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A Generalization of Some Forms of g-Irresolute Functions

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Abstract. In this paper, by using gm-closed sets [27], we obtain the unified definitions and properties for g-continuity, gs-continuity, gp-continuity, αg-continuity, γg-continuity and gsp-continuity.

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1. Introduction

The concept of generalized closed (briefly g-closed) sets in topological spaces was introduced by Levine [20] in 1970. These sets were also considered by Dunham [15] and Dunham and Levine [16]. The notion of αg-closed [12] (resp. gs-closed [11],

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gp-closed [6], gb-closed or γg-closed [18], gsp-closed or $g\beta$-closed [14]) sets is introduced and investigated. In 1981, Munshy and Bassan [25] introduced the notion of generalized continuous (briefly g-continuous) functions which are called in [7] as g-irresolute functions. Furthermore, the notion of gs-irresolute [11] (resp. gp-irresolute [6], αg-irresolute [12], gb-irresolute [3], gsp-irresolute [32]) functions is introduced.

Recently, the present authors [29], [30] have introduced the notions of m-structures, m-spaces and M-continuity. In [27], the first author introduced the notion of generalized m-closed (briefly gm-closed) sets and tried to unify certain types of modifications of g-closed sets such as stated above. In this paper, by using gm-closed sets, we obtain the unified definitions and properties for g-irresoluteness, gs-irresoluteness, gp-irresoluteness, αg-irresoluteness, gb-irresoluteness and gsp-irresoluteness.

2. Preliminaries

Let (X,τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. We recall some generalized open sets in topological spaces.

Definition 1. Let (X,τ) be a topological space. A subset A of X is said to be

(1) *α-open* [26] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,

(2) *semi-open* [19] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,

(3) *preopen* [22] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,

(4) *β-open* [1] or *semi-preopen* [4] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$,

(5) *γ-open* [18] or *b-open* [5] if $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$.

The family of all α-open (resp. semi-open, preopen, β-open, γ-open) sets in (X,τ) is denoted by $\alpha(X)$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\beta(X)$ or $\mathrm{SPO}(X)$, $\gamma(X)$ or $\mathrm{BO}(X)$).

Definition 2. Let (X,τ) be a topological space. A subset A of X is said to be *α-closed* [23] (resp. *semi-closed* [10], *preclosed* [22], *β-closed* [1] or *semi-preclosed* [4], *γ-closed* [18] or *b-closed* [5]) if the complement of A is α-open (resp. semi-open, preopen, β-open, γ-open).

Definition 3. Let (X,τ) be a topological space and A a subset of X. The intersection of all α-closed (resp. semi-closed, preclosed, β-closed, γ-closed) sets of X containing A is called the *α-closure* [23] (resp. *semi-closure* [10], *preclosure* [17], *β-closure* [2] or *semi-preclosure* [4], *γ-closure* [18] or *b-closure* [5]) of A and is denoted by $\alpha\mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$ or $\mathrm{spCl}(A)$), $\mathrm{Cl}_{\gamma}(A)$ or $\mathrm{bCl}(A)$).

Definition 4. Let (X,τ) be a topological space and A a subset of X. The union of all α-open (resp. semi-open, preopen, β-open, γ-open) sets of X contained in A is called the *α-interior* [23] (resp. *semi-interior* [10], *preinterior* [17], *β-interior* [2] or *semi-preinterior* [4], *γ-interior* [18] or *b-interior* [5]) of A and is denoted by $\alpha\mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, ${}_{\beta}\mathrm{Int}(A)$ or $\mathrm{spInt}(A)$), $\mathrm{Int}_{\gamma}(A)$ or $\mathrm{bInt}(A)$).

3. Minimal structures and m-continuity

Definition 5. Let X be a nonempty set and $\mathscr{P}(X)$ the power set of X. A subfamily m_X of $\mathscr{P}(X)$ is called a *minimal structure* (briefly *m-structure*) on X [29], [30] if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be *m_X-open* and the complement of an m_X-open set is said to be *m_X-closed*.

Remark 1. Let (X,τ) be a topological space. Then the family $\alpha(X)$ is a topology finer than τ. The families $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\beta(X)$, and $\gamma(X)$ are all m-structures on X.

Definition 6. Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the *m_X-closure* of A and the *m_X-interior* of A are defined in [21] as follows:

(1) $\mathrm{mCl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,

(2) $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 2. Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\gamma(X)$), then we have

(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_\beta\mathrm{Cl}(A)$, $\mathrm{Cl}_\gamma(A)$),

(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_\beta\mathrm{Int}(A)$, $\mathrm{Int}_\gamma(A)$).

Lemma 1. (Maki et al. [21]). *Let X be a nonempty set and m_X a minimal structure on X. For subsets A and B of X, the following properties hold:*

(1) $\mathrm{mCl}(X - A) = X - \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X - A) = X - \mathrm{mCl}(A)$,

(2) If $(X - A) \in m_X$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m_X$, *then* $\mathrm{mInt}(A) = A$,

(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,

(4) If $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,

(5) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,

(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 2. (Popa and Noiri [29]). *Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then* $x \in \mathrm{mCl}(A)$ *if and only if* $U \cap A \neq \emptyset$ *for every* $U \in m_X$ *containing x.*

Definition 7. An m-structure m_X on a nonempty set X is said to have *property $\mathscr{B}$* [21] if the union of any family of subsets belong to m_X belongs to m_X.

