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On ψgs-Functions in Bitopological Spaces

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Abstract. A subset A of a bitopological space (X, τ_1, τ_2) is called an (i, j)-ψgs-closed set if (i, j)-$\psi cl(A) \subseteq U$ whenever $A \subseteq U$, U is (i, j)-semi-open in (X, τ_1, τ_2). In this work, the properties of this set are considered to investigate the concepts of ψgs-functions in bitopological spaces. Specifically, this study establishes some properties and provides characterizations of ψgs-open and ψgs-closed functions, ψgs-continuous functions, and ψgs-irresolute functions in bitopological spaces.

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1. Introduction

Topology is a branch of mathematics that studies geometric properties and spatial relations unaffected by the continuous changes in the shape or size of objects. A topological space is a set equipped with a topology, which is a collection of open sets satisfying certain axioms related to union, intersection, and inclusion of sets.

To deepen the understanding and extend the scope of topological concepts, the notion of bitopological spaces was introduced. A bitopological space is a generalization of topological spaces, where two different topologies are defined on the same underlying set. For instance, (X, τ_1, τ_2) is a bitopological space where X is a nonempty set and τ_1 and τ_2 are two different topologies.

Many concepts have been investigated in bitopological spaces. One of these is the concept of functions. Functions in bitopological spaces refer to the mappings between two bitopological spaces. For instance, $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a function in bitopological spaces where (X, τ_1, τ_2) and (Y, σ_1, σ_2) are two bitopological spaces. Important functions in bitopological spaces include open and closed functions, continuous functions, and irresolute functions.

Over the years, many researchers have introduced different types of functions in bitopological spaces. Noiri and Popa in [7] studied some properties of weakly open functions in

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bitopological spaces and obtained further characterizations. Subsequently, the properties and characterizations of weakly β-continuous functions in bitopological spaces were investigated by Tahiliani [10]. In 2012, Mukundhan and Nagaveni in [6] introduced and studied two new types of functions in bitopological spaces called (i,j)-quasi semi weakly g^*-open and (i,j)-quasi semi weakly g^*-closed functions. They investigated some properties and proved equivalent statements. In the same year, Khedr and Al-Saadi in [3] introduced and investigated the notions of a new class of g-closed functions and a class of semi-generalized closed functions in bitopological spaces. They further studied the properties of generalized semi closed and semi-generalized closed functions in bitopological spaces. In 2014, Mahmood and Hamdi in [4] created a special type of open and closed functions in bitopological spaces, namely quasi $(1,2)^*$ b-open functions and quasi $(1,2)^*$ b-closed functions. They gave some properties and equivalent statements of these concepts. In 2017, Sarma in [9] introduced the notion of weakly b-open functions in bitopological spaces, established some properties of this function, and investigated the relationships with some other types of spaces. Additionally, another type of function has been studied by Kadham and Hassan in [2] namely,the λ-continuous function. Furthermore, a generalization of λ-continuous functions in bitopological spaces called weakly λ-continuous functions, has been investigated by Moosa Meera, et. al in [5]. They studied several properties of weakly λ-continuous functions and obtained several characterizations. In 2021, Sivanthi and Leevathi in [8] introduced rg-continuous functions and rg-irresolute functions using rg-closed sets and characterized some of their properties. Recently, Atewi et.al in [1] introduced the concepts of ω-continuous functions in bitopological spaces and further characterized these concepts.

With all these concepts in mind, the author is motivated to define and introduce (i,j)-ψgs-open and (i,j)-ψgs-closed functions, (i,j)-ψgs-continuous functions, and (i,j)-ψgs-irresolute functions using (i,j)-ψgs-closed sets in bitopological spaces, and intends to investigate its properties and characterizations.

The findings of this study could serve as a resource for future research and possible applications. This may encourage other mathematics enthusiasts to discover more results and establish new research directions for further study.

