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Almost Near (τ_1, τ_2)-continuity for Multifunctions

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Abstract. This paper presents new classes of multifunctions called upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations and some properties concerning upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions are established.

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1. Introduction

In 1968, Singal and Singal [56] introduced the concept of almost continuous functions as a generalization of continuity. Popa [46] defined almost quasi-continuous functions as a generalization of almost continuity and quasi-continuity [42]. Munshi and Bassan [43] studied the notion of almost semi-continuous functions. Maheshwari et al. [40] introduced the concept of almost feebly continuous functions as a generalization of almost continuity. In 1984, Malghan and Hanchinamani [41] introduced the concept of N-continuous functions. Noiri and Ergun [44] investigated some characterizations of N-continuous functions. Ekici [34] introduced and studied the concept of nearly continuous multifunctions as a generalization of semi-continuous multifunctions and N-continuous functions. Viriyapong and Boonpok [65] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [12]. Dungthaisong et al. [33] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [32] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost

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(Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were presented in [57], [60], [16], [49], [25], [11], [8], [10], [4], [1], [2], [26], [23] and [18], respectively. Srisarakham et al. [58] introduced and studied the concept of faintly (τ_1, τ_2)-continuous functions. Kong-ied et al. [39] introduced and investigated the notion of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [31] introduced and studied the concept of weakly quasi (τ_1, τ_2)-continuous functions. Thongmoon et al. [63] introduced and investigated the notion of rarely (τ_1, τ_2)-continuous functions.

In 2004, Ekici [35] introduced and investigated the notion of almost nearly continuous multifunctions as a generalization of nearly continuous multifunctions and almost continuous multifunctions [47]. In 2009, Noiri and Popa [45] introduced and studied the notion of almost nearly m-continuous multifunctions as multifunctions from a set satisfying some minimal conditions into a topological spaces. Carpintero et al. [30] introduced and studied the notion of nearly ω-continuous multifunctions as a weaker form of nearly continuous multifunctions. Moreover, several characterizations and some properties concerning $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, almost quasi (τ_1, τ_2)-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions, c-quasi (τ_1, τ_2)-continuous multifunctions and s-$(\tau_1, \tau_2)p$-continuous multifunctions were established in [5], [28], [66], [3], [7], [17], [24], [6], [21], [20], [15], [9], [19], [22], [36], [13], [27], [59], [14], [52], [38], [62], [53], [51], [37], [50] and [70], respectively. Rosas et al. [54] introduced and studied upper almost nearly continuous multifunctions and lower almost nearly continuous multifunctions using notions of topological ideals. Rychlewicz [55] introduced and studied the notion of nearly quasi-continuous multifunctions as a generalization of almost nearly continuous multifunctions and almost quasi continuous multifunctions [48]. In this paper, we introduce the concepts of upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions. We also investigate several characterizations of upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [29] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [29] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [29] of A and is denoted by $\tau_1\tau_2$-Int(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [64] (resp. $(\tau_1, \tau_2)s$-*open* [5], $(\tau_1, \tau_2)p$-*open* [5], $(\tau_1, \tau_2)\beta$-*open* [5]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [69] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed (resp. $(\tau_1, \tau_2)s$-closed, $\alpha(\tau_1, \tau_2)$-closed) sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* [68] (resp. $(\tau_1, \tau_2)s$-*closure* [5], $\alpha(\tau_1, \tau_2)$-*closure* [67]) of A and is denoted by (τ_1, τ_2)-pCl(A) (resp. (τ_1, τ_2)-sCl(A), $\alpha(\tau_1, \tau_2)$-Cl(A)). The union of all $(\tau_1, \tau_2)p$-open (resp. $(\tau_1, \tau_2)s$-open, $\alpha(\tau_1, \tau_2)$-open) sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* [68] (resp. $(\tau_1, \tau_2)s$-*interior* [5], $\alpha(\tau_1, \tau_2)$-*interior* [67]) of A and is denoted by (τ_1, τ_2)-pInt(A) (resp. (τ_1, τ_2)-sInt(A), $\alpha(\tau_1, \tau_2)$-Int(A)). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$-*closed* [61] if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a finite subcover.

