EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 16, No. 1, 2023, 18-28
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Resolving Domination in Graphs Under Some Binary Operations

Gerald B. Monsanto[1,*], Helen M. Rara[2]

[1] *College of Teacher Education, Arts and Sciences, Visayas State University Villaba, 6537 Villaba, Leyte, Philippines*
[2] *Department of Mathematics and Statistics, College of Science and Mathematics, Center of Graph Theory, Algebra, and Analysis-Premier Research Institute of Science and Mathematics, Mindanao State University-Iligan Institute of Technology, 9200 Iligan City, Philippines*

Abstract. In this paper, we investigate the concept of resolving dominating set in a graph. In particular, we characterize the resolving dominating sets in the join, corona and lexicographic product of two graphs and determine the resolving domination number of these graphs.

2020 Mathematics Subject Classifications: 05C69

Key Words and Phrases: Resolving dominating set, resolving domination number, join, corona, lexicographic product

1. Introduction

All graphs considered in this study are finite, simple, and undirected connected graphs, that is, without loops and multiple edges. For some basic concepts in Graph Theory, we refer readers to [5].

Let $G = \big(V(G), E(G)\big)$ be a connected graph. The *open neighborhood* of $v \in V(G)$ is $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. Any element u of $N_G(v)$ is called a *neighbor* of v. The *closed neighborhood* of $v \in V(G)$ is $N_G[v] = N_G(v) \cup \{v\}$. Thus, the degree of $v \in V(G)$ is given by $deg_G(v) = |N_G(v)|$. Customarily, for $S \subseteq V(G)$, $N_G(S) = \bigcup_{v\in S} N_G(v)$ and $N_G[S] = \bigcup_{v\in S} N_G[v]$.

A nonempty set $S \subseteq V(G)$ is a *dominating set* in graph G if $N_G[S] = V(G)$. Otherwise, we say S is a *non-dominating set* of G. The *domination number* of a graph G, denoted by $\gamma(G)$, is given by $\gamma(G) = \min|S| : S$ *is a dominating set of* G. If S is a dominating set of G and if $|S| = \gamma(G)$, then S is called a *minimum dominating set* or a γ-set of G.

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v16i1.4643
Email addresses: gerald.monsanto@vsu.edu.ph (G. Monsanto), helen.rara@g.msuiit.edu.ph (H. Rara)

The distance $d_G(u,v)$ in G of two vertices u,v is the length of a shortest *u-v path* in G.

A vertex x of a connected graph G is said to *resolve two vertices* u and v of G if $d_G(x,u) \neq d_G(x,v)$. For an ordered set $W = \{x_1, \ldots, x_k\} \subseteq V(G)$ and a vertex v in G, the *k-vector* $r_G(v/W) = \big(d_G(v,x_1), d_G(v,x_2), \ldots, d_G(v,x_k)\big)$ is called the *representation* of v with respect to W. The set W is a *resolving set* for G if and only if no two distinct bvertices of G have the same representation with respect to W. The metric dimension of G, denoted by, $dim(G)$, is the minimum cardinality over all resolving sets of G. A resolving set of cardinality $dim(G)$ is called a *basis.*

Slater [10] brought in the notion of locating sets and its minimum cardinality as locating number. The same concept was also introduced by Harary and Melter [5] but using the terms resolving sets and metric dimension to refer to locating sets and locating number, respectively. Some variations of locating sets and resolving sets are studied in [2, 6–9, 11, 12].

Let G be a connected graph. A set $S \subseteq V(G)$ is a *locating set* of G if for every two distinct vertices u and v of $V(G) \setminus S$, $N_G(u) \cap S \neq N_G(v) \cap S$. The *locating number* of G, denoted by $\ln(G)$, is the smallest cardinality of a locating set of G. A locating set of G of cardinality $\ln(G)$ is referred to as an ln-set of G.

Let G be a connected graph. A set $S \subseteq V(G)$ is a *strictly locating set* of G if it is a locating set of G and $N_G(u) \cap S \neq S$, for all $u \in V(G) \setminus S$. The *strictly locating number* of G, denoted by $sln(G)$, is the smallest cardinality of a strictly locating set of G. A strictly locating set of G of cardinality $sln(G)$ is referred to as *sln-set* of G.

