EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 17, No. 2, 2024, 1228-1243
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

The Stochastic Transportation Problem with Imprecise Data Using Lomax Distribution

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Abstract. The stochastic transportation problem with imprecise data is a probabilistic chance-constrained programming (CCP) problem in which the objective function is fuzzy and the supply and demand are random. Three models for the STP with ID with mixed-type restrictions that follow the Lomax distribution (LD) are created in this research. Optimising the transportation cost in FTP under probabilistic mixed constraints is the goal of the research project. To do this, the probabilistic mixed constraints are transformed into deterministic form using the LD, and the cost coefficient of the fuzzy objective function is changed with alpha cut representation. Numerical examples are presented to demonstrate the suggested models.

2020 Mathematics Subject Classifications: 90B36, 90C70

Key Words and Phrases: Stochastic Transportation Problem, Imprecise, Lomax, Mixed constraints

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1. Introduction

Making decisions is essential in many different fields. The main goal of the transportation problem (TP) is to reduce the cost of transferring goods and materials between producers and consumers, enabling the manufacturer to better satisfy consumer demands. An uneven transportation issue with mixed constraints (TPMC) arises when there is a noticeable expansion or reduction in both the capacity of a supply and the need of a demand in a typical transportation system. Many researchers TP with mixed constraints introduced first by Brigden [7], had proposed different models by V. Adlakha [2], A. Das, [11], S. Agarwal [3],S. Gupta, [19], V. Vidhya, [32], Rashid, [31], and extended multiobjectived in fuzzy by Gupta [20]. The terms for TP that are modelled under such circumstances are fuzzy transportation problems (FTP) and stochastic transportation problems (STP). Comparision of the TP with different authors defined on Gessesse [16], Jerbi [21], Al Qahtani et al. [30], Nasseri and Bavandi [28], Dutta et al. [13], Mahapatra et al. [26], Agrawal and Ganesh [5], Das and Lee

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DOI: https://doi.org/10.29020/nybg.ejpam.v17i2.5162

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The Lomax distribution is utilised extensively these days in numerous domains, including decision-making. This paper's goal is to use the SFTPMC model to optimise the overall cost of transportation under ambiguous situations. Generally speaking, fuzzy or probability theory is used to characterise uncertainty. However, because it requires sufficient knowledge, using fuzzy theory or probability theory with stochastic to show every indeterminacy is not always feasible. In actuality, the uncertainty theory is applied because low-frequency occurrences happen in our day-to-day lives. It becomes difficult to schedule an appropriate transport plan under these conditions in order to reduce the overall expenditures. This study offers a useful paradigm that the decision making may use to handle some unpredictable variables without compromising customer reliability. Agrawal [4], Aruna Chalam [9], Giri et al. [18][19], Acharya et al. [1], Gessesse [17], Maity et al. [27] were solved the STP articles that involve both fuzziness and randomness. This study might be expanded to include additional areas where plans or decisions must be made under unknown circumstances.

There are four sections to this paper: section 2 provides an overview of the fundamental terms and concepts, while section 3 presents the suggested lomax distribution, the mathematical formulation of the SFTPMC, the steps involved in solving it, and a description of the suggested strategy supported by numerical examples. Moreover, the conclusion in section 4 signifies the end of the paper.

1.1. Research Gap and Motivation

SFTPMC was used to discover and resolve uncertainty in linear programming problems (LPPs). Fuzzy transposition problems (FTP) are unique circumstances in which many academics have devised various algorithms and ranking functions to turn fuzzy data into crisp data in order to handle the FTP. Neutrosophic Transportation Problems (NTP) have recently been proposed as a way to use optimization techniques with unknown and indeterminate variables. Furthermore, many researchers have proposed Fuzzy Optimization Techniques (FOT), the Single-valued Trapezoidal Neutrosophic Transportation Problem (SVTNTP) [22], the Commercial Traveler Problem (CTP) [29], and a two-stage conventional transportation model to distribute relief aid to victims in uncertain scenarios [15] and multi-objective stochastic solid transportation problem (MOSSTP) uncertainity with weibull distribution [12]. In this paper proposed the SFTPMC, the stochastic transportation problem with imprecise data using the Lomax distribution provides a valuable tool for addressing the challenges of transportation planning and logistics in an uncertain environment, and it helps decision makers make more accurate and robust decisions by considering the stochastic nature of various factors and incorporating the Lomax distribution for modeling imprecise data.