Lemma 1. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) (τ_1, τ_2)-*sCl*$(A) = \tau_1\tau_2$-*Int*$(\tau_1\tau_2$-*Cl*$(A)) \cup A$ [5];

(2) (τ_1, τ_2)-*sInt*$(A) = \tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Int*$(A)) \cap A$ [52].

Lemma 2. *Let (X, τ_1, τ_2) be a bitopological space. If V is a $\tau_1\tau_2$-open set of X having $\mathscr{N}(\tau_1, \tau_2)$-closed complement, then $\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(V))$ is a $(\tau_1, \tau_2)r$-open set having $\mathscr{N}(\tau_1, \tau_2)$-closed complement.*

Proof. It is obvious that $\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(V))$ is a $(\tau_1, \tau_2)r$-open set. Let us denote $K = X - \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(V))$. Of course, $K \subseteq X - V$. Let $\{U_\gamma \mid \gamma \in \Gamma\}$ be a $\tau_1\tau_2$-open cover of the set K. Then, $\{U_\gamma \mid \gamma \in \Gamma\} \cup (X - K)$ is a $\tau_1\tau_2$-open cover of the set $X - V$. Thus, there exist indexes $\gamma_1, \gamma_2, ..., \gamma_k$ such that

$$X - V \subseteq \bigcup_{i=1}^{k} \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(U_{\gamma_i})) \cup \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(X - K)).$$

Since $\tau_1\tau_2$-Int($\tau_1\tau_2$-Cl($X - K$)) = $\tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)), $\tau_1\tau_2$-Int($\tau_1\tau_2$-Cl($X - K$)) $\cap K = \emptyset$. It was shown that $K \subseteq \bigcup_{i=1}^{k} \tau_1\tau_2$-Int($\tau_1\tau_2$-Cl($U_{\gamma_i}$)). The proof of $\mathscr{N}(\tau_1, \tau_2)$-closedness of the set K is finished.

Lemma 3. *Let (X, τ_1, τ_2) be a bitopological space. If V is a $(\tau_1, \tau_2)p$-open set of X having $\mathscr{N}(\tau_1, \tau_2)$-closed complement, then $\tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) is a $(\tau_1, \tau_2)r$-open set having $\mathscr{N}(\tau_1, \tau_2)$-closed complement.*

Proof. It is evident that $\tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) is a $(\tau_1, \tau_2)r$-open set. Since V is $(\tau_1, \tau_2)p$-open, $V \subseteq \tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) and hence $X - \tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) $\subseteq X - V$. By the hypothesis, $X - V$ is $\mathscr{N}(\tau_1, \tau_2)$-closed and $X - \tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) is $(\tau_1, \tau_2)r$-closed. Thus, it follows from Lemma 2 that $X - \tau_1\tau_2$-Int($\tau_1\tau_2$-Cl(V)) is $\mathscr{N}(\tau_1, \tau_2)$-closed.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and lower almost nearly (τ_1, τ_2)-continuous multifunctions

In this section, we introduce the notions of upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions. Furthermore, several characterizations of upper almost nearly (τ_1, τ_2)-continuous multifunctions and lower almost nearly (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper almost nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl($V$)). A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper almost nearly (τ_1, τ_2)-continuous if F is upper almost nearly (τ_1, τ_2)-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost nearly (τ_1, τ_2)-continuous at $x \in X$;

(2) $x \in \tau_1\tau_2$-Int(F^+($\sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl(V)))) for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(3) $x \in \tau_1\tau_2$-Int(F^+((σ_1, σ_2)-sCl(V))) for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(4) $x \in \tau_1\tau_2$-*Int*$(F^+(V))$ *for each* $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *containing* $F(x)$ *and having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(5) for each $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *containing* $F(x)$ *and having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $F(U) \subseteq V$*.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$. Thus, we have $x \in U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ and hence $x \in \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$.