A connected graph G of order $n \geq 3$ is *point distinguishing* if for any two distinct vertices u and v of G, $N_G(u) \neq N_G(v)$ [3]. It is *totally point determining* if for any two distinct vertices u and v of G, $N_G(u) \neq N_G(v)$ and $N_G[u] \neq N_G[v]$ [13].

Brigham et al. [1] defined a *resolving dominating set* as a set S of vertices of a connected graph G that is both resolving and dominating. The cardinality of a minimum resolving dominating set is called the *resolving domination number* of G and is denoted by $\gamma_R(G)$. A resolving dominating set of cardinality $\gamma_R(G)$ is called a γ_R-set of G.

Canoy and Malacas [11] defined a *locating-dominating* (resp. *strictly locating-dominating*) as a locating (resp. strictly locating) subset S of $V(G)$ which is also dominating set in a connected graph G. The minimum cardinality of a locating-dominating (resp. strictly locating-dominating) set in G, denoted by $\gamma_L(G)$ (resp. (γ_{SL})), is called the *L-domination* (resp. *SL-domination*) number of G. Any L-dominating (resp. SL-dominating) set of cardinality $\gamma_L(G)$ (resp. $\gamma_{SL}(G)$) is then referred to as a *γ_L-set* (*γ_{SL}-set*) of G.

Let G be a connected graph. A set $S \subseteq V(G)$ is *strictly resolving dominating set* of G if it is a resolving dominating set of G and $N_G \cap S \neq S$ for $u \in V(G) \setminus S$. The *strictly resolving dominating number* of G, denoted by $\gamma_{SR}(G)$, is the smallest cardinality of a strictly resolving dominating set of G. A strictly resolving dominating set of G of cardinality $\gamma_{SR}(G)$ is referred to as *γ_{SR}-set* of G.

This study aims to define and characterize the resolving dominating sets in the join, corona and lexicographic product of graphs and determine their corresponding resolving domination number.

2. Preliminary Results

Remark 1. *Every locating-dominating set of a connected graph G is a resolving dominating set and every resolving dominating set is a dominating set of G. Hence,*

$$\gamma(G) \leq \gamma_R(G) \leq \gamma_L(G).$$

Example 1. Consider the graph G in Figure 1 and let $S = \{v_1, v_5\}$. Observe that $N_G[S] = V(G)$, that is, S is a dominating set in G. Moreover, S is a resolving set since the representation of each vertex in G, with respect to S is unique: $r_G(v_1/S) = (0,1)$, $r_G(v_2/S) = (1,2)$, $r_G(v_3/S) = (2,1)$, $r_G(v_4/S) = (1,1)$ and $r_G(v_5/S) = (1,0)$. Hence, $\gamma_R(G) \leq |S| = 2$. Since any singleton is not a resolving set, $\gamma_R(G) = 2$. Also, S is a locating-dominating set of a graph G since $N_G(v_2) \cap S = \{v_1\}, N_G(v_3) \cap S = \{v_5\}$, and $N_G(v_4) \cap S = \{v_1, v_5\}$. Thus, $\gamma_L(G) = 2$.