Remark 3. If (X, τ) is a topological space, then $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$ and $\gamma(X)$ have property $\mathscr{B}$,

Lemma 3. (Popa and Noiri [30]). *Let X be a nonempty set and m_X an m-structure on X satisfying property $\mathscr{B}$. For a subset A of X, the following properties hold:*

(1) $A \in m_X$ *if and only if* $\mathrm{mInt}(A) = A$*,*

(2) A *is* m_X*-closed if and only if* $\mathrm{mCl}(A) = A$*,*

(3) $\mathrm{mInt}(A) \in m_X$ *and* $\mathrm{mCl}(A)$ *is* m_X*-closed.*

Definition 8. A function $f : (X, m_X) \to (Y, m_Y)$ is said to be *M-continuous* at a point $x \in X$ [30] if for each $x \in X$ and each $V \in m_Y$ containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$. A function $f : (X, m_X) \to (Y, m_Y)$ is said to be *M-continuous* if it has this property at each point $x \in X$.

Theorem 1. *For a function* $f : (X, m_X) \to (Y, m_Y)$*, the following properties are equivalent:*

(1) f is M-continuous at $x \in X$*;*

(2) $x \in \mathrm{mInt}(f^{-1}(V))$ *for every* $V \in m_Y$ *containing f(x);*

(3) $x \in f^{-1}(\mathrm{mCl}(f(A)))$ *for every subset A of X with* $x \in \mathrm{mCl}(A)$*;*

(4) $x \in f^{-1}(\mathrm{mCl}(B))$ *for every subset B of Y with* $x \in \mathrm{mCl}(f^{-1}(B))$*;*

(5) $x \in \mathrm{mInt}(f^{-1}(B))$ *for every subset B of Y with* $x \in f^{-1}(\mathrm{mInt}(B))$*;*

(6) $x \in f^{-1}(K)$ *for every* m_Y*-closed set K of Y such that* $x \in \mathrm{mCl}(f^{-1}(K))$.

Proof. (1) $\Rightarrow$ (2): Let $V \in m_Y$ containing $f(x)$. Then, there exists $U \in m_X$ containing x such that $f(U) \subset V$. Thus $x \in U \subset f^{-1}(V)$. Since $U \in m_X$, we have $x \in \mathrm{mInt}(f^{-1}(V))$.

(2) $\Rightarrow$ (3): Let A be any subset of X. Let $x \in \mathrm{mCl}(A)$ and $V \in m_Y$ containing $f(x)$. Then $x \in \mathrm{mInt}(f^{-1}(V))$. There exists $U \in m_X$ such that $x \in U \subset f^{-1}(V)$. Since $x \in \mathrm{mCl}(A)$, by Lemma 2, $U \cap A \neq \emptyset$ and $\emptyset \neq f(U \cap A) \subset f(U) \cap f(A) \subset V \cap f(A)$. Since $V \in m_Y$ containing $f(x)$, $f(x) \in \mathrm{mCl}(f(A))$ and hence $x \in f^{-1}(\mathrm{mCl}(f(A))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y and $x \in \mathrm{mCl}(f^{-1}(B))$, then by (3) $x \in f^{-1}(\mathrm{mCl}(f(f^{-1}(B)))) \subset f^{-1}(\mathrm{mCl}(B))$. Hence, we have $x \in f^{-1}(\mathrm{mCl}(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $x \notin \mathrm{mInt}(f^{-1}(B))$. Then $x \in X - \mathrm{mInt}(f^{-1}(B)) = \mathrm{mCl}(X - f^{-1}(B)) = \mathrm{mCl}(f^{-1}(Y - B))$. By (4), we have $x \in$

$f^{-1}(\mathrm{mCl}(Y-B)) = f^{-1}(Y-\mathrm{mInt}(B)) = X - f^{-1}(\mathrm{mInt}(B))$. Hence, $x \notin f^{-1}(\mathrm{mInt}(B))$.

(5) $\Rightarrow$ (6): Let K be any m_Y-closed set of Y such that $x \notin f^{-1}(K)$. Then $x \in X - f^{-1}(K) = f^{-1}(Y-K) = f^{-1}(\mathrm{mInt}(Y-K))$ because $Y-K$ is m_Y-open. By (5), $x \in \mathrm{mInt}(f^{-1}(Y-K)) = \mathrm{mInt}(X - f^{-1}(K)) = X - \mathrm{mCl}(f^{-1}(K))$. Hence $x \notin \mathrm{mCl}(f^{-1}(K))$.

(6) $\Rightarrow$ (2): Let $x \in X$ and $V \in m_Y$ containing $f(x)$. Suppose that $x \notin \mathrm{mInt}(f^{-1}(V))$. Then $x \in X - \mathrm{mInt}(f^{-1}(V)) = \mathrm{mCl}(X - f^{-1}(V)) = \mathrm{mCl}(f^{-1}(Y-V))$. By (6), $x \in f^{-1}(Y-V) = X - f^{-1}(V)$. Hence $x \notin f^{-1}(V)$. This contraries to the hypothesis.