2. Preliminaries

A collection τ of subsets of a nonempty set X is a *topology* on X if it satisfies the conditions: (i) $\varnothing, X \in \tau$, (ii) $\{M_\omega : \omega \in \Omega\} \subseteq \tau$ implies $\cup_{\omega\in\Omega} M_\omega \in \tau$, and (iii) $A, B \in \tau$ implies $A \cap B \in \tau$. If τ is a topology on X, then (X,τ) is called a *topological space*, and the elements of τ are called *τ-open* (or simply open) sets. A subset F of X is said to be *τ-closed* (or simply closed) if its complement $X \smallsetminus F$ is open. The *interior* of A, denoted by $int(A)$, is the union of all open sets contained in A. That is, $int(A) = \bigcup \{O \in \tau : O \subseteq A\}$. The *closure* of A, denoted by $cl(A)$, is the intersection of all closed sets containing A. That is, $cl(A) = \bigcap \{F \subseteq X : F \text{ is closed and } A \subseteq F\}$.

A set X endowed with two topologies, τ_1 and τ_2, is called a *bitopological space* (abbreviated as BTS) and denoted as (X, τ_1, τ_2). An open set in a BTS is denoted by *τ_i-open*, where $i \in \{1, 2\}$. The *interior* and *closure* of a subset A of X in a BTS are written as $int_i(A)$ and $cl_i(A)$, respectively.

The following definitions in BTS, as introduced in [11], are pertinent to this study.

Definition 1. A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j)-ψ *generalized semi-closed* (briefly, (i, j)-ψgs*-closed*) set if (i, j)-$\psi cl(A) \subseteq U$ whenever $A \subseteq U$, U is (i, j)-semi-open in $(X, \tau_1, \tau_2), i, j \in \{1, 2\}$ and $i \neq j$.

Definition 2. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. An element $x \in A$ is called (i, j)-ψgs*-interior point of* A if there exists an (i, j)-ψgs-open set O such that $x \in O \subseteq A$. The set of all (i, j)-ψgs-interior points of A is called the (i, j)-ψgs*-interior of* A and is denoted by (i, j)-ψgs-$int(A)$.

Definition 3. Let $A \subseteq X$. Then $x \in X$ is (i, j)-ψgs*-adherent* to A if $V \cap A \neq \varnothing$ for every (i, j)-ψgs-open set V containing x. The set of all (i, j)-ψgs-adherent points of A is called the (i, j)-ψgs*-closure* of A and is denoted by (i, j)-ψgs-$cl(A)$.

The following results from [11] are crucial for demonstrating certain findings in this study.

Corollary 1. *Let (Y, σ_i) be a topological space and (Y, σ_1, σ_2) be a bitopological space. Then every σ_i-closed set is (i, j)-ψgs-closed set.*

Remark 1. *Let (X, τ_1, τ_2) be a bitopological space and $A, B \subseteq X$. Then the following hold:*

(i) (i, j)-ψgs-$int(A) \subseteq A$;

(ii) (i, j)-ψgs-$int(A)$ is (i, j)-ψgs-open set; and

(iii) If $B \subseteq A$ such that B is (i, j)-ψgs-open set, then $B \subseteq (i, j)$-ψgs-$int(A)$.

Theorem 1. *Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. A set A is (i, j)-ψgs-open set, if and only if (i, j)-ψgs-$int(A) = A$.*

Remark 2. *Let (X, τ_1, τ_2) be a bitopological space and $A, B \subseteq X$. Then the following hold:*

(i) $A \subseteq (i, j)$-ψgs-$cl(A)$;

(ii) (i, j)-ψgs-$cl(A)$ is (i, j)-ψgs-closed set; and

(iii) If $A \subseteq B$ such that B is (i, j)-ψgs-closed set, then (i, j)-ψgs-$cl(A) \subseteq B$.

Theorem 2. *Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. A set A is (i, j)-ψgs-closed set, if and only if (i, j)-ψgs-$cl(A) = A$.*

3. ψgs-open function in BTS

In this section ψgs-open function is introduced in BTS and some of its properties are investigated.

Definition 4. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be an (i, j)-*ψ-generalized semi open* (briefly, (i, j)-*ψgs-open*) function if for every τ_i-open set A in X, $f(A)$ is (i, j)-ψgs-open set in Y, where $i \in \{1, 2\}$.

