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Upper and Lower Near (τ_1, τ_2)-continuity

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Abstract. This paper presents new classes of multifunctions called upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions. Furthermore, some characterizations of upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions are established.

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1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Weaker and stronger forms of open sets play an important role in the generalization of different forms of continuity. Using different forms of open sets, several authors have introduced and investigated various types of continuity for functions and multifunctions. Carnahan [30] introduced the notion of N-closed sets in topological spaces. Noiri [44] studied several properties of N-closed sets and some separation axioms. The concept of N-continuous functions was introduced by Malghan and Hanchinamani [43]. Noiri and Ergun [45] investigated some characterizations of N-continuous functions. Viriyapong and Boonpok [61] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [12]. Dungthaisong et al. [36] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [35] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost

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(Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions and faintly (τ_1, τ_2)-continuous functions were presented in [54], [57], [16], [48], [25], [11], [8], [10], [4], [1], [2], [26], [23], [18] and [55], respectively. Chiangpradit et al. [33] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions. Kong-ied et al. [42] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Thongmoon et al. [59] introduced and investigated the notion of rarely (τ_1, τ_2)-continuous functions.

In 2003, Ekici [37] introduced and studied the concept of nearly continuous multifunctions as a generalization of semi-continuous multifunctions and N-continuous functions. Moreover, Ekici [38] introduced and investigated the notion of almost nearly continuous multifunctions as a generalization of nearly continuous multifunctions and almost continuous multifunctions [47]. Furthermore, several characterizations and some properties concerning $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, almost quasi (τ_1, τ_2)-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions, c-quasi (τ_1, τ_2)-continuous multifunctions and s-$(\tau_1, \tau_2)p$-continuous multifunctions were established in [5], [28], [62], [3], [7], [17], [24], [6], [21], [20], [15], [9], [19], [22], [39], [13], [27], [56], [14], [51], [41], [58], [52], [50], [40], [49] and [64], respectively. Noiri and Popa [46] introduced and studied the notion of almost nearly m-continuous multifunctions as multifunctions from a set satisfying some minimal conditions into a topological spaces. Carpintero et al. [31] introduced and studied the notion of nearly ω-continuous multifunctions as a weaker form of nearly continuous multifunctions. Rosas et al. [53] introduced and studied upper and lower almost nearly continuous multifunctions using notions of topological ideals. In this paper, we introduce the concepts of upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions. We also investigate several characterizations of upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [29] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [29] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [29] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [29] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.

(3) $\tau_1\tau_2\text{-}Cl(A)$ is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2\text{-}Cl(A)$.

(5) $\tau_1\tau_2\text{-}Cl(X - A) = X - \tau_1\tau_2\text{-}Int(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [29] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [60] (resp. $(\tau_1, \tau_2)s$-*open* [5], $(\tau_1, \tau_2)p$-*open* [5], $(\tau_1, \tau_2)\beta$-*open* [5]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [63] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$-*closed* if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a finite subcover. Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [60] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [60] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [60] if $(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [60] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2. [60] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2$-Cl$(A) = (\tau_1, \tau_2)\theta$-Cl$(A)$.

(2) $(\tau_1, \tau_2)\theta$-Cl(A) is $\tau_1\tau_2$-closed in X.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x\in A}F(x)$.

3. Upper and lower nearly (τ_1, τ_2)-continuous multifunctions

In this section, we introduce the notions of upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations of upper nearly (τ_1, τ_2)-continuous multifunctions and lower nearly (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper nearly (τ_1, τ_2)-continuous if F is upper nearly (τ_1, τ_2)-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper nearly (τ_1, τ_2)-continuous at $x \in X$;

(2) $x \in \tau_1\tau_2$-Int$(F^+(V))$ for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(3) $x \in F^-(\sigma_1\sigma_2$-Cl$(B))$ for each subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure such that $x \in \tau_1\tau_2$-Cl$(F^-(B))$;

(4) $x \in \tau_1\tau_2$-Int$(F^+(B))$ for each subset B of Y such that $Y - \sigma_1\sigma_2$-Int(B) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $x \in F^+(\sigma_1\sigma_2$-Int$(B))$.

