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Topologies on Hyper BCK-Algebra $[0, 1]$

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Abstract. In this paper, we introduce the definition of a hyper operation $*$ on the set [0,1], and with this hyper operation, we will show that [0,1] is a hyper BCK-algebra. We also investigate the topologies that will be formulated with $B_R([0,1])$ and $B_L([0,1])$ and show some topology of $R_[0,1](A)$ and $L_[0,1](A)$. Furthermore, we investigate a basis for the intersection of topologies $\tau_R(H)$ and $\tau_L(H)$.

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1. Introduction

In 1966, Imai and Isèki [1] introduced the concept of BCK-algebra as a generalization of the concept of set-theoretic difference and propositional calculi. The study of algebraic hyperstructure theory (or multialgebras) was introduced in 1934 by F. Marty [2] at the 8^th Congress of Scandinavian Mathematics. Since then it becomes the interest of many researchers. Recently, Jun, et al.,[4] proposed hyperstructure theory on BCK-algebras and they were able to prove that a hyper BCK-algebra is a generalization of a BCK-algebra.

In the paper of Patangan and Canoy [3, 4], they defined the sets $R_H(A) = \{x \in H : a \ll x, \forall a \in A\} = \{x \in H : 0 \in a * x, \forall a \in A\}$ and $L_H(A) = \{x \in H : x \ll a, \forall a \in A\} = \{x \in H : 0 \in x * a, \forall a \in A\}$ by the right applications of hyperorder on H, respectively. They showed that $\mathcal{B}_R(H)$ consisting of the sets $R_H(A)$, is a basis for some topology $\tau_R(H)$ on a hyper BCK-algebra via right application of hyperorder. Also, $\mathcal{B}_L(H)$ consisting of the sets $L_H(A)$, is a basis for some topology $\tau_L(H)$ on a hyper BCK-algebra via left application of hyperorder.

This paper is motivated by the work of Patangan and Canoy [3] on A Topology on a Hyper BCK-Algebra, as published in JP Journal of Algebra, Number Theory and Applications. In this paper, we define a hyperoperation $*$ on the set [0, 1], and with this operation, we will show that [0, 1] is a hyper BCK-algebra. We investigate the basis for intersection of topologies $\tau_R(H)$ and $\tau_L(H)$. We also investigate the topologies that will be formulated or generated with bases $\mathcal{B}_R([0, 1])$ and $\mathcal{B}_L([0, 1])$ and their intersection.

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2. Known Results

Definition 1. [5] Let $\mathcal{P}(H)$ be the power set of a nonempty set H. Consider $\mathcal{P}(H)^* = \mathcal{P}(H) \setminus \{\emptyset\}$. A **hyperoperation** on a nonempty set H is a function $*$: $H \times H \to \mathcal{P}(H)^*$. The image of $(x, y) \in H \times H$ under $*$ is denoted by $x * y$. If $x \in H$ and A, B are nonempty subsets of H, then we define

(i) $A * B = \bigcup_{a\in A,\, b\in B} a * b$;

(ii) $A * x = A * \{x\}$; and,

(iii) $x * B = \{x\} * B$.

Definition 2. [5] Let $x, y \in H$ and $A, B \subseteq H$. Then

(i) $x \ll y$ if and only if $0 \in x * y$; and

(ii) $A \ll B$ if and only if for any $a \in A$, there exist $b \in B$ such that $a \ll b$. We call "$\ll$" a hyperorder on H.

Definition 3. [6] A hyper BCK-algebra is a nonempty set H endowed with a hyperoperation "$*$" and a constant 0 satisfying the following axioms: for all $x, y, z \in H$,

(i) $(x * z) * (y * z) \ll x * y$,

(ii) $(x * y) * z = (x * z) * y$,

(iii) $x * H \ll x$,

(iv) $x \ll y$ and $y \ll x$ imply $x = y$.

Proposition 1. [6] In a hyper BCK-algebra $(H, *, 0)$, the condition (iii) of Definition 3 is equivalent to the condition $x * y \ll \{x\}$ for all $x, y \in H$.

Theorem 1. [7] Let $\mathcal{B} \subseteq \tau$. The following two properties of $\mathcal{B}$ are equivalent:

(1) $\mathcal{B}$ is a basis for τ.