Theorem 3. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-ψgs-open function if and only if for every τ_i-open set A in X*

$$f(int_i(A)) = (i, j)\text{-}\psi gs\text{-}int(f(A)).$$

Proof. Let f be (i, j)-ψgs-open function and A be τ_i-open in X. It follows that $f(A)$ is (i, j)-ψgs-open set in Y where $i, j \in \{1, 2\}$. Since A is τ_i-open, $int_i(A) = A$, and so $f(int_i(A)) = f(A)$. Note that $f(A)$ is (i, j)-ψgs-open set, it follows that

$$(i, j)\text{-}\psi gs\text{-}int(f(A)) = f(A),$$

and hence $f(int_i(A)) = (i, j)$-ψgs-$int(f(A))$. Conversely, suppose

$$f(int_i(A)) = (i, j)\text{-}\psi gs\text{-}int(f(A))$$

and let A be τ_i-open set in X. Then $int_i(A) = A$, and so $f(int_i(A)) = f(A)$. It follows that, (i, j)-ψgs-$int(f(A)) = f(A)$. Thus, $f(A)$ is (i, j)-ψgs-open set by Theorem 1. Hence, by Definition 4, f is (i, j)-ψgs-open function. □

Theorem 4. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-ψgs-open function if and only if for any subset B of Y and for any τ_i-closed set A of X containing $f^{-1}(B)$ there exists an (i, j)-ψgs-closet set C of Y containing B such that $f^{-1}(C) \subseteq A$.*

Proof. Let f be (i, j)-ψgs-open function, $B \subseteq Y$, and A be τ_i-closed set of X containing $f^{-1}(B)$. Take $C = Y \setminus f(X \setminus A)$ and note that $f^{-1}(B) \subseteq A$. These imply that $B \subseteq C$. Since f is (i, j)-ψgs-open function and $X \setminus A$ is τ_i-open set, $f(X \setminus A)$ is (i, j)-ψgs-open in Y. Hence C is (i, j)-ψgs-closed set of Y. Moreover, $f^{-1}(C) \subseteq A$. Conversely, let G be τ_i-open set in X. Take $B = Y \setminus f(G)$. Then $X \setminus G$ is τ_i-closed set in X such that $f^{-1}(B) \subseteq X \setminus G$. By hypothesis, there exists (i, j)-ψgs-closed set C of Y containing B such that $f^{-1}(C) \subseteq X \setminus G$. Thus, $f(G) \subseteq Y \setminus C$. Note that $B \subseteq C$, and so $Y \setminus C \subseteq Y \setminus B = f(G)$. Now, $f(G) \subseteq Y \setminus C$ and $Y \setminus C \subseteq f(G)$. Hence, $f(G) = Y \setminus C$, which is (i, j)-ψgs-open set in Y. Therefore, f is (i, j)-ψgs-open function. □

Theorem 5. *Let $(X, \tau_1, \tau_2), (Y, \mu_1, \mu_2)$, and (Z, σ_1, σ_2) be three bitopological spaces. If $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is τ_i-open function and $g : (Y, \mu_1, \mu_2) \to (Z, \sigma_1, \sigma_2)$ is (i, j)-ψgs-open function, then $g \circ f : (X, \tau_1, \tau_2) \to (Z, \sigma_1, \sigma_2)$ is (i, j)-ψgs-open function.*

Proof. Let f be τ_i-open function. Then $f(A)$ is τ_i-open in Y for every τ_i-open set A in X. Since g is (i,j)-ψgs-open function, it follows that $g(f(A)) = (g \circ f)(A)$ is (i,j)-ψgs-open set in Z. Hence $g \circ f$ is (i,j)-ψgs-open function. □

4. ψgs-closed function in BTS

In this section ψgs-closed function is presented in BTS and some of its properties are explored.

Definition 5. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be an *(i,j)-ψ-generalized semi closed* (briefly, *(i,j)-ψgs-closed*) function if for every τ_i-closed set H in X, $f(H)$ is (i,j)-ψgs-closed set in Y, where $i \in \{1, 2\}$.