Proof. (1) ⇒ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement and $x \in F^+(V)$. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq V$. Thus, $x \in U \subseteq F^+(V)$. Since U is $\tau_1\tau_2$-open, we have $x \in \tau_1\tau_2$-Int$(F^+(V))$.

(2) ⇒ (3): Let B be any subset of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2$-Cl(B) is $\sigma_1\sigma_2$-closed and $Y - \sigma_1\sigma_2$-Cl(B) is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Suppose that $x \notin F^-(\sigma_1\sigma_2$-Cl$(B))$. Then, we have

$$x \in X - F^-(\sigma_1\sigma_2\text{-Cl}(B)) = F^+(Y - \sigma_1\sigma_2\text{-Cl}(B))$$

and hence $F(x) \subseteq Y - \sigma_1\sigma_2\text{-Cl}(B)$. Since $Y - \sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, by (2) we have

$$\begin{aligned} x \in \tau_1\tau_2\text{-Int}(F^+(Y - \sigma_1\sigma_2\text{-Cl}(B))) &= \tau_1\tau_2\text{-Int}(X - F^-(\sigma_1\sigma_2\text{-Cl}(B))) \\ &= X - \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(B))) \\ &\subseteq X - \tau_1\tau_2\text{-Cl}(F^-(B)). \end{aligned}$$

Thus, $x \notin \tau_1\tau_2\text{-Cl}(F^-(B))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y such that $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Suppose that $x \notin \tau_1\tau_2\text{-Int}(F^+(B))$. Then, we have

$$x \in X - \tau_1\tau_2\text{-Int}(F^+(B)) = \tau_1\tau_2\text{-Cl}(X - F^+(B)) = \tau_1\tau_2\text{-Cl}(F^-(Y - B))$$

and by (3), $x \in F^-(\sigma_1\sigma_2\text{-Cl}(Y - B)) = F^-(Y - \sigma_1\sigma_2\text{-Int}(B)) = X - F^+(\sigma_1\sigma_2\text{-Int}(B))$. Thus, $x \notin F^+(\sigma_1\sigma_2\text{-Int}(B))$.

(4) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - \sigma_1\sigma_2\text{-Int}(V) = Y - V$ which is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $x \in F^+(\sigma_1\sigma_2\text{-Int}(V))$. By (4), we have $x \in \tau_1\tau_2\text{-Int}(F^+(V))$. Therefore, there exists a $\tau_1\tau_2$-open set U of X containing x such that $x \in U \subseteq F^+(V)$. Thus, $F(U) \subseteq V$. This shows that F is upper nearly (τ_1, τ_2)-continuous at x.

Definition 2. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower nearly (τ_1, τ_2)-continuous if F is lower nearly (τ_1, τ_2)-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower nearly (τ_1, τ_2)-continuous at $x \in X$;

(2) $x \in \tau_1\tau_2\text{-Int}(F^-(V))$ for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(3) $x \in F^+(\sigma_1\sigma_2\text{-Cl}(B))$ for each subset B of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure such that $x \in \tau_1\tau_2\text{-Cl}(F^+(B))$;

(4) $x \in \tau_1\tau_2\text{-Int}(F^-(B))$ for each subset B of Y such that $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $x \in F^-(\sigma_1\sigma_2\text{-Int}(B))$.

Proof. The proof is similar to that of Theorem 1.

Theorem 3. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper nearly (τ_1, τ_2)-continuous;

(2) $F^+(V)$ is $\tau_1\tau_2$-open in X for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(3) $F^-(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y;

(4) $\tau_1\tau_2$-Cl$(F^-(B)) \subseteq F^-(\sigma_1\sigma_2$-Cl$(B))$ for every subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;

(5) $F^+(\sigma_1\sigma_2$-Int$(B)) \subseteq \tau_1\tau_2$-Int$(F^+(B))$ for every subset B of Y such that $Y - \sigma_1\sigma_2$-Int(B) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement and $x \in F^+(V)$. Then, we have $F(x) \subseteq V$. By Theorem 1, $x \in \tau_1\tau_2$-Int$(F^+(V))$. Thus, $F^+(V) \subseteq \tau_1\tau_2$-Int$(F^+(V))$ and hence $F^+(V)$ is $\tau_1\tau_2$-open in X.

