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Exploring Ideals: An Analysis Through Rough Set Theory with Examples

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Abstract. Rough set theory, a powerful mathematical framework, excels in addressing uncertainty and imprecision. In this study, we explore the analysis and provide examples of ideals and prime ideals within imprecise scenarios, utilizing rough set theory to quantify and navigate the inherent imprecision and roughness within these algebraic structures. This research contributes to our comprehension of how rough set theory effectively manages imprecision in algebraic contexts.

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1. Introduction

Real-world problems are inherently plagued by imprecision, necessitating the employment of mathematical tools capable of handling uncertainty. Various mathematical theories, including fuzzy set theory, soft set theory, and rough set theory, have emerged to address these imprecise scenarios. Among these, rough set theory stands out due to its unique feature of requiring only the dataset itself, without any prior information, to analyze uncertainty. In the realm of mathematics and its practical applications, the study of ideals and prime ideals assumes a foundational role in understanding algebraic structures. These concepts, originating from abstract algebra, possess far-reaching implications across various domains, including number theory, ring theory, and algebraic geometry. However, when confronted with real-world data and imprecise information, traditional algebraic methods may fall short in providing meaningful insights. This is where Rough Set Theory, a branch of mathematics tailored for handling uncertainty and imprecision, comes into play.

Rough Set Theory, introduced by Polish mathematician Zdzisław Pawlak in the 1980s, provides a mathematical framework designed specifically to grapple with imprecision and

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uncertainty in data [14]. At its core, Rough Set Theory deals with approximations and object classification based on available information. The central concept in Rough Set Theory is the notion of a rough set, characterized by two sets: the lower approximation (containing certain information) and the upper approximation (containing potential information). These sets are used to define boundaries and make decisions when faced with imprecise data.

Developments in rough set theory, spanning advancements in pure mathematics, the establishment of algebraic foundations, and practical applications, have been rapidly progressing since its inception. Pawlak's seminal work [16] laid the primary algebraic groundwork for rough sets, culminating in the exploration of various algebraic properties and theories in this field [6]. This paper references several related works to underscore the significance of rough set theory. Pawlak himself emphasized the importance of rough sets [15]. Concepts such as rough groups, rough subgroups, and their properties have been thoroughly investigated in [12]. Q. Xiao and Z. Zhang introduced the concepts of rough prime ideals and rough fuzzy prime ideals [20]. Fuzzy ideals within a ring have been meticulously explored by T. Mukherjee and M. Sen [13]. B. Davvaz explores into roughness based on fuzzy ideals and roughness within rings [7, 8]. Kuroki has conducted research on rough ideals in semigroups [11]. A.K. Sinha and A. Prakash have meticulously examined the algebraic properties of rough set theory [18, 19]. Recent extensions of rough set theory can be found in [10, 17, 21].

The article by Hosny et al. [9] explores how rough set theory can be enhanced using ideals and maximal right neighborhoods. The study focuses on minimizing boundary regions and increasing classification accuracy, particularly in medical cases such as COVID-19 and heart disease diagnosis. Similarly, the work by Al-shami et al. [5] introduces an advanced extension of rough set theory by incorporating $\mathbb{I}_k$-neighborhoods, which combine neighborhood systems and ideal structures. This approach enhances decision-making accuracy and is applied to analyze data related to Chikungunya disease. Their results highlight the effectiveness of these models in reducing uncertainty, aligning with the current article's focus on the algebraic foundations of ideals within rough set theory.

Further, Al-shami et al. [1] develop a novel decision-making framework that uses rough set theory with new approximation models based on basic-minimal neighborhoods. Their application to heart failure diagnosis achieves 100% accuracy, significantly improving upon previous methods. This work complements the present study by expanding the connection between rough set theory and topological structures, reinforcing the practical utility of ideals-based methods in medical applications. These recent developments, alongside the current article, underscore the potential of ideals-based rough set theory in enhancing decision-making accuracy and addressing uncertainties [3] in various fields, especially in medical data analysis. Recent advancements in rough set theory, including concepts like somewhere dense sets [2], containment neighborhoods, and supra-topological frameworks [4], have improved accuracy and decision-making, particularly in medical diagnostics [1]. Future research could explore these enhanced rough set models in complex applications, such as broader clinical contexts and other data-driven environments, to strengthen decision frameworks under uncertain data conditions.