(2) $\Rightarrow$ (1): Let $V \in m_Y$ containing $f(x)$. By (2), $x \in \mathrm{mInt}(f^{-1}(V))$ and hence there exists $U \in m_X$ containing x such that $x \in U \subset f^{-1}(V)$. Therefore, $f(U) \subset V$ and f is M-continuous at x.

For a function $f : (X, m_X) \to (Y, m_Y)$, we define $D_M(f)$ as follows:

$$D_M(f) = \{x \in X : f \text{ is not } M\text{-continuous at } x\}.$$

Theorem 2. *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties hold:*

$$\begin{aligned} D_M(f) &= \textstyle\bigcup_{G \in m_Y} \{f^{-1}(G) - \mathrm{mInt}(f^{-1}(G))\} \\ &= \textstyle\bigcup_{B \in \mathscr{P}(Y)} \{f^{-1}(\mathrm{Int}(B)) - \mathrm{mInt}(f^{-1}(B))\} \\ &= \textstyle\bigcup_{B \in \mathscr{P}(Y)} \{\mathrm{mCl}(f^{-1}(B)) - f^{-1}(\mathrm{mCl}(B))\} \\ &= \textstyle\bigcup_{A \in \mathscr{P}(X)} \{\mathrm{mCl}(A) - f^{-1}(\mathrm{mCl}(f(A)))\} \\ &= \textstyle\bigcup_{K \in \mathscr{F}} \{\mathrm{mCl}(f^{-1}(K)) - f^{-1}(K)\}, \end{aligned}$$

where $\mathscr{F}$ is the family of m_Y-closed sets of Y.

Proof. We show only the first equality because the proofs of the others are similar to the first one. Let $x \in D_M(f)$. By Theorem 1, there exists $V \in m_Y$ such that $f(x) \in V$ and $x \notin \mathrm{mInt}(f^{-1}(V))$. Therefore, we have $x \in f^{-1}(V) - \mathrm{mInt}(f^{-1}(V)) \subset \bigcup_{G \in m_Y} \{f^{-1}(G) - \mathrm{mInt}(f^{-1}(G))\}$. Conversely, let $x \in \bigcup_{G \in m_Y} \{f^{-1}(G) - \mathrm{mInt}(f^{-1}(G))\}$. There exists $V \in m_Y$ such that $x \in f^{-1}(V) - \mathrm{mInt}(f^{-1}(V))$. By Theorem 1, $x \in D_M(f)$.

Theorem 3. (Popa and Noiri [29]). *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*

(1) f is M-continuous;

(2) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every* $V \in m_Y$*;*

(3) $f(\mathrm{mCl}(A)) \subset \mathrm{Cl}(f(A))$ *for every subset A of X;*

(4) $\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}(\mathrm{mCl}(B))$ *for every subset B of Y;*

(5) $f^{-1}(\mathrm{Int}(B)) \subset \mathrm{mInt}(f^{-1}(B))$ *for every subset B of Y;*

(6) $\mathrm{mCl}(f^{-1}(K)) = f^{-1}(K)$ *for every* m_Y*-closed set K of Y.*

Corollary 1. (Popa and Noiri [29]). *For a function* $f : (X, m_X) \to (Y, m_Y)$*, where* m_X *has property* $\mathscr{B}$*, the following properties are equivalent:*

(1) f is M-continuous;

(2) $f^{-1}(V)$ *is* m_X*-open for every* $V \in m_Y$*;*

(3) $f^{-1}(F)$ *is* m_X*-closed in X for every* m_Y*-closed set F of Y.*

Definition 9. A function $f : (X, m_X) \to (Y, m_Y)$ is said to be M^**-continuous* [24] if $f^{-1}(V)$ is m_X-open for each m_Y-open set V of Y.

Remark 4. (1) If $f : (X, m_X) \to (Y, m_Y)$ is M^*-continuous, then it is M-continuous. By Example 3.4 of [24], an M-continuous function may not be M^*-continuous.

(2) If m_X has property $\mathscr{B}$, then M-continuity and M^*-continuity are equivalent.

4. gm-closed sets and gM-continuity

Definition 10. Let (X, τ) be a topological space. A subset A of X is said to be

(1) *g-closed* [20] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(2) *αg-closed* [12] if $\alpha\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(3) *gs-closed* [11] if $\mathrm{sCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(4) *gp-closed* [6] if $\mathrm{pCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(5) *gb-closed* or *γg-closed* [18] if $\mathrm{bCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(6) *gsp-closed* [14] or *gβ-closed* if $\mathrm{spCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

Definition 11. A subset A of a topological space is said to be *g-open* (resp. *gs-open*, *gp-open*, *αg-open*, *gb-open*, *gsp-open*) if $X - A$ is g-closed (resp. gs-closed, gp-closed αg-closed, gb-closed, gsp-closed).

The family of all g-open (resp. gs-open, gp-open, αg-open, gb-open, gsp-open) sets of X is denoted by $\mathrm{GO}(X)$ (resp. $\mathrm{GSO}(X)$, $\mathrm{GPO}(X)$, $\alpha\mathrm{GO}(X)$, $\mathrm{GBO}(X)$, $\mathrm{GSPO}(X)$).