2. Preliminaries

Let we recall first of all the fuzzy set theory and triangular fuzzy number and alpha-cut concepts used in this paper. Now a days the imprecise information modelling are done by using fuzzy concepts [33].

Definition 1. (Fuzzy set). *Let X be a crisp set and let $\tilde{A} \subseteq F$ be a fuzzy set where F is a fuzzy space. The fuzzy set $\tilde{A}$ is defined on crisp set X with a membership function $\mu_{\tilde{A}}$, can be expressed as follows*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$$

where $\mu_{\tilde{A}}$:$X \to [0,1]$.

Definition 2. (Triangular Fuzzy Number). *A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{a}}$ is given below*

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{if } a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

Definition 3. (α - cut). *The cut or level of a fuzzy set A is a crisp set defined by*

$$A_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}, \quad 0 < \alpha < 1.$$

A triangular fuzzy number (a,b,c) can be represented as an interval number form as follows.

$$(a, b, c) = [a + (b - a)\alpha, c - (c - b)\alpha]$$

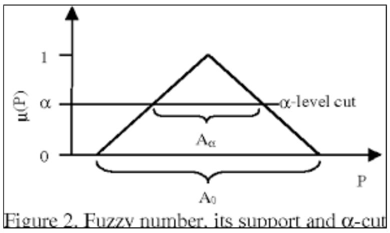

Figure 1: α-cut

Definition 4. (Linear Membership Function). *[6] A linear membership function can be defined as*

$$\mu_R(X) = \begin{cases} 0 & \text{if } x_{ij} < \underline{x}_{ij} \\ \frac{\bar{x}_{ij}-x_{ij}}{\bar{x}_{ij}-\underline{x}_{ij}} & \text{if } \underline{x}_{ij} < x_{ij} < \bar{x}_{ij} \\ 1 & \text{if } x_{ij} > \bar{x}_{ij} \end{cases}$$

In order to transform the fuzzy system to a deterministic set, the alpha cut representation using linear membership function is

$$\frac{\bar{x}_{ij} - x_{ij}}{\bar{x}_{ij} - \underline{x}_{ij}} = \alpha$$

such that

$$x_{ij} = (1 - \alpha)\bar{x}_{ij} + \alpha \underline{x}_{ij}, \ \forall \alpha \in [0, 1]$$

Definition 5. (Feasible solution). *Any set of* $\{x_{ij} \geq 0, i = 1, 2, ..., m; j = 1, 2, ...n\}$ *that satisfies all the constraints is called a feasible solution to the problem.*

Definition 6. (Optimal solution). *A feasible solution to the problem which minimizes the total shipping cost is called an optimal solution to the problem.*

2.1. Lomax Distribution

The Lomax distribution, also known as the Pareto Type II distribution [25], is a probability distribution commonly used to model heavy-tailed and skewed data.

Definition 7. (Lomax Distribution). *[23] [24] The Lomax distribution expressed as the random variable X has parameters scale parameter* β *and shape parameter* α *as*

$$X \sim Lomax(\beta, \alpha),$$

A Lomax random variable X with scale parameter β *and shape parameter* α *has probability density function*

$$f(x) = \frac{\alpha}{\beta}(1 + \frac{x}{\beta})^{-(\alpha+1)}, x > 0$$

for $\beta > 0, \alpha > 0$.
The cumulative distribution function is as follows by

$$F(x) = 1 - (1 + \tfrac{x}{\beta})^{-\alpha}$$

where α *and* β *are the shape and scale parameters, respectively, and* $x > 0, \beta > 0, \alpha > 0$.