(2) For each $G \in \tau$ and each $x \in G$ there is a $U \in \mathcal{B}$ with $x \in U \subseteq G$.

Theorem 2. [7] Let $X \neq \emptyset$. A class of subsets $\mathcal{B}$ of X is a basis for some topology τ on X if it satisfies the following:

(i) $\mathcal{B}$ covers X, and

(ii) For each $x \in \mathcal{U}_\alpha \cap \mathcal{U}_\beta$, there exists $\mathcal{U} \in \mathcal{B}$ such that $x \in \mathcal{U} \subseteq \mathcal{U}_\alpha \cap \mathcal{U}_\beta$ where $\mathcal{U}_\alpha, \mathcal{U}_\beta \in \mathcal{B}$.

Definition 4. [7] A space X is said to be connected if X is not the union of two disjoint open sets. Otherwise, it is said to be disconnected.

Definition 5. [3] Let H be a hyper BCK-algebra and $A \subseteq H$. Then the set $R_H(A)$ is defined as $R_H(A) = \{x \in H : a \ll x \text{ for all } a \in A\} = \{x \in H : 0 \in a * x \text{ for all } a \in A\}$. If $A = \{a\}$, then we write $R_H(\{a\}) = R_H(a)$.

Definition 6. [3] An element a of H of a hyper BCK-algebra H is called a **hyperatom** if $x \ll a$ implies $x = 0$ or $x = a$ for all $x \in H$. Denote $A(H)$ the set of all hyperatoms of H and $A^*(H) = A(H) \setminus \{0\}$. Obviously, $0 \in A(H)$. If each element of H is a hyperatom, then H is said to be hyperatomic, that is, $A^*(H) = H \setminus \{0\}$. A hyper BCK-algebra H is called **ordered** if the hyperorder "$\ll$" is transitive.

Proposition 2. [3] Let A and B be subsets of H. the the following hold:

(i) $R_H(\emptyset) = H$.

(ii) If $A \subseteq B$, then $R_H(B) \subseteq R_H(A)$.

(iii) If H is an ordered hyper BCK-algebra, then $R_H(R_H(A)) \subseteq R_H(A)$.

Theorem 3. [3] Let H be a hyper BCK-algebra then $\mathcal{B}_R(H) = \{R_H(A) : A \subseteq H\}$ is a basis for some topology in H.

Remark 1. [3] Let A and B be nonempty subsets of H. $R_H(A) \cap R_H(B) = R_H(A \cup B)$.

Definition 7. [4] Let H be a hyper BCK-algebra and $A \subseteq H$. Then the set $L_H(A)$ is defined as $L_H(A) = \{x \in H : x \ll a, \text{ for all } a \in A\} = \{x \in H : 0 \in x * a, \text{ for all } a \in A\}$. If $A = \{a\}$, the we write $L_H(\{a\}) = L_H(a)$.

Theorem 4. [8] The set $[0, 1]$, together with the binary operation "$*$", is a BCK-algebra.

3. A Hyperoperation on [0, 1]

On the set $[0, 1]$, we define "$*$" as follows: $x * y = \{x - y\}$, if $x > y$ and $x * y = \{0\}$ if $x \leq y$.

Lemma 1. *For each $x, y, z \in [0, 1]$, $(x * z) * (y * z) \ll x * y$.*

Proof. Case 1: $x \geq y$
If $y \leq z \leq x$ then $x - z \geq 0$ so that

$$\begin{aligned}(x * z) * (y * z) &= \{x - z\} * \{0\} \\ &= \{x - z - 0\} \\ &= \{x - z\}\end{aligned}$$

while $x*y = \{x-y\}$ so that $(x*z)*(y*z) = \{x-z\} \ll \{x-y\}$ for $0 \in \{x-z\}*\{x-y\} = \{0\}$, since $x - z \leq x - y$.
If $z \leq y \leq x$ then $x - y \leq x - z$ so that

$$\begin{aligned}(x*z)*(y*z) &= \{x-z\}*\{y-z\}\\ &= \{x-z-(y-z)\}\ for\ x-z \geq y-z\\ &= \{x-y\}\\ &= x*y\end{aligned}$$

