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Grey wolves attack process for the Pareto optimal front construction in the multiobjective optimization

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Abstract. We propose a new metaheuristic, HmGWOGA-MO, for solving multiobjective optimization problems operating with a population of solutions. The method is a hybridization of the HmGWOGA method, which is a single objective optimization method, and the ϵ-constraint approach, which is an aggregation technique. The ϵ-constraint technique is one of the best ways to transform a problem with many objective functions into a single objective problem because it works even if the problem has any kind of Pareto optimal front. Previously, the HmGWOGA method was designed to optimize a positive single-objective function without constraints. The obtained solutions are good. That is why, in this current work, we combined have it with the ϵ-constraint approach for the resolution of multiobjective optimization problems. Our new method proceeds by transforming a given multiobjective optimization problem with constraints into an unconstrained optimization of a single objective function. With the HmGWOGA method, five different test problems with varying Pareto fronts have been successfully solved, and the results are compared with those of NSGA-II regarding convergence towards the Pareto front and the distribution of solutions on the Pareto front. This numerical study indicates that HmGWOGA-MO is the best choice for solving a multiobjective optimization problem when convergence is the most important performance parameter.

2020 Mathematics Subject Classifications: 90C29, 90C30, 49M37

Key Words and Phrases: Multiobjective optimization, Metaheuristics, Pareto optimality

1. Introduction

Considering its intervention in several areas sensitive to development, multiobjective optimization is one of the most important tools for finding solutions to the problems of everyday life. Indeed, the modeling of most economic, management, social life, decision-making and industrial problems results in multiobjective formulations, which is why there

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is interest in their resolution. If the number of functions or variables becomes too large, these formulations become difficult to solve. Recall that a problem related to multiobjective optimization can be modeled without loss of generality in the following form :

$$\min_{x \in X} F(x) = (f_1(x), f_2(x), \ldots, f_K(x)); \tag{MOP}$$

where $X = \{x \in \mathbb{R}^n, g_j(x) \leq 0, j = \overline{j,q}\}$ is called the set of eligible points and the $f_i, i = \overline{1,K}$ the objective functions, which are, in general, in conflicting. The result of this situation is that there is no global optimal solution for this type of problem. There are better compromise solutions or Pareto optimal solutions, hence the following definition would be more appropriate[2]:

Definition 1. *Let $x \in X$ be an eligible point of MOP. $x \in X$ is called:*

(i) weakly Pareto optimal solution if there is no other point $x' \in X$ such as $f_i(x') < f_i(x),\ i = \overline{1,K}$;

(ii) Pareto optimal solution if there is no other solution $x' \in X$ such as $f_i(x') \leq f_i(x),\ i = \overline{1,K}$ and at least one $k \in \{1, 2, \cdots, K\}, f_k(x') < f_k(x)$.

Finding solutions for these types of problems is a challenge for current methods. They can be divided into two groups, such as the exact methods and the approximation methods. The first group of methods are inefficient in the case of a problem with a large size and/or nonlinear; and the second group of methods forms with heuristics and metaheuristics, which have difficulty in satisfying both the convergence and distribution of the produced solutions. This is why many researchers today are focusing their efforts on resolving these types of issues.

Many recent studies have focused on solving multiobjective optimization problems using metaheuristics. Among the numerous methods implemented, the majority is based on genetic algorithms. In 1994, N. Srinivas and Deb Kalyanmoy[21] proposed an evolutionary non-dominated sorting algorithm based on genetic algorithms (NSGA). In 2002, Deb et al.[3] proposed NSGA-II, a method for solving multiobjective problems. In 2006, Demin Lei et al.[11] also proposed in a method of solving planning problems. In 2008, Hamidreza Eskandari and Christopher D. Geiger[7] proposed an approach to solving multiobjective problems that is costly in terms of calculation time. In 2011, Kounhinir Somé et al.[20] proposed a new non-stochastic metaheuristic for solving multiobjective nonlinear optimization problems. Similarly, Yujun Zheng et al.[26] proposed a taboo-based method for solving planning problems. All of these methods are for multiobjective case, but in general, they are built for single objective case at the beginning. For example, in 2014, Seyedali et al.[21] implemented an effective single objective optimization heuristic based on the grey wolf hunting principle, the Grey Wolf Optimizer(GWO) algorithm. Which is used by many works such as the resolution of problem with large data by Long et al.[13]; the solving of adjustable parametric problems by Long Wen[22] and the search of global optimum of positive function by combining GWO algorithm with genetic algorithm. W.

