**EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS**
Vol. 16, No. 1, 2023, 271-285
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

# On the study of rainbow antimagic connection number of corona product of graphs

Brian Juned Septory[1,3], Liliek Susilowati[1,*], Dafik[2,3], Veerabhadraiah Lokesha[4], Gnaneswaran Nagamani[5]

[1] *Mathematics Department, Faculty of Science and Technology, Airlangga University, Surabaya, Indonesia*
[2] *Mathematics Education Study Program, Faculty of Teacher Training and Education, University of Jember, Jember, Indonesia*
[3] *Pusat Unggulan Ipteks-Perguruan Tinggi, Combinatorics and Graph, CGANT, University of Jember, Jember, Indonesia*
[4] *Mathematics Department, Vijayanagara Sri Krishnadevaraya University, Bellary, India*
[5] *Department of Mathematic The Gandhigram Rural Institute, Tamil Nadu, India*

---

**Abstract.** Given that a graph $G = (V, E)$. By an edge-antimagic vertex labeling of graph, we mean assigning labels on each vertex under the label function $f : V \to \{1, 2, \dots, |V(G)|\}$ such that the associated weight of an edge $uv \in E(G)$, namely $w(xy) = f(x) + f(y)$, has distinct weight. A path $P$ in the vertex-labeled graph $G$ is said to be a rainbow path if for every two edges $xy, x'y' \in E(P)$ satisfies $w(xy) \neq w(x'y')$. The function $f$ is called a rainbow antimagic labeling of $G$ if for every two vertices $x$ and $y$ of $G$, there exists a rainbow $x - y$ path. When we assign each edge $xy$ with the color of the edge weight $w(xy)$, thus we say the graph $G$ admits a rainbow antimagic coloring. The rainbow antimagic connection number of $G$, denoted by $rac(G)$, is the smallest number of colors induced from all edge weight of antimagic labeling. In this paper, we will study the $rac(G)$ of the corona product of graphs. By the corona product of graphs $G$ and $H$, denoted by $G \odot H$, we mean a graph obtained by taking a copy of graph $G$ and $n$ copies of graph $H$, namely $H_1, H_2, ..., H_n$, then connecting vertex $v_i$ from the copy of graph $G$ to every vertex on graph $H_i, i = 1, 2, 3, \dots, n$. In this paper, we show the exact value of the rainbow antimagic connection number of $T_n \odot S_m$ where $T_n \in \{P_n, S_n, S_{n,p}, F_{n,3}\}$.

**2020 Mathematics Subject Classifications**: 05C15, 05C78

**Key Words and Phrases**: Antimagic labeling, Rainbow connection, Rainbow antimagic connection number, Corona product of graphs.

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v16i1.4520

*Email addresses:* `liliek-s@fst.unair.ac.id` (L. Susilowati), `brianseptory95@gmail.com` (B. J. Septory), `d.dafik@unej.ac.id` (Dafik), `mathematics@vskub.ac.in` (V. Lokesha), `nagamanigru@gmail.com` (G. Nagamani)

 

## 1. Introduction

Given two any graphs $G$ and $H$. The corona operation of two graphs $G$ and $H$, denoted by $G \odot H$, is a graph obtained by taking a copy of graph $G$ and $n$ copies of graph $H$ namely $H_1, H_2, ..., H_n$ then connecting vertex $v_i$ from the copy of graph $G$ to every vertex on graph $H_i, i = 1, 2, 3, \ldots, n$, see [8, 12] for detail.

The rainbow antimagic coloring defined in above abstract is a combination of the rainbow coloring and antimagic labeling concepts. Rainbow coloring was first introduced by Chartrand *et al.* in 2008 [7]. Suppose that $G$ is a nontrivially connected graph and $c$ is the edge coloring of $G$. A $u - v$ path of $G$, if no two edges of the $u - v$ path are the same color is called a rainbow path. The $c$ edge coloring, if for every vertex $u, v \in V(G)$ there is a rainbow path $u - v$ is called a rainbow connection. There are a lot of results related to the rainbow connection, see [17] and [18].

**Proposition 1.** *[21] Let $G$ be a connected graph of size $m$. The rainbow connection number $rc(G) = m$ if and only if $G$ is a tree.*

Other extension of rainbow coloring study is rainbow vertex coloring introduced in [16]. Some results on rainbow vertex connection number can see in [19], [23]. Furthermore, we also have other version of rainbow coloring study, called rainbow total coloring, see [14] and [24]. The complete survey of rainbow connection number can be found in [20].

Meanwhile, graph labeling was first introduced by Wallis *et al.* in 2001 [25]. Hartsfield and Ringel in 1990 [13] introduced an antimagic labeling of a graph $G$ with edges is a bijection function $f : E(G) \rightarrow \{1, 2, ..., |E(G)|\}$ and $w(v) = \Sigma_{e \in E(v)} f(e)$, and $E(v)$ is the set of edges incidence to $v$, for vertex $u, v \in V(G)$, $w(u) \neq w(v)$. Some results of antimagic labeling have been extensively studied by Baca *et al.* in [2–4]. Furthermore, Dafik *et al.* in 2021 [10] also contributed some results on antimagic labeling. Moreover, there are some other results related to antimagic labeling, see [6] and [9].

The concept of combining the graph coloring and the graph labeling was initiated by Arumugam *et al.* in 2017 [1]. He defined that for a bijection $f : E(G) \rightarrow \{1, 2, ...|E(G)|\}$ and $w(v) = \Sigma_{e \in E(v)} f(e)$, and $E(v)$ is the set of edges incidence to $v$, for each $v \in V(G)$. The bijection $f$ is called local antimagic labeling if for every two adjacent vertices $u, v \in V(G), w(u) \neq w(v)$. Thus, each local antimagic labeling is a vertex coloring at $G$ with vertex $v$ colored with $w(v)$. When we consider the chromatic number of the local antimagic labeling, this notion is called a local antimagic coloring. Motivated by this combination, Dafik *et al.* in 2021 [11] initiates to study a rainbow antimagic coloring of graph.

## 2. Rainbow Antimagic Coloring

Based on the description above, Dafik *et al.* in 2021 [11] have obtained the lower bound of the $rac(G)$ and given some relevan results too.

**Proposition 2.** *[11] For any connected graph $G$, $rac(G) \geq rc(G)$.*

**Theorem 1.** *[11] Let $G$ be any connected graph. Let $rc(G)$ and $\Delta(G)$ be the rainbow connection number of $G$ and the maximum degree of $G$, $rac(G) \geq \max\{rc(G), \Delta(G)\}$.*

**Theorem 2.** *[11] Let $G$ be a connected graph of diameter $diam(G) \leq 2$. Let $f$ be any bijective function from $V(G)$ to the set $\{1, 2, \ldots, |V(G)|\}$, there exists a rainbow path $u-v$.*

**Theorem 3.** *[11] For $T_m$, being any tree of order $m \geq 3$, $rac(T_m) = m - 1$.*

Some initial results for rainbow antimagic coloring have been found in [5], [11], [15] and [22].