This introduction lays the groundwork for a comprehensive investigation into the application of rough set theory to analyze ideals and prime ideals within imprecise scenarios. It underscores the necessity of such an approach in addressing real-world problems characterized by uncertainty and imprecision.

2. Preliminaries

In this section, we present a foundational understanding of rough set theory with key algebraic properties.

Let U represent a universal set, and let θ denote an equivalence relation on U. The equivalence class of an element $x \in U$ is defined as the set of elements related to x in U, denoted as $[x]_\theta$. The pair (U, θ), where U is non-empty, and θ is an equivalence relation on U, constitutes an approximation space. In this context, rough approximation [16] in (U, θ) is represented by a mapping $Apr : P(U) \to P(U) \times P(U)$, defined for every $X \in P(U)$ as follows:

$$Apr(X) = (\theta_-(X), \theta^-(X)),$$

where $\theta_-(X) = \{x \in U \,|\, [x]_\theta \subseteq X\}$ and $\theta^-(X) = \{x \in U \,|\, [x]_\theta \cap X \neq \emptyset\}$. These sets, $\theta_-(X)$ and $\theta^-(X)$, are known as the lower and upper approximations of set X within (U, θ), respectively. The accuracy of rough set can be measured as [16]:

$$\alpha_\theta(X) = \frac{\text{card}\Big(\theta_-(X)\Big)}{\text{card}\Big(\theta^- X\Big)},$$

where $X \neq \emptyset$. The accuracy measure $\alpha_\theta(X)$ quantifies the degree of completeness of our knowledge about the set X. It holds that $0 \leq \alpha_\theta(X) \leq 1$, and when $\alpha_\theta(X) = 1$, the θ-borderline region of X is empty. The θ-roughness of X [16] is defined as:

$$\rho_\theta(X) = 1 - \alpha_\theta(X) = 1 - \frac{\text{card}\Big(\theta_-(X)\Big)}{\text{card}\Big(\theta^-(X)\Big)}.$$

It's worth noting that lower roughness of a subset indicates a better approximation.

In this paper, we consider R as a ring with standard operations. Let I represent an ideal of R, and X (where $X \neq emptyset$) be a subset of R. We define $I_-(X)$ and $I^-(X)$ as follows:

$$I_-(X) = \{x \in R \,|\, (x + I) \subseteq X\}$$

$$I^-(X) = \{x \in R \,|\, (x + I) \cap X \neq \emptyset\},$$

representing the lower and upper approximations of set X concerning the ideal I, respectively. Additionally, we introduce notations for R as a ring and ρ as a congruence relation on R:

$$\rho_-(A) = \{x \in R \,|\, [x]_\rho \subseteq A\}$$

$$\rho^{-}(A) = \{x \in R \mid [x]_\rho \cap A \neq \emptyset\}$$

A congruence relation on R is termed complete if $[a]_\rho[b]_\rho = [ab]_\rho$ for any $a, b \in R$. We also present key propositions and theorems relevant to our study:

Proposition 1. *([7], Proposition 3.4) Let I be an ideal of R, and A, B non-empty subsets of R. Then*

$$I^{-}(A) \cdot I^{-}(B) = I^{-}(A \cdot B).$$

Proposition 2. *([7], Proposition 3.5) Let I be an ideal of R, and A, B non-empty subsets of R. Then*

$$I_{-}(A) \cdot I_{-}(B) = I_{-}(A \cdot B).$$

Theorem 1. *(Theorem 3.12, [7]) Let I and J be two ideals of the ring R. Then $I^{-}(J)$ is an ideal of R.*

Theorem 2. *(Theorem 3.13, [7]) Let I and J be two ideals of the ring R. Then $I_{-}(J)$ is an ideal of R.*

These propositions and theorems establish essential properties and relationships within our research domain.