One specific aspect that has gained attention in stochastic transportation modeling is the involvement of the Lomax distribution. The real motivation behind incorporating the Lomax distribution into stochastic transportation modeling lies in its ability to capture the variability and uncertainty in transportation parameters such as supply and demand. By using the Lomax distribution, researchers aim to better represent the range of possible values for these parameters and incorporate their probabilistic nature into the modeling process. By doing so, they can obtain more realistic and robust transportation models that account for the inherent uncertainty in the system. Additionally, the Lomax distribution provides flexibility in handling mixed-type restrictions and fuzzy objective functions, allowing for a more comprehensive analysis of the transportation problem under uncertain conditions.

Probabilistic programming is a mathematical programming approach that is utilized when some or all of the model parameters are random and follow a probability distribution. Charnes and Cooper [10] established the chance-constrained programming technique for individual probabilistic constraints. Rasha [14] revealed how to turn chance constraints into similar deterministic linear constraints in the lomax distribution, as presented here, offers equal deterministic constraints for individual and joint constraints.

The goal of the paper is to optimize the transportation cost in the Stochastic Transportation Problem under probabilistic mixed constraints. To achieve this, transforms the probabilistic mixed constraints into a deterministic form using the Lomax distribution. Overall, the real motivation of Lomax distribution involvement in stochastic transportation modeling is to enhance the accuracy and reliability of transportation models by incorporating the probabilistic nature of transportation parameters and capturing the variability and uncertainty present in real-world transportation systems.

3. Main results

3.1. Formulation of the Stochastic Transportation Problem by Using Lomax Distribution

The following is our definition of the transportation issue with mixed constraints. Let us assume that there are m origins, O_i, {i = 1,2,,,,,,m} divided into sets I_1, I_2, I_3, such that the origin $O_i(i \in I_1)$ must distribute at least at a_i supply units, $O_i(i \in I_2)$ must distribute precisely at a_i supply units, and $O_i(i \in I_3)$ may distribute at most at a_i supply units. Assume also that there are n destinations, D_j {j=1,2,...,n}, divided into sets J_1, J_2, J_3, where $D_j(j \in J_1)$ is required to receive a minimum of b_j units of demand, $D_j(j \in J_2)$ an exact amount of b_j units of demand, and $D_j(j \in J_3)$ a maximum of b_j units of demand.

The main goal is to reduce the overall cost of shipping, where c_{ij} represents the cost of shipping from $O_i to D_j$ and x_{ij} represents the amount of shipping from $O_i to D_j$. All requests and supply are considered to be non-negative.

Applying the constraints in the proposed TP model to the deterministic constraints is required to obtain the quantiles of a probability distribution function in a closed form. The fact that the Lomax distribution has the quantiles in their closed form is also another incentive to use it.

Mathematically, the transportation issue with mixed constraints may be expressed as follows:

$$Minimize z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to constraints,

$$P(\sum_{j=J} x_{ij} \geq a_i) \geq P(a_i), \qquad i \in I_1 = 1, 2, ...m_1$$

$$P(\sum_{j=J} x_{ij} = a_i) \geq P(a_i), \qquad i \in I_2 = m_1 + 1, m_1 + 2, ...m_2$$

$$P(\sum_{j=J} x_{ij} \leq a_i) \geq P(a_i), \qquad i \in I_3 = m_2 + 1, m_2 + 2, ...m$$

$$P(\sum_{i=I} x_{ij} \geq b_j) \geq P(b_j), \qquad j \in J_1 = 1, 2, ...n_1$$

$$P(\sum_{i=I} x_{ij} = b_j) \geq P(b_j), \qquad j \in J_2 = n_1+1, n_1+2, ...n_2$$

$$P(\sum_{i=I} x_{ij} \leq b_j) \geq P(b_j), \qquad j \in J_3 = n_2+1, n_2+2, ...n$$

and

$$x_{ij} \geq 0, i \in I, j \in J$$

where $P(a_i)$ and $P(b_j)$ are the probabilities of the random variables of supply and demand respecively and also which follows the lomax distribution. The parameters of the lomax distribution for supply a_i has shape parameter α_{a_i} and scale parameter β_{a_i}. Like wise the parameters of the lomax distribution for demand b_j has shape parameter α_{b_j} and scale parameter β_{b_j}.

