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Selection of Quality Gemstones Based on Fuzzy Analytical Hierarchy Process

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Abstract. The suggestion in this paper tackles the uncertainty of choosing the best analysis and selection of high-quality gemstones based on multiple specific criteria of quantitative and qualitative nature. In this paper, we use the fuzzy analytic hierarchy process (F-AHP) in order to make the right decision to know the quality of the gemstone. This method is based on an effective algorithm through comparisons between the characteristics of the stones and their weight. F-AHP applies in this work to select one criterion from five criteria (specific gravity, color, clarity, cleavage, and Hardness). From the outcome, we note the hat standard of gem size is not always the best standard as seen by some researchers, and we found that small-sizzled stones are sometimes better effective and more flexible than others. Through this study, we note that the method helps reduce bias in decision-making, and method the comparisons are converted into numerical values that are processed and compared within with hierarchy that is related to the fuzzy logic with the property of uncertainty. All computations are applied by MATLAB language.

2020 Mathematics Subject Classifications: 90B50, 62A86

Key Words and Phrases: Multi-criteria, fuzzy analytical hierarchy process, decision-making, Gemstones.

1. Introduction

It is difficult for decision-makers to know the best decision-making using fuzzy numbers, and the reason for this is due to the presence of uncertainty in the formation, so many decision-makers resort to using fuzzy terms or linguistic terms to express the best in decision-making which is always used in addressing the information [24]. The fuzzy triple number has been applied in many fields such as forecasting [8, 14] representing and evaluating risks, and representing space [7, 23]. Many researchers use the similarity between triangles for fuzzy numbers [10–12, 19, 20] and other studies on midpoints to

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compare the effect of the methods used or the proposed methods. Some studies deal with decision-making problems in choosing high-quality gemstones according to pre-defined criteria. Numerous studies conducted using the hierarchical process method and liking it with fuzzy logic [4, 14, 15]. Several related studies were recently conducted by researchers including applications in numerical analysis, fuzzy cones, oscillation theory, thermal science and image processing can be found here [1, 3, 5, 6, 9, 13, 17, 21, 25]. Scientist L. A. Zadeh was considered the first to propose the idea of a fuzzy number as a convex group and an ordinary fuzzy group [2, 8, 22]. As part of fuzzy set theory, fuzzy numbers are introduced which take the form of a set of real numbers with a correlated organic function. These numbers can be used to accurately represent linguistic scales that include ambiguities and uncertainties in the human mind. Fuzzy AHP is simply an extension of the original AHP with human preferences recorded as fuzzy numbers, so the resulting comparison matrix also consists of fuzzy numbers.

2. Materials and Methods

When using the problems solving method, the data is collected and then arranged in hierarchical form is considered an initial method for making the right decision. Therefore, the method of the Analytical Hierarchy Process (AHP) developed to support for decision is called the best model to get the best comparison of the associated criteria [8, 18]. There are several tools and criteria for decision-making to achieve maximum goals such as linear programming of goals and binary ordering [24]. The multi-criteria hierarchical decision analysis method is considered one of the most important methods used in decision analysis in the case of certainty for decision problems that include multiple criteria and qualitative factors that cannot be included in additional discussion analysis Data validity is the main entrance to the AHP method. However, there is a property of uncertainty. For good decision making sometimes a single criterion is not an effective metric. For this, methods with multiple criteria are taken to make an irreversible decision. Further studies have been conducted on the topics of stock classification and consideration of multiple criteria. Several studies have been conducted to develop the Analytical Hierarchy Process (AHP) through a common criteria matrix with another standard. Although this approach is interesting, it does come with some limitations.

In decision support, the direct impact on the accuracy of the data and all the results we obtained is shown the evaluation where, in this research, the method of the analytical hierarchy process has been combined with the fuzzy Logic theory to become the method as the fuzzy analytical hierarchy process, where the fuzzy analytical process is used similarly to AHP method, whereas the fuzzy AHP or F-AHP method [4, 16].F-AHP determines to reach the best decision-making method [24, 26].

Step (1): Define the problem statement.

According to the criteria that are used in this research, in order to choose the model for gemstones through (heavy, hardness, cut, quality, and clarity). The weight of the stone has a Qirat and the unit is called Mohs because of the name of the first person to do clarity the level is divided into vvs1, vvs2, vs1, vs2, i1, i2, etc. the color level can be given by B, A, AA, AAA, . . . , etc.

Step (2): find a comparison, matrix.

Through the date of criteria stored in Table 1, we must create a comparison matrix to get the consistency framework in addition to that information of comparison to analyze the priority date from the gemstone certificate. To define pairwise comparisons, we use the following equations:

$$\alpha_{ij} = \frac{\lambda_i}{\lambda_j}, i, j = 1, 2, \ldots, n, \tag{1}$$

where α_{ij} is the ratio of weights from i, j, λ_i are the weights to 1 , and in representing the number of criteria Table (1) contains the comparison of the criteria's of gemstone.