3. Rough Set Analysis of Ideals: Theory, Applications, and Illustrated Examples

In this section, we explore the fundamental theorems related to ideals, with a specific focus on upper and lower approximations. To aid in comprehension, we provide concrete examples that illustrate these theorems. We also explore the concept of rough ideals, offering a precise definition to facilitate understanding. Further, we present additional theorems and illustrative examples to enhance the practical application of these concepts.

Theorem 3. *Let ρ be a congruence relation on a ring R. Suppose A is a subset of ring R,and if A is an ideal of ring R, then the upper approximation of A, denoted as $\rho^{-}(A)$, is also an ideal of ring R.*

Proof. Assume that A is an ideal of ring R, which implies that $RA \subseteq A$ and $AR \subseteq A$. Note that $\rho^{-}(R) = R$. By Proposition 1, we have:

$$R\,\rho^{-}(A) = \rho^{-}(R)\rho^{-}(A) \subseteq \rho^{-}(RA) \subseteq \rho^{-}(A)$$
$$\rho^{-}(A)R = \rho^{-}(A)\rho^{-}(R) \subseteq \rho^{-}(AR) \subseteq \rho^{-}(A)$$

This implies that $\rho^{-}(A)$ is an ideal of R, making it an upper rough ideal of ring R. It's important to note that the converse of this theorem does not hold in general.

Example 1. *Let's consider* $R = \mathbb{Z}_{12}$*,* $I = \{0, 6\}$ *as a congruence relation on* $\mathbb{Z}_{12}$*, and* $A = \{0, 4, 8\}$ *as a subset of* $\mathbb{Z}_{12}$*. In this case,* $I^-(A) = \{0, 2, 4, 6, 8, 10\}$ *is an ideal of* $\mathbb{Z}_{12}$*. However, the lower approximation is an empty set, i.e.,* $I_-(A) = \emptyset$*. Now, consider another subset,* $B = \{0, 2, 4, 6, 8, 10\}$ *in* $\mathbb{Z}_{12}$*. Here,* $I^-(B) = \{0, 2, 4, 6, 8, 10\}$ *is also an ideal of* $\mathbb{Z}_{12}$*, and its lower approximation is* $I_-(B) = \{0, 6\}$*. The roughness* $\rho(B) = 0.67$*, indicating the degree of completeness of our knowledge about set* B*.*

Theorem 4. *Let* ρ *be a complete congruence relation on a ring* R*, and* A *be an ideal of* R*. If the lower approximation* $\rho_-(A)$ *is non-empty, then it is also an ideal of ring* R*.*

Proof. Consider A as an ideal of ring R, implying $RA \subseteq A$ and $AR \subseteq A$. Note that $\rho_-(R) = R$. By Proposition 2, we have:

$$R\,\rho_-(A) = \rho_-(R)\rho_-(A) \subseteq \rho_-(RA) \subseteq \rho_-(A)$$

This indicates that $\rho_-(A)$ is an ideal of R, and it represents a lower rough ideal.

Definition 1. *Suppose* ρ *is a congruence relation on a ring* R*, and* $A \subseteq R$*. We define* $\rho(A) = (\rho_-(A), \rho^-(A))$ *as a rough ideal of* R *if both the lower approximation* $\rho_-(A)$ *and the upper approximation* $\rho^-(A)$ *are ideals of the ring* R*.*

Example 2. *Let's take* $R = \mathbb{Z}_{12}$*,* $I = \{0, 4\}$ *as an ideal of* R*, and* $A = \{0, 4, 6\}$ *as a subset of* R*. In this case,* $I_-(A) = \{0, 4\}$ *and* $I^-(A) = \{0, 2, 4, 6\}$ *are the lower and upper approximations of* A *concerning the congruence relation* I*. Since both* $I_-(A)$ *and* $I^-(A)$ *are ideals of* R*, this implies that* $I(A) = (I_-(A), I^-(A))$ *is a rough ideal of* $\mathbb{Z}_{12}$*. The roughness of* A *is calculated as* $\rho(A) = 0.5$*.*