The following cases are to be considered:
(i) Only a_i, i = 1, 2, . . . , m follows LD.
(ii) Only b_j, j = 1, 2, . . . , n follows LD.
(iii) Both a_i and b_j, i=1,2,.....m and j=1,2,....n follows LD.

Case 1.

Only a_i follows LD

For

$$P(\sum_{j=1}^{n} x_{ij} \geq a_i) \geq P(a_i), i \in I_1,$$

$$P(a_i \leq \sum_{j=1}^{n} x_{ij}) \geq P(a_i), i \in I_1$$

Let us consider $\sum_{j=1}^{n} x_{ij} = \delta_{ai}$ and $a_i \geq \xi_{ai}$then

$$P(a_i \leq \delta_{ai}) \geq P(a_i), i \in I_1$$

Now by using Lomax distribution PDF, integrating from

$$\int_{\xi_{ai}}^{\delta_{ai}} \frac{\alpha_{ai}}{\beta_{ai}}(1+\frac{a_i-\xi_{ai}}{\beta_{ai}})^{-(\alpha_{ai}+1)} da_i \geq P(a_i)$$

$$[-(1+\frac{a_i-\xi_{ai}}{\beta_{ai}})^{-\alpha_{ai}}]_{\xi_{ai}}^{\delta_{ai}} \geq P(a_i)$$

$$1+\frac{\delta_{ai}-\xi_{ai}}{\beta_{ai}} \leq -(P(a_i)-1)^{-\frac{1}{\alpha_{ai}}}$$

$$\delta_{ai} \leq \xi_{ai} - \beta_{ai}[1+(P(a_i)-1)^{-\frac{1}{\alpha_{ai}}}]$$

$$\sum_{j=1}^{n} x_{ij} \leq \xi_{ai} - \beta_{ai}[1+(P(a_i)-1)^{-\frac{1}{\alpha_{ai}}}]$$

Remarks.

(i) For

$$P(\sum_{j=1}^{n} x_{ij} \leq a_i) \geq P(a_i), i \in I_3,$$

then

$$\sum_{j=1}^{n} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j)+1)^{-\frac{1}{\alpha_{bj}}} - 1]$$

(ii) For

$$P(\sum_{j=1}^{n} x_{ij} = a_i) \geq P(a_i), i \in I_2,$$

then

$$\sum_{j=1}^{n} x_{ij} = \xi_{ai} - \beta_{ai}[1 + (P(a_i)-1)^{-\frac{1}{\alpha_{ai}}}]$$

We get the same deterministic result for both inequality types for supply and demand constraints by using lomax distribution to select the probability value at the 50% level in the supply and demand inequality constraint.

Case 2.
Only b_j follows LD
For

$$P(\sum_{i=1}^{m} x_{ij} \leq b_j) \geq P(b_j), j \in J_3,$$

$$P(b_j \geq \sum_{i=1}^{m} x_{ij}) \geq P(b_j), j \in J_3$$

Let us consider $\sum_{i=1}^{m} x_{ij} = \delta_{b_j}$ and $b_j \geq \xi_{bj}$then

$$P(b_j \leq \delta_{bj}) \geq P(b_j), j \in J_3$$

Now by using Lomax distribution PDF, integrating from

$$\int_{\delta_{bj}}^{\xi_{bj}} \frac{\alpha_{bj}}{\beta_{bj}}(1 + \frac{b_j - \xi_{bj}}{\beta_{bj}})^{-(\alpha_{bj}+1)} db_j \geq P(b_j)$$