Table 1: Gemstones Criterions Data

Rubi	Gravity Of specific	Color	Clarity	Cleavage	Hardness
1	2.74ct	A	I1	Moderate	9 months
2	1.98ct	AA	S11	Good	9 months
3	4.65ct	A	VS	Excellent	9 months
4	5.12ct	AAA	VVS	Excellent	9 months
5	2.88ct	B	S12	moderate	9 months

Where gemstone cleavage is defined as any break of a crystal along specific flat surfaces and and describe the degrees of divisive of gemstone to the directions of the crystals: poor (or weak), fair (moderate), good and excellent.

It is difficult to divide stones that do not have cleavage, while they are divided into stones with perfect cleavage.

Table 2: Comparison matrix of criteria

Criteria	Gravity of specific	Color	Clarity	Cleavage	Hardness
Gravity of specific	1	7	2	3	4
Color	$\frac{1}{7}$	1	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$
Clarity	$\frac{1}{2}$	5	1	2	3
Cleavage	$\frac{1}{3}$	3	$\frac{1}{2}$	1	$\frac{1}{2}$
Hardness	$\frac{1}{4}$	4	$\frac{1}{3}$	2	1

The form of the matrithe x by dividing every value of column j and row i with the greater value of column j.

$$\alpha_{ij} = \frac{\alpha_{ij}}{\max \alpha_{ij}}, \quad \forall i, j, \tag{2}$$

We get the matrix normalization from Table 2 obtained:

$$\begin{bmatrix} 0.4492 & 0.3495 & 0.4959 & 0.3601 & 0.4571 \\ 0.0642 & 0.0499 & 0.0496 & 0.0396 & 0.0286 \\ 0.2246 & 0.2496 & 0.2479 & 0.2401 & 0.3429 \\ 0.1497 & 0.1498 & 0.124 & 0.12 & 0.0571 \\ 0.1123 & 0.1997 & 0.0826 & 0.2401 & 0.1143 \end{bmatrix}$$

We get the priority

$$\text{Priority } = \begin{pmatrix} 0.4297 & 0.0460 & 0.26 & 0.1171 & 0.1422 \end{pmatrix} \tag{3}$$

Table 3: fuzzy evaluation matrix with triangular numbers

Criteria	Specific gravity	Color	Clarity	Cleavage	Hardness
Specific gravity	$(1,1,1)$	$(6,7,8)$	$(1,2,3)$	$(2,3,4)$	$(3,4,5)$
Color	$(1/8,1/7,1/6)$	$(1,1,1)$	$(1/6,1/5,1/4)$	$(1/4,1/3,1/2)$	$(1/5,1/4,1/3)$
Clarity	$(1/3,1/2,1)$	$(4,5,3)$	$(1,1,1)$	$(1,2,3)$	$(2,3,4)$
Cleavage	$(1/4,1/3,1/2)$	$(2,3,4)$	$(1/3,1/2,1)$	$(1,1,1)$	$(1/3,1/2,1)$
Hardness	$(1/4,1/3,1/2)$	$3,4,5)($	$(1/4,1/3,1/2)$	$(1,2,3)$	$(1,1,1)$

Table (3) contains the criterion of gemstones (specific gravity, color, clarity, cleavage, and hardness), with triangular numbers based on Table (2).

Now, to find the eigenvector of criteria, we use the equation:

$$\lambda_i = \frac{\hat{\alpha}_i}{n}, \quad \forall i, \tag{4}$$

where λ_i is the eigenvector, $\widehat{\alpha}_i$ the sum of the matrix normalization values and it is divided by (n) (n is number the r of criteria). We can find the largest eigenvector $\lambda maks$ which obtained by:

$$\lambda maks = \left(\sum GM_{i1-ni} X \bar{X}_1\right) + \cdots + \left(\sum GM_{in-ni} X \bar{X}_n\right) \tag{5}$$

$$\lambda maks = 5.1662$$

$$\text{and } CI = \frac{\lambda maks - n}{n-1} \tag{6}$$

$$CI = 0.0415$$

$$\text{and } RI = 1.12$$

$$CR = \frac{CI}{RI} = \frac{0.0415}{1.12} = 0.03705$$

We get the results eigenvector of criteria of gemstones in the Table (4) below:

Table 4: Eigenvector of criteria

Specific gravity	2.786
Color	0.298
Clarity	1.7180
Cleavage	0.7594
Hardness	0.9221
	Sum = 6.4852

Step (3): consistency check

The comparison of consistency indicator in the random generator (RI) as the results following in Table (5):

Table 5: Ratio index

n	1	2	3	4	5	6	7	8	9
R_I	0	0	0.58	0.90	1.120	1.2400	1.320	1.4200	1.4500

Where the value of the Ratio index relies on the matrix order.