Theorem 5. *If* ρ *and* λ *are congruence relations on a ring* R*, and* $\rho \subset \lambda$*, and if* A *is a non-empty subset of* R*, then*

$$(\rho \cap \lambda)^-(A) = \rho^-(A) \cap \lambda^-(A).$$

Proof. Since ρ and λ are congruence relations on a ring R, this implies that $\rho \cap \lambda$ is also a congruence relation on ring R. For any $c \in (\rho \cap \lambda)^-(A)$, we can deduce that $c \in \rho^-(A)$ and $c \in \lambda^-(A)$. This establishes that

$$(\rho \cap \lambda)^-(A) \subseteq \rho^-(A) \cap \lambda^-(A).$$

Conversely, for $c \in \rho^-(A) \cap \lambda^-(A)$, we can show that $c \in (\rho \cap \lambda)^-(A)$, which leads to the conclusion that

$$\rho^-(A) \cap \lambda^-(A) \subseteq (\rho \cap \lambda)^-(A).$$

Hence, we have proven

$$(\rho \cap \lambda)^-(A) = \rho^-(A) \cap \lambda^-(A).$$

Example 3. *Consider $R = \mathbb{Z}_{12}$ as the ring, $J = \{0, 6\}$ and $K = \{0, 2, 4, 6, 8, 10\}$ as congruence relations on $\mathbb{Z}_{12}$. It's clear that J and K are ideals on R. In this scenario, J is a subset of K. Let $B = \{0, 1, 2, 5\}$ as a subset of $\mathbb{Z}_{12}$. By calculating the respective classes, we obtain the lower approximations:*

$$J_-(B) = \{0, 1, 2, 5, 6, 7, 8, 11\}$$
$$K_-(B) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$
$$(J \cap K)_-(B) = \{0, 1, 2, 5, 6, 7, 8, 11\}$$

The intersection of $J_-(B)$ and $K_-(B)$ equals $(J \cap K)_-(B)$, showing that the lower approximation of B under congruence relation $(J \cap K)$ is $(J \cap K)_-(B)$.

Theorem 6. *If ρ and λ are congruence relations on a ring R, and A is a non-empty subset of R, then*

$$(\rho \cap \lambda)_-(A) = \rho_-(A) \cap \lambda_-(A).$$

Proof. The proof for this theorem follows a similar logic to the previous theorem, with the focus on the lower approximations.

Example 4. *Let's consider $R = \mathbb{Z}_{12}$ as the ring, $I = \{0, 4, 8\}$, and $K = \{0, 2, 4, 6, 8, 10\}$ as congruence relations on $\mathbb{Z}_{12}$. We note that I and K are ideals on $\mathbb{Z}_{12}$. In this case, I is a subset of K. Let $A = \{1, 3, 5, 7, 9, 11\}$ as a subset of $\mathbb{Z}_{12}$. By calculating the classes for I and K, we obtain the lower approximations:*

$$I_-(A) = \{1, 3, 5, 7, 9, 11\}$$
$$K_-(A) = \{1, 3, 5, 7, 9, 11\}$$
$$(I \cap K)_-(A) = \{1, 3, 5, 7, 9, 11\}$$

In this case, the intersection of $I_-(A)$ and $K_-(A)$ equals $(I \cap K)_-(A)$, showing that the lower approximation of A under congruence relation $(I \cap K)$ is $(I \cap K)_-(A)$.

Definition 2. *Let R be a ring in the usual sense. An ideal A of a ring R is termed a prime ideal of the ring R if, for any $x, y \in R$, $xy \in A$ implies that either $x \in A$ or $y \in A$.*

Now, let ρ be a congruence relation on a ring R. We define a subset I of ring R as a lower rough prime ideal of ring R if $\rho_-(I)$ is a prime ideal of ring R, and an upper rough prime ideal of ring R if $\rho^-(I)$ is a prime ideal of ring R. We then denote $\rho(I) = (\rho_-(I), \rho^-(I))$ as a rough prime ideal of R.