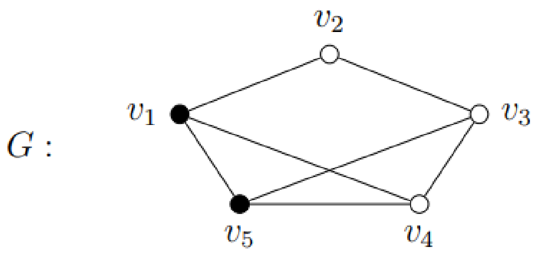

Figure 1: A graph G with $\gamma(G) = \gamma_R(G) = \gamma_L(G) = 2$

Example 2. Consider the graph G in Figure 2. Let $S_1 = \{x, y, z\}$. Observe that $N_G[S_1] = V(G)$, that is, S_1 is a dominating set in G. Hence, $\gamma(G) = 3$. Moreover, let $S_2 = \{w_1, w_2, w_3, w_4\}$. Then $r_G(x/S_2) = (1,1,1,2)$, $r_G(y/S_2) = (1,1,1,4)$, $r_G(z/S_2) = (2,2,2,1)$, $r_G(w_1/S_2) = (0,2,2,3)$, $r_G(w_2/S_2) = (2,0,2,3)$, $r_G(w_3/S_2) = (2,2,0,3)$, and $r_G(w_4/S_2) = (3,2,2,0)$. Thus, S_2 is a resolving dominating set of a graph G. Hence, $\gamma_R(G) = 4$. Furthermore, let $S_3 = \{x, w_1, w_2, w_3, w_4\}$. Then S_3 is a locating-dominating set of a graph G since $N_G(y) \cap S_3 = \{w_1, w_2, w_3\}$ and $N_G(z) \cap S_3 = \{w_4\}$. Thus, $\gamma_L(G) = 5$.

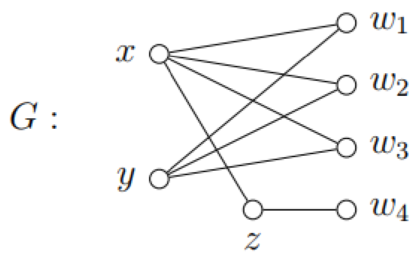

Figure 2: A graph G with $\gamma(G) = 3, \gamma_R(G) = 4,$ and $\gamma_L(G) = 5$

Remark 2. *Every resolving dominating set of a connected graph G is a resolving set of G. Thus, $dim(G) \leq \gamma_R(G)$.*

Example 3. Consider the graph G in Figure 3. The set $S = \{v_1, v_2\}$ is a resolving set. Since no single vertex constitutes a resolving set for G, it follows that W is a minimum resolving set. Hence, $dim(G) = 2$. Moreover, S is a resolving dominating set of a graph G since $N_G[S] = V(G)$. Thus, $\gamma_R(G) = 2$.

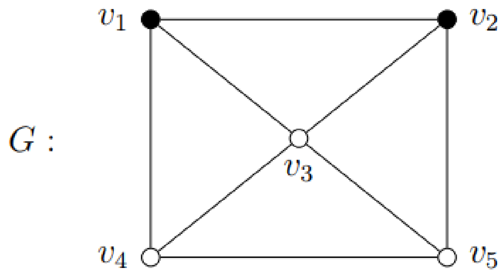

Figure 3: A graph G with $dim(G) = 2 = \gamma_R(G)$

Example 4. Consider the graph G in Figure 4. Let $S_1 = \{v, y\}$ is a resolving set of a graph G since the representation of each vertex in G, with respect to S_1 is unique: $r_G(x/S_1) = (2,2)$, $r_G(u_1/S_1) = (2,1)$, $r_G(u_2/S_1) = (1,2)$, $r_G(u_3/S_1) = (1,3)$, $r_G(z/S_1) = (2,3)$, $r_G(v/S_1) = (0,3)$, and $r_G(y/S_1) = (3,0)$. Hence, $dim(G) = 2$. Moreover, let $S_2 = \{u_1, u_2, u_3\}$. Then S_2 is a resolving dominating set of a graph G. Thus, $\gamma_R(G) = 3$.

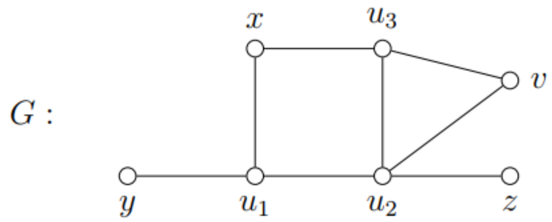

Figure 4: A graph G with $dim(G) = 2$ and $\gamma_R(G) = 3$

Proposition 1. [1] *Let G be a connected graph of order $n \geq 2$, then*

(*i*) $\gamma_R(P_3) = 2$ *and for* $n \geq 4$, $\gamma_R(P_n) = \lceil \frac{n}{3} \rceil$

(*ii*) *For* $n \geq 3$, $\gamma_R(C_n) = \lceil \frac{n}{3} \rceil$ *if* $n \neq 6$ *and* $\gamma_R(C_6) = 3$.