**Theorem 4.** *[22]For any integer $m \geq 3$, $rac(K_{2,m}) = m + 1$.*

## 3. Results

In this section, we will show the rainbow antimagic connection number of $T_n \odot S_m$ where $T_n \in \{P_n, S_n, S_{n,p}, F_{n,3}\}$. Our strategy is firstly determined the rainbow antimagic connection number of $K_1 + S_m$. Secondly, establish the lower bound of $rac(T_n \odot S_m)$. Finally, we show the exact values of $rac(P_n \odot S_m)$, $rac(S_n \odot S_m)$, $rac(S_{n,p} \odot S_m)$ and $rac(F_{n,3} \odot S_m)$.

**Theorem 5.** *For $m \geq 3$, $rac(K_1 + S_m) = m + 2$.*

*Proof.* The graph $K_1 + S_m$ is a connected graph with vertex set $V(K_1 + S_m) = \{a_1\} \cup \{b_1\} \cup \{x_j, 1 \leq j \leq m\}$, and the edge set $E(K_1 + S_m) = \{a_1b_1\} \cup \{a_1x_j, b_1x_j, 1 \leq j \leq m\}$. The cardinality of $|V(K_1 + S_m)| = m + 2$ and the cardinality of $|E(K_1 + S_m)| = 2nm + 1$. Based on this definition, the graph $K_1 + S_m$ has $\Delta(K_1 + S_m) = m + 1$.

To prove $rac(K_1 + S_m)$, first we have to show the lower bound of $rac(K_1 + S_m)$. Based on Theorem 1, we have $rac(G) \geq \max\{rc(G), \Delta(G)\} = m + 1$. Since, the construction of vertex labeling with the function $f : V(K_1 + S_m) \to \{1, 2, \ldots |V(K_1 + S_m)|\}$ is a bijective function, assigning the most possible label for $a_1, b_1$ gives $w(a_1b_1)$ must be different with other edge weights. The rest of the labels are considered to be a rainbow antimagic coloring of complete bipartite graph $K_{2,m}$. Refer to Theorem 4, $rac(K_{2,m}) = \Delta(K_{2,m}) + 1 = m + 1$. Since $\Delta(K_1 + S_m) = rac(K_{2,m})$, and apart from edge $a_1b_1$, the graph $K_1 + S_1$ is $K_{2,m}$, it implies that $rac(K_1 + S_m) \geq max\{rc(G), \Delta(G)\} = \Delta(K_1 + S_m)$. However, if $rac(K_1 + S_m) \geq \Delta(K_1 + S_m)$ then there is a conflict, since we need to include the edge $a_1b_1$, thus it must be $m + 1 + 1 = \Delta(K_1 + S_m) + 1$. It concludes that $rac(K_1 + S_m) \geq \Delta(K_1 + S_m) + 1 = m + 2$.

Secondly, we have to show the upper bound of $rac(K_1+S_m)$. Define the vertex labeling $f : V(K_1 + S_m) \to \{1, 2, ..., m + 2\}$ as follows.

$$f(a_1) = 1,$$
$$f(b_1) = 2,$$
$$f(x_i) = i + 2, \quad \text{for } 1 \leq i \leq m.$$

The edge weights of the above vertex labeling $f$ can be presented as

$$w(a_1b_1) = 3,$$

$$w(a_1x_i) = i + 3, \quad \text{for } 1 \leq i \leq m$$
$$w(b_1x_i) = i + 4 \quad \text{for } 1 \leq i \leq m.$$

It is easy to see that the above edge weight will induce a rainbow antimagic coloring of graph $K_1+S_m$. By this set of edge weight, we can easily calculate that the number of color $w(a_1b_1)$ is 1. The sets $w(a_1x_i) = \{4, 5, 6, 7, \dots, m + 3\}$ and $w(b_1x_i) = \{5, 6, 7, \dots, m + 4\}$, thus the number of distinct colors of $w(a_1x_i) \cup w(b_1x_i)$ is $m+1$. It implies the edge weights of $f : V(K_1 + S_m) \to \{1, 2, ..., m + 2\}$ induces a rainbow antimagic coloring of $1 + m + 1$ colors. Therefore $rac(K_1 + S_m) \leq m + 2$. Combining the two bounds, we have the exact value of $rac(K_1 + S_m) = m + 2$.

The next step, we need to evaluate the existence of rainbow path of $K_1 + S_m$. Since $diam(K_1 + S_m) = 2$, based on Theorem 2, there exists a rainbow $u - v$ path for any two vertices $u, v \in V(K_1 + S_m)$. It completes the proof.

**Lemma 1.** *Let $T_n \odot S_m$ be a coronation of tree with order $n$ and star graph with $m \geq 3$. The lower bound of $rac(T_n \odot S_m) \geq rac(T_n) + m + 2$.*

*Proof.* The graph $T_n \odot S_m$ is the corona product of two graphs $T_n$ and $S_m$. It is obtained by taking one copy of $T_n$ and $|V(T_n)|$ copies of $S_m$ and joining the $i$-th vertex of $T_n$ to every vertex in the $i$-th copy of $S_m$. By this definition, it implies the graph $T_n \odot S_m$ contains $|V(T_n)|$ copies of $K_1 + S_m$, see Figure 1. Thus, to obtain the $rac(T_n \odot S_m)$, we need to consider the $rac(K_1+S_m)$ and $rac(T_n)$. Based on Theorem 5, we have $rac(K_1+S_m) = m+2$. Based on Theorem 3, we have $rac(T_n) = n - 1$, since $E(T_n) = -1$, for $uv, u'v' \in E(T_n)$ has a different colors. Thus, it implies that $rac(T_n \odot S_m) \geq rac(T_n) + m + 2$.

**Theorem 6.** *For odd integers $n \geq 3$ and $m \geq 3$, $rac(P_n \odot S_m) = n + m + 1$.*

*Proof.* The graph $P_n \odot S_m$ is a connected graph with vertex set $V(P_n \odot S_m) = \{x_{0i}, y_i, 1 \leq i \leq n\} \cup \{y_{ij}, 1 \leq i \leq n\ , 1 \leq j \leq m\}$ and edge set $E(P_n \odot S_m) = \{x_{0i}x_{0i+1}, 1 \leq i \leq n - 1\} \cup \{x_{0i}y_i, 1 \leq i \leq n\} \cup \{x_{0i}y_{ij}, y_iy_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$. The cardinality of $|V(P_n \odot S_m)| = 2n + nm$ and the cardinality of $|E(P_n \odot S_m)| = 2n + 2nm - 1$.