(2) $\Rightarrow$ (3): This follows from Lemma 1.

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. It follows from Lemma 1 that

$$V = \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)) = (\sigma_1, \sigma_2)\text{-sCl}(V).$$

(4) $\Rightarrow$ (5): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (4), $x \in \tau_1\tau_2\text{-Cl}(F^+(V))$ and therefore there exists a $\tau_1\tau_2$-open set U of X such that $x \in U \subseteq F^+(V)$; hence $F(U) \subseteq V$.

(5) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By Lemma 2, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is a $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Thus by (5), there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$. This shows that F is upper almost nearly (τ_1, τ_2)-continuous.

Definition 2. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be lower almost nearly* (τ_1, τ_2)*-continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$ *and having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $\sigma_1\sigma_2$-*Int*$(\sigma_1\sigma_2$-*Cl*$(V)) \cap F(z) \neq \emptyset$ *for each* $z \in U$*. A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be lower almost nearly* (τ_1, τ_2)*-continuous if* F *is lower almost nearly* (τ_1, τ_2)*-continuous at each point* x *of* X*.*

Theorem 2. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower almost nearly* (τ_1, τ_2)*-continuous at* $x \in X$*;*

(2) $x \in \tau_1\tau_2$-*Int*$(F^-(\sigma_1\sigma_2$-*Int*$(\sigma_1\sigma_2$-*Cl*$(V))))$ *for each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$ *and having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(3) $x \in \tau_1\tau_2$-*Int*$(F^-((\sigma_1, \sigma_2)$-*sCl*$(V)))$ *for each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that*

$$F(x) \cap V \neq \emptyset$$

and having $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(4) *$x \in \tau_1\tau_2$-Int$(F^-(V))$ for each (σ_1, σ_2)r-open set V of Y such that $F(x) \cap V \neq \emptyset$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(5) *for each (σ_1, σ_2)r-open set V of Y such that $F(x) \cap V \neq \emptyset$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that*

$$F(z) \cap V \neq \emptyset$$

for every $z \in U$.

Proof. The proof is similar to that of Theorem 1.

Theorem 3. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper almost nearly (τ_1, τ_2)-continuous;*

(2) *$F^+(V) \subseteq \tau_1\tau_2$-Int$(F^+(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$ for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(3) *$\tau_1\tau_2$-Cl$(F^-(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(K)))) \subseteq F^-(K)$ for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y;*

(4) *$\tau_1\tau_2$-Cl$(F^-(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(B))))) \subseteq F^-(\sigma_1\sigma_2$-Cl$(B))$ for every every subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;*

(5) *$F^+(\sigma_1\sigma_2$-Int$(B)) \subseteq \tau_1\tau_2$-Int$(F^+(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(B)))))$ for every every subset B of Y such that $Y - \sigma_1\sigma_2$-Int(B) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed;*

(6) *$F^+(V)$ is $\tau_1\tau_2$-open in X for each (σ_1, σ_2)r-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(7) *$F^-(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed and (σ_1, σ_2)r-closed set K of Y.*

Proof. (1) ⇒ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$ having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement and $x \in F^+(V)$. Then, $F(x) \subseteq V$. By Theorem 1, we have

$$x \in \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$$

and hence $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$.

(2) ⇒ (3): Let K be any $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y. Then, $Y - K$ is a $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (2), we have

$$\begin{aligned} X - F^-(K) &= F^+(Y - K) \\ &\subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K)))) \\ &= \tau_1\tau_2\text{-Int}(X - F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \end{aligned}$$

$$= X - \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))).$$

Thus, $\tau_1\tau_2$-Cl$(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq F^-(K)$.