Proposition 2. *For any connected graph G of order $n \geq 2$, $1 \leq \gamma_R(G) \leq n-1$. Moreover,*

(*i*) $\gamma_R(G) = 1$ *if and only if* $G = P_2$ *and*

(*ii*) $\gamma_R(G) = n - 1$ *if and only if* $G = K_n$ *or* $K_{1,n-1}$.

Proof: Suppose that $\gamma_R(G) = 1$, say $W = \{v\}$ is a minimum resolving dominating set of G. Since G is connected and non-trivial, there exists $x \in V(G) \setminus \{v\}$ such that $xv \in E(G)$. If $|V(G)| = 2$, then $G = K_2 = P_2$. Therefore, (i) holds.

Suppose $G \neq K_n$. Let $V(G) \setminus \{v\} = W$ is γ_R-set. Let $w \in N_G(v)$. Consider that $|N_G(v)| = 1$.

Claim: $G = \langle w \rangle + \overline{K}_{n-1} \cong K_{1,n-1}$.

Suppose there exists $u \in V(G) \setminus \{v, w\}$ such that $uv \notin E(G)$. Let $W = V(G) \setminus \{u, v\}$. Clearly, W is a dominating set of G. Since $v \in N_G(w)$ and $u \in N_G(w)$, $r(u/W) \neq r(v/W)$. Hence, W is a resolving dominating set of G and $\gamma_R(G) \leq n-2$, a contradiction. Thus, $w \in N_G(z)$ for all $z \in V(G) \setminus \{w\}$. Next, suppose there exist distinct vertices $a, b \in V(G) \setminus \{w, v\}$ such that $ab \in E(G)$. Let $W_1 = V(G) \setminus \{a, v\}$. Then W_1 is a dominating set of G. Since $w \in N_G(v)$ and $b \notin N_G(v)$, $r(w/W_1) \neq r(b/W_1)$. Hence, W_1 is a resolving dominating set of G. Thus, $\gamma_R(G) \leq |W_1| = n-2$, a contradiction. Therefore, $ab \notin E(G)$. Accordingly, $G = \langle w \rangle + \overline{K}_{n-1} \cong \overline{K}_{1,n-1}$.

For the converse, suppose $G = K_n$ or $G = K_{1,n-1}$. Then $\gamma_R(G) = n-1$. Hence, (ii) holds. □

3. Resolving Domination in the Join of Graphs

The *join* of two graphs G and H is the graph $G + H$ with vertex set $V(G+H) = V(G) \overset{\bullet}{\cup} V(H)$ and edge set $E(G+H) = E(G) \overset{\bullet}{\cup} E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.

Theorem 1. [9] *Let G and H be non-trivial connected graphs. A set $W \subseteq V(G+H)$ is a resolving set of $G+H$ if and only if $W = W_G \cup W_H$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.*

Theorem 2. [4] *Let G and H be connected graphs. Then $C \subseteq V(G+H)$ is a dominating set in $G+H$ if and only if at least one of the following is true:*

(*i*) *$C \cap V(G)$ is a dominating set in G.*

(*ii*) *$C \cap V(H)$ is a dominating set in H.*

(*iii*) *$C \cap V(G) \neq \varnothing$ and $C \cap V(H) \neq \varnothing$.*

Theorem 3. *Let G and H be non-trivial connected graphs. A set $W \subseteq V(G+H)$ is a resolving dominating set of $G+H$ if and only if W is a locating-dominating set of $G+H$.*

Proof: Suppose that W is a resolving dominating set of $G+H$. Then W is a resolving set of $G+H$. By Theorem 1, $W = W_G \cup W_H$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set. Since W is a dominating set of $G+H$, W_G and W_H are also dominating sets of G and H, respectively. By Theorem 1, W is a locating-dominating set of $G+H$.