To prove the rainbow antimagic connection number of $rac(P_n \odot S_m)$, first we have to show that the lower bound of $rac(P_n \odot S_m)$. Based on Lemma 1, we have $rac(P_n \odot S_m) \geq rac(P_n) + m + 2$. Since $rac(P_n) = n - 1$, thus, $rac(P_n \odot S_m) \geq n + m + 1$.
Secondly, we have to show the upper bound of $rac(P_n \odot S_m)$. Define the vertex labeling $f : V(P_n \odot S_m) \to \{1, 2, ..., 2n + nm\}$ as follows.

$$f(x_{0i}) = \left\lfloor \frac{n}{2} \right\rfloor + i, \quad \text{for } 1 \leq i \leq n$$
$$f(y_i) = \begin{cases} \lfloor \frac{n}{2} \rfloor + i + n, & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \\ i - \lceil \frac{n}{2} \rceil\ , & \text{for } \lceil \frac{n}{2} \rceil\ + 1 \leq i \leq n, \end{cases}$$
$$f(y_{ij}) = \begin{cases} 2n + jn - i - 1, & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil, 1 \leq j \leq m \\ 2n + jn - i + 4, & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n, 1 \leq j \leq m, \end{cases}$$

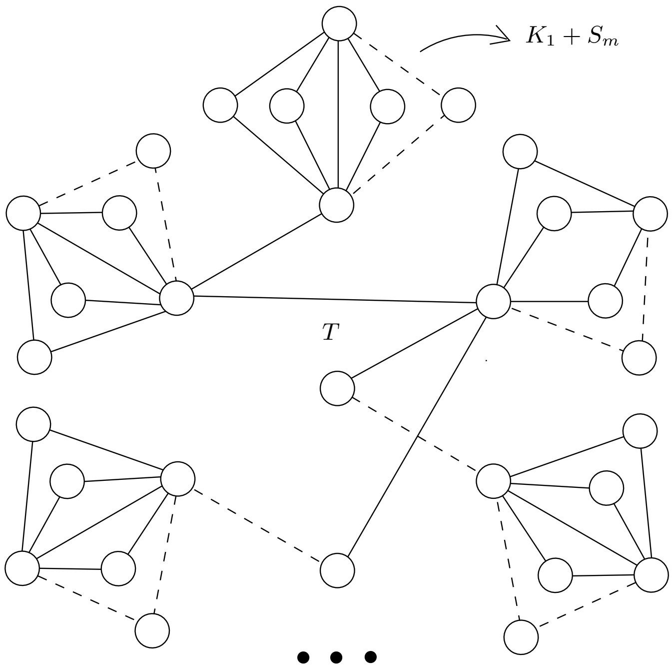


Figure 1: The illustration of graph $T_n \odot S_m$.

The edge weights of the above vertex labeling $f$ can be presented as: for $1 \leq i \leq n-1$, $w(x_{0i}x_{0i+1}) = n + 2i$, and for $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $1 \leq j \leq m$

$$\begin{aligned} w(x_{0i}y_i) &= 2n + 2i - 1, \\ w(x_i y_{ij}) &= 2n + jn + 1, \\ w(y_i y_{ij}) &= 3n + jn + 1. \end{aligned}$$

and for $\lceil \frac{n}{2} \rceil + 1 \leq i \leq n$, $1 \leq j \leq m$,

$$\begin{aligned} w(x_{0i}y_i) &= 2i - 1, \\ w(x_i y_{ij}) &= 3n + jn + 1, \\ w(y_i y_{ij}) &= 2n + jn + 1. \end{aligned}$$

Based on Theorem 3, $rac(P_n) = n - 1$. Since $E(P_n) = n - 1$, for $u, v \in E(P_n)$ has a different colors. Therefore, the sum of the weights of graph $P_n$ is $n - 1$. Based on Theorem 5, we have $rac(K_1 + S_m) = m + 2$. Based on the description above, we have that the distinct weight of graph $(P_n \odot S_m)$ is $n + m + 1$. It implies the edge weights of $f : V(P_n \odot S_m) \to \{1, 2, ..., 2n + nm\}$ induces a rainbow antimagic coloring of $m + n + 1$ colors. Thus $rac(P_n \odot S_m) \leq n + m + 1$. Comparing the two bounds, we have the exact value of $rac(P_n \odot S_m) = n + m + 1$.

The next step, evaluate to prove the existence of a rainbow $u - v$ path $P_n \odot S_m$. Based on the definition of the graph $P_n \odot S_m$, then the graph $P_n \odot S_m$ contains one graph $P_n$ and $|V(P_n)|$ copies of $K_1 + S_m$, so that we can evaluate the rainbow $u - v$ path of the graph

$P_n \odot S_m$ by evaluating the rainbow $u-v$ path on the graph $P_n$ and the graph $K_1+S_m$. Since $diam(K_1+S_m)=2$, based on Theorem 2, there is a rainbow $u-v$ path for every $u,v \in V(K_1+S_m)$. Based on Theorem 3, $rac(P_n)=n-1$. Since $P_n$ has $n-1$ edges, there is a rainbow $u-v$ path for every $u,v \in V(P_n)$. Therefore, according to the explanation, it can be seen that there is a rainbow $u-v$ path for every $u,v \in V(P_n \odot S_m)$.

Rainbow antimagic coloring of graph $P_n \odot S_m$ can be seen in Figure 2.

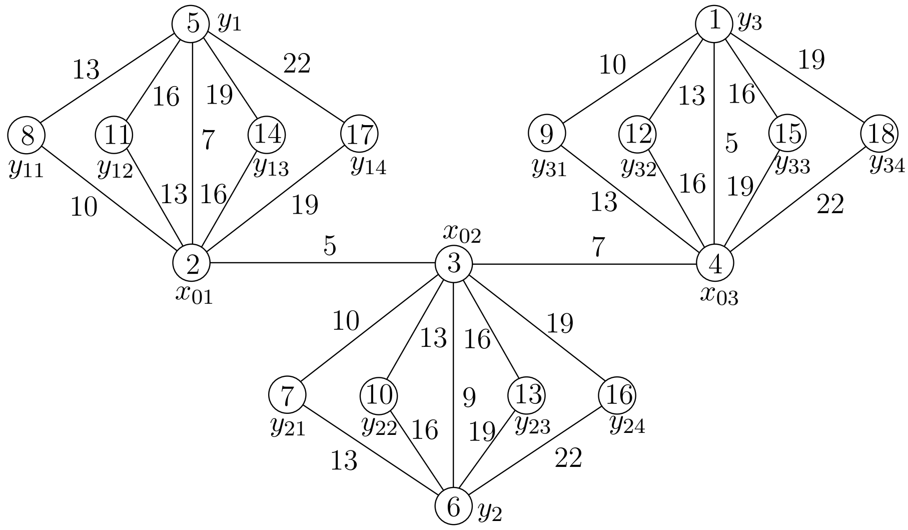


Figure 2: Rainbow antimagic coloring of graph $P_3 \odot S_4$.