(3) $\Rightarrow$ (4): Let B be any subset of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2$-Cl(B) is a $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed set of Y. Thus by (3),

$$\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(B)).$$

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $Y - \sigma_1\sigma_2$-Int(B) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Since $Y - \sigma_1\sigma_2$-Int(B) is $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Then by (4), we have

$$\begin{aligned} F^+(\sigma_1\sigma_2\text{-Int}(B)) &= X - F^-(Y - \sigma_1\sigma_2\text{-Int}(B)) \\ &= X - F^-(\sigma_1\sigma_2\text{-Cl}(Y - B)) \\ &\subseteq X - \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - B))))) \\ &= X - \tau_1\tau_2\text{-Cl}(F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))) \\ &= \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))). \end{aligned}$$

(5) $\Rightarrow$ (6): Let V be any $(\sigma_1, \sigma_2)r$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - \sigma_1\sigma_2$-Int(V) is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Thus by (5), we have

$$F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(V))$$

and hence $F^+(V)$ is $\tau_1\tau_2$-open in X.

(6) $\Rightarrow$ (7): Let K be any $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $(\sigma_1, \sigma_2)r$-closed set of Y. Then, $Y - K$ is a $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (6), we have $F^+(Y - K) = X - F^-(K)$ is $\tau_1\tau_2$-open in X and hence $F^-(K)$ is $\tau_1\tau_2$-closed in X.

(7) $\Rightarrow$ (1): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $(\sigma_1, \sigma_2)r$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed. By (7), $F^-(Y - V) = X - F^+(V)$ is $\tau_1\tau_2$-closed in X. Thus, $F^+(V)$ is $\tau_1\tau_2$-open in X. Then, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq V$. It follows from Theorem 1 that F is upper almost nearly (τ_1, τ_2)-continuous at x. This shows that F is upper almost nearly (τ_1, τ_2)-continuous.

Theorem 4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower nearly almost (τ_1, τ_2)-continuous;

(2) $F^-(V) \subseteq \tau_1\tau_2$-Int$(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(3) $\tau_1\tau_2$-Cl$(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq F^+(K)$ for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y;

(4) $\tau_1\tau_2$-Cl$(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(B))$ for every every subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;

(5) $F^-(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(B)))))$ *for every every subset* B *of* Y *such that* $Y - \sigma_1\sigma_2\text{-}Int(B)$ *is* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed;*

(6) $F^-(V)$ *is* $\tau_1\tau_2$*-open in* X *for each* $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(7) $F^+(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed and* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Theorem 3.

Corollary 1. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper almost nearly* (τ_1, τ_2)*-continuous if* $F^-(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed set* K *of* Y*.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $(\sigma_1, \sigma_2)r$-closed. By the hypothesis,

$$X - F^+(V) = F^-(Y - V)$$

is $\tau_1\tau_2$-closed in X and hence $F^+(V)$ is $\tau_1\tau_2$-open in X. It follows from Theorem 3 that F is upper almost nearly (τ_1, τ_2)-continuous.

Corollary 2. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is lower almost nearly* (τ_1, τ_2)*-continuous if* $F^+(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Corollary 1.

Theorem 5. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper almost nearly* (τ_1, τ_2)*-continuous;*

(2) $\tau_1\tau_2\text{-}Cl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every every* $(\sigma_1, \sigma_2)\beta$*-open set* V *of* Y *having the* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed* $\sigma_1\sigma_2$*-closure;*

(3) $\tau_1\tau_2\text{-}Cl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every every* $(\sigma_1, \sigma_2)s$*-open set* V *of* Y *having the* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed* $\sigma_1\sigma_2$*-closure;*

(4) $F^+(V) \subseteq \tau_1\tau_2\text{-}Int(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V))))$ *for every every* $(\sigma_1, \sigma_2)p$*-open set* V *of* Y *having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement.*

Proof. (1) ⇒ (2): Let V be any $(\sigma_1, \sigma_2)\beta$-open set of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed in Y. Since F is upper almost nearly (τ_1, τ_2)-continuous, by Theorem 3 we have $F^-(\sigma_1\sigma_2\text{-Cl}(V))$ is $\tau_1\tau_2$-closed in X. Thus, $\tau_1\tau_2\text{-Cl}(F^-(V)) \subseteq \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(V))) = F^-(\sigma_1\sigma_2\text{-Cl}(V))$.