Thus, $(x*z)*(y*z) = \{x-y\} \ll x*y$ for $0 \in \{x-y\}*\{x-y\} = \{0\}$.
If $y \leq x \leq z$ then $(x*z)*(y*z) = \{0\}*\{0\} = \{0\}$, $x*y = \{x-y\}$ and $0 \in 0*\{x-y\} = \{0\}$.
Thus, $(x*z)*(y*z) = \{0\} \ll \{x-y\} = x*y$.
Case 2: If $x \leq y$ then $x*y = \{0\}$.
If $x \leq z \leq y$ then $(x*z)*(y*z) = \{0\}*\{y-z\} = \{0\} \ll \{0\} = x*y$ for $0 \in 0*0 = \{0\}$.
If $z \leq x \leq y$ then $(a*z)*(y*z) = \{x-z\}*\{y-z\} = \{0\}$ for $x-z \leq y-z$ so that$(x*z)*(y*z) = \{0\} \ll \{0\} = x*y$ for $0 \in 0*0 = \{0\}$.
If $x \leq y \leq z$ then $(x*z)*(y*z) = \{0\}*\{0\} = \{0\} \ll \{0\} = x*y$.

■

Lemma 2. *For all* $x, y, z \in [0,1]$, $(x*y)*z = (x*z)*y$.

Proof. Case 1: If $x \geq y$.
If $y \leq z \leq x$, $(x*y)*z = \{x-y\}*z = \begin{cases}\{0\}, & \text{if} \quad x-y \leq z,\\ \{x-y-z\}, & \text{if} \quad x-y > z\end{cases}$
while
$(x*z)*y = \{x-z\}*y = \begin{cases}\{x-z-y\}, & \text{if} \quad x-z \geq y,\\ \{0\}, & \text{if} \quad x-z \leq y\end{cases}$
Note that when the result of $(x*y)*z$ is $\{0\}$, it happens when $x-y < z$ and it is just the same as $x-z < y$. Also if the result is $\{x-y-z\}$, it happens when $x-y > z$ and it is just the same as $x-z > y$. Thus, $(x*y)*z = (x*z)*y$.
If $z \leq y \leq x$ then

$$(x*y)*z = \{x-y\}*z = \begin{cases}\{x-y-z\}, & \text{if} \quad x-y \geq z\\ \{0\}, & \text{if} \quad x-y < z\end{cases}$$

$$\begin{aligned}(x*z)*y &= \{x-z\}*y\\ &= \begin{cases}\{x-z-y\}, & \text{if} \quad x-z \geq y\\ \{0\}, & \text{if} \quad x-z < y\end{cases}\\ &= \begin{cases}\{x-y-z\}, & \text{if} \quad x-y \geq z\\ \{0\}, & \text{if} \quad x-y < z\end{cases}\\ &= (x*y)*z\end{aligned}$$

If $y \leq x \leq z$ then $(x*y)*z = \{x-y\}*z = \{0\}$ for $x-y < z$ and

$$\begin{aligned}(x*z)*y &= \{0\}*y \\ &= \{0\} \text{ for } 0 \leq y \\ &= (x*y)*z\end{aligned}$$

Case 2: If $x < y$
If $x \leq z < y$, then $(x*y)*z = \{0\}*z = \{0\}$ and $(x*z)*y = \{0\}*y = \{0\} = (x*y)*z$.
If $z \leq x \leq y$, then $(x*y)*z = \{0\}*z = \{0\}$ and

$$\begin{aligned}(x*z)*y &= \{x-z\}*y \\ &= \{0\} \text{ for } x-z < y \\ &= (x*y)*z\end{aligned}$$

If $x \leq y \leq z$ then $(x*y)*z = \{0\}*z = \{0\}$ and $(x*z)*y = \{0\}*y = \{0\} = (x*y)*z$.
Thus $(x*y)*z = (x*z)*y$.

■

Lemma 3. *For each $x \in [0,1]$, $x*[0,1] \ll \{x\}$.*

Proof. : Case 1: If $x \geq y$, then $x*y = \{x-y\}$ and that $x-y < x$ so that $x-y \ll x$ for $0 \in x-y*x = \{0\}$.
Case 2: If $x < y$, then $x*y = \{0\} \ll \{x\}$ for $0 \in 0*x = \{0\}$.
In any case, for all $y \in H$, $x*y \ll \{x\}$. Thus $x*[0,1] \ll \{x\}$.