O. Sawadogo et al.[16] provided this work in 2019. The method called modified hybrid grey wolf optimizer and genetic algorithm (HmGWOGA) is the result. Many other works have been done, inspiring on the GWO after this works. In 2020, Shubham et al.[8] proposed Enhanced leadership-inspired grey wolf optimizer for global optimization problem. In 2021, Wei et al.[24] published their works on Path Planning of UAV Based on Improved Adaptive Grey Wolf Optimization Algorithm. In 2022, Zeynab et al.[10] suggested a New Enhanced Hybrid Grey Wolf Optimizer (GWO) Combined with Elephant Herding Optimization (EHO) Algorithm for Engineering Optimization; and Sofora et al.[1] presented a paper on a Condition-based Grey Wolf Optimizer Algorithm for Global Optimization Problems.

Many methods for resolving multiobjective optimization transform the objectives of the problem into a global objective through the use of a scalar aggregation approach or a non-scalar aggregation approach[12, 19, 23]. The first case occurs when weights are used to prioritize each objective that requires the decision maker's intervention, and the second case does not use weights. The ϵ-approach[2, 14] is one of this second class. It was developed by Haimes et al.[9] and has caught our attention because of its ability to give all the best Pareto optimal solutions.

In this work, we propose a new metaheuristic for solving multiobjective optimization problems that is a hybridization of the HmGWOGA method, the Lagrangian penalty function, and the ϵ-constraint approach. It is called Multiobjective Optimizer based on Green Wolves Attack Technic (HmGWOGA-MO). Our approach involves transforming an initial constrained multiobjective optimization problem into an unconstrained single objective optimization problem. Some theoretical convergence results are obtained through theorem proofs, and numerical performances are proved through the solving of five test problems taken from the literature [3]. Among these problems, we have the case where the Pareto front is convex, concave, or discontinuous. The results from these problems show the effectiveness of our method.

Our work will be presented as follows: Section 2 is devoted to the preliminary; Section 3 to the presentation of our results; and Section 4 will be dedicated to the conclusion.

2. Preliminaries

2.1. ε-constraint approach

The technique of the ϵ-constraint is to choose only one of the objective functions to optimize, and make the other one into constraints. Those will be added to the initial constraints. This transformation results in a single objective function problem. Let us consider the problem (MOP) and use $f_p,\ p \in \{1, 2, \cdots, K\}$ as a priority objective function.

The final formulation is presented below[5]:

$$\left[\begin{array}{l}\min f_p(x);\\ \text{subject to}\\ f_i(x)\leq \epsilon_i,\ \forall i=\overline{1,K},\ i\neq p;\\ x\in X;\end{array}\right. \tag{P_{ϵ_p}}$$

where $\epsilon_p = (\epsilon_1, \cdots, \epsilon_{p-1}, \epsilon_{p+1}, \cdots, \epsilon_k)^T \in \mathbb{R}^{k-1}$ where $p \in \{1, \cdots, K\}$. The set of eligible points of this problem is defined by $X_p^\epsilon = \left\{x \in X : f_i(x) \leq \epsilon_i, i = \overline{1,K},\ i \neq p\right\}$. Note that ϵ_p is chosen in such a way that $X_p^\epsilon \neq \emptyset$.

Theorem 1. *[5] x^* is an optimal solution for problem (P_{ϵ_p}) if and only x^* is a Pareto optimal solution of problem (MOP) with $\epsilon_i = f_i(x^*)$, $i = \overline{1,K}$, $i \neq p$.*

Proof. See in [5].■

2.2. Lagrangian penalty function

Many types of penalty functions are found in the literature[19, 23, 25], which aim to transform a constrained optimization problem into an unconstrained optimization problem. In this work, we use the Lagrangian penalty function to convert our problem (P_{ϵ_p}) into one with no constraints. This function is described as follows:

$$L(x) = f_p(x) + \eta\Big[\sum_{i\neq p}^{K}\big(f_i(x) - \epsilon_i + |f_i(x) - \epsilon_i|\big) + \sum_{j=1}^{q}\big(g_j(x) + |g_j(x)|\big)\Big];$$

where η is taken large enough. the following formulation of the initial problem:

$$\left\{\begin{array}{l}\min L(x);\\ x\in X_p^\epsilon.\end{array}\right. \tag{$P_{\epsilon_p}^L$}$$

