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Real Spectra in Logarithmic model PT-symmetry operators: Iso-spectra in Logarithmic PT-symmetry

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Abstract. We reflect real spectra of new logarithmic model PT-symmetry operators with singular and non-singular in nature. We also notice the iso-spectral nature between inverted and non-inverted PT-symmetry potentials. Present numerical result give good agreement with previous results.

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1. Introduction

Real spectra in quantum operators are confined to Hermiticity($H = H^{\dagger}$) as well as PT-symmetry [1]($[H, PT] = 0$).Here P stands for parity operator having the properties: $PxP^{-1} = -x$; $PpP^{-1} = -p$. Similarly T stands for the time reversal operator having the properties$TxT^{-1} = x$;$TpT^{-1} = -p$ and $TT^{-1} = -i$. In the Hermiticity(more precisely self-adjoint operator), it ia possible to find two Hamiltonians, which are iso-spectral to each other[2]. For example[2,3]

$$h^{(1)} = p^2 + V_0(1 - e^{2|x|/a}) \tag{1}$$

and

$$h^{(2)} = p^2 + v_0(1 - e^{-2|x|/a}) \tag{2}$$

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If one clearly analyzes one is scattering in nature and the other is confining nature [2,3].

Till now no such models are reflected in nature. In a recent paper Bender etal [4] have suggested a new class of logarithmic PT-symmetry potentials as

$$h_1 = p^2 + x^4 \log(ix) \tag{3}$$

$$h_2 = p^2 - x^4 \log(ix) \tag{4}$$

$$h_3 = p^2 - x^4 \log(x^2) \tag{5}$$

and reflected energy spectrum of H_1 only. Further authors reported analytical calculation of energy level using WKB approach[4]

$$\frac{E_n}{[\log E_n)]^{1/3}} \sim \left[\frac{\Gamma(7/4)(n+1/2)\sqrt{\pi}}{\Gamma(5/4)\sqrt{2}}\right]^{4/3} \tag{6}$$

does not give encouraging results when compared with numerical results. This motivates the present author to calculate spectra of $H_{1,2}$ and suggest new models on logarithmic potentials. Apart from this aim is to find out whether iso-spectral Hamiltonians are possible in PT-symmetry operators.

2. Logarithmic new models

Here we consider different models as follows
Quadratic Logarithmic model $V(x) = -x^2 \log(\frac{i}{x})$
The Hamiltonian considered here is

$$H_1^{quadratic} = p^2 - x^2 \log(\frac{i}{x}) \tag{7}$$

Quartic inverted model $V(x) = -x^4 \log(\frac{i}{x})$
Here we consider the Hamiltonian

$$H_2^{quartic} = p^2 - x^4 \log(\frac{i}{x}) \tag{8}$$

Below we present few energy levels

3. Logarithmic models

Here we consider the recently prposed models

$$h_1 = p^2 + \lambda x^4 \log(ix) \tag{9}$$

$$h_2 = p^2 - \lambda x^4 \log(ix) \tag{10}$$

$$h_3 = p^2 - \lambda x^4 \log(x^2) \tag{11}$$

and present complete spectra in table-2 for $\lambda = 1$.

TABLE.I: PT-symmetry inverted logarithmic model potentials

n	H_1 Present	H_2
0	1.326 591 6	1.249 087 3
3	9.173 294 3	13.738 280 4
6	17.734 002 2	31.665 810 8
9	26.633 530 1	52.993 926 2
12	76.113 329 2	76.974 762 5

4. Method of calculation

Here we use matrix diagonalisation method [5] to solve the eigenvalue relation

$$H|\Psi >= E|\Psi > \tag{12}$$

where

$$|\Psi >= \sum A_m|m > \tag{13}$$

Here $|m>$ satisfy the relation

$$[p^2 + x^2]|m >= (2m + 1)|m > \tag{14}$$

Numerical results obtained using MDM are tabulated in table.1.

5. Conclusion

In this report, we present numerical convergent energy levels of new model PT-invariant Hamltonians using matrix diagonalisation method[5]. Further we feel the present method can be used confidently to realize real spectra study in similar Hamiltonians of interest. Lastly we the spectra of H_2 and h_1 are the same. In brief

$$V(x) = x^4 \log(ix) \rightarrow V(x) = -x^4 \log(\frac{i}{x}) \tag{15}$$

Hence these two potentials can be considered as iso-spectral models in PT-symmetry. Lastly we do not find any numerical results to present in table-1 for a comparison with the present numericals. Interested readers can consider other values of λ.

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TABLE.2: PT-symmetry logarithmic model

n	h_1 Present	h_1Previous [4]	Previous (WKB)[4]
0	1.249 08	1.249 09	0.546 27
3	13.738 27	13.738 3	7.314 80
6	31.665 82	31.665 8	16.697 9
9	52.993 79	52.993 9	27.695 6
12	76.976 08	76.974 8	39.932 4
n	h_2 Present	Previous[4]	Previous(WKB)[4]
0	0.109 1		
1	6,959 6		
2	8.257 1		
3	18.039 4		
n	h_3 Present	Previous[4]	Previous(WKB)[4]
0	0.025 4		
1	4.977 7		
2	9.237 1		
3	16.478 6		

References

[1] C.Bender and S.Boettcher Real spectra in non-Hermitian Hamiltonian having PT-symmetry, Phys.Rev.Lett,(1997),**80**,5243-5246.

[2] H.F.Jones,Comment on: Solvable model bound states in the continuum (BIC)in One dimension(2019, 94, 105214),Phys.Scr,(2021),**96**,087001.(see ref-2)

[3] Z.Ahmed and H.F.Jones,Scattering states and bound states of exponential potentials, arxiv:2102:06095v1(see ref-3).

[4] C.Bender,A.Felski,S.P.Klevansky and S.Sarkar,PT-symmetry and Renormalisation in Quantum Field Theory,arxiv:2103.14864v1.

[5] B.Rath,Real spectra in some negative potentials: Linear and nonlinear one dimensional PT-invariant quantum systems,Eur.Phys.Journal.Plus.(2021), **136**,493.