The converse follows immediately from Theorem 1 and Theorem 2(*iii*). □

Theorem 4. *Let G and H be non-trivial connected graphs. A set $W \subseteq V(G+H)$ is a resolving dominating set of $G+H$ if and only if $W = W_G \cup W_H$ where $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set.*

Proof: Suppose that W is a resolving dominating set of $G+H$. Then W is a resolving set of $G+H$. By Theorem 1, $W_G = W \cup V(G)$ where $W_G \subseteq V(G)$ and $W_H \subseteq V(H)$ are locating sets of G and H, respectively, where W_G or W_H is a strictly locating set. Since W is a dominating set of $G+H$, W_G and W_H are also dominating sets of G and H, respectively. By Theorem 3, W is a locating-dominating set of $G+H$.

Conversely, let $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$ be locating sets of G and H, respectively, and W_G or W_H is a strictly locating set of $G+H$. By Theorem 2, W is a dominating set of $G+H$. Let $u, v \in V(G+H) \setminus W$ with $u \neq v$. Consider the following cases:

Case 1. $u, v \in V(G)$

Since W_G is a locating set of G, $N_G(u) \cap W_G \neq N_G(v) \cap W_G$. Hence, $r_{G+H}(u/W) \neq r_{G+H}(v/W)$.

Case 2. $u, v \in V(H)$

The proof is similar to case 1.

Case 3. $u \in V(G)$ and $v \in V(H)$

$$r_{G+H}(u/W) = \big(d_{G+H}(u, w_1), \ldots, d_{G+H}(u, w_n), 1, 1, \ldots, 1\big) \text{ and}$$

$$r_{G+H}(v/W) = \big(1, 1, \ldots, 1, d_{G+H}(v, u_1), \ldots, d_{G+H}(v, u_n)\big)$$

Suppose there exists $j \in \{1, 2, \ldots, n\}$ such that $d_{G+H}(u, w_j) \neq 1$ or there exists $k \in \{1, 2, \ldots, m\}$ such that $d_{G+H}(v, u_k) \neq 1$. Hence, $r_{G+H}(u/W) \neq r_{G+H}(v/W)$.

Therefore, W is a resolving set of $G+H$. Accordingly, W is a resolving dominating set of $G+H$. □

Corollary 1. *Let G and H be non-trivial connected graphs. Then*

$$\gamma_R(G+H) = min\left\{sln(H) + ln(G), sln(G) + ln(H)\right\}.$$

Proof: Let W be a minimum resolving dominating set in $G+H$. Let $W_G = V(G) \cap W$ and $W_H = V(H) \cap W$. By Theorem 4, W_G and W_H are locating sets in G and H, respectively, where W_G or W_H is a strictly locating set. If W_G is strictly locating, then

$$\begin{aligned} sln(G) + ln(H) &\leq |W_G| + |W_H| \\ &= |W| \\ &= \gamma_R(G+H). \end{aligned}$$

If W_H is strictly locating, then

$$\begin{aligned} sln(G) + ln(H) &\leq |W_H| + |W_G| \\ &= |W| \\ &= \gamma_R(G+H). \end{aligned}$$

Thus, $\gamma_R(G+H) = min\left\{sln(H) + ln(G), sln(G) + ln(H)\right\}$. Next, suppose that $sln(G) + ln(H) \leq sln(H) + ln(G)$.

Let W_G be a minimum strictly locating set of G and W_H be a minimum locating set of H. Then $W = W_G \cup W_H$ is a resolving dominating set of $G + H$ by Theorem 4. Hence, $\gamma_R(G + H) \leq |W| = |W_G| + |W_H| = sln(G) + ln(H)$. Therefore,

$$\gamma_R(G + H) = min\{sln(H) + ln(G), sln(G) + ln(H)\}. \quad \square$$

Theorem 5. [11] Let G be a connected graph of order $n \geq 2$.

(*i*) If $ln(G) < sln(G)$, then $1 + ln(G) = sln(G)$.

(*ii*) If $ln(G) < \gamma_L(G)$, then $1 + ln(G) = \gamma_L(G)$.

(*iii*) If $sln(G) < \gamma_{SL}(G)$, then $1 + sln(G) = \gamma_{SL}(G)$.