**Theorem 7.** *For odd integers $n \geq 3$ and $m \geq 3$, $rac(S_n \odot S_m) = n+m+2$.*

*Proof.* The graph $S_n \odot S_m$ is a connected graph with vertex set $V(S_n \odot S_m) = \{x_0\} \cup \{x_{0i}, 1 \leq i \leq n\} \cup \{y_i, 1 \leq i \leq n+1\} \cup \{y_{ij}, 1 \leq i \leq n+1, 1 \leq j \leq m\}$ and edge set $E(S_n \odot S_m) = \{x_0x_i, 1 \leq i \leq n\} \cup \{x_0y_{n+1}\} \cup \{x_{0i}y_i, 1 \leq i \leq n\} \cup \{x_0y_{n+1j}, y_{n+1}y_{n+1j}, 1 \leq j \leq m\} \cup \{x_{0i}y_i, y_iy_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$. The cardinality of $|V(S_n \odot S_m)| = 2n+nm+2$ and the cardinality of $|E(S_n \odot S_m)| = 2nm+2n+2m+1$.

To prove the rainbow antimagic connection number of $rac(S_n \odot S_m)$, first we have to show that the lower bound of $rac(S_n \odot S_m)$. Based on Lemma 1, we have $rac(S_n \odot S_m) \geq rac(S_n)+m+2$. Since $rac(S_n)=n$, thus, $rac(S_n \odot S_m) \geq n+m+2$.

Secondly, we have to show the upper bound of $rac(S_n \odot S_m)$. Define the vertex labeling $f: V(S_n \odot S_m) \to \{1,2,\dots 2n+nm+2\}$ as follows.

$$f(x_0) = n+1$$

$$f(x_i) = \begin{cases} n+i+1 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ i & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \\ i & \text{for } i = n+1 \end{cases}$$

$$f(y_i) = \begin{cases} i & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ n+i+1 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \\ 2n+2 & \text{for } i = n+1, \end{cases}$$

$$f(y_{ij}) = 2n + jn - i + j + 3, \quad \text{for } 1 \le i \le n+1, 1 \le j \le m,$$

The edge weights of the above vertex labeling $f$ can be presented as

$$w(x_0x_{0i}) = \begin{cases} 2n+i+2 & \text{for } 1 \le i \le n \text{ and } i \text{ is odd} \\ n+i+1 & \text{for } 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$
$$w(x_0y_{n+1}) = 3n+3$$
$$w(x_iy_i) = n+2i+1, \quad \text{for } 1 \le i \le n.$$
$$w(x_0y_{n+1j}) = 2n+jn+j+3, \quad \text{for } 1 \le j \le m$$
$$w(x_{0i}y_{ij}) = \begin{cases} 3n+jn+j+4 & \text{for } 1 \le i \le n, 1 \le j \le m \text{ and } i \text{ is odd} \\ 2n+jn+j+3 & \text{for } 1 \le i \le n, 1 \le j \le m \text{ and } i \text{ is even} \end{cases}$$
$$w(y_iy_{ij}) = \begin{cases} 2n+jn+j+3 & \text{for } 1 \le i \le n+1, 1 \le j \le m \text{ and } i \text{ is odd} \\ 3n+jn+j+4 & \text{for } 1 \le i \le n+1, 1 \le j \le m \text{ and } i \text{ is even} \\ 3n+jn+j+4 & \text{for } i = n+1, \end{cases}$$

Based on Theorem 3, $rac(S_n) = n$. Since $E(S_n) = n$, for $u, v \in E(S_n)$ has a different colors. Therefore, the sum of the weights of graph $S_n$ is $n$. Based on Theorem 5, we have $rac(K_1 + S_m) = m + 2$. Based on the description above, we have that the distinct weight of graph $(S_n \odot S_m)$ is $n+m+2$. It implies the edge weights of $f : V(S_n \odot S_m) \to \{1, 2, ..., 2n + nm + 2\}$ induces a rainbow antimagic coloring of $m+n+2$ colors. Thus $rac(S_n \odot S_m) \le n+m+2$. Comparing the two bounds, we have the exact value of $rac(S_n \odot S_m) = n+m+2$.

The next step, evaluate to prove the existence of a rainbow $u-v$ path $S_n \odot S_m$. Based on the definition of the graph $S_n \odot S_m$, then the graph $S_n \odot S_m$ contains one graph $S_n$ and $|V(S_n)|$ copies of $K_1 + S_m$, so that we can evaluate the rainbow $u-v$ path of the graph $S_n \odot S_m$ by evaluating the rainbow $u-v$ path on the graph $S_n$ and the graph $K_1 + S_m$. Since $diam(K_1 + S_m) = 2$, based on Theorem 2, there is a rainbow $u-v$ path for every $u, v \in V(K_1 + S_m)$. Based on Theorem 3, $rac(S_n) = n$. Since $S_n$ has $n$ edges, there is a rainbow $u-v$ path for every $u, v \in V(S_n)$. Therefore, according to the explanation, it can be seen that there is a rainbow $u-v$ path for every $u, v \in V(S_n \odot S_m)$.

Rainbow antimagic coloring of graph $S_n \odot S_m$ can be seen in Figure 3.

**Theorem 8.** *For $n = 2$, $m \ge 3$ and odd integers $p \ge 3$, $rac(S_{n,p} \odot S_m) = m + p + 5$.*

*Proof.* The graph $S_{n,p} \odot S_m$ is a connected graph with vertex set $V(S_{n,p} \odot S_m) = \{x, y, b, z\} \cup \{x_i, x_{0i}, 1 \le i \le 2\} \cup \{y_j, y_{0j}, 1 \le j \le p\} \cup \{b_k, z_k, 1 \le k \le m\} \cup \{x_{ik}, 1 \le i \le 2, 1 \le k \le m\} \cup \{y_{jk}, 1 \le j \le p, 1 \le k \le m\}$ and edge set $E(S_{n,p} \odot S_m) = \{xy, xb, yz\} \cup \{xx_i, x_ix_{0i}, 1 \le i \le 2\} \cup \{yy_j, y_jy_{0j}, 1 \le j \le p\} \cup \{xb_k, bb_k, yz_k, zz_k, 1 \le i \le m\} \cup \{x_ix_{ik}, x_{0i}x_{ik}, 1 \le i \le 2, 1 \le k \le m\} \cup \{y_jy_{jk}, y_{0j}y_{jk}, 1 \le j \le p, 1 \le k \le m\}$. The cardinality of $|V(S_{n,p} \odot S_m)| = 2p + 4m + pm + 8$ and the cardinality of $|E(S_{n,p} \odot S_m)| = 2p + 8m + 2pm + 7$.