$$[-(1 + \frac{b_j - \xi_{bj}}{\beta_{bj}})^{-\alpha_{bj}}]_{\delta_{bj}}^{\xi_{bj}} \geq P(b_j)$$

$$[1 + \frac{\delta_{bj} - \xi_{bj}}{\beta_{bj}}]^{-\alpha_{bj}} \geq P(b_j) + 1$$

$$\delta_{bj} \geq \xi_{bj} + \beta_{bj}[(P(b_j)+1)^{-\frac{1}{\alpha_{bj}}} - 1]$$

$$\sum_{i=1}^{m} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j)+1)^{-\frac{1}{\alpha_{bj}}} - 1]$$

Remarks.

(i) For

$$P(\sum_{i=1}^{m} x_{ij} \leq b_j) \geq P(b_j), j \in J_1,$$

then

$$\sum_{i=1}^{m} x_{ij} \geq \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}]$$

(ii) For

$$P(\sum_{i=1}^{m} x_{ij} = b_j) \geq P(b_j), j \in J_2,$$

then

$$\sum_{i=1}^{m} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1]$$

Case 3.

Both a_i and b_j follows LD By using the above cases (i) and (ii) we follows.

3.2. Formulation of the Modulations of the Stochastic Transportation Problem with the Imprecise Data by Using Lomax Distribution

The nature of the parameters (cost, supply, and demand) changes in many real-life scenarios, making it difficult for decision to make the best choice. It is possible to manage this scenario with fuzzy and random variables. We treat restrictions as random variables and cost as triangular fuzzy variables in our model. Depending on the circumstances around decision, there may or may not be uncertainty regarding supply or demand limits. As a result, we develop three STP models depending on the degree of uncertainty in the requirements.

Here the objective function can be used the alpha cut to transform the provided triangular fuzzy cost of problem into an equal deterministic cost.

$$Minimize z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}((1 - \hat{\alpha})\bar{x_{ij}} + \hat{\alpha}\underline{x_{ij}}) \tag{1}$$

The following models are to be considered:

(i) Only a_i, i = 1, 2, . . . , m follows uncertainty.

(ii) Only b_j, j = 1, 2, . . . , n follows uncertainty.

(iii) Both a_i, i = 1, 2, . . . , m and b_j, j = 1, 2, . . . , n follow uncertainty.

Model 1.

Only a_i follows uncertainty For modulation of the STP with the imprecise data by using LD is used for probabilistic only for supply constraints and demand constraints are certain.

The mathematics formulation can be represented as follows as

$$Minimizez = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}((1-\hat{\alpha})\bar{x_{ij}} + \hat{\alpha}\underline{x_{ij}})$$

subject to constriants,

$$\sum_{j=1}^{n} x_{ij} \leq \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], i \in I_1 \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], i \in I_2 \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], i \in I_3 \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, j \in J_1 \tag{5}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j \in J_2 \tag{6}$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j, j \in J_3 \tag{7}$$

and

$$x_{ij} \geq 0$$

Model 2.
Only b_j follows uncertainty For modulation of the STP with the imprecise data by using LD is used for probabilistic only for demand constraints and supply constraints are certain.

The mathematics formulation can be represented as follows as

$$Minimizez = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}((1-\hat{\alpha})\bar{x_{ij}} + \hat{\alpha}\underline{x_{ij}})$$

subject to constriants,

$$\sum_{j=1}^{n} x_{ij} \geq a_i, i \in I_1 \tag{8}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i \in I_2 \tag{9}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i, i \in I_3 \tag{10}$$

$$\sum_{i=1}^{m} x_{ij} \leq \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], j \in J_1 \tag{11}$$

$$\sum_{i=1}^{m} x_{ij} = \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], j \in J_2 \tag{12}$$

$$\sum_{i=1}^{m} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], j \in J_3 \tag{13}$$

and

$$x_{ij} \geq 0$$

Model 3.