Theorem 2. *Let $x^* \in X_p^\epsilon$ be an eligible point. If x^* is the global minimum of the problem $(P_{\epsilon_p}^L)$ then x^* also is the global minimum of the problem P_{ϵ_p}.*

Proof. Let us suppose that x^* is the global minimum of the problem $(P_{\epsilon_p}^L)$ then: $\forall x \in X_p^\epsilon,\ L(x^*) \leq L(x)$. That allows to have : $f_p(x^*) + \eta\sum\Big(f_i(x^*) - \epsilon_i + |f_i(x^*) - \epsilon_i|\Big) + \sum_{j=1}^{q}\big(g_j(x^*) + |g_j(x^*)|\big) \leq f_p(x) + \eta\sum\Big(f_i(x) - \epsilon_i + |f_i(x) - \epsilon_i|\Big) + \sum_{j=1}^{q}\big(g_j(x) + |g_j(x)|\big)$. By using the definition of the set X_p^ϵ, $\forall x \in X_p^\epsilon$ and $\forall i \in \{1, 2, \cdots, K\}, i \neq p$, $f_i(x) - \epsilon_i + |f_i(x) - \epsilon_i| = 0$, and $\forall j \in \{1, 2, \cdots, q\}, g_j(x) + |g_j(x)| = 0$. Then, we have that $f_p(x^*) \leq f_p(x)$, and since $x^* \in X_p^\epsilon$ this can be written as $f_i(x^*) \leq \epsilon_i$, hence x^* realizes the global minimum of the problem P_{ϵ_p}. ■

2.3. HmGWOGA method

2.3.1. Description

The HmGWOGA algorithm is a combination of the Grey Wolf Optimizer(GWO) algorithm and the NSGA-II. It is a good method for solving single-objective optimization problems with positive objectives, such as [16]. The intent of the designers of this method is to apply the selection, crossover, and mutation operators to determine a good family of solutions to which the algorithm GWO will be applied. The GWO method works with a population of initial solutions and is based on the hunting techniques of the grey wolves. The family of grey wolves is organized into four levels, of which the first level is positioned by the appointed leader (α) who is assisted by the wolf (β) at the second level. On the third level are wolves (δ) and on the fourth level are wolves (ω). In hunting, this hierarchy is respected, which makes wolves (α) the best hunting solution, followed by wolves (β) and so on. The wolves initiate the pursuit, encircle the prey, and torment it until they immobilize it. They will attack at that moment. Mathematically, encirclement is modeled as follows[16, 17, 22]:

$$\begin{cases} \overrightarrow{D}(i) = |\overrightarrow{C}.\overrightarrow{X}_p(i) - \overrightarrow{X}(i)| \\ \overrightarrow{X}(i+1) = \overrightarrow{X}_p(i) - \overrightarrow{A}.\overrightarrow{D}(i) \end{cases} \tag{1}$$

where i denote the number of iterations, $\overrightarrow{A} = 2a\vec{r}_1 - a$, $\overrightarrow{C} = 2\vec{r}_2$, a is a coefficient whose is decreased relative to iterations. It is defined by[16, 17, 22]:

$$a = 2\left(1 - \frac{i^d}{MaxInter^d}\right) \tag{2}$$

where i is the current iteration, d the space dimension, MaxInter is the maximal number of iterations. $\overrightarrow{X}_p$ is the vector given the position of the prey, $\overrightarrow{X}$ the vector given the position of green wolves, $\vec{r}_1$ and $\vec{r}_2$ are random vectors belong in $[0, 1]$.

When $|\overrightarrow{A}| < 1$, then the wolf (α) converges toward the prey to attack it as presented in the figure1.a and when $|\overrightarrow{A}| > 1$ the wolf (α) is looking for a prey as shown in the figure1.b.

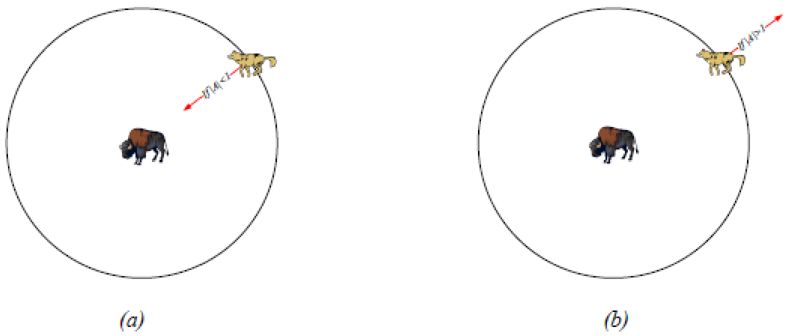

Figure 1: Search for or attack on prey[15].