Example 5. *Consider $R = \mathbb{Z}_{12}$ as a ring, with $I = \{0, 6\}$ and subsets $A = \{0, 1, 2, 5, 6, 8\}$ and $B = \{0, 3, 4, 6, 9\}$. In this context, I is an ideal of ring $\mathbb{Z}_{12}$. A is a subset of $\mathbb{Z}_{12}$, and it's evident that I is a congruence relation in $\mathbb{Z}_{12}$. Now, we define the lower approximation of A with respect to the congruence relation I as $I_-(A) = \{x \in \mathbb{Z}_{12} | (x + I) \subseteq A\}$ and the upper approximation as $I^-(A) = \{x \in \mathbb{Z}_{12} | (x + I) \cap A \neq \emptyset\}$.*

For this example, the classes of I are $\{2, 8\}, \{4, 10\}, \{3, 9\}, \{1, 7\}, \{6, 0\}, \{5, 11\}$ in $\mathbb{Z}_{12}$. Consequently, $I_-(A) = \{0, 2, 6, 8\}$ and $I^-(A) = \{0, 1, 2, 5, 6, 7, 8, 11\}$. This implies that $I_-(A)$ and $I^-(A)$ are prime ideals of $\mathbb{Z}_{12}$, demonstrating that $Apr(A) = (I_-(A), I^-(A))$ is a rough prime ideal. The roughness of set A is calculated as $\rho(A) = 0.5$.

Example 6. *Let's consider the same ring $R = \mathbb{Z}_{12}$, but this time with the congruence relation $I = \{0, 4, 8\}$ and the subset $B = \{0, 1, 3, 4, 6, 8, 10\}$. Here, I is an ideal over ring R, and B is any subset of R. By defining the lower approximation of B with respect to congruence relation I, we get $I_-(B) = \{x \in R | x + I \subseteq B\}$ and the upper approximation is $I^-(B) = \{x \in R | (x + I) \cap B \neq \emptyset\}$.*

In this scenario, the classes of ρ are $\{0, 4, 8\}, \{1, 5, 9\}, \{2, 6, 10\}, \{3, 7, 11\}$ within $\mathbb{Z}_{12}$. Thus $\rho_-(B) = \{0, 2, 4, 6, 8, 10\}$ and $\rho^-(B) = \{0, 2, 3, 4, 6, 7, 8, 11\}$. This indicates that $\rho-(B)$ and $\rho^-(B)$ are prime ideals of $R = \mathbb{Z}_{12}$, suggesting that $\rho(B) = (\rho_-(B), \rho^-(B))$ is a prime ideal of $R = \mathbb{Z}_{12}$.

4. Conclusion

In conclusion, traditional algebraic interpretations of ideals and prime ideals are grounded in precise mathematical definitions, often struggling to accommodate the inherent imprecision and uncertainty encountered in real-world data. In contrast, Rough Set Theory offers a refreshing perspective, introducing a high degree of flexibility by considering lower and upper approximations. These rough interpretations bridge the gap between classical algebraic structures and practical applications, providing a versatile mathematical framework that excels in scenarios where precision remains elusive.

The fusion of abstract algebraic concepts with the adeptness of Rough Set Theory in handling imprecision and uncertainty opens up a promising avenue for further research and practical applications. The exploration of algebraic concepts within rough sets has proven to be captivating. We explained the concepts of upper and lower approximations of ideals within the context of congruence relations, shedding light on the notion of rough ideals with examples. We also investigated the study of upper and lower prime ideals, examining their associated roughness and bridging the theory and applications. These findings are poised to make significant contributions to the mathematical foundation of rough set theory. They not only extend our understanding of algebraic structures within rough set theory but also pave the way for further explorations and applications within this intriguing field. The mathematical interpretations of rough ideals and rough prime ideals empower us to adapt and extend traditional algebraic concepts effectively, providing the tools needed to tackle imprecise information efficiently. This, enables us to gain deeper insights and develop solutions applicable in various domains where uncertainty and data imperfections prevail. As mathematical tools continue to evolve to meet the demands of the modern world, Rough Set Theory stands as a testament to the adaptability and versatility of mathematics in addressing real-world challenges.

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Author contributions

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Conflict of interest

The authors state no conflict of interest.

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