Theorem 6. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces and $H \subseteq X$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-ψgs-closed function if and only if for every τ_i-closed set H in X*

$$f(cl_i(H)) = (i,j)\text{-}\psi gs\text{-}cl(f(H)).$$

Proof. Let f be (i,j)-ψgs-closed function. It follows that $f(H)$ is (i,j)-ψgs-closed set in Y for every τ_i-closed set H in X. Since H is τ_i-closed, $cl_i(H) = H$, and so $f(cl_i(H)) = f(H)$. Also, since $f(H)$ is (i,j)-ψgs-closed set, (i,j)-ψgs-$cl(f(H)) = f(H)$, and hence $f(cl_i(H)) = (i,j)$-ψgs-$cl(f(H))$. Conversely, suppose

$$f(cl_i(H)) = (i,j)\text{-}\psi gs\text{-}cl(f(H))$$

for every τ_i-closed set H in X. Then $cl_i(H) = H$, and so $f(cl_i(H)) = f(H)$. It follows that, (i,j)-ψgs-$cl(f(H)) = f(H)$.Thus, $f(H)$ is (i,j)-ψgs-closed set by Theorem 2. Consequently, by Definition 5, f is (i,j)-ψgs-closed function. □

Theorem 7. *Let $(X, \tau_1, \tau_2), (Y, \mu_1, \mu_2)$, and (Z, σ_1, σ_2) be three bitopological spaces. If $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is τ_i-closed function and $g : (Y, \mu_1, \mu_2) \to (Z, \sigma_1, \sigma_2)$ is (i,j)-ψgs-closed function, then $g \circ f : (X, \tau_1, \tau_2) \to (Z, \sigma_1, \sigma_2)$ is (i,j)-ψgs-closed function.*

Proof. Suppose f be τ_i-closed function. Then $f(H)$ is τ_i-closed in Y for every τ_i-closed set H in X. Since g is (i,j)-ψgs-closed function, $g(f(H))$ is (i,j)-ψgs-closed set in Z. Hence $g \circ f$ is (i,j)-ψgs-closed function. □

5. ψgs-continuous function in BTS

In this section ψgs-continuous function is defined in BTS and some of its properties are established. Moreover, equivalent statements involving ψgs-continuous function, ψgs-open, and ψgs-closed functions are provided.

Definition 6. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be an (i, j)-ψ *generalized semi continuous* (briefly, (i, j)-ψgs*-continuous*) function if the inverse image of each σ_i-closed set in Y is (i, j)-ψgs-closed set in X, where $i \in \{1, 2\}$.

Theorem 8. *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces and* $A \subseteq X$. *If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i, j)-ψgs*-continuous, then*

$$f((i, j)\text{-}\psi gs\text{-}cl(A)) \subseteq cl_i(f(A)).$$

Proof. Let f be (i, j)-ψgs-continuous function and $A \subseteq X$. Then $f(A) \subseteq Y$. Note that $f(A) \subseteq cl_i(f(A))$, and so $A \subseteq f^{-1}(cl_i(f(A)))$. Also, note that $cl_i(f(A))$ is σ_i-closed in Y, and so $f^{-1}(cl_i(f(A)))$ is (i, j)-ψgs-closed set in X. Since $A \subseteq f^{-1}(cl_i(f(A)))$ and $f^{-1}(cl_i(f(A)))$ is (i, j)-ψgs-closed set, (i, j)-ψgs-$cl(A) \subseteq f^{-1}(cl_i(f(A)))$, by Remark 2(iii). Thus, $f((i, j)$-ψgs-$cl(A)) \subseteq cl_i(f(A))$. □

Theorem 9. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ *is* (i, j)-ψgs*-continuous in BTS if and only if the inverse image of every* μ_i*-open set in* Y *is a* (i, j)-ψgs*-open set in* X.