Definition 12. Let (X, τ) be a topological space and A a subset of X. The intersection of all g-closed (resp. αg-closed, gs-closed, gp-closed, gsp-closed, gb-closed) sets of X containing A is called the *g-closure* [15] (resp. *αg-closure*, *gs-closure*, *gp-closure*, *gsp-closure*, *gb-closure*) of A and is denoted by $\mathrm{Cl}_g(A)$ (resp. $\alpha\mathrm{Cl}_g(A)$, $\mathrm{sCl}_g(A)$, $\mathrm{pCl}_g(A)$, $\mathrm{spCl}_g(A)$, $\mathrm{bCl}_g(A)$).

Definition 13. Let (X, τ) be a topological space and A a subset of X. The union of all g-open (resp. αg-open, gs-open, gp-open, gsp-open, gb-open) sets of X contained in A is called the *g-interior* [9] (resp. *αg-interior*, *gs-interior*, *gp-interior*, *gsp-interior*, *gb-interior*) of A and is denoted by $\mathrm{Int}_g(A)$ (resp. $\alpha\mathrm{Int}_g(A)$, $\mathrm{sInt}_g(A)$, $\mathrm{pInt}_g(A)$, $\mathrm{spInt}_g(A)$, $\mathrm{bInt}_g(A)$).

Remark 5. Let (X, τ) be a topological space and A a subset of X.

(1) Then, $\mathrm{GO}(X)$, $\mathrm{GSO}(X)$, $\mathrm{GPO}(X)$, $\alpha\mathrm{GO}(X)$ and $\mathrm{GSPO}(X)$ are all m-structures on X. Hence, if we put $m_X = \mathrm{GO}(X)$ (resp. $\alpha\mathrm{GO}(X)$, $\mathrm{GSO}(X)$, $\mathrm{GPO}(X)$, $\mathrm{GSPO}(X)$), then we have

(i) $\mathrm{mCl}(A) = \mathrm{Cl}_g(A)$ (resp. $\alpha\mathrm{Cl}_g(A)$, $\mathrm{sCl}_g(A)$, $\mathrm{pCl}_g(A)$, $\mathrm{spCl}_g(A)$),

(ii) $\mathrm{mInt}(A) = \mathrm{Int}_g(A)$ (resp. $\alpha\mathrm{Int}_g(A)$), $\mathrm{sInt}_g(A)$, $\mathrm{pInt}_g(A)$, $\mathrm{spInt}_g(A)$).

(2) If $m_X = \mathrm{GO}(X)$, then by Lemma 1 we obtain the results established in Theorem 2.1 (4), (5) and Theorem 2.8 (2), (3), (5), (6) in [9]. By Lemma 2, we obtain the result established in Theorem 2.1 (4) in [9].

(3) The m-structures $\mathrm{GO}(X)$, $\mathrm{GSO}(X)$, $\mathrm{GPO}(X)$, $\alpha\mathrm{GO}(X)$, $\mathrm{GSPO}(X)$ and $\mathrm{GBO}(X)$ do not have property $\mathscr{B}$, in general.

Definition 14. Let (X, τ) be a topological space and m_X an m-structure on X. A subset A of X is said to be *generalized m-closed* (briefly gm-$closed$) [27] if $\mathrm{mCl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

The complement of a gm-closed set is said to be gm-$open$. The family of all gm-open sets of a topological space (X, τ) is denoted by $\mathrm{GMO}(X)$. Obviously, $\mathrm{GMO}(X)$ is an m-structure on X and is called a *gm-structure* on X.

Remark 6. Let (X, τ) be a topological space and m_X an m-structure on X. We put $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{SPO}(X)$, $\mathrm{BO}(X)$). Then, a gm-closed set is a g-closed (resp. gs-closed, gp-closed, αg-closed, gsp-closed, gb-closed) set.

Definition 15. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *g-irresolute* [7] or *g-continuous* [25] (resp. *gs-irresolute* [11], *gp-irresolute* [6], *αg-irresolute* [12], *gsp-irresolute* [32], *gb-irresolute* [3]) if $f^{-1}(K)$ is a g-closed (resp. gs-closed, gp-closed, αg-closed, gsp-closed, gb-closed) in X for every g-closed (resp. gs-closed, gp-closed, αg-closed, gsp-closed, gb-closed) set K of Y.

Definition 16. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) *gM-continuous* at a point $x \in X$ if $f : (X, \mathrm{GMO}(X)) \to (Y, \mathrm{GMO}(Y))$ is M-continuous at a point $x \in X$. The function $f : (X, \tau) \to (Y, \sigma)$ is said to be *gM-continuous* if it is gM-continuous at each point $x \in X$.

(2) *gM-irresolute* if $f : (X, \mathrm{GMO}(X)) \to (Y, \mathrm{GMO}(Y))$ is M^*-continuous.

Remark 7. (1) Every gM-irresolute function is gM-continuous.

(2)If $m_X = \mathrm{GO}(X)$ (resp. $\mathrm{GSO}(X)$, $\mathrm{GPO}(X)$, $\alpha\mathrm{GO}(X)$, $\mathrm{GSPO}(X)$, $\mathrm{BO}(X)$), $m_Y = \mathrm{GO}(Y)$ (resp. $\mathrm{GSO}(Y)$, $\mathrm{GPO}(Y)$, $\alpha\mathrm{GO}(Y)$, $\mathrm{GSPO}(Y)$, $\mathrm{BO}(Y)$) and $f : (X, \tau) \to (Y, \sigma)$ is gM-irresolute, then f is g-irresolute (resp. gs-irresolute, gp-irresolute, αg-irresolute, gsp-irresolute, gb-irresolute).