(2) ⇒ (3): The proof is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1,\sigma_2)p$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Then by Lemma 3, $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1,\sigma_2)r$-open set having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Then, $Y-\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1,\sigma_2)r$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set. Therefore, $Y-\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1,\sigma_2)s$-open set having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure. By (3), we have

$$\begin{aligned} X-\tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) &= \tau_1\tau_2\text{-Cl}(F^-(Y-\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y-\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &= X-F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq X-F^+(V) \end{aligned}$$

and hence $F^+(V)\subseteq\tau_1\tau_2$-Int$(F^+(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$.

(4) $\Rightarrow$ (1): Let V be any $(\sigma_1,\sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Then, V is a $(\sigma_1,\sigma_2)p$-open set having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. By (4), we have $F^+(V)\subseteq\tau_1\tau_2$-Int$(F^+(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))=\tau_1\tau_2$-Int$(F^+(V))$ and hence $F^+(V)$ is $\tau_1\tau_2$-open in X. Thus by Theorem 3, F is upper almost nearly (τ_1,τ_2)-continuous.

Theorem 6. *For a multifunction $F:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is lower almost nearly (τ_1,τ_2)-continuous;

(2) $\tau_1\tau_2$-Cl$(F^+(V))\subseteq F^+(\sigma_1\sigma_2$-Cl$(V))$ for every every $(\sigma_1,\sigma_2)\beta$-open set V of Y having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure;

(3) $\tau_1\tau_2$-Cl$(F^+(V))\subseteq F^+(\sigma_1\sigma_2$-Cl$(V))$ for every every $(\sigma_1,\sigma_2)s$-open set V of Y having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure;

(4) $F^-(V)\subseteq\tau_1\tau_2$-Int$(F^-(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$ for every every $(\sigma_1,\sigma_2)p$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement.

Proof. The proof is similar to that of Theorem 5.

Lemma 4. *For a bitopological space (X,τ_1,τ_2), the following properties hold:*

(1) $\alpha(\tau_1,\tau_2)$-Cl$(U)=\tau_1\tau_2$-Cl(U) for every $(\tau_1,\tau_2)\beta$-open set U of X;

(2) (τ_1,τ_2)-pCl$(U)=\tau_1\tau_2$-Cl(U) for every $(\tau_1,\tau_2)s$-open set U of X.

Corollary 3. *For a multifunction $F:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is upper almost nearly (τ_1,τ_2)-continuous;

(2) $\tau_1\tau_2$-Cl$(F^-(V))\subseteq F^-(\alpha(\sigma_1,\sigma_2)$-Cl$(V))$ for every every $(\sigma_1,\sigma_2)\beta$-open set V of Y having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure;

(3) $\tau_1\tau_2$-*Cl*$(F^-(V)) \subseteq F^-((\sigma_1,\sigma_2)$-*pCl*$(V))$ *for every every* $(\sigma_1,\sigma_2)s$-*open set* V *of* Y *having the* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed* $\sigma_1\sigma_2$-*closure.*

Corollary 4. *For a multifunction* $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$*, the following properties are equivalent:*

(1) F *is lower almost nearly* (τ_1,τ_2)-*continuous;*

(2) $\tau_1\tau_2$-*Cl*$(F^+(V)) \subseteq F^+(\alpha(\sigma_1,\sigma_2)$-*Cl*$(V))$ *for every every* $(\sigma_1,\sigma_2)\beta$-*open set* V *of* Y *having the* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed* $\sigma_1\sigma_2$-*closure;*

