset$, by assumption. Thus, $i_1(G) \in \tilde{\mu}_1$. Since $G \subset c_2(E)$ we have $H \cap E \neq \emptyset$ for every $H \in \tilde{\sigma}_2$ such that $H \subset i_1(G)$ which is a contradiction to hypothesis. For, $H \in \tilde{\sigma}_2$ which implies $H \subset c_2(i_2(H))$. Since μ_2 is a strong generalized topology, $i_2(H) \in \tilde{\mu}_2$. Here $i_2(H) \subset i_1(G) \subset c_2(E)$. This implies $i_2(H) \cap c_2(E) \neq \emptyset$ which implies that $i_2(H) \cap E \neq \emptyset$, by Lemma 2. Thus, $H \cap E \neq \emptyset$. Therefore, $E \in (2,1)^\star - \mathcal{N}(X)$.

Theorem 37. *Let (X, μ_1, μ_2) be a BGTS which has the $\mathcal{I}_S$-property. Then $(s,v)^\star - \mathcal{N}(X) \subset (v,s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.*

Proof. Assume that, (X, μ_1, μ_2) satisfy the $\mathcal{I}_S$-property. Let $Q \in (s, v)^\star - \mathcal{N}(X)$ where $s, v = 1, 2$ and $s \neq v$. By hypothesis and Theorem 26, $Q \in (s, v) - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$. Also, $(s, v) - \mathfrak{S}(X) \subset (v, s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$. Therefore, $Q \in (v, s)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Theorem 38. *Let (X, μ_1, μ_2) be a BGTS. If $\mu_v \subset \mu_s, \mu_v$ is a sGT and $Q \in (s, v)^\star - \mathfrak{S}(X)$, then Q is a μ_v-strongly nowhere dense set where $s, v = 1, 2$ and $s \neq v$.*

Proof. Assume that, $\mu_v \subset \mu_s, \mu_v$ is a sGT and $Q \in (s, v)^\star - \mathfrak{S}(X)$ where $s, v = 1, 2$ and $s \neq v$. Take $s = 1$ and $v = 2$. Then $Q \in (1, 2)^\star - \mathfrak{S}(X); \mu_2 \subset \mu_1$ and μ_2 is a sGT. Let $H \in \tilde{\mu}_2$. Then $H \in \tilde{\mu}_1$. By assumption, there is $K \in \tilde{\sigma}_2$ such that $K \subset H$ and $K \cap Q = \emptyset$. Since $K \in \tilde{\sigma}_2$ we have $K \subset c_2(i_2(K))$. This implies $i_2(K) \neq \emptyset$, since μ_2 is a sGT which implies that $i_2(K) \in \tilde{\mu}_2$. Take $B = i_2(K)$. Thus, there is $B \in \tilde{\mu}_2$ such that $B \subset H$ and $B \cap Q = \emptyset$. Hence Q is a μ_2-strongly nowhere dense set in X.
By similar arguments, we can prove the result for the case $s = 2$ and $v = 1$.

Theorem 39. *Let (X, μ_1, μ_2) be a BGTS which has the $\mathcal{I}_S$-property. If μ_v is a sGT and $D \in (s, v)^\star - \mathfrak{S}(X)$, then D is a μ_s-strongly nowhere dense set where $s, v = 1, 2$; $s \neq v$.*

Proof. We give the detailed proof only for $s = 2$ and $v = 1$. Assume that, the bigeneralized topological space (X, μ_1, μ_2) satisfy the $\mathcal{I}_S$-property and μ_1 is a strong generalized topology. Let D be $(2, 1)^\star$-strongly nowhere dense set and $G \in \tilde{\mu}_2$. Then there is a set $P \in \tilde{\sigma}_1$ such that $P \subset G$ and $P \cap D = \emptyset$. Since $P \in \tilde{\sigma}_1$ we have $i_{\mu_1}(P) \neq \emptyset$, by our assumption. Take $J = i_{\mu_1}(P)$. Then $J \in \tilde{\mu}_1$. Here $G \in \tilde{\mu}_2, J \in \tilde{\mu}_1$ and $J \cap G \neq \emptyset$. By our assumption, $i_{\mu_2}(J \cap G) \neq \emptyset$. Take $E = i_{\mu_2}(J \cap G)$. Then $E \in \tilde{\mu}_2$. Thus, there exists $E \in \tilde{\mu}_2$ such that $E \subset G$ and $E \cap D = \emptyset$. Hence D is μ_2-strongly nowhere dense in X.

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