The position of wolves α, β and δ are individually adjusted according to prey and those of wolves ω follows the principle of hierarchy. The mathematical modeling of positions of these three wolves is [13, 15–17]:

$$\begin{cases} \overrightarrow{D}_\alpha(i) = |\overrightarrow{C}_1.\overrightarrow{X}_\alpha(i) - \overrightarrow{X}(i)|, \\ \overrightarrow{D}_\beta(i) = |\overrightarrow{C}_2.\overrightarrow{X}_\beta(i) - \overrightarrow{X}(i)|, \\ \overrightarrow{D}_\delta(i) = |\overrightarrow{C}_3.\overrightarrow{X}_\delta(i) - \overrightarrow{X}(i)|. \end{cases} \tag{3}$$

where $\overrightarrow{C}_1, \overrightarrow{C}_2$ and $\overrightarrow{C}_3$ are random vectors. $X_\alpha, X_\beta, X_\delta$ are respectively the positions of α, β and δ. The new best position of wolves, which is the optimal solution, is:

$$X(i+1) = 0.7 \times X_1(i) + 0.2 \times X_2(i) + 0.1 \times X_3(i) \tag{4}$$

where:

$$\begin{cases} \overrightarrow{X}_1(i) = \overrightarrow{X}_\alpha(i) - \overrightarrow{A}_1.\overrightarrow{D}_\alpha(i) \\ \overrightarrow{X}_2(i) = \overrightarrow{X}_\beta(i) - \overrightarrow{A}_2.\overrightarrow{D}_\beta(i) \\ \overrightarrow{X}_3(i) = \overrightarrow{X}_\delta(i) - \overrightarrow{A}_3.\overrightarrow{D}_\delta(i) \end{cases} \tag{5}$$

2.3.2. Advantages

In practice the HmGWOGA method has given better results (it is faster and more convergent) compared to the initial method GWO on mono-objective optimization problems[16].

2.4. Performance study

In general, when evaluating a new method, many things are considered. It concerns the convergence of obtained solutions toward the true Pareto front and their distribution on the true Pareto front. It is therefore more interesting to use problems for which the true Pareto front is known. After the numerical resolution of some test problems, the generational distance γ and spread metric Δ will be calculated.

Generational distance γ . It allows us to measure the distance between a set of obtained solutions and the analytic Pareto front. It is denotedγ and defined by[2, 18]:

$$\gamma = \frac{1}{n}\left(\sum_{i=1}^{n} d_i^p\right)^{\frac{1}{p}}; \tag{6}$$

where we will take p =2; d_i is the distance between the solution i and the nearest solution that belongs to the analytical front; n is the number of approached solutions that we have got. When its value is close to zero, the method has good convergence.

Spread Δ . It provides the degree of distribution of obtained solution on true Pareto front[4]. A method is good if the value of this metric is close to zero.

$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{|Q|} |d_i - \overline{d}|}{\sum_{m=1}^{M} d_m^e + |Q|\overline{d}};$$

where the d_i is the Euclidean distance between the neighboring solutions with the mean value $\overline{d}$. The parameter d_m^e is the distance between the extreme solutions of the front obtained by the method and the true Pareto front [6].

Test Problems . We have selected five test problems from the literature to evaluate the performance of our new method. Among them, there are with the Pareto front being convex, concave, or discontinuous. All the test problems are bi-objective optimization problems from [3, 4]. Here is the list of test problems we have solved.