(3) $\tau_1\tau_2$-*Cl*$(F^+(V)) \subseteq F^+((\sigma_1,\sigma_2)$-*pCl*$(V))$ *for every every* $(\sigma_1,\sigma_2)s$-*open set* V *of* Y *having the* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed* $\sigma_1\sigma_2$-*closure.*

Theorem 7. *For a multifunction* $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$*, the following properties are equivalent:*

(1) F *is upper almost nearly* (τ_1,τ_2)-*continuous;*

(2) for each $x \in X$ *and for every* $\sigma_1\sigma_2$-*closed and* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed set* K *of* Y *such that* $x \in F^+(Y-K)$*, there exists a* $\tau_1\tau_2$-*closed set* H *of* X *such that* $x \in X-H$ *and* $F^-(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(K))) \subseteq H$*;*

(3) $F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ *is* $\tau_1\tau_2$-*open in* X *for every* $\sigma_1\sigma_2$-*open set* V *of* Y *having* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed complement;*

(4) $F^-(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(K)))$ *is* $\tau_1\tau_2$-*closed in* X *for every* $\sigma_1\sigma_2$-*closed and* $\mathscr{N}(\sigma_1,\sigma_2)$-*closed set* K *of* Y*.*

Proof. (1) ⇒ (2): Let $x \in X$ and K be any $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set of Y such that $x \in F^+(Y-K)$. Then, $Y-K$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Since F is upper almost nearly (τ_1,τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y-K))) = X - F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))).$$

It is clear that $H = X - U$ is $\tau_1\tau_2$-closed in X and $F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \subseteq H$.

(2) ⇒ (1): The proof is similar to the proof (1) ⇒ (2).

(1) ⇒ (3): Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement and $x \in F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Then, we have $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is a $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Thus by (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Since U is $\tau_1\tau_2$-open, we have $x \in \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ and hence

$$F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))).$$

Thus, $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-open in X.

(3) ⇒ (1): The proof is clear.

(3) $\Rightarrow$ (4): Let K be any $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed set of Y. Then, $Y - K$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (3), $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K)))$ is $\tau_1\tau_2$-open in X. Since $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K)) = Y - \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))$, it follows that

$$\begin{aligned} F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K))) &= F^+(Y - \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= X - F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))). \end{aligned}$$

Thus, $F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $\tau_1\tau_2$-closed in X.

(4) $\Rightarrow$ (3): It can be obtained similarly as (3) $\Rightarrow$ (4).

Theorem 8. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost nearly (τ_1, τ_2)-continuous;

(2) for each $x \in X$ and for every $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y such that $x \in F^-(Y - K)$, there exists a $\tau_1\tau_2$-closed set H of X such that $x \in X - H$ and $F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \subseteq H$;

(3) $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;

(4) $F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y.

Proof. The proof is similar to that of Theorem 7.

Recall that a net (x_γ) in a topological space (X, τ) is said to be *eventually* in the set $U \subseteq X$ if there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$. A net (x_γ) is called (τ_1, τ_2)-*converge to a point* x if for every $\tau_1\tau_2$-open set V containing x, there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in V$ for all $\gamma \geq \gamma_0$.

Theorem 9. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper almost nearly (τ_1, τ_2)-continuous if and only if for each $x \in X$ and for each net (x_γ) which (τ_1, τ_2)-converges to x in X and for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $x \in F^+(V)$, the net (x_γ) is eventually in $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$.*

Proof. Let (x_γ) be a net which (τ_1, τ_2)-converges to x in X and V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $x \in F^+(V)$. Since F is upper almost nearly (τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Since (x_γ) (τ_1, τ_2)-converges to x, it follows that there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$. Therefore,

$$x_\gamma \in U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$$

for all $\gamma \geq \gamma_0$. Thus, the net (x_γ) is eventually in $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$.