Definition 17. Let (X,τ) be a topological space and $\mathrm{GMO}(X)$ a gm-structure on X. For a subset A of X, the *gm-closure* of A and the *gm-interior* of A are defined as follows:

(1) $\mathrm{mCl}_g(A) = \cap\{F : A \subset F, X - F \in \mathrm{GMO}(X)\}$,

(2) $\mathrm{mInt}_g(A) = \cup\{U : U \subset A, U \in \mathrm{GMO}(X)\}$.

By Definition 16 and Theorem 3, we obtain the following theorem and corollary.

Theorem 4. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is gM-continuous;

(2) $f^{-1}(V) = \mathrm{mInt}_g(f^{-1}(V))$ for every gm-open set V of Y;

(3) $\mathrm{mCl}_g(f^{-1}(F)) = f^{-1}(F)$ for every gm-closed set F of Y;

(4) $\mathrm{mCl}_g(f^{-1}(B)) \subset f^{-1}(\mathrm{mCl}_g(B))$ for every subset B of Y;

(5) $f(\mathrm{mCl}_g(A)) \subset \mathrm{mCl}_g(f(A))$ for every subset A of X;

(6) $f^{-1}(\mathrm{mInt}_g(B)) \subset \mathrm{mInt}_g(f^{-1}(B))$ for every subset B of Y.

Corollary 2. *For a function $f : (X,\tau) \to (Y,\sigma)$, where GMO(X) has property $\mathscr{B}$, the following properties are equivalent:*

(1) f is gM-continuous;

(2) $f^{-1}(V)$ is gm-open for every gm-open set V of Y;

(3) $f^{-1}(F)$ is gm-closed for every gm-closed set F of Y.

Let (X,τ) be a topological space and $\mathrm{GMO}(X)$ a gm-structure on X. For a function $f : (X,\tau) \to (Y,\sigma)$, we denote by $D_{gM}(f)$ the set of all points of X at which the function f is not gM-continuous. Then by Definition 16 and Theorem 4, we obtain the following theorem.

Theorem 5. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties hold:*

$$
\begin{aligned}
D_{gM}(f) &= \bigcup_{G\in \mathrm{GMO}(Y)}\{f^{-1}(G) - \mathrm{mInt}_g(f^{-1}(G))\} \\
&= \bigcup_{B\in \mathscr{P}(Y)} \{f^{-1}(\mathrm{mInt}_g(B)) - \mathrm{mInt}_g(f^{-1}(B))\} \\
&= \bigcup_{B\in \mathscr{P}(Y)} \{\mathrm{mCl}_g(f^{-1}(B)) - f^{-1}(\mathrm{mCl}_g(B))\}
\end{aligned}
$$

$$= \bigcup_{A \in \mathscr{P}(X)} \{\mathrm{mCl}_g(A) - f^{-1}(\mathrm{mCl}_g(f(A)))\}$$
$$= \bigcup_{K \in \mathscr{F}g} \{\mathrm{mCl}_g(f^{-1}(K)) - f^{-1}(K)\},$$

where $\mathscr{F}_g$ is the family of gm-closed sets of Y.

Definition 18. Let (X, m_X) be an m-space and A a subset of X. The *m_X-frontier* of A, mFr(A), [30] is defined by $\mathrm{mFr}(A) = \mathrm{mCl}(A) \cap \mathrm{mCl}(X - A) = \mathrm{mCl}(A) - \mathrm{mInt}(A)$.

If (X, τ) is a topological space and GMO(X) is a gm-structure on X, then $\mathrm{gmFr}(A) = \mathrm{mCl}_g(A) \cap \mathrm{mCl}_g(X - A) = \mathrm{mCl}_g(A) - \mathrm{mInt}_g(A)$.

Theorem 6. *The set of all points of X at which a function $f : (X, m_X) \to (Y, m_Y)$ is not M-continuous is identical with the union of the m-frontiers of the inverse images of m_Y-open sets containing f(x).*

Proof. Suppose that f is not M-continuous at $x \in X$. There exists an m_Y-open set V of Y containing $f(x)$ such that $U \cap (X - f^{-1}(V)) \neq \emptyset$ for every m_X-open set U containing x. By Lemma 2, we have $x \in \mathrm{mCl}(X - f^{-1}(V))$. On the other hand, we have $x \in f^{-1}(V)$ and hence $x \in \mathrm{mFr}(f^{-1}(V))$.

Conversely, suppose that f is M-continuous at $x \in X$. Then, for any m_Y-open set V of Y containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$; hence $U \subset f^{-1}(V)$. Therefore, we have $x \in U \subset \mathrm{mInt}(f^{-1}(V))$. This contradicts to the fact that $x \in \mathrm{mFr}(f^{-1}(V))$.