Code	formulation	Number of variables	Pareto front kind
$PL1$	$\begin{cases} \min f_1(x) = x^2 \\ \min f_2(x) = (x-2)^2 \\ x \in [-5, 5] \end{cases}$	1	Convex
$PL2$	$\begin{cases} \min f_1(x) = x_1 \\ \min f_2(x) = \dfrac{1+x_2}{x_1} \\ x = (x_1, x_2) \in [0.1, 1] \times [0.0, 5] \end{cases}$	2	Convex
$PL3$	$\begin{cases} \min f_1(x) = x_1 \\ \min f_2(x) = g(x) \times \left(1 - \sqrt{\dfrac{f_1(x)}{g(x)}}\right) \\ g(x) = 1 + \dfrac{9}{n-1} \times \sum_{i=2}^{n} x_i \\ x = (x_1, x_2, \cdots, x_{10}) \in [0,1]^{10} \end{cases}$	10	convex
$PL4$	$\begin{cases} \min f_1(x) = x_1 \\ \min f_2(x) = g(x) \times \left(1 - \left(\dfrac{f_1(x)}{g(x)}\right)^2\right) \\ g(x) = 1 + \dfrac{9}{n-1} \times \sum_{i=2}^{n} x_i \\ x = (x_1, x_2, \cdots, x_{10}) \in [0,1]^{10} \end{cases}$	10	concave
$PL5$	$\begin{cases} \min f_1(x) = x_1 \\ \min f_2(x) = g(x) \times h(x) \\ g(x) = 1 + \dfrac{9}{n-1} \times \sum_{i=2}^{n} x_i \\ h(x) = 1 - \sqrt{\dfrac{f_1(x)}{g(x)}} - \dfrac{f_1(x)}{g(x)} sin(10\pi f_1(x)) \\ x = (x_1, x_2, \cdots, x_{10}) \in [0,1]^{10} \end{cases}$	10	discontinuous

3. Main results

3.1. HmGWOGA-MO Method

3.1.1. Description

The principle of method HmGWOGA-MO consists in transforming a multiobjective problem subject to constraints into a single objective problem without constraints, which is then resolved by algorithm HmGWOGA. The transformation from a constrained to an unconstrained single objective optimization is achieved by successive use of the ϵ-constraint approach and the Lagrangian penalty function. We obtain the following formulation :

$$\begin{cases} \min L(x) \\ x \in X_p^{\epsilon} \end{cases} \tag{7}$$

where $L(x) = f_p(x) + \eta \sum \Big(f_i(x) - \epsilon_i + |f_i(x) - \epsilon_i| \Big), i \neq p$. These stages are presented in the Section 2.1 and Section 2.2.

The single objective without constraint problem is finally solved by the HmGWOGA algorithm. The HmGWOGA-MO algorithm has six main stages: aggregation, penalization, and resolution.

3.1.2. Algorithm

1. Applying the ϵ-constraint approach.
2. Applying the Lagrangian penalty function.
3. Initialize parameters N, d, lb, ub, MaxIter, pm, sigma.
4. Generate the initial N-size population.
5. While $k < MaxIter$
    a. Evaluate the fitness of each solution.
    b. Applying genetic operators.
    c. Applying elitism.
    d. for i from 1 to N
       Adapt positions of alpha, beta and delta wolves.
       End For.
    e. For i from 1 to N
       Adapt the positions of the solutions, using equations (3) and (5);
       Adjust the position of the optimum using equation (4);
       End For.

   end While
6. Back to the position and fitness of wolf α.

with N the size of the population, d the number of variables, lb and ub the lower and upper bounds of the optimum search interval, n the number of initial population of solutions, pm is the probability of mutation, and σ the standard deviation of the Gauss law used in the mutation.

3.2. Numerical Results

3.2.1. Pareto front representation

For each test problem, it is necessary to determine the value of ϵ_1. In practice, we have chosen f_2 as the priority objective function and computed $\epsilon_1 \in [\min f_1, \max f_1]$. In each figure, we have the obtained solutions with our method HmGWOGA-MO, those of NSGA-II, and the true Pareto front.

Table 1: Graphical results of test problems

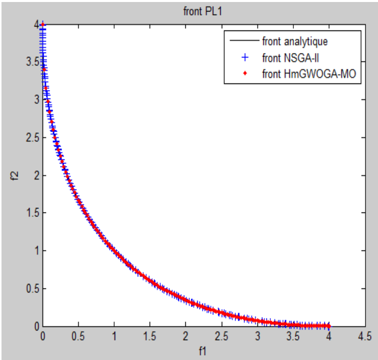

Figure 2: Pareto front of PL1 (convex)

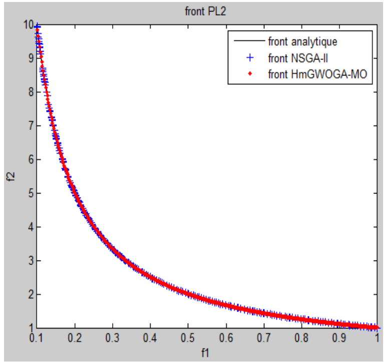

Figure 3: Pareto front of PL2 (convex)