Corollary 2. *Let G be a non-trivial connected graph and let K_n be a complete graph of order $n \geq 2$. Then $\gamma_R(G + K_n) = sln(G) + n - 1$.*

Proof: Note that $\gamma_R(K_n) = n - 1$ and $sln(K_n) = n$. From Corollary 1, $\gamma_R(G + K_n) = min\{sln(H) + ln(G), sln(G) + ln(H)\}$. By Theorem 5, $sln(G) - 1 \leq ln(G)$. Therefore,

$$\gamma_R(G + K_n) = min\{sln(G) + n - 1, ln(G) + n\} = sln(G) + n - 1. \quad \square$$

Theorem 6. [11] *Let H be a non-trivial connected graph and let $K_1 = \langle v \rangle$. Then $W \subseteq V(H)$ is a locating-dominating set of $H + K_1$ if and only if either $v \notin W$ and W is a strictly locating dominating set of H or $W = \{v\} \cup W_H$, where W_H is a locating set of H.*

Theorem 7. [9] *Let H be a non-trivial connected graph and let $K_1 = \langle v \rangle$. Then $W \subseteq V(H)$ is a resolving set of $H + K_1$ if and only if either $v \notin W$ and W is a strictly locating set of H or $W = \{v\} \cup W_H$, where W_H is a locating set of H.*

The next result follows immediately from Theorem 6 and Theorem 7.

Theorem 8. *Let H be a non-trivial connected graph and let $K_1 = \langle v \rangle$. Then $W \subseteq V(H)$ is a resolving dominating set of $H + K_1$ if and only if either $v \notin W$ and W is a strictly resolving dominating set of H or $W = \{v\} \cup W_H$, where W_H is a locating set of H.*

Corollary 3. *Let H be a non-trivial connected graph. Then*

$$\gamma_R(H + K_1) = min\{\gamma_{SR}(H), ln(H) + 1\}.$$

4. Resolving Domination in the Corona of Graphs

The *corona* of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining every vertex of the ith copy of H to the ith vertex of G. For $v \in V(G)$, denoted by H^v the copy of H whose vertices are attached one by one to the vertex v. Subsequently, denote by $v + H^v$ the subgraph of the corona $G \circ H$ corresponding to the join $\langle \{v\} \rangle + H^v, v \in V(G)$.

Theorem 9. [9] *Let G and H be non-trivial connected graphs. Then $W \subseteq V(G \circ H)$ is a resolving set of $G \circ H$ if and only if $W \cap V(H^v) \neq \varnothing$ for all $v \in V(G)$ and $W = A \cup B$, where $A \subseteq V(G)$ and*

$$B = \cup\{B_v : v \in V(G) \text{ and } B_v \text{ is a locating set of } H^v\}.$$

Theorem 10. *Let G and H be non-trivial connected graphs. Then $W \subseteq V(G \circ H)$ is a resolving dominating set of $G \circ H$ if and only if $W \cap V(H^v) \neq \varnothing$ for every $v \in V(G)$ and $W = A \cup B \cup D$, where $A \subseteq V(G)$,*

$$B = \cup\{B_v : v \in A \text{ and } B_v \text{ is a locating set of } H^v\} \text{ and}$$

$$D = \cup\{D_u : u \notin A \text{ and } D_u \text{ is a locating-dominating set of } H^v\}.$$

Proof: Let W be a resolving dominating set of $G \circ H$. Then by Theorem 9, $W \cap V(H^v) \neq \varnothing$ for any $v \in V(G)$. Since W is a resolving set, $G = A \cup B^*$, where $A \subseteq V(G)$ and $B^* = \cup\{B_v : v \in V(G)$ and B_v is a locating set of $H^v\}$ by Theorem 9. Let $B = \cup\{B_v : v \in A\}$ and $D = \cup\{B_u : u \in V(G) \setminus A\}$. Since W is a dominating set, it follows that B_u is a dominating set for each $u \in V(G) \setminus A$.