To prove the rainbow antimagic connection number of $rac(S_{n,p} \odot S_m)$, first we have to show that the lower bound of $rac(S_{n,p} \odot S_m)$. Based on Lemma 1, we have $rac(S_{n,p} \odot S_m) \ge$

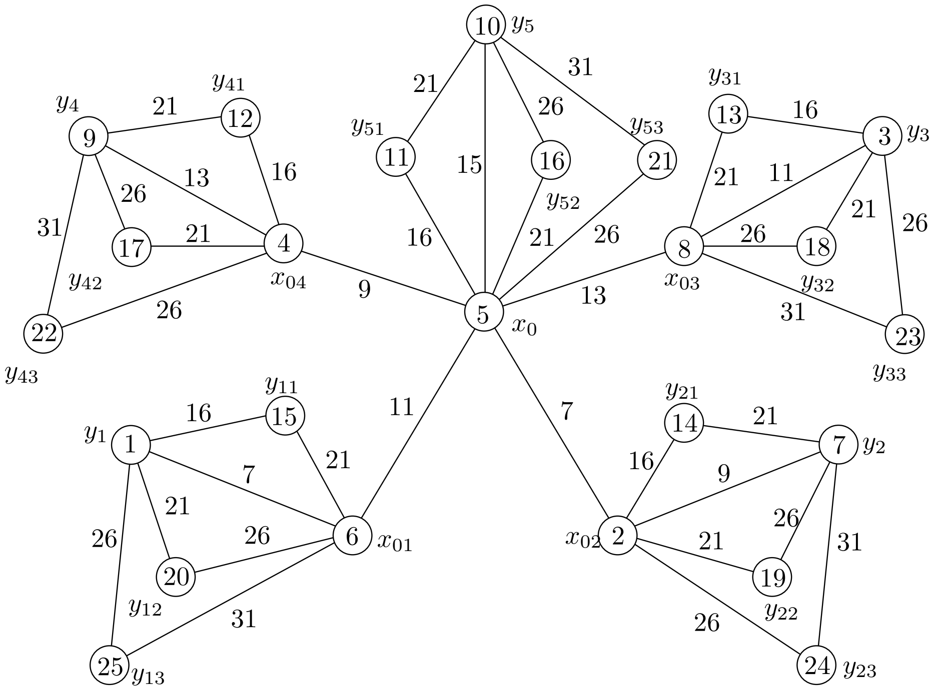

Figure 3: Rainbow antimagic coloring of graph $S_4 \odot S_3$.

$rac(S_{n,p}) + m + 2$. Since $rac(S_{n,p}) = p + 3$, thus, $rac(S_{n,p} \odot S_m) \geq m + p + 5$.
Secondly, we have to show the upper bound of $rac(S_{n,p} \odot S_m)$. Define the vertex labeling $f : V(S_{n,p} \odot S_m) \to \{1, 2, \ldots 2p + 4m + pm + 8\}$ as follows.

$$f(x) = p + 2$$
$$f(y) = p + 3$$
$$f(x_i) = \begin{cases} p+5 & \text{for } i = 1 \\ 2p+8 & \text{for } i = 2 \end{cases}$$
$$f(y_j) = \begin{cases} 2j+1 & \text{for } 1 \leq j \leq \lfloor \frac{p}{2} \rfloor \\ 2j+5 & \text{for } \lceil \frac{p}{2} \rceil + 1 \leq j \leq p \end{cases}$$
$$f(x_{0i}) = \begin{cases} 1 & \text{for } i = 1, \\ p+4 & \text{for } i = 2 \end{cases}$$
$$f(y_{0j}) = \begin{cases} p+2j+5 & \text{for } 1 \leq j \leq \lfloor \frac{p}{2} \rfloor \\ 2j+1-p & \text{for } \lceil \frac{p}{2} \rceil \leq j \leq p \end{cases}$$
$$f(b) = 2p + 6$$
$$f(z) = 2p + 7$$
$$f(x_{ik}) = \begin{cases} k(p+4) + 2p + 8 & \text{for } , i = 1, 1 \leq k \leq m, \\ k(p+4) + p + 5 & \text{for } , i = 2, 1 \leq k \leq m, \end{cases}$$
$$f(z_k) = k(p+4) + p + 6 \quad \text{for } 1 \leq k \leq m,$$
$$f(b_k) = k(p+4) + p + 7 \quad \text{for } 1 \leq k \leq m,$$

$$f(y_{jk}) = \begin{cases} k(p+4) - 2j + 2p + 8 & \text{for } 1 \le j \le \left\lfloor \frac{p}{2} \right\rfloor, 1 \le k \le m, \\ k(p+4) - 2j + 3p + 8 & \text{for } \lceil \frac{p}{2} \rceil, 1 \le k \le m. \end{cases}$$

The edge weight of the above vertex labeling $f$ can be presented as

$$w(xy) = 2p + 5$$

$$w(xx_i) = \begin{cases} 2p + 7 & \text{for } i = 1 \\ 3p + 10 & \text{for } i = 2 \end{cases}$$

$$w(yy_j) = \begin{cases} p + 2j + 4 & \text{for } 1 \le j \le \left\lfloor \frac{p}{2} \right\rfloor \\ p + 2j + 8 & \text{for } \lceil \frac{p}{2} \rceil, 1 \le k \le m \end{cases}$$

$$w(x_i x_{0i}) = \begin{cases} p + 6 & \text{for } i = 1 \\ 3p + 12 & \text{for } i = 2 \end{cases}$$

$$w(xb) = 3p + 8$$

$$w(yz) = 3p + 10$$

$$w(y_j y_{0j}) = \begin{cases} p + 4j + 6 & \text{for } 1 \le j \le \left\lfloor \frac{p}{2} \right\rfloor \\ 4j + 6 - p & \text{for } \lceil \frac{p}{2} \rceil, 1 \le k \le m \end{cases}$$

$$w(x_i x_{ik}) = k(p+4) + 3p + 13, \quad \text{for } 1 \le \ \le 2, 1 \le k \le m$$

$$w(x_{0i} x_{ik}) = k(p+4) + 2p + 9, \quad \text{for } 1 \le \ \le 2, 1 \le k \le m$$

$$w(xb_k) = k(p+4) + 2p + 9, \quad \text{for } 1 \le k \le m$$

$$w(bb_k) = k(p+4) + 3p + 13, \quad \text{for } 1 \le k \le m$$

$$w(yz_k) = k(p+4) + 2p + 9, \quad \text{for } 1 \le k \le m$$

$$w(zz_k) = k(p+4) + 3p + 13, \quad \text{for } 1 \le k \le m$$

$$w(y_j y_{jk}) = \begin{cases} k(p+4) + 2p + 9 & \text{for } 1 \le j \le \left\lfloor \frac{p}{2} \right\rfloor, 1 \le k \le m, \\ k(p+4) + 3p + 13 & \text{for } \lceil \frac{p}{2} \rceil, 1 \le k \le m. \end{cases}$$

$$w(y_{0j} y_{jk}) = \begin{cases} k(p+4) + 3p + 13 & \text{for } 1 \le j \le \left\lfloor \frac{p}{2} \right\rfloor, 1 \le k \le m, \\ k(p+4) + 2p + 9 & \text{for } \lceil \frac{p}{2} \rceil, 1 \le k \le m. \end{cases}$$