Proof. Let f be (i, j)-ψgs-continuous function and G be μ_i-open set in Y. Then $Y \setminus G$ is μ_i-closed set in Y. By assumption, $f^{-1}(Y \setminus G) = X \setminus f^{-1}(G)$ is (i, j)-ψgs-closed set in X. Hence, $f^{-1}(G)$ is (i, j)-ψgs-open set in X. Conversely, let B be μ_i-open set in Y such that $f^{-1}(B)$ is (i, j)-ψgs-open set in X. Then $X \setminus f^{-1}(B) = f^{-1}(Y \setminus B)$ is (i, j)-ψgs-closed set in X for every μ_i-closed set $Y \setminus B$ in Y. Hence f is (i, j)-ψgs-continuous function. □

Theorem 10. *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces and* $A \subseteq X$. *If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i, j)-ψgs*-continuous, then*

$$int_i(f(B)) \subseteq f((i, j)\text{-}\psi gs\text{-}int(B)).$$

Proof. Let f be (i, j)-ψgs-continuous function and $B \subseteq X$. Then $f(B) \subseteq Y$. Now, $int_i(f(B)) \subseteq f(B)$, and so $f^{-1}(int_i(f(B))) \subseteq B$. Note that $int_i(f(B))$ is σ_i-open in Y, and so $f^{-1}(int_i(f(B)))$ is (i, j)-ψgs-open set in X by Theorem 9. Since $f^{-1}(int_i(f(B))) \subseteq B$ and $f^{-1}(int_i(f(B)))$ is (i, j)-ψgs-open set, $f^{-1}(int_i(f(B))) \subseteq (i, j)$-$\psi gs$-$int(B)$ by Remark 1(iii). Thus,

$$int_i(f(B)) \subseteq f((i, j)\text{-}\psi gs\text{-}int(B)).$$

□

Theorem 11. *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces. Then the following statements are equivalent.*

(i) $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i, j)-ψgs*-continuous function;*

(ii) f^{-1} *is* (i, j)-ψgs*-open function; and*

(iii) f^{-1} *is* (i,j)*-*ψgs*-closed function.*

Proof.

$(i) \implies (ii)$: Let f be (i,j)-ψgs-continuous function. We want to show that $f^{-1} : (Y, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is (i,j)-ψgs-open function. Now, let A be σ_i-open set in Y. Since f is (i,j)-ψgs-continuous function, by Theorem 9, $f^{-1}(A)$ is (i,j)-ψgs-open set in X. Hence f^{-1} is (i,j)-ψgs-open function.

$(ii) \implies (iii)$: Suppose f^{-1} is (i,j)-ψgs-open function and B a σ_i-closed in Y. Then $Y \setminus B$ is σ_i-open in Y, and so $X \setminus f^{-1}(B) = f^{-1}(Y \setminus B)$ is (i,j)-ψgs-open set in X since f^{-1} is (i,j)-ψgs-open function. It follows that $f^{-1}(B)$ is (i,j)-ψgs-closed set in X, and thus f^{-1} is (i,j)-ψgs-closed function.

$(iii) \implies (i)$: Assume f^{-1} is (i,j)-ψgs-closed function and C be σ_i-closed set in Y. Then, by assumption, $f^{-1}(C)$ is (i,j)-ψgs-closed set in X. Thus, by Definition 6, f is (i,j)-ψgs-continuous function. □

6. ψgs-irresolute function in BTS

In this section, the ψgs-irresolute function is introduced and defined within BTS, with several of its properties established. Furthermore, a characterization of the ψgs-irresolute function is presented.

Definition 7. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-ψ *generalized semi irresolute* (briefly, (i,j)-ψgs*-irresolute*) function if the inverse image of each (i,j)-ψgs-closed set in Y is (i,j)-ψgs-closed set in X, where $i \in \{1,2\}$.