For the converse, suppose $W = A \cup B \cup D$, where A, B and D are the sets possesing the properties described. Then by Theorem 9, W is a resolving set of $G \circ H$. Since D_u is a dominating set of H^u for each $u \notin W$, W is a resolving dominating set of $G \circ H$. □

Remark 3. [11] *For any connected graph G, $ln(G) \leq \gamma_L(G) \leq \gamma_{SL}(G)$.*

Corollary 4. *Let G and H be non-trivial connected graphs with $|V(G)| = n$. Then $\gamma_R(G \circ H) = n \cdot \gamma_L(H)$.*

Proof: Let W be a minimum resolving dominating set in $G \circ H$. Then $W = A \cup B \cup D$ where A, B and D are the sets described in Theorem 10. By Remark 3 and Theorem 5(ii), it follows that

$$\begin{aligned} \gamma_R(G \circ H) = |W| &= |A| + |B| + |D| \\ &\geq |A| + |A| \cdot ln(H) + \big(n - |A|\big) \cdot \gamma_L(H) \\ &= |A|\big(1 + ln(H)\big) + \big(n - |A|\big) \cdot \gamma_L(H) \\ &= |A| \cdot \gamma_L(H) + \big(n - |A|\big) \cdot \gamma_L(H) \\ &= n \cdot \gamma_L(H). \end{aligned}$$

Now, let F be a minimum locating dominating set of H. For each $v \in V(G)$, pick $F_v \subseteq V(H^v)$ with $\langle F_v \rangle \cong \langle F \rangle$. Then $W = \bigcup_{v \in V(G)} F_v$ is a resolving dominating set of $G \circ H$ by Theorem 10. Hence,

$$\gamma_R(G \circ H) \leq |W| = n \cdot \gamma_L(H).$$

Therefore, $\gamma_R(G \circ H) = n \cdot \gamma_L(H)$. □

5. Resolving Domination in the Lexicographic Product of Graphs

The *lexicographic product* of graphs G and H, denoted by $G[H]$, is the graph with vertex set $V(G[H]) = V(G) \times V(H)$ such that $(v,a)(u,b) \in E(G[H])$ if and only if either $uv \in E(G)$ or $u = v$ and $ab \in E(H)$.

Theorem 11. [9] *Let G and H be non-trivial connected graphs with $\Delta(H) \leq |V(H)| - 2$. Then $W = \bigcup_{x\in S} \left[\{x\} \times T_x\right]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a resolving set of $G[H]$ if and only if W is a locating set of $G[H]$.*

Theorem 12. [4] *Let G and H be non-trivial be connected graphs. Then $C \subseteq V(G[H])$ is a dominating set in $G[H]$ if and only if $W = \bigcup_{x\in S} \left[\{x\} \times T_x\right]$ and either*

(i) S is a total dominating set in G or

(ii) S is a dominating set in G and T_x is a dominating set in H for every $x \in S \setminus N_G(s)$.

Theorem 13. *Let G and H be non-trivial connected graphs with $\Delta(H) \leq |V(H)| - 2$. Then $W = \bigcup_{x\in S} \left[\{x\} \times T_x\right]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a resolving dominating set of $G[H]$ if and only if W is a locating-dominating set of $G[H]$.*

Proof: Suppose W is a resolving dominating set of $G[H]$. Then by Theorem 11, W is a locating-dominating set of $G[H]$.

The converse follows from Theorem 11 and Theorem 12. □

Theorem 14. [11] *Let G and H be non-trivial connected graphs with $\Delta(H) \leq |V(H)| - 2$. Then $W = \bigcup_{x\in S} \left[\{x\} \times T_x\right]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a locating-dominating set of $G[H]$ if and only if*

(i) $S = V(G)$;

(ii) T_x is a locating set of H for every $x \in V(G)$;

(iii) T_x or T_y is strictly locating of H whenever x and y are adjacent vertices of G with $N_G[x] = N_G[y]$; and

(iv) T_x or T_y is (locating) dominating of H whenever x and y are nonadjacent vertices of G with $N_G(x) = N_G(y)$.

The next result follows immediately from Theorem 13 and Theorem 14.

Theorem 15. *Let G and H be non-trivial connected graphs with $\Delta(H) \leq |V(H)| - 2$. Then $W = \bigcup_{x\in S} \left[\{x\} \times T_x\right]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a resolving dominating set of $G[H]$ if and only if*

(i) $S = V(G)$;

(ii) T_x *is a locating set of* H *for every* $x \in V(G)$*;*

(iii) T_x *or* T_y *is strictly locating of* H *whenever* x *and* y *are adjacent vertices of* G *with* $N_G[x] = N_G[y]$*; and*

(iv) T_x *or* T_y *is (locating) dominating of* H *whenever* x *and* y *are nonadjacent vertices of* G *with* $N_G(x) = N_G(y)$*.*

The following is a direct consquence of Theorem 15.