(2) $\Rightarrow$ (3): The proof follows immediately from the fact that $F^+(Y - B) = Y - F^-(B)$ for every subset B of Y.

(3) $\Rightarrow$ (4): Let B be any subset of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2$-Cl(B) is $\sigma_1\sigma_2$-closed and by (3), $F^-(\sigma_1\sigma_2$-Cl$(B))$ is $\tau_1\tau_2$-closed in X. Thus,

$$F^-(B) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(B)) = \tau_1\tau_2\text{-Cl}(\sigma_1\sigma_2\text{-Cl}(B))$$

and hence $\tau_1\tau_2$-Cl$(F^-(B)) \subseteq F^-(\sigma_1\sigma_2$-Cl$(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $Y - \sigma_1\sigma_2$-Int(B) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Then by (4), we have

$$\begin{aligned} X - \tau_1\tau_2\text{-Int}(F^+(B)) &= \tau_1\tau_2\text{-Cl}(X - F^+(B)) \\ &= \tau_1\tau_2\text{-Cl}(F^-(Y - B)) \\ &\subseteq \tau_1\tau_2\text{-Cl}(F^-(Y - \sigma_1\sigma_2\text{-Int}(B))) \\ &\subseteq F^-(Y - \sigma_1\sigma_2\text{-Int}(B)) \\ &= X - F^+(\sigma_1\sigma_2\text{-Int}(B)). \end{aligned}$$

Thus, $F^+(\sigma_1\sigma_2$-Int$(B)) \subseteq \tau_1\tau_2$-Int$(F^+(B))$.

(5) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Thus by (5),

$$x \in F^+(V) = F^+(\sigma_1\sigma_2\text{-Int}(V)) \subseteq \tau_1\tau_2\text{-Int}(F^+(V)).$$

By Theorem 1, F is upper nearly (τ_1, τ_2)-continuous at x. This shows that F is upper nearly (τ_1, τ_2)-continuous.

Theorem 4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F *is lower nearly* (τ_1, τ_2)*-continuous;*

(2) $F^-(V)$ *is* $\tau_1\tau_2$*-open in* X *for each* $\sigma_1\sigma_2$*-open set* V *of* Y *having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(3) $F^+(K)$ *is* $\tau_1\tau_2$*-open in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed and* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $\tau_1\tau_2\text{-}Cl(F^+(B)) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y *having the* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed* $\sigma_1\sigma_2$*-closure;*

(5) $F^-(\sigma_1\sigma_2\text{-}Cl(B)) \subseteq \tau_1\tau_2\text{-}Int(F^-(B))$ *for every subset* B *of* Y *such that* $Y - \sigma_1\sigma_2\text{-}Int(B)$ *is* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed.*

Proof. The proof is similar to that of Theorem 3.

Corollary 1. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper nearly* (τ_1, τ_2)*-continuous if* $F^-(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed set* K *of* Y*.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. By the hypothesis, $F^-(Y - V) = X - F^+(V)$ is $\tau_1\tau_2$-closed in X and hence $F^+(V)$ is $\tau_1\tau_2$-open in X. It follows from Theorem 3 that F is upper nearly (τ_1, τ_2)-continuous.

Corollary 2. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is lower nearly* (τ_1, τ_2)*-continuous if* $F^+(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Corollary 1.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)*-regular* [32] if for each $\tau_1\tau_2$-closed set F and each point $x \in X - F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Theorem 5. *Let* (Y, σ_1, σ_2) *be a* (σ_1, σ_2)*-regular space. For a multifunction*

$$F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2),$$

the following properties are equivalent:

(1) F *is upper nearly* (τ_1, τ_2)*-continuous;*

(2) $F^-((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ *is* $\tau_1\tau_2$*-closed in* X *for every subset* B *of* Y *such that*

$$(\sigma_1, \sigma_2)\theta\text{-}Cl(B)$$

is $\mathscr{N}(\sigma_1, \sigma_2)$*-closed;*

(3) $F^-(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed and* $(\sigma_1, \sigma_2)\theta$*-closed set* K *of* Y*;*

(4) $F^+(V)$ is $\tau_1\tau_2$-open in X for each $(\sigma_1,\sigma_2)\theta$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement.