Based on Theorem 3, $rac(S_{n,p}) = p + 3$. Since $E(S_{n,p}) = p + 3$, for $u, v \in E(S_{n,p})$ has a different colors. Therefore, the sum of the weights of graph $S_{n,p}$ is $p + 3$. Based on Theorem 5, we have $rac(K_1 + S_m) = m + 2$. Based on the description above, we have that the distinct weight of graph $(S_{n,p} \odot S_m)$ is $m + p + 5$. It implies the edge weights of $f : V(S_{n,p} \odot S_m) \to \{1, 2, ..., 2p + 4m + pm + 8\}$ induces a rainbow antimagic coloring of $m + n + 2$ colors. Thus $rac(S_{n,p} \odot S_m) \le m + p + 5$. Comparing the two bounds, we have the exact value of $rac(S_{n,p} \odot S_m) = m + p + 5$.

The next step, evaluate to prove the existence of a rainbow $u - v$ path $S_{n,p} \odot S_m$. Based on the definition of the graph $S_{n,p} \odot S_m$, then the graph $S_{n,p} \odot S_m$ contains one graph $S_{n,p}$ and $|V(S_{n,p})|$ copies of $K_1 + S_m$, so that we can evaluate the rainbow $u - v$ path of the graph $S_{n,p} \odot S_m$ by evaluating the rainbow $u - v$ path on the graph $S_{n,p}$ and the graph $K_1 + S_m$. Since $diam(K_1 + S_m) = 2$, based on Theorem 2, there is a rainbow

$u-v$ path for every $u, v \in V(K_1+S_m)$. Based on Theorem 3, $rac(S_{n,p}) = n+p+1$. Since $S_{n,p}$ has $n+p+1$ edges, there is a rainbow $u-v$ path for every $u, v \in V(S_{n,p})$. Therefore, according to the explanation, it can be seen that there is a rainbow $u-v$ path for every $u, v \in V(S_{n,p} \odot S_m)$.

Rainbow antimagic coloring of graph $S_{n,p} \odot S_m$ can be seen in Figure 4.

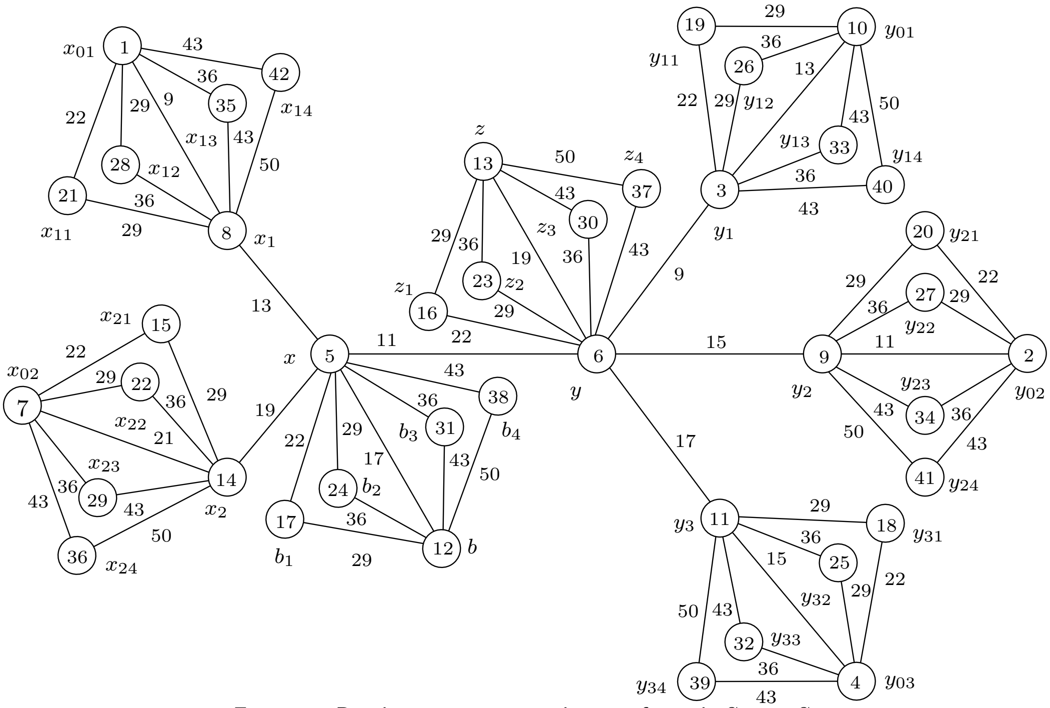


Figure 4: Rainbow antimagic coloring of graph $S_{2,3} \odot S_4$.

**Theorem 9.** *For odd integers $n \geq 3$ and $m \leq 3$, $rac(F_{n,3} \odot S_m) = 3n+m+1$.*

*Proof.* The graph $F_{n,3} \odot S_m$ is a graph with $V(F_{n,3} \odot S_m) = \{x_i, y_i, z_i, x_{0i}, y_{0i}, z_{0i} 1 \leq i \leq n\} \cup \{x_{ij}, y_{ij}, z_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$ and edge set $E(F_{n,3} \odot S_m) = \{x_i x_{i+1}, 1 \leq i \leq n-1\} \cup \{x_i y_i, y_i z_i, x_i x_{0i}, y_i y_{0i}, z_i z_{0i}, 1 \leq i \leq n\} \cup \{x_i x_{ij}, x_{0i} x_{ij}, y_i y_{ij}, y_{0i} y_{ij}, z_i z_{ij}, z_{0i} z_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$. The cardinality of $|V(F_{n,3} \odot S_m)| = 6n+3nm$ and the cardinality of $|E(F_{n,3} \odot S_m)| = 6n+6nm-1$.