Corollary 5. *Let* G *be a connected totally point determining graph and let* H *be a non-trivial connected graph. Then* $W = \bigcup\limits_{x\in S}\big[\{x\} \times T_x\big]$ *is a minimum resolving dominating set of* $G[H]$ *if and only if* $S = V(G)$ *and* T_x *is a minimum locating set of* H *for every* $x \in V(G)$*.*

Corollary 6. *Let* G *be a connected totally point determining graph and let* H *be a non-trivial connected graph. Then* $\gamma_R\big(G[H]\big) = |V(G)| \cdot ln(H)$*.*

Proof: Let $W = \bigcup_{x\in S}(\{x\}\ \text{x}\ T_x)$ be a minimum resolving dominating set of $G[H]$. Then $S = V(G)$ and T_x is a minimum locating set in H for every $x \in V(G)$, by Corollary 5. Therefore, $\gamma_R(G[H]) = |V(G)| \cdot ln(H)$. □

6. Conclusion

This study did introduce the concept of resolving domination under some binary operations. Let G and H be non-trivial connected graphs. It is shown that the resolving domination number in the corona of two graphs is $n \cdot \gamma_L(H)$, the resolving domination number in the join of two graphs is the $min\,\{sln(H) + ln(G), sln(G) + ln(H)\}$, and the resolving domination number of the lexicographic product of graphs G and H is $|V(G)| \cdot ln(H)$. The parameter can be investigated further for graphs under other binary operations.

Acknowledgements

This research is funded by the Commission on Higher Education (CHED) and Mindanao State University-Iligan Institute of Technology, Philippines. Also, the authors would like to recognize the efforts of the anonymous reviewers whose suggestions and recommendations contributed to the big improvement of the paper.

References

[1] R.C. Brigham, G. Chartrand, R.D. Dutton, and P. Zhang. Resolving domination in graphs. *Mathematica Bohemica*, 1:25–36, 2003.

[2] J.M. Cabaro and H.M. Rara. On 2-resolving sets in the join and corona of graphs. *European Journal of Pure and Applied Mathematics*, 14(3):773–782, 2021.

[3] D. Geoffrey. Nuclei for totally point determining graphs. *Discrete Mathematics*, 21:145–162, 1978.

[4] C. Go and Jr. S.R. Canoy. Domination in the corona and join of graphs. *International Mathematical Forum*, 6(16):763–771, 2011.

[5] F. Harary. *Graph Theory.* Addison-Wesley Publishing Company, USA, 1969.

[6] G. Monsanto and H. Rara. On strong resolving domination in the join and corona of graphs. *European Journal of Pure and Applied Mathematics*, 13(1):170–179, 2020.

[7] G. Monsanto and H. Rara. Resolving restrained domination in graphs. *European Journal of Pure and Applied Mathematics*, 14(3):829–841, 2021.

[8] G. Monsanto and H. Rara. Movable resolving domination in graphs. *Discrete Mathematics, Algorithms and Applications*, 14(6):2250016, 2022.

[9] G. Monsanto and H. Rara. Resolvings sets in graphs. *Italian Journal of Pure and Applied Mathematics*, 47:862–871, 2022.

[10] P. Slater. Dominating and Reference Sets in a Graph. *Journal of Mathematics and Physical Sciences*, 22(4):445–455, 1988.

[11] Jr. S.R. Canoy and G.A. Malacas. Locating-dominating sets in a graph. *Applied Mathematical Sciences*, 8(88):4381–4388, 2014.

[12] Jr. S.R. Canoy and S.A. Omega. Locating sets in a graph. *Applied Mathematical Sciences*, 9(60):2957–2964, 2015.

[13] D.P. Summer. Point determination in graphs. *Discrete Mathematics*, 5:1979–1987, 1973.