■

Lemma 4. *For all $x, y \in [0,1]$ $x \ll y$ and $y \ll x$ implies $x = y$.*

Proof. : Suppose $x \ll y$ and $y \ll x$. Then

$$0 \in x*y = \begin{cases} \{x-y\}, & \text{if} \quad x > y \\ \{0\}, & \text{if} \quad x \leq y \end{cases}$$

and

$$0 \in y*x = \begin{cases} \{y-x\}, & \text{if} \quad y > x \\ \{0\}, & \text{if} \quad y \leq x \end{cases}$$

This implies that $x*y = \{0\} = y*x$. Hence, $x \leq y$ and $y \leq x$ implies $x = y$.

■

Theorem 5. $([0,1], *, 0)$ *is a hyper BCK-algebra.*

Proof. : It follows immediately from Lemmas 1, 2, 3 and 4.

■

4. The topology $\tau_R[0,1]$

By Definition 5, $R_H(A) = \{x \in H : a \ll x,\ \forall\, a \in A\} = \{x \in H : 0 \in a * x,\ \forall\, a \in A\}$ and considering the hyper BCK-algebra $[0,1]$. Note that from now on $H = [0,1]$, we have the following example.

Example 1. Consider $A = \left\{\frac{1}{2}, \frac{1}{3}\right\}$. Then

$$\begin{aligned} R_H(A) &= \{x \in [0,1] : 0 \in a * x,\ \forall\, a \in A\} \\ &= \left\{x \in [0,1] : 0 \in a * x,\ \forall\, a \in \left\{\frac{1}{2}, \frac{1}{3}\right\}\right\} \\ &= \left\{x \in [0,1] : 0 \in \frac{1}{2} * x \ and\ 0 \in \frac{1}{3} * x\right\} \end{aligned}$$

Note that for 0 to be in $\frac{1}{2} *\ x$, $\frac{1}{2} *\ x = \{0\}$ this implies $x \geq \frac{1}{2}$ or $\left[\frac{1}{2}, 1\right]$. Also for $0 \in \frac{1}{3} *\ x$, $x \geq \frac{1}{3}$ or $\left[\frac{1}{3}, 1\right]$. Since $R_H(A) = \bigcap_{a\in A} R_H(a)$ this implies that $R_H(\{\frac{1}{2}, \frac{1}{3}\}) = [\frac{1}{2}, 1] \cap [\frac{1}{3}, 1] = [\frac{1}{2}, 1]$. Therefore, $R_H\left(\{\frac{1}{2}, \frac{1}{3}\}\right) = \left[\frac{1}{2}, 1\right]$.

Theorem 6. *Let $A \subseteq\ H$, $R_H(A) = [sup\ A, 1] \cap H$. In particular,*

$$R_H(A) = \begin{cases} H & if\ A = \varnothing \\ [sup\ A, 1] & if\ A \neq \varnothing. \end{cases}$$

.

Proof. : Note that $x \in R_H(A)$ if and only if $0 \in a * x$ for all $a \in A$. Now, $0 \in a * x$ for all $a \in A$ if and only if $a \leq x$ for all $a \in A$, that is, $x \in H$ is an upperbound of A. Hence, $x \in R_H(A)$ if and only if $x \in [sup\ A, 1] \cap H$. This shows that $R_H(A) = [sup\ A, 1] \cap H$. If $A = \varnothing$, then $sup\ A = -\infty$ and so $R_H(A) = H$. Otherwise, $sup\ A \in H$ implying that $R_H(A) = [sup\ A, 1]$.

■

Example 2. Consider $A = (\frac{1}{8}, \frac{1}{2})$. Then the $sup\ A = \frac{1}{2}$. Thus, $R_H(A) = [\frac{1}{2}, 1]$.

The next result follows from Theorem 6.