Theorem 12. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i,j)*-*ψgs*-irresolute in BTS if and only if the inverse image of every* (i,j)*-*ψgs*-open set in* Y *is a* (i,j)*-*ψgs*-open set in* X*.*

Proof. Let f be (i,j)-ψgs-irresolute function and let H be (i,j)-ψgs-open set in Y. Then $Y \setminus H$ is (i,j)-ψgs-closed set in Y. By assumption, $f^{-1}(Y \setminus H) = X \setminus f^{-1}(H)$ is (i,j)-ψgs-closed in X. Hence, $f^{-1}(H)$ is (i,j)-ψgs-open set in X. Conversely, let B be (i,j)-ψgs-closed set in Y. Then, $Y \setminus B$ is (i,j)-ψgs-open set in Y. Since the inverse image of every (i,j)-ψgs-open set in Y is a (i,j)-ψgs-open set in X, $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$ is (i,j)-ψgs-open in X. Thus, $f^{-1}(B)$ is (i,j)-ψgs-closed in X, consequently, f is (i,j)-ψgs-irresolute function. □

Theorem 13. *Let* (X, τ_1, τ_2) *and* (Y, σ_1, σ_2) *be two bitopological spaces. If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i,j)*-*ψgs*-irresolute, then* f *is* (i,j)*-*ψgs*-continuous.*

Proof. Let f be (i,j)-ψgs-irresolute function and A be σ_i-closed set in Y. By Corollary 1, A is (i,j)-ψgs-closed set in Y. Since f is (i,j)-ψgs-irresolute function, $f^{-1}(A)$ is (i,j)-ψgs-closed set in X. Therefore, f is (i,j)-ψgs-continuous. □

Theorem 14. *If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\mu_1,\mu_2)$ are (i,j)-ψgs-irresolute functions, then $g\circ f:(X,\tau_1,\tau_2)\to(Z,\mu_1,\mu_2)$ is (i,j)-ψgs-irresolute.*

Proof. Let A be (i,j)-ψgs-closed set in Z. Since g is (i,j)-ψgs-irresolute function, $g^{-1}(A)$ is (i,j)-ψgs-closed set in Y. Moreover, $f^{-1}(g^{-1}(A))$ is (i,j)-ψgs-closed set in X since f is (i,j)-ψgs-irresolute function. Note that $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$. Therefore, $g\circ f$ is (i,j)-ψgs-irresolute. □

Theorem 15. *If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\mu_1,\mu_2)$ are (i,j)-ψgs-irresolute functions, then $g\circ f:(X,\tau_1,\tau_2)\to(Z,\mu_1,\mu_2)$ is (i,j)-ψgs-continuous.*

Proof. Let B be μ_i-closed set in Z. By Corollary 1, B is (i,j)-ψgs-closed set in Z. Since g is (i,j)-ψgs-irresolute function, $g^{-1}(B)$ is (i,j)-ψgs-closed set in Y. Also, since f is (i,j)-ψgs-irresolute function, $f^{-1}(g^{-1}(B))=(g\circ f)^{-1}(B)$ is (i,j)-ψgs-closed set in X. Therefore, $g\circ f$ is (i,j)-ψgs-continuous. □

Theorem 16. *If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (i,j)-ψgs-irresolute function and $g:(Y,\sigma_1,\sigma_2)\to(Z,\mu_1,\mu_2)$ is (i,j)-ψgs-continuous function, then*

$$g\circ f:(X,\tau_1,\tau_2)\to(Z,\mu_1,\mu_2) \text{ is } (i,j)\text{-}\psi gs\text{-continuous.}$$

Proof. Let V be μ_i-closed set in Z. Then $g^{-1}(V)$ is (i,j)-ψgs-closed set in Y since g is (i,j)-ψgs-continuous function. It follows that $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$ is (i,j)-ψgs-closed set in X since f is (i,j)-ψgs-irresolute function. Therefore, $g\circ f$ is (i,j)-ψgs-continuous. □

7. Conclusion

In this paper, the author defined and introduced (i,j)-ψgs-open and (i,j)-ψgs-closed functions, (i,j)-ψgs-continuous functions, and (i,j)-ψgs-irresolute functions using (i,j)-ψgs-closed sets in bitopological spaces. The properties and characterizations of these functions were investigated in detail. The results of this study are purely theoretical; therefore, further research into the practical applications of these findings is recommended.

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