Conversely, suppose that F is not upper almost nearly (τ_1, τ_2)-continuous. Then, there exists a point x of X and a $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement with $x \in F^+(V)$ such that $U \not\subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ for each $\tau_1\tau_2$-open set U of X containing x. Let $x_U \in U$ and $x_U \notin F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ for each $\tau_1\tau_2$-open set U of X containing x. Then, for each $\tau_1\tau_2$-neighbourhood net (x_U), (x_U) (τ_1, τ_2)-converges to x, but (x_U) is not eventually in $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. This is a contradiction. Thus, F is upper almost nearly (τ_1, τ_2)-continuous.

Theorem 10. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower almost nearly (τ_1, τ_2)-continuous if and only if for each $x \in X$ and for each net (x_γ) which (τ_1, τ_2)-converges to x in X and for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $x \in F^-(V)$, the net (x_γ) is eventually in $F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$.*

Proof. The proof is similar to that of Theorem 9.

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, a multifunction

$$\text{Cl}F_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is defined in [29] as follows: $\text{Cl}F_{\circledast}(x) = \sigma_1\sigma_2\text{-Cl}(F(x))$ for each $x \in X$.

Definition 3. [29] *A subset A of a bitopological space (X, τ_1, τ_2) is said to be:*

(1) $\tau_1\tau_2$-paracompact if every cover of A by $\tau_1\tau_2$-open sets of X is refined by a cover of A which consists of $\tau_1\tau_2$-open sets of X and is $\tau_1\tau_2$-locally finite in X;

(2) $\tau_1\tau_2$-regular if for each $x \in A$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V of X such that $x \in V \subseteq \tau_1\tau_2\text{-}Cl(V) \subseteq U$.

Lemma 5. [29] *If A is a $\tau_1\tau_2$-regular $\tau_1\tau_2$-paracompact set of a bitopological space (X, τ_1, τ_2) and U is a $\tau_1\tau_2$-open neighbourhood of A, then there exists a $\tau_1\tau_2$-open set V of X such that $A \subseteq V \subseteq \tau_1\tau_2\text{-}Cl(V) \subseteq U$.*

Lemma 6. [29] *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\tau_1\tau_2$-regular and $\tau_1\tau_2$-paracompact for each $x \in X$, then $ClF^+_{\circledast}(V) = F^+(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Theorem 11. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-paracompact and $\sigma_1\sigma_2$-regular for each $x \in X$. Then, F is upper almost nearly (τ_1, τ_2)-continuous if and only if $G : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper almost nearly (τ_1, τ_2)-continuous, where G denote $ClF_{\circledast}$.*

Proof. Suppose that F is upper almost nearly (τ_1, τ_2)-continuous. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-connected complement. It follows from Lemma 6 and Theorem 3 that $G^+(V) = F^+(V)$ is $\tau_1\tau_2$-open in X. By Theorem 3, G is upper almost nearly (τ_1, τ_2)-continuous.

Conversely, suppose that G is upper almost nearly (τ_1, τ_2)-continuous. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-connected complement. By Lemma 6 and Theorem 3, $F^+(V) = G^+(V)$ is $\tau_1\tau_2$-open in X. Thus by Theorem 3, F is upper almost nearly (τ_1, τ_2)-continuous.

Lemma 7. [29] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, $ClF^-_{\circledast}(V) = F^-(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Theorem 12. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower almost nearly (τ_1, τ_2)-continuous if and only if $G : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower almost nearly (τ_1, τ_2)-continuous, where G denote $ClF_{\circledast}$.*

Proof. By using Lemma 7 this can be shown similarly as in Theorem 11.