Corollary 1. *Let $a \in [0,1]$. Then $R_{[0,1]}(a) = [a, 1]$.*

Proposition 3. *$(H, *, 0)$ is not hyperatomic. In particular, $A(H) = 0$.*

Proof. Clearly, $0 \in H$. Let $a \in H \setminus \{0\}$, i.e., $a \in (0,1]$. Since $\frac{a}{2} < a$, $\frac{a}{2} * a = \{0\}$. Here, we find that $0 \in \frac{a}{2}$ but $\frac{a}{2} \notin \{0, a\}$. Thus $a \notin A(H)$, showing that $A(H) = \{0\}$. Therefore, $(H, *, 0)$ is not hyperatomic.

■

Corollary 2. $\mathcal{B}_R([0,1]) = \{[r,1] : r \in [0,1]\}$.

Let $\tau_R(H)$ be the topology generated by $\mathcal{B}_R(H)$. That is, for each $G \in \tau_R(H)$, $G = \bigcup \mathcal{B}_i \quad, i \in K \subseteq \mathcal{B}_R(H)$. Note that for any $a \in [0,1]$, the set $(a,1]$ is open for $(a,1] = \bigcup_{n=1}^{\infty} \left[a + \frac{1}{n}, 1\right]$.

Example 3. *The following are open set in* $[0,1]$*:*

- *Sets of the form* $(r,1]$ *for* $(r,1] = \bigcup_{n=1}^{\infty} \left[r + \frac{1}{n}, 1\right]$
- $\{1\}$ *is open for* $\{1\} = [1,1]$

Example 4. *The following are closed sets in* $[0,1]$*:*

- $[0,r)$ *for* $[0,r) = [r,1]^c$
- $[0,r]$ *for* $[0,r] = (r,1]^c$*,* $r \in [0,1]$

Thus, the next theorem follows:

Theorem 7. *In a hyper BCK-algebra* $[0,1]$*,* $\tau_R([0,1]) = \{\varnothing, \ [0,1], (r,1], [r,1] : \ r \in [0,1]\}$.

Proof. Let $G \in \tau_R([0,1])$, and $G \neq \varnothing$. Then $G = \bigcup_{i \in K \subseteq \mathcal{B}_R([0,1])} \mathcal{B}_i$. Thus, $\mathcal{B}_i$ are of the form $[r_i, 1]$, $G = \bigcup [r_i, 1] = [r,1], r_0 = inf\{r_i\}$ and $(r,1] = \bigcup_{n=1}^{\infty} \left[r + \frac{1}{n}, 1\right]$. Thus, $\tau_R([0,1]) = \{\varnothing, \ [0,1], (r,1], [r,1] : \ r \in [0,1]\}$.

■

Theorem 8. $[0,1]$ *with topology* $\tau_R([0,1])$ *is connected.*

Proof. Suppose $[0,1]$ is disconnected. Then, there exist disjoint open sets A, B such that $[0,1] = A \cup B$. Since A is open, $A = [r,1]$ or $(r,1]$. If $A = [r,1]$ then $B = [0,r)$. If $A = (r,1]$ then $B = [0,r]$. Whether $B = [0,r)$ or $[0,r]$, B is not open. This contradicts the statement that A and B are open sets. Therefore, by Definition 4, $[0,1]$ is connected.

■

5. The topology $\tau_L[0,1]$

Example 5. *Consider the hyper BCK-algebra* $[0,1]$ *and* $A = \{\frac{1}{2}, \frac{1}{3}\}$.

$$L_H(A) = \{x \in [0,1] : 0 \in x * a, \ \forall \ a \in A\}$$

$$= \left\{ x \in [0,1] : 0 \in x * a, \ \forall \, a \in \{\frac{1}{2}, \frac{1}{3}\} \right\}$$
$$= \left\{ x \in [0,1] : 0 \in x * \frac{1}{2} \ and \ 0 \in x * \frac{1}{3} \right\}$$

Note that for 0 *to be in* $x * \frac{1}{2}$, $x * \frac{1}{2} = \{0\}$ *implies* $x \leq \frac{1}{2}$ *or* $\left[0, \frac{1}{2}\right]$. *Also for* $0 \in x * \frac{1}{3}$, $x * \frac{1}{3} = \{0\}$ *which implies* $x \leq \frac{1}{3}$ *or* $\left[0, \frac{1}{3}\right]$. *For these two to hold, Therefore* $L_H(\{\frac{1}{2}, \frac{1}{3}\}) = \left[0, \frac{1}{3}\right]$.