Step (4): For triangular Furry Number (TFN), we use the F - AHP scale for Triangular fuzzy number containing three values (l, m, u) where the lowest value, middle value, and highest value, respectively, see Figure (1)

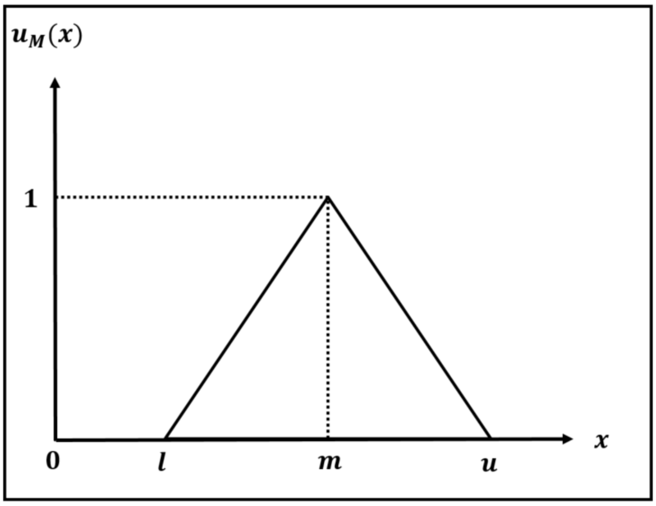

Figure 1: Membership function of Triangular Fuzzy Number.

TFN uses in this paper through (l, m, u) and membership function M_m as follows:

$$\mu_m(x) = \begin{pmatrix} \frac{x}{m-L} - \frac{L}{m-L}, & x \in [l\, m] \\ \\ \frac{x}{m-u} - \frac{u}{m-u} & x \in [m\ \ u] \end{pmatrix}$$

Let (l_1, m_1, u_1) and (l_2, m_2, u_2), and the fuzzy arithmetic operations are defined as:

- Addition: $(1_1,\ \mathrm{m}_1, \mathrm{u}_1) + (\mathrm{l}_2,\ \mathrm{m}_2, \mathrm{u}_2) = (l_1 + \mathrm{l}_2 \quad \mathrm{m}_1 + \mathrm{m}_2 \quad \mathrm{u}_1 + \mathrm{u}_2)$
- Multiplication: $(1_1,\ \mathrm{m}_1, \mathrm{u}_1) \cdot (\mathrm{l}_2,\ \mathrm{m}_2, \mathrm{u}_2) = (1_1 1_2,\ \mathrm{m}_1\ \mathrm{m}_2, \mathrm{u}_1\mathrm{u}_2)$
- Inverse: $(1_1,\ \mathrm{m}_1, \mathrm{u}_1)^{-1} \approx (\lambda l_1, \lambda \mathrm{m}_1, \lambda \mathrm{u}_1)$

Step (5): Calculating weights of fuzzy vector

(TFN) is of the most senses technique to calculate weights from comparison matrices (TFN) required, we have $X = \{x_1, x_2, \ldots x_n\}$ is object set, and $G = \{g_1, g_2, \ldots g_n\}$ goal set for each object set can be analysis procedure of each g_i.

Can be calculated value of S_i as:

$$S_i = \sum_{j=1}^{m} M_{qi}^{j} \times \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{qi}^{j} \right]^{-1} \tag{7}$$

$$\text{Where } \sum_{j=1}^{m} M_{qi}^{j} = \left(\sum_{j=1}^{m} lj, \sum_{j=1}^{m} mj, \sum_{k=1}^{m} uj \right). \tag{8}$$

In the original method of AHP, we use the scale $1 \ldots 9$ to calculate the final weights through the (TFN).

Fuzzy weight with the others fuzzy and we used the lower set of this probability (possibility) for each scale of I we obtain as:

$$\begin{aligned} V\left(M_2 \geq M_1\right) &= hgt\left(M_1 \cap M_2\right) \\ &= M_{m2}(d) \\ &= 1 \quad 1l \quad m_2 \geq m_1 \\ &= 0 \quad 1l \quad l_1 \geq u_2 \\ &\frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} \text{ otherwise} \end{aligned}$$

See Figure (2):