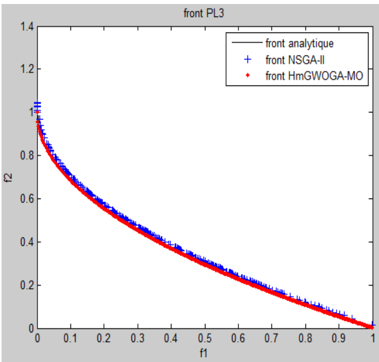

Figure 4: Pareto front of PL3 (convex)

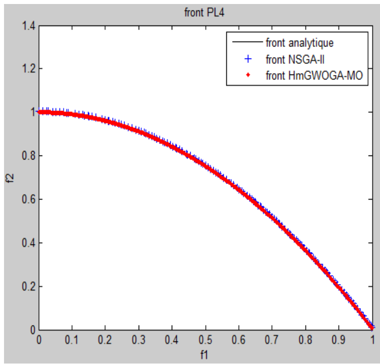

Figure 5: Pareto front of PL4 (concave)

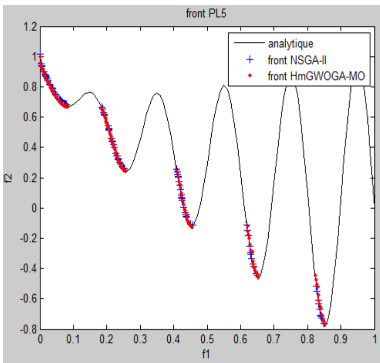

Figure 6: Pareto front of PL5 (discontinuous)

According to the figures in the Table 1, all of the solutions obtained by HmGWOGA-MO are on the the analytic Pareto front.

3.2.2. Performance index

The following table shows the value of convergence for HmGWOGA-MO and NSGA-II on the five test problems.

Table 2: Generational distance γ

		PL1	PL2	PL3	PL4	PL5
NSGA-II	$\overline{\gamma}$	1.1385e-3	8.865e-4	3.3482e-2	7.2391e-2	1.1450e-1
	σ_γ^2	4.4800e-7	3.8300e-8	4.7500e-3	3.1689e-2	7.9400e-3
HmGWOGA-MO	$\overline{\gamma}$	8.0946e-5	4.5495e-5	8.4609e-5	8.2169e-5	0.1313e-5
	σ_γ^2	5.4916e-14	6.0050e-12	6.1712e-13	6.5544e-13	5.1550e-11

The following table shows the values of the distribution of HmGWOGA-MO and NSGA-II on the five test problems.

Table 3: Spread Δ

		PL1	PL2	PL3	PL4	PL5
NSGA-II	$\overline{\Delta}$	0.4235	0.6788	0.3903	0.4307	0.7385
	σ_Δ^2	0.0011	0.0060	0.0018	0.0047	0.0197
HmGWOGA-MO	$\overline{\Delta}$	1.6758	0.9447	0.2823	0.3125	1.1032
	$\sigma_\Delta^2(e-5)$	9.4360	5.8090	0.6710	4.3750	7.5360

3.3. Discussions

Table 2 shows the mean and variance of the convergence metric. It is evident from the results that the values obtained with two methods on the five test problems are very close to zero. However, HmGWOGA-MO gives the best values on all five problem tests. That proves a good convergence of the HmGWOGA-MO method compared to the NSGA-II method on these test problems.

Table 3 presents the mean and variance of the diversity metric. The values obtained with two methods are close to zero. We found that HmGWOGA-MO is better than NSGA-II on problems PL3 and PL4, and that NSGA-II is better on the others. Therefore, none of the method dominates the other in terms of the distribution of obtained solutions.

We can conclude that HmGWOGA-MO is the best choice for solving multiobjective optimization problems when the convergence criterion is priority.

4. Conclusion

In this work, we introduced a new metaheuristic for solving multiobjective optimization problems. Named HmGWOGA-MO, it is a combination of an algorithm from Grey Wolves Optimizer and the ϵ-constraint approach. The optimality condition of the obtained solutions is proven through a theorem. In addition, the good convergence and distribution of obtained are proved by the computing of two parameters relative to these performance notions. Through the five test problems, we found that our method is better than NSGA-II in terms of convergence.

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