Corollary 3. *Let (X, τ) (resp. (Y, σ)) be a topological space and* GOM*(X) (resp.* GOM*(Y)) a gm-structure on X (resp. Y). Then, the set of all points at $x \in X$ which a function $f : (X, \tau) \to (Y, \sigma)$ is not gM-continuous is identical with the union of the gm-frontiers of the inverse images of gm-open sets containing f(x).*

Proof. This follows immediately from Theorem 6.

5. Some properties of gM-continuity

In this section, we use gm-open sets and gm-closed sets in order to obtain some properties of gm-T_2 spaces and the preservation theorems of gm-compact spaces and gm-connected spaces. Furthermore, we investigate some properties of strongly m-closed graphs.

Definition 19. An m-space (X, m_X) is said to be m-T_2 [29] if for any distinct points x, y, there exist $U, V \in m_X$ such that $x \in U, y \in V$, and $U \cap V = \emptyset$.

Remark 8. (1) Let (X, τ) be a topological space, then (X, τ) is said to be gm-T_2 if the m-space $(X, \text{GMO}(X))$ is m-T_2.

(2) If $\text{GMO}(X) = \text{GO}(X)$ (resp. $\text{GSO}(X)$, $\text{GPO}(X)$, $\alpha\text{GO}(X)$ $\text{GBO}(X)$, $\text{GSPO}(X)$) and (X, τ) is mg-T_2, then (X, τ) is said to be g-T_2 [8] (resp. gs-T_2, gp-T_2, αg-T_2, gb-T_2, gsp-T_2).

Lemma 4. (Popa and Noiri [29]). *If $f : (X, m_X) \to (Y, m_Y)$ is an M-continuous injection and (Y, m_Y) is m-T_2, then (X, m_X) is m-T_2.*

Theorem 7. *If $f : (X, \tau) \to (Y, \sigma)$ is a gM-continuous injection and (Y, σ) is a gm-T_2-space, then (X, τ) is gm-T_2.*

Proof. The proof follows from Remark 8 and Lemma 4.

Corollary 4. *If $f : (X, \tau) \to (Y, \sigma)$ is a gM-irresolute injection and (Y, σ) is a gm-T_2-space, then (X, τ) is gm-T_2.*

Definition 20. An m-space (X, m_X) is said to be *m-compact* [29] if every cover of X by sets of m_X has a finite subcover.

A subset K of an m-space (X, m_X) is said to be *m-compact* [29] if every cover of K by subsets of m_X has a finite subcover.

Remark 9. (1) If (X,τ) is a topological space and $(X, \text{GMO}(X))$ is m-compact, then (X,τ) is said to be *gm-compact*.

(2) If $\text{GMO}(X) = \text{GO}(X)$ (resp. $\text{GSO}(X)$, $\text{GPO}(X)$, $\alpha\text{GO}(X)$), then we obtain the definition of GO-compactness [7] (resp. GSO-compactness [11], GPO-compactness [6], αGO-compactness [12]).

Lemma 5. (Popa and Noiri [29]). *If a function $f : (X, m_X) \to (Y, m_Y)$ is M-continuous and K is an m-compact set of X, then f(K) is m-compact.*

Theorem 8. *If $f : (X,\tau) \to (Y,\sigma)$ is a gM-continuous function and K is a gm-compact set of X, then f(K) is gm-compact.*

Proof. The proof follows from Definition 20 and Lemma 5.

Corollary 5. *If $f : (X,\tau) \to (Y,\sigma)$ is a gM-irresolute function and K is a gm-compact set of X, then f(K) is gm-compact.*

Remark 10. If $\text{GMO}(X) = \text{GO}(X)$ (resp. $\text{GSO}(X)$, $\text{GPO}(X)$, $\alpha\text{GO}(X)$) and $\text{GMO}(Y) = \text{GO}(Y)$ (resp. $\text{GSO}(Y)$, $\text{GPO}(Y)$, $\alpha\text{GO}(Y)$), then by Corollary 5 we obtain the result established in Proposition 9(ii) of [7] (resp. Proposition 5.5(iii) of [11], Theorem 5.5(iii) of [6], Proposition 4.3(iii) [12]).

Definition 21. An m-space (X, m_X) is said to be *m-connected* [29] if X cannot be written as the union of two nonempty disjoint m_X-open sets.

Remark 11. Let (X,τ) be a topological space and $\text{GMO}(X)$ a gm-structure on X, then

(1) (X,τ) is said to be *gm-connected* if X cannot be written as the union of two nonempty disjoint gm-open sets.

(2) If $\text{GMO}(X) = \text{GO}(X)$ (resp. $\alpha\text{GO}(X)$), then we obtain the definition of GO-connected spaces [7] (resp. αGO-connected spaces [12]).

Lemma 6. *If $f : (X, m_X) \to (Y, m_Y)$ is an M^*-continuous surjection and (X, m_X) is m-connected, then (Y, m_Y) is m-connected.*

Proof. Suppose that (Y, m_Y) is not m-connected. Then there exist nonempty m_Y-open sets V_1 and V_2 such that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = Y$. Hence we have $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. Since f is an M^*-continuous surjection, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty m_X-open sets. Therefore, (X, m_X) is not m-connected. This is a contradiction and hence (Y, m_Y) is m-connected.

Theorem 9. *If $f : (X, \tau) \to (Y, \sigma)$ is a gM-irresolute surjection and (X, τ) is gm-connected, then (Y, σ) is gm-connected.*

Proof. The proof follows from Definition 21, Remark 11 and Lemma 6.