Proof. (1) ⇒ (2): Let B be any subset of Y such that $(\sigma_1,\sigma_2)\theta$-Cl$(B)$ is $\mathscr{N}(\sigma_1,\sigma_2)$-closed. Then, $(\sigma_1,\sigma_2)\theta$-Cl$(B)$ is $\mathscr{N}(\sigma_1,\sigma_2)$-closed and $\sigma_1\sigma_2$-closed. Thus by Theorem 3, $F^-((\sigma_1,\sigma_2)\theta\text{-Cl}(B))$ is $\tau_1\tau_2$-closed in X .

(2) ⇒ (3): Let K be any $\mathscr{N}(\sigma_1,\sigma_2)$-closed and $(\sigma_1,\sigma_2)\theta$-closed set of Y. Then, we have $K = (\sigma_1,\sigma_2)\theta$-Cl$(K)$ is $\mathscr{N}(\sigma_1,\sigma_2)$-closed and by (2), $F^-(K)$ is $\tau_1\tau_2$-closed in X.

(3) ⇒ (4): Let V be any $(\sigma_1,\sigma_2)\theta$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1,\sigma_2)$-closed and $(\sigma_1,\sigma_2)\theta$-closed. By (3), $F^-(Y - V) = X - F^+(V)$ is $\tau_1\tau_2$-closed in X and hence $F^+(V)$ is $\tau_1\tau_2$-open in X.

(4) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Since (Y,σ_1,σ_2) is (σ_1,σ_2)-regular, V is $(\sigma_1,\sigma_2)\theta$-open in Y and having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. By (4), we have $F^+(V)$ is $\tau_1\tau_2$-open in X and by Theorem 3, F is upper nearly (τ_1,τ_2)-continuous.

Theorem 6. *Let (Y,σ_1,σ_2) be a (σ_1,σ_2)-regular space. For a multifunction*

$$F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2),$$

the following properties are equivalent:

(1) F is lower nearly (τ_1,τ_2)-continuous;

(2) $F^+((\sigma_1,\sigma_2)\theta\text{-}Cl(B))$ is $\tau_1\tau_2$-closed in X for every subset B of Y such that

$$(\sigma_1,\sigma_2)\theta\text{-}Cl(B)$$

is $\mathscr{N}(\sigma_1,\sigma_2)$-closed;

(3) $F^+(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1,\sigma_2)$-closed $(\sigma_1,\sigma_2)\theta$-closed set K of Y;

(4) $F^-(V)$ is $\tau_1\tau_2$-open in X for each $(\sigma_1,\sigma_2)\theta$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement.

Proof. The proof is similar to that of Theorem 5.

4. Some results on near (τ_1,τ_2)-continuity

Recall that a bitopological space (X,τ_1,τ_2) is said to be (τ_1,τ_2)-T_2 [34] if for any pair of distinct points x, y in X, there exist disjoint $\tau_1\tau_2$-open sets U and V of X containing x and y, respectively.

Definition 3. *A bitopological space (X,τ_1,τ_2) is called $\mathscr{N}(\tau_1,\tau_2)$-normal if for each disjoint $\tau_1\tau_2$-closed sets K and H of X, there exist $\tau_1\tau_2$-open sets U and V having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complements such that $K \subseteq U$, $H \subseteq V$ and $U \cap V = \emptyset$.*

Theorem 7. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper nearly (τ_1, τ_2)-continuous multifunction satisfying the following conditions:*