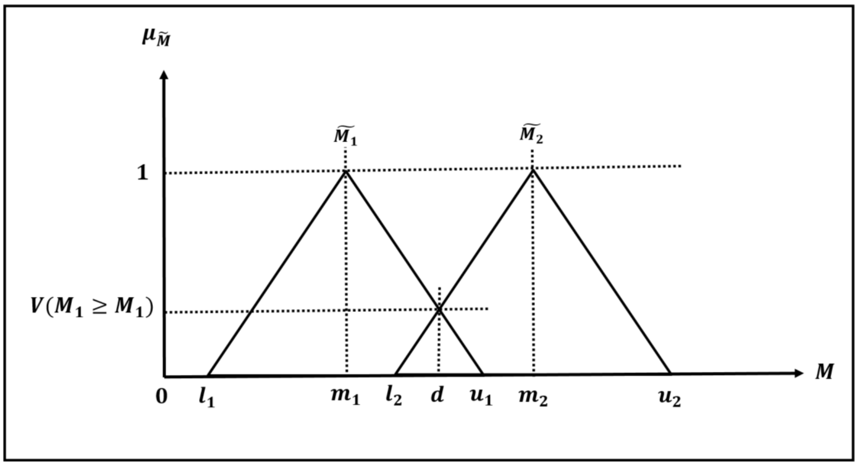

Figure 2: Degree of possibility.

We note the degree of the possibility of the fuzzy number is convex, and it is given as:

$$\begin{aligned} &\mathrm{V}\left(\mathrm{M} \geq \mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_\mathrm{k}\right) = \\ &\mathrm{V}\left(\mathrm{M} \geq \mathrm{M}_1\right) \text{ and } \left(\mathrm{M} \geq \mathrm{M}_2\right), \ldots., \left(\mathrm{M} \geq \mathrm{M}_\mathrm{k}\right) \\ &= \min\left(\mathrm{M} \geq \mathrm{M}_\mathrm{i}\right), \mathrm{i} \neq 1 \ldots, \mathrm{k} \\ &\text{assuming} \quad W_i^l = \min V\left(M_i \geq M_k\right) \end{aligned}$$

The weight value is given by:

$$W^l = W_l^l, W_2^l, \ldots, W_n^l$$

Step (6): Ranking and selection of decision after using the alternative value Si.

$$Si = \begin{bmatrix} 0.1727 & 0.2446 & 0.4336 \\ 0.2732 & 0.354 & 0.2668 \\ 0.01988 & 0.3109 & 0.5447 \\ 0.0779 & 0.0903 & 0.1390 \end{bmatrix}$$

Where $Si(i = 1, 2, \ldots, n)$ is n elements of the matrix. Also, we get the normalized weights vector as the following:

The vector of normalized weights is:

$$w = \left(d\left(A_1\right), d\left(A_2\right), \ldots, d\left(A_n\right)\right)^T, \tag{9}$$

where w is a non-fuzzy number, we can be getting the final weights of each criterion by multiplying the criteria with the matrix. We also obtain the matrix of fuzzy written as:

fuzzy matrix

$$= \begin{bmatrix} 0.25 & 2 & 0.33 & 1 & 0.33 & 0.33 & 4 & 1 & 1 & 2 & 0.12 & 1 & 0.16 & 0.25 & 0.20 & 1 & 6 & 1 & 2 & 3 \\ 0.33 & 3 & 0.50 & 1 & 0.50 & 0.50 & 5 & 1 & 2 & 3 & 0.14 & 1 & 0.20 & 0.33 & 0.25 & 1 & 7 & 2 & 3 & 4 \\ 0.50 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 4 & 0.16 & 1 & 0.25 & 0.50 & 0.33 & 1 & 8 & 3 & 4 & 5 \end{bmatrix}$$

Then the matrix of V can be written as:

$$V = \begin{bmatrix} 0.5941 & 1 & 1 & 0 \\ 0.7799 & 1 & 0.8623 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Table 6: calculation results of criteria

Criteria	Specific Gubi	Color	Clarity	Cleavage	Hardness	Final weights
Rubi 1	0.5034	0.421	0.3492	0.3721	0.25	0.5941
Rubi 2	0.1806	0.4966	0.5681	0.4819	0.25	1
Rubi 3	0.2913	0.0457	0.0822	0.17	0.25	0.8623
Rubi 4	0	0.0511	0	0	0.25	0
Rubi 5	0	0	0	0	0.25	0
Sum	0.9753	1.0144	0.999	1.024	1.25	

3. Conclusions

This paper summarizes the criterion of a system that can help us to choose and evaluate the selection of high-quality gemstones by F-AHP. The accuracy of one step is shown by no error when applying the method of F-AHP in MATLAB language. The model of high-quality Rubi 2 with a weight equal to one has the optimal weight by comparing the standards of gemstones. While the model of high-quality Rubi 3 with a weight of (0.8623) is the second selection and the third is Rubi 1 with (0.5941) while Rubi 4, and Rubi 5 have weights (0).

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