Both a_i and b_j follows uncertainty For modulation of the STP with the imprecise data by using LD is used for probabilistic both supply and demand constraints.

The mathematics formulation can be represented as follows as

$$Minimize z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}((1-\hat{\alpha})\bar{x_{ij}} + \hat{\alpha}\underline{\mathrm{x}_{ij}})$$

subject to constriants,

$$\sum_{j=1}^{n} x_{ij} \leq \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], i \in I_1 \tag{14}$$

$$\sum_{j=1}^{n} x_{ij} = \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], i \in I_2 \tag{15}$$

$$\sum_{j=1}^{n} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], i \in I_3 \tag{16}$$

$$\sum_{i=1}^{m} x_{ij} \leq \xi_{ai} - \beta_{ai}[1 + (P(a_i) - 1)^{-\frac{1}{\alpha_{ai}}}], j \in J_1 \tag{17}$$

$$\sum_{i=1}^{m} x_{ij} = \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], j \in J_2 \tag{18}$$

$$\sum_{i=1}^{m} x_{ij} \geq \xi_{bj} + \beta_{bj}[(P(b_j) + 1)^{-\frac{1}{\alpha_{bj}}} - 1], j \in J_3 \tag{19}$$

and

$$x_{ij} \geq 0.$$

Example 1. *This section offers an example to show the effectiveness and applicability. By taking from the weibull distribution [8]. There are three factories and four repositories, and the coal plant produces a homogeneous output. The manufacturing capacity of coal plant A is precisely a_1 units, the manufacturing capacity of coal plant B is at least a_2 units, and the manufacturing capacity of coal plant C is the greatest amount of a_3 units. Similarly, demand capacity for repository 1 is at least b_1 units; demand capacity for repository 2 is at most b_2 units; and demand capacity for repository 3 is at least b_3 units. Repository 4 can hold precisely b_4 units in demand. If the cost of transportation for each unit from every coal plant to every deposit is c_{ij}, it is in the form of imprecise data*

	1	2	3	4	a$_i$
A	(0,0.5,1)	(2,4,6)	(1.5,2,3)	(2,4,5)	$=a_1$
B	(3,5,7)	(1.5,2,3)	(0,0.5,1)	(4,5.8,6)	$\geq a_2$
C	(7,8.5,9)	(2.5,3,4)	(3,4,5)	(2,3,4)	$\leq a_3$
b_j	$\geq b_1$	$\leq b_2$	$\geq b_3$	$= b_4$	

Table 1: Fuzzy Data

i.e., TFN by using the linear membership function of alpha cut representation it will be convert into crisp data. By using these formula with alpha value as "0", it represent in given table into crisp data,

$$x_{ij} = ((1-\hat{\alpha})\bar{x_{ij}} + \hat{\alpha}\underline{x_{ij}})\forall[0,1]$$

	1	2	3	4	a$_i$
A	1	6	3	5	$=a_1$
B	7	3	1	6	$\geq a_2$
C	9	4	5	4	$\leq a_3$
b_j	$\geq b_1$	$\leq b_2$	$\geq b_3$	$= b_4$	

Table 2: Crisp data

Here the following arbitrary nominal values of certain constants are supplied as a_1 = 20, a_2 = 16 a_3 = 25, and demand as b_1 = 11, b_2 = 13, b_3= 17, b_4= 14 in the following sections. Furthermore, probabilities are given as P_{a1} = 0.50, P_{a2}= 0.96, P_{a3}= 0.95, P_{b1}= 0.26, P_{b2}= 0.29, P_{b3}= 0.25, P_{b4}= 0.28. Since a_i and b_j are presumed to follow Lomax distribution, the distinct values for the parameters are ξ_{a1} = 19, ξ_{a2} =13, ξ_{a3}= 24, ξ_{b1}= 10, ξ_{b2} = 11, $\xi - b3$= 16, ξ_{b4} = 13 and also the scale parameter β_{ai} and β_{bj} both are as 2 and the shape parameter α_{ai} and α_{bj} are 3 as taken. Now the modulations of the stochastic transportation problem with the imprecise data by using lomax distribution as

Model-1. *For modulation of the STP with the imprecise data by using LD is used for probabilistic only for supply constraints and demand constraints are certain.*

Then By using the Lingo software, we obtain the optimal transportation cost as 92.52 and x_{11}= 11, x_{14}= 8.52, x_{23}= 17, x_{34}=5.48 and unit flow as 42.