(1) $F(x)$ is $\sigma_1\sigma_2$-closed in Y for each $x \in X$,

(2) $F(x) \cap F(y) = \emptyset$ for each distinct points $x, y \in X$, and

(3) (Y, σ_1, σ_2) is an $\mathscr{N}(\sigma_1, \sigma_2)$-normal space,

then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Proof. Let x and y be distinct points of X. Then, we have $F(x) \cap F(y) = \emptyset$. Since $F(x)$ and $F(y)$ are $\sigma_1\sigma_2$-closed and (Y, σ_1, σ_2) is $\mathscr{N}(\sigma_1, \sigma_2)$-normal, there exist disjoint $\sigma_1\sigma_2$-open sets U and V having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $F(x) \subseteq U$ and $F(y) \subseteq V$. By Theorem 3, $F^+(U)$ and $F^+(V)$ are $\tau_1\tau_2$-open in X containing x and y, respectively, such that $F^+(U) \cap F^+(V) = \emptyset$. This shows that (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Theorem 8. *Let (X, τ_1, τ_2) be a bitopological space. If for each pair of distinct points x and x' in X, there exists a multifunction F from (X, τ_1, τ_2) into an $\mathscr{N}(\sigma_1, \sigma_2)$-normal space (Y, σ_1, σ_2) satisfying the following conditions:*

(1) $F(x)$ and $F(x')$ are $\sigma_1\sigma_2$-closed in Y,

(2) F is upper nearly (τ_1, τ_2)-continuous at x and x', and

(3) $F(x) \cap F(x') = \emptyset$,

then (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Proof. Let x and x' be distinct points of X. Then, we have $F(x) \cap F(x') = \emptyset$. Since $F(x)$ and $F(x')$ are $\sigma_1\sigma_2$-closed and (Y, σ_1, σ_2) is $\mathscr{N}(\sigma_1, \sigma_2)$-normal, there exist disjoint $\sigma_1\sigma_2$-open sets V and V' having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $F(x) \subseteq V$ and $F(x') \subseteq V'$. Since F is upper nearly (τ_1, τ_2)-continuous at x and x', there exist $\tau_1\tau_2$-open sets U and U' of X containing x and x', respectively, such that $F(U) \subseteq V$ and $F(U') \subseteq V'$. This implies that $U \cap U' = \emptyset$. Thus, (X, τ_1, τ_2) is (τ_1, τ_2)-T_2.

Definition 4. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-dense on X if $\tau_1\tau_2$-$Cl(A) = X$.*

Theorem 9. *Let (X, τ_1, τ_2) be a bitopological space and (Y, σ_1, σ_2) be an $\mathscr{N}(\sigma_1, \sigma_2)$-normal space. If the following four conditions are satisfied:*

(1) $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper nearly (τ_1, τ_2)-continuous,

(2) $G : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper nearly (τ_1, τ_2)-continuous,

(3) $F(x)$ and $G(x)$ are $\sigma_1\sigma_2$-closed in Y for each $x \in X$, and

(4) $A = \{x \in X \mid F(x) \cap G(x) \neq \emptyset\}$,

then A is $\tau_1\tau_2$-closed. If $F(x) \cap G(x) \neq \emptyset$ for each point x in a $\tau_1\tau_2$-dense set D of X, then $F(x) \cap G(x) \neq \emptyset$ for each point $x \in X$.

Proof. Suppose that $x \notin A$. Then, $F(x) \cap G(x) = \emptyset$. Since $F(x)$ and $G(x)$ are $\sigma_1\sigma_2$-closed and (Y, σ_1, σ_2) is $\mathscr{N}(\sigma_1, \sigma_2)$-normal, there exist $\sigma_1\sigma_2$-open sets V and W in Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $F(x) \subseteq V$, $G(x) \subseteq W$ and $V \cap W = \emptyset$. Since F is upper nearly (τ_1, τ_2)-continuous at x, there exists a $\tau_1\tau_2$-open set U' of X containing x such that $F(U') \subseteq V$. Since G is upper nearly (τ_1, τ_2)-continuous at x, there exists a $\tau_1\tau_2$-open set U'' of X containing x such that $F(U'') \subseteq W$. Now set $U = U' \cap U''$, then U is $\tau_1\tau_2$-open in X and $U \cap A = \emptyset$. Thus, $x \notin \tau_1\tau_2\text{-Cl}(A)$ and hence $A = \tau_1\tau_2\text{-Cl}(A)$. This shows that A is $\tau_1\tau_2$-closed. On the other hand, if $F(x) \cap G(x) \neq \emptyset$ on a $\tau_1\tau_2$-dense set D of X, then we have $X = \tau_1\tau_2\text{-Cl}(D) \subseteq \tau_1\tau_2\text{-Cl}(A) = A$. Thus, $F(x) \cap G(x) \neq \emptyset$ for each $x \in X$.

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