Remark 12. If GMO(X) = GO(X), then we obtain the result established in Proposition 13 of [7].

Definition 22. A function $f : (X, m_X) \to (Y, m_Y)$ is said to have a *strongly m-closed graph* (resp. *m-closed graph*) [29] if for each $(x, y) \in (X \times Y) - \mathrm{G}(f)$, there exist $U \in m_X$ containing x and $V \in m_Y$ containing y such that $[U \times \mathrm{mCl}(V)] \cap \mathrm{G}(f) = \emptyset$ (resp. $[U \times V] \cap \mathrm{G}(f) = \emptyset$).

Remark 13. Let (X, τ) (resp. (Y, σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y). A function $f : (X, \tau) \to (Y, \sigma)$ is said to have a *strongly gm-closed graph* (resp. *gm-closed graph*) if for each $(x, y) \in (X \times Y) - \mathrm{G}(f)$, there exist $U \in \mathrm{GMO}(X)$ containing x and $V \in \mathrm{GMO}(Y)$ containing y such that $[U \times \mathrm{mCl}_g(V)] \cap \mathrm{G}(f) = \emptyset$ (resp. $[U \times V] \cap \mathrm{G}(f) = \emptyset$).

Lemma 7. (Popa and Noiri [29]). *A function $f : (X, m_X) \to (Y, m_Y)$ is M-continuous and (Y, m_Y) is m-T_2, then f has a strongly m-closed graph.*

Theorem 10. *Let (X,τ) (resp. (Y,σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y). If a function $f:(X,\tau)\to(Y,\sigma)$ is gM-continuous and (Y,σ) is gm-T_2, then f has a strongly gm-closed graph.*

Proof. The proof follows from Definition 22, Remark 13 and Lemma 7.

Corollary 6. *If a function $f:(X,\tau)\to(Y,\sigma)$ is gM-irresolute and (Y,σ) is gm-T_2, then f has a strongly gm-closed graph.*

Remark 14. If (Y,σ) is g-T_2 (resp. gs-T_2, gp-T_2, αg-T_2, gb-T_2, gsp-T_2) and $f:(X,\tau)\to(Y,\sigma)$ is a g-irresolute (resp. gs-irresolute, gp-irresolute, αg-irresolute, gb-irresolute, gsp-irresolute) function, then $G(f)$ is strongly g-closed (resp. strongly gs-closed, strongly gp-closed, strongly αg-closed, strongly gb-closed, strongly gsp-closed).

Lemma 8. (Popa and Noiri [29]). *If $f:(X,m_X)\to(Y,m_Y)$ is a surjective function with a strongly m-closed graph, then (Y,m_Y) is m-T_2.*

Theorem 11. *Let (X,τ) (resp. (Y,σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y). If $f:(X,\tau)\to(Y,\sigma)$ is a surjective function with a strongly gm-closed graph, then (Y,σ) is gm-T_2.*

Proof. The proof follows from Definition 22 and Lemma 8.

Remark 15. If $f:(X,\tau)\to(Y,\sigma)$ is a surjective function with a strongly g-closed (resp. strongly gs-closed, strongly gp-closed, strongly αg-closed, strongly gb-closed, strongly gsp-closed), then Y is g-T_2 (resp. gs-T_2, gp-T_2, αg-T_2, gb-T_2, gsp-T_2).

Lemma 9. (Popa and Noiri [29]). *Let $f:(X,m_X)\to(Y,m_Y)$ be a function, where m_X has property $\mathscr{B}$. If f is an M-continuous surjection with an m-closed graph, then (X,m_X) is m-T_2.*

Theorem 12. *Let (X,τ) (resp. (Y,σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y) and* GMO(X) *a gm-structure satisfying property $\mathscr{B}$. If $f:(X,\tau)\to(Y,\sigma)$ is a gM-continuous surjection with a gm-closed graph, then X is gm-T_2.*

Proof. The proof follows from Definition 22 and Lemma 9.

Corollary 7. *If a function $f:(X,\tau)\to(Y,\sigma)$ is a gM-irresolute surjection with a gm-closed graph and* GMO(X) *has property $\mathscr{B}$, then (X,τ) is gm-T_2.*

Definition 23. Let A a subset of an m-space (X,m_X). A point $x\in X$ is called an m_θ-adherent point of A [31] if $\mathrm{mCl}(U)\cap A\neq\emptyset$ for every m_X-open set U containing x. The set of all m_θ-adherent points of A is called the *m_θ-closure* of A and is denoted by $\mathrm{mCl}_\theta(A)$. If $A=\mathrm{mCl}_\theta(A)$, then A is said to be *m_θ-closed*. The complement of a m_θ-closed set is said to be *m_θ-open*. The union of all m_θ-open sets contained in A is called the *m_θ-interior* of A and is denoted by $\mathrm{mInt}_\theta(A)$.

Remark 16. Let A be a subset of a topological space (X,τ) and m_X an m-structure on X. If $m_X=\tau$ (resp. SO(X), PO(X)), then $\mathrm{mCl}_\theta(A)=\mathrm{Cl}_\theta(A)$ [33] (resp. $\mathrm{sCl}_\theta(A)$ [13], $\mathrm{pCl}_\theta(A)$ [28]).