To prove the rainbow antimagic connection number of $rac(F_{n,3} \odot S_m)$, first we have to show that the lower bound of $rac(F_{n,3} \odot S_m)$. Based on Lemma 1, we have $rac(F_{n,3} \odot S_m) \geq rac(F_{n,3}) + m + 2$. Since $rac(F_{n,3}) = 3n-1$, thus, $rac(F_{n,3} \odot S_m) \geq 3n+m+1$. Secondly, we have to show the upper bound of $rac(F_{n,3} \odot S_m)$. Define the vertex labeling $f : V(F_{n,3} \odot S_m) \to \{1, 2, ..., 6n+3nm\}$ as follows.

$$f(x_i) = \begin{cases} 3i + \left\lfloor \frac{n}{2} \right\rfloor + n & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 3i + n + \left\lfloor \frac{n}{2} \right\rfloor - 2 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(y_i) = 3i + n + \left\lfloor \frac{n}{2} \right\rfloor - 1, \text{ for } 1 \leq i \leq n$$

$$f(z_i) = \begin{cases} 3i + n + \lfloor \frac{n}{2} \rfloor - 2 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 3i + \lfloor \frac{n}{2} \rfloor + n & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(x_{0i}) = \begin{cases} 3i + \lfloor \frac{n}{2} \rfloor + 4n & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n - 2 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil - 2 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n - 2 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil - 2 & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

$$f(y_{0i}) = \begin{cases} 3i + 4n + \lfloor \frac{n}{2} \rfloor - 1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \\ 3i - \lceil \frac{n}{2} \rceil - n - 1 & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \end{cases}$$

$$f(z_{0i}) = \begin{cases} 3i + \lfloor \frac{n}{2} \rfloor + 4n - 2 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil - 2 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n - 1 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n - 2 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil - 2 & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 3i + \lfloor \frac{n}{2} \rfloor + 4n - 1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ 3i - n - \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

$$f(x_{ij}) = \begin{cases} k(3n) + 5n + 1 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ k(3n) + 7n + 1 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ k(3n) + 5n + 3 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ k(3n) + 7n + 3 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ k(3n) + 5n + 1 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ k(3n) + 7n + 1 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ k(3n) + 5n + 3 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ k(3n) + 7n + 3 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

$$f(y_{ij}) = \begin{cases} k(3n) + 4n + \lceil \frac{n}{2} \rceil + 2 - 3i & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil, 1 \leq\leq m \\ k(3n) + 7n + \lceil \frac{n}{2} \rceil + 2 - 3i & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

$$f(z_{ij}) = \begin{cases} k(3n) + 5n + 3 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ k(3n) + 7n + 3 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ k(3n) + 5n + 2 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ k(3n) + 7n + 1 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ k(3n) + 5n + 3 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ k(3n) + 7n + 3 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ k(3n) + 5n + 2 - 3i - \lfloor \frac{n}{2} \rfloor & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ k(3n) + 7n + 1 - 3i + \lceil \frac{n}{2} \rceil & \text{for } \lceil \frac{n}{2} \rceil + 1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

The edge weights of the above vertex labeling $f$ can be presented as

$$w(x_ix_{i+1}) = 3n + 6i, \text{ for } 1 \leq i \leq n-1$$

$$w(x_iy_i) = \begin{cases} 6i+2n+2(\lfloor \frac{n}{2} \rfloor)-1 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 6i+2n+2(\lfloor \frac{n}{2} \rfloor)-4 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$w(y_iz_i) = \begin{cases} 6i+2n+2(\lfloor \frac{n}{2} \rfloor)-3 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 6i+2n+2(\lfloor \frac{n}{2} \rfloor)-2 & \text{for } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$w(x_ix_{0i}) = \begin{cases} 6i+2(\lfloor \frac{n}{2} \rfloor)+5n & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 6i-1 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-4 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 6i-5 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 6i-1 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-4 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ 6i-5 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

$$w(y_iy_{0i}) = \begin{cases} 6i+5n+\lfloor \frac{n}{2} \rfloor-2 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \\ 6i-3 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \end{cases}$$

$$w(z_iz_{0i}) = \begin{cases} 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-4 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 6i-5 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 3 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-1 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 6i-3 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } i \text{ is even}, n \equiv 3 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-4 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 6i-5 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } i \text{ is odd}, n \equiv 1 \mod 4 \\ 6i+2(\lfloor \frac{n}{2} \rfloor)+5n-1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \\ 6i-3 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } i \text{ is even}, n \equiv 1 \mod 4 \end{cases}$$

$$w(x_ix_{ij}) = \begin{cases} k(3n)+6n+1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } n \equiv 3 \mod 4 \\ k(3n)+9n+1 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } n \equiv 3 \mod 4 \\ k(3n)+6n+1 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } n \equiv 1 \mod 4 \\ k(3n)+9n+1 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } n \equiv 1 \mod 4 \end{cases}$$

$$w(x_{0i}x_{ij}) = \begin{cases} k(3n)+9n+1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } n \equiv 3 \mod 4 \\ k(3n)+6n+1 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } n \equiv 3 \mod 4 \\ k(3n)+9n+1 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } n \equiv 1 \mod 4 \\ k(3n)+6n+1 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } n \equiv 1 \mod 4 \end{cases}$$

$$w(y_iy_{ij}) = \begin{cases} k(3n)+6n+1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil, 1 \leq \leq m \\ k(3n)+9n+1 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

$$w(z_iz_{ij}) = \begin{cases} k(3n)+6n+1 & \text{for } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \text{ and } n \equiv 3 \mod 4 \\ k(3n)+9n+1 & \text{for } \lceil \frac{n}{2} \rceil \leq i \leq n \text{ and } n \equiv 3 \mod 4 \\ k(3n)+6n+1 & \text{for } 1 \leq i \leq \lceil \frac{n}{2} \rceil \text{ and } n \equiv 1 \mod 4 \\ k(3n)+9n+1 & \text{for } \lceil \frac{n}{2} \rceil+1 \leq i \leq n \text{ and } n \equiv 1 \mod 4 \end{cases}$$

$$w(z_{0i}z_{ij}) = \begin{cases} k(3n)+9n+1 & \text{for } 1 \le i \le \lfloor \frac{n}{2} \rfloor \text{ and } n \equiv 3 \mod 4 \\ k(3n)+6n+1 & \text{for } \lceil \frac{n}{2} \rceil \le i \le n \text{ and } n \equiv 3 \mod 4 \\ k(3n)+9n+1 & \text{for } 1 \le i \le \lceil \frac{n}{2} \rceil \text{ and } n \equiv 1 \mod 4 \\ k(3n)+6n+1 & \text{for } \lceil \frac{n}{2} \rceil + 1 \le i \le n \text{ and } n \equiv 1 \mod 4 \end{cases}$$

Based on Theorem 3, $rac(F_{n,3}) = 3n-1$. Since $E(F_{n,3}) = 3n-1$, for $u,v \in E(F_{n,3})$ has a different colors. Therefore, the sum of the weights of graph $F_{n,3}$ is $3n-1$. Based on Theorem 5, we have $rac(K_1+S_m) = m+2$. Based on the description above, we have that the distinct weight of graph $(F_{n,3} \odot S_m)$ is $3n+m+1$. It implies the edge weights of $f : V(F_{n,3} \odot S_m) \to \{1,2,...,6n+3nm\}$ induces a rainbow antimagic coloring of $3n+m+1$ colors. Thus $rac(F_{n,3} \odot S_m) \le 3n+m+1$. Comparing the two bounds, we have the exact value of $rac(F_{n,3} \odot S_m) = 3n+m+1$.