Model-2. *For modulation of the STP with the imprecise data by using LD is used for probabilistic only for demand constraints and supply constraints are certain.*

	1	2	3	4	a_i
A	1	6	3	5	=19.52
B	7	3	1	6	≥ 16.85
C	9	4	5	4	≤ 23.6
b_j	≥ 11	≤ 13	≥ 17	$= 14$	

Table 3: Only a_i are uncertain

	1	2	3	4	a_i
A	1	6	3	5	=20
B	7	3	1	6	≥ 16
C	9	4	5	4	≤ 25
b_j	≥ 10.15	≤ 10.83	≥ 16.2	$= 12.84$	

Table 4: Only b_j are uncertain

Then By using the Lingo software, we obtain the optimal transportation cost as 87.56 and x_{11}= 10.15, x_{14}= 9.85, x_{23}= 16.2, x_{34}=2.99 and unit flow as 39.19.

Model-3. *For modulation of the STP with the imprecise data by using LD is used for probabilistic both supply and demand constraints.*

	1	2	3	4	a_i
A	1	6	3	5	=19.52
B	7	3	1	6	≥ 16.85
C	9	4	5	4	≤ 23.6
b_j	≥ 10.15	≤ 10.83	≥ 16.2	$= 12.84$	

Table 5: Both a_i and b_j are uncertain

Then By using the Lingo software, we obtain the optimal transportation cost as 87.08 and x_{11}= 10.15, x_{14}= 9.37, x_{23}= 16.2, x_{34}=3.47 and unit flow as 39.19.

3.3. Result and Discussion with comparison

The ideal results are compared to different distributions for the SFTPMC problem minimizes transportation costs relative to prior techniques. The table below illustrates the comparison of findings.

The table 6 shows the results obtained from different distribution functions. It justified the lomax distribution of the approach which attains very minimum when compare with other different distributions.

Distribution	Optimization Method	Model 1	Model 2	Model 3
Weibull	Optimal Transportation cost	93.67	95.75	96.42
Normal		102.68	105.8	107.4
Poisson		98.45	100.2	99.89
Lomax		92.52	87.56	87.08
Weibull	Units flow	42	42.84	42.84
Normal		49	49.85	49.85
Poisson		45	45.6	45.6
Lomax		42	39.19	39.19

Table 6: Comparing results with different distributions

4. Conclusion

This article presents a methodology for solving an SFTPMC that has probabilistic constraints together with the lomax distribution and a fuzzy integer for the objective function's cost coefficient. Alpha cut representation is used to transform the fuzzy objective value into a corresponding consistent objective function, and LD is used to transform each stochastic constraint into an analogous deterministic constraint. The Lomax distribution is a crucial aspect of stochastic transportation modeling, as it captures variability and uncertainty in transportation parameters like supply and demand. This paper aim to represent these values realistically and incorporate their probabilistic nature, resulting in more robust models and more comprehensive analysis under uncertain conditions.Additionally, three SFTPMC models—models 1, 2, and 3—are constructed, and Lingo software has been used to determine each model's ideal answer.A numerical example illustrating the models' performance is provided.

Due to this issue, SFTPMC is essential in many circumstances involving managerial decision-making, such as the planning of several intricate resource allocation issues in the context of industrial production, where supply and demand are essentially random variables. This model will function as an effective tool for the best planning in such circumstances. When compared with different distributions, the LD optimum solution is superior in this instance, due to minimum.

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