Lemma 10. (Popa and Noiri [31]). *Let A be a subset of an m-space (X,m_X). Then the following properties hold:*

(1) If A is m_X-open in X, then $\mathrm{mCl}_\theta(A)=\mathrm{mCl}(A)$*,*

(2) If m_X has property $\mathscr{B}$, then $\mathrm{mCl}_\theta(A)$ *is m_X-closed in X for every subset A of X.*

Definition 24. An m-space (X,m_X) is said to be *m-regular* [31] if for each m_X-closed set F of X and each point $x\notin F$, there exist disjoint m_X-open sets U and V such that $x\in U$ and $F\subset V$.

Lemma 11. (Popa and Noiri [31]). *Let (X,m_X) be an m-regular m-space. Then the following properties hold:*

(1) $\mathrm{mCl}_\theta(A) = \mathrm{mCl}(A)$ *for every subset A of X,*

(2) Every m_X*-open set is* m_θ*-open.*

Theorem 13. *Let* (Y, m_Y) *be an m-regular m-space and* m_Y *have property* $\mathscr{B}$. *For a function* $f : (X, m_X) \to (Y, m_Y)$*, the following properties are equivalent:*

(1) f is M-continuous;

(2) $f^{-1}(\mathrm{mCl}_\theta(B)) = \mathrm{mCl}(f^{-1}(\mathrm{mCl}_\theta(B)))$ *for every subset B of Y;*

(3) $f^{-1}(K) = \mathrm{mCl}(f^{-1}(K))$ *for every* m_θ*-closed set K of Y;*

(4) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every* m_θ*-open set V of Y.*

Proof. (1) ⇒ (2): Let B be any subset of Y. Then, by Lemma 10 $\mathrm{mCl}_\theta(B)$ is m_Y-closed in Y. By Theorem 3, we obtain $f^{-1}(\mathrm{mCl}_\theta(B)) = \mathrm{mCl}(f^{-1}(\mathrm{mCl}_\theta(B)))$.

(2) ⇒ (3): Let K be an m_θ-closed set of Y. Then $\mathrm{mCl}_\theta(K) = K$. Then by (2) we obtain $f^{-1}(K) = \mathrm{mCl}(f^{-1}(K))$.

(3) ⇒ (4): Let V be an m_θ-open set of Y. Then $Y - V$ is m_θ-closed and $f^{-1}(Y - V) = \mathrm{mCl}(f^{-1}(Y - V))$. Therefore, $X - f^{-1}(V) = X - \mathrm{mInt}(f^{-1}(V))$. Hence we obtain $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$.

(4) ⇒ (1): Let V be any m_Y-open set of Y. Since Y is m-regular, by Lemma 11 V is m_θ-open and by (4) we have $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$. By Theorem 1, f is M-continuous.

Theorem 14. *Let* (Y, m_Y) *be m-regular and let* m_X *and* m_Y *have property* $\mathscr{B}$. *For a function* $f : (X, m_X) \to (Y, m_Y)$*, the following properties are equivalent:*

(1) f is M-continuous;

(2) $f^{-1}(\mathrm{mCl}_\theta(B))$ *is* m_X*-closed for every subset B of Y;*

(3) $f^{-1}(K)$ *is* m_X*-closed for every* m_θ*-closed set K of Y;*

(4) $f^{-1}(V)$ *is* m_X*-open for every* m_θ*-open set V of Y.*

Proof. The proof follows from Theorem 13 and Lemma 3.

Let (X,τ) be a topological space and GMO(X) a gm-structure on X. For a subset A of X, we denote the gm-θ-closure of A by $\mathrm{gmCl}_\theta(A)$. If $A = \mathrm{gmCl}_\theta(A)$, then A is said to be gm_θ-$closed$. The complement of a gm_θ-closed set is said to be gm_θ-$open$.

By Theorems 13 and 14, we obtain the following theorems:

Theorem 15. *Let (X,τ) (resp. (Y,σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y) and let* GMO(Y) *be gm-regular and have property $\mathscr{B}$. For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is gM-continuous;

(2) $f^{-1}(\mathrm{gmCl}_\theta(B)) = \mathrm{mCl}_g(f^{-1}(\mathrm{gmCl}_\theta(B)))$ for every subset B of Y;

(3) $f^{-1}(K) = \mathrm{mCl}_g(f^{-1}(K))$ for every gm_θ-closed set K of Y;

(4) $f^{-1}(V) = \mathrm{mInt}_g(f^{-1}(V))$ for every gm_θ-open set V of Y.

Theorem 16. *Let (X,τ) (resp. (Y,σ)) be a topological space and GMO(X) (resp. GMO(Y)) a gm-structure on X (resp. Y), where* GMO*(X) and* GMO*(Y) have property $\mathscr{B}$, and let* GMO(Y) *be gm-regular. For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is gM-continuous;

(2) $f^{-1}(\mathrm{gmCl}_\theta(B))$ is gm-closed for every subset B of Y;

(3) $f^{-1}(K)$ is gm-closed for every gm_θ-closed set K of Y;

(4) $f^{-1}(V)$ is gm-open for every gm_θ-open set V of Y.

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