The next step, evaluate to prove the existence of a rainbow $u-v$ path $F_{n,3} \odot S_m$. Based on the definition of the graph $F_{n,3} \odot S_m$, then the graph $F_{n,3} \odot S_m$ contains one graph $F_{n,3}$ and $|V(F_{n,3})|$ copies of $K_1+S_m$, so that we can evaluate the rainbow $u-v$ path of the graph $F_{n,3} \odot S_m$ by evaluating the rainbow $u-v$ path on the graph $F_{n,3}$ and the graph $K_1+S_m$. Since $diam(K_1+S_m) = 2$, based on Theorem 2, there is a rainbow $u-v$ path for every $u,v \in V(K_1+S_m)$. Based on Theorem 3, $rac(F_{n,3}) = 3n-1$. Since $F_{n,3}$ has $3n-1$ edges, there is a rainbow $u-v$ path for every $u,v \in V(F_{n,3})$. Therefore, according to the explanation, it can be seen that there is a rainbow $u-v$ path for every $u,v \in V(F_{n,3} \odot S_m)$.

## 4. Concluding Remarks

We have studied the rainbow antimagic coloring of the corona product on a graph with a star graph. Based on the results we have the exact value of the rainbow antimagic connection number of graph $T_n \odot S_m$ where $T_n$ is path $P_n$, star $S_n$, double star $S_{n,p}$ and fire craker $F_{n,3}$. However, if $T_n$ is not a tree graph, it is still difficult to determine the exact value of the rainbow antimagic connection number. Therefore, this study raises an open problem.

Determine the exact value of the rainbow antimagic connection number of graph $G \odot S_m$ where $G$ is not tree.

## Acknowledgements

We would like to earnestly acknowledge the sincere efforts and valuable guidance given by the research teams of PUI-PT Combinatorics and Graph, CGANT, University of Jember, Indonesia and the postgraduate program researchers of Airlangga University.

## References

[1] S Arumugam, K Premalatha, M Bača, and A Semaničová-Feňovčíková. Local antimagic vertex coloring of a graph. *Graphs and Combinatorics*, 33(2):275–285, 2017.

[2] M Bača, E T Baskoro, S Jendrol, and M Miler. Antimagic labelings of hexagonal plane maps. *Utilitas mathematica*, 66:231–238, 2004.

[3] M Bača, Y Lin, and M Miler. Antimagic labelings of grids. *Utilitas mathematica*, 72:65–75, 2007.

[4] M Bača, M Miler, and J Ryan. Antimagic labelings of disjoint union of s-crowns. *Utilitas mathematica*, 79:193–205, 2009.

[5] H S Budi, Dafik, I M Tirta, I H Agustin, and A I Kristiana. On rainbow antimagic coloring of graphs. volume 1832, page 012016, 2021.

[6] F Chang, Y C Liang, Z Pan, and X Zhu. Antimagic labeling of reguler graphs. *Journal of Graph Theory*, 82(4):339–349, 2016.

[7] G Chartrand, G L Johns, K A Mckeon, and P Zhang. Rainbow connection in graphs. *Math. Bohemica*, 133:85–98, 2008.

[8] G Chartrand, L Lesniak, and P Zhang. *Graphs & Digraphs.* Taylor & Francis Group, New York, 2016.

[9] D W Cranston. Reguler bipartite graphs are antimagic. *Journal of Graph Theory*, 60(3):173–182, 2009.

[10] Dafik, M Miler, J Ryan, and M Bača. Antimagic labeling of the union of two stars. *Australasian Journal of combinatorics*, 42:35–44, 2018.

[11] Dafik, F Susanto, R Alfarisi, B J Septory, I H Agustin, and M Venkatachalam. On rainbow antimagic coloring of graphs. *Advanced Mathematical Models and Aplication*, 6(3):278–291, 2021.

[12] R Frucht and F Harary. On the corona of two graphs. *Aequationes Math*, 4:322–325, 1970.

[13] N Hartsfield and G Ringel. *Pearls in Graph Theory.* Academic Press, San Diego, 1990.

[14] M S Hasan, Slamin, Dafik, I H Agustin, and R Alfarisi. On the total rainbow connection of the wheel related graphs. *Journal of Physics: Conf. Series*, 1008:012954, 2018.

[15] J C Joedo, Dafik, A I Kristiana, I H Agustin, and R Nisviasari. On the rainbow antimagic coloring of vertex almagamation of graphs. *Journal of Physics: Conf. Series*, 2157:012014, 2022.

[16] M Krivelevich and R Yuster. The rainbow connection of a graph is (at most) reciprocal to its minimum degree. *J. Graph Theory*, 63(3):185–191, 2010.

[17] H Li, X Li, and S Liu. Rainbow connection of graphs with diameter 2. *Discrete Mathematics*, 312(8):1453–1457, 2012.

[18] H Li, X Li, and Y Sun. Rainbow connection of graphs with diameter 3. *Discussiones Mathematicae Graph Theory*, 37(2):141–154, 2017.

[19] X Li and Y Shi. On the rainbow vertex connection. *Graph Theory*, 33:307–313, 2013.

[20] X Li and Y Sun. An updated survey on rainbow connections of graphs. *Theory and Application of Graph*, 0(1):Article 3, 2017.

[21] I Schiermeyer. Bounds for the rainbow connection number of graph. *Discussiones Mathematicae Graph Theory*, 31:387–395, 2011.

[22] B J Septory, M I Utoyo, Dafik, B Sulistiyono, and I H Agustin. On rainbow antimagic coloring of special graphs. *Journal of Physics: Conference Series*, 1836:012016, 2021.

[23] D N S Simamora and A N M Salman. The rainbow (vertex) connection number of pencil graphs. *Procedia Computer Science*, 74:138–142, 2010.

[24] Y Sun. On rainbow total coloring of a graph. *Discrete Applied Mathematics*, 194:171–177, 2015.

[25] W D Wallis. *Magic graphs.* Springer Science & Business Media, Boston, 2001.