The proof of the following theorems are analogous to that topology $\tau_R[0,1]$.

Theorem 9. *In the hyper BCK-algebra* $[0,1]$, $L_H(a) = [0,a]$.

Theorem 10. *Let* $A \subseteq [0,1]$. $L_{[0,1]}(A) = [0,r]$ *where* $r = inf A$.

Theorem 11. *In the hyper BCK-algebra* $[0,1]$, $\mathcal{B}_L([0,1]) = \{[0,r] : r \in [0,1]\}$.

The following examples are open and closed sets in $[0,1]$.

Example 6. *The following are open set in* $[0,1]$:

- $[0,a)$ *for* $[0,a) = \bigcup_{n=1}^{\infty} \left[0, a - \frac{1}{n}\right]$
- $\{0\}$ *for* $\{0\} = [0,0]$

Example 7. *The following are closed set in* $[0,1]$:

- $(a,1]$ for $(a,1]^c = [0,a]$ *is open*
- $[a,1]$ for $[a,1]^c = [0,a)$ *is open*
- $(0,1]$ *for* $(0,1] = \{0\}^c$

Thus we have the following result.

Theorem 12. *In a hyper BCK-algebra* $[0,1]$, $\tau_L([0,1]) = \{\varnothing, [0,1], [0,r], [0,r) : r \in [0,1]\}$.

Proof. Let $G \in \tau_L([0,1])$, and $G \neq \varnothing$. Then $G = \bigcup_{i \in K \subseteq \mathcal{B}_L([0,1])} \mathcal{B}_i$. Thus, $\mathcal{B}_i$ are of the form $[0,r]$, $G = \bigcup [0, r_i] = [0,r], r_0 = inf\{r_i\}$. Thus, $\tau_L([0,1]) = \{\varnothing, [0,1], [0,r], [0,r) : r \in [0,1]\}$.

■

Corollary 3. $\tau_L([0,1])$ *is not a subspace of* $\mathbb{R}$ *with the usual topology.*

Theorem 13. $[0,1]$ *with topology* $\tau_L([0,1])$ *is connected.*

Proof. Suppose $[0,1]$ is disconnected. Then, there exist disjoint open sets A, B such that $[0,1] = A \cup B$. Since A is open, $A = [0,r]$ or $[0,r)$. If $A = [0,r]$ then $B = [r,1]$. If $A = [0,r)$ then $B = [r,1]$. Whether $B = [r,1]$ or $[r,1]$, B is not open. This contradicts the statement that A and B are open sets. Therefore, by Definition 4, $[0,1]$ is connected.

■

Theorem 14. *In a hyper BCK-algebra* $[0,1]$, $\tau_R([0,1]) \cap \tau_L([0,1])$ *is the indiscrete topology on* $[0,1]$.

Proof. In $[0,1]$, $\mathcal{B}_R(H) = \{[r,1] : r \in [0,1]\}$ and $\mathcal{B}_L(H) = \{[0,p] : p \in [0,1]\}$. Suppose there exixts $G \in [\tau_R(H) \cap \tau_L(H)] \setminus \{\varnothing, H\}$. Let $x \in G$. Then there exist $a \in (0,1]$ and $b \in [0,1)$ such that $x \in [a,1] \subseteq G$ and $x \in [0,b] \subseteq G$. This implies that $a \leq b$; hence, $[a,1] \cup [0,b] = [0,1] \subseteq G$. This is a contradiction to our assumption of set G. Therefore, $\tau_R(H) \cap \tau_L(H)$ is the indiscrete topology on H.

■

6. Conclusion

The paper explores the structure and topology of a hyper BCK-algebra $[0,1]$. Two topologies, $\tau_R([0,1])$ and $\tau_L([0,1])$, are constructed based on right and left applications of hyperorder, respectively. The paper demonstrates the properties of these topologies, including the formation of bases, open and closed sets, and connectedness. The intersection of these topologies results in the indiscrete topology on $[0,1]$, highlighting a unique convergence of the two structures. This study builds on previous work in algebraic hyperstructures and provides a foundational exploration of topologies within hyper BCK-algebras.

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