4. Several characterizations

The $\tau_1\tau_2$-*frontier* [26] of a subset A of a bitopological space (X, τ_1, τ_2), denoted by $\tau_1\tau_2$-fr(A), is defined by

$$\tau_1\tau_2\text{-fr}(A) = \tau_1\tau_2\text{-Cl}(A) \cap \tau_1\tau_2\text{-Cl}(X - A) = \tau_1\tau_2\text{-Cl}(A) - \tau_1\tau_2\text{-Int}(A).$$

Theorem 13. *The set of all points x of X at which a multifunction*

$$F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is not upper almost nearly (τ_1, τ_2)-continuous is identical with the union of the $\tau_1\tau_2$-frontier of the upper inverse images of $(\sigma_1, \sigma_2)r$-open sets containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement.

Proof. Let x be a point of X at which F is not upper almost nearly (τ_1, τ_2)-continuous. Then, by Theorem 1 there exists a $(\sigma_1, \sigma_2)r$-open set V of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $U \cap (X - F^+(V)) \neq \emptyset$ for every $\tau_1\tau_2$-open set U of X containing x. Thus, $x \in \tau_1\tau_2\text{-Cl}(X - F^+(V))$. On the other hand, we have

$$x \in F^+(V) \subseteq \tau_1\tau_2\text{-Cl}(F^+(V))$$

and hence $x \in \tau_1\tau_2\text{-fr}(F^+(V))$.

Conversely, suppose that V is a $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $x \in \tau_1\tau_2\text{-fr}(F^+(V))$. If F is upper almost nearly (τ_1, τ_2)-continuous at $x \in X$. Then by Theorem 1, we have $x \in \tau_1\tau_2\text{-Int}(F^+(V))$. This is a contradiction and hence F is not upper almost nearly (τ_1, τ_2)-continuous at x.

Theorem 14. *The set of all points x of X at which a multifunction*

$$F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is not lower almost nearly (τ_1, τ_2)-continuous is identical with the union of the $\tau_1\tau_2$-frontier of the lower inverse images of $(\sigma_1, \sigma_2)r$-open sets meeting $F(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement.

Proof. The proof is similar to that of Theorem 13.

Recall that a subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [29] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed.

Definition 4. [29] *A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-connected if X cannot be written as the union of two disjoint nonempty $\tau_1\tau_2$-open sets.*

Definition 5. *A bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$-connected if X cannot be written as the union of two disjoint nonempty $\tau_1\tau_2$-open sets having $\mathscr{N}(\tau_1, \tau_2)$-closed complements.*

Theorem 15. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper or lower almost nearly (τ_1, τ_2)-continuous surjective multifunction such that $F(x)$ is $\sigma_1\sigma_2$-connected for each $x \in X$ and (X, τ_1, τ_2) is $\tau_1\tau_2$-connected, then (Y, σ_1, σ_2) is $\mathscr{N}(\sigma_1, \sigma_2)$-connected.*

Proof. Suppose that (Y, σ_1, σ_2) is not $\mathscr{N}(\sigma_1, \sigma_2)$-connected. There exist nonempty $\sigma_1\sigma_2$-open sets U and V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $U \cap V = \emptyset$ and $U \cup V = Y$. Since $F(x)$ is $\sigma_1\sigma_2$-connected for each $x \in X$, either $F(x) \subseteq U$ or $F(x) \subseteq V$. If $x \in F^+(U \cup V)$, then $F(x) \subseteq U \cup V$ and hence $x \in F^+(U) \cup F^+(V)$. Moreover, since F is surjective, there exist x and y in X such that $F(x) \subseteq U$ and $F(y) \subseteq V$; hence $x \in F^+(U)$ and $y \in F^+(V)$. Therefore, we obtain the following:

(1) $F^+(U) \cup F^+(V) = X$;

(2) $F^+(U) \cap F^+(V) = \emptyset$;

(3) $F^+(U) \neq \emptyset$ and $F^+(V) \neq \emptyset$.

Next, we show that $F^+(U)$ and $F^+(V)$ are $\tau_1\tau_2$-open in X. (i) Let F be upper almost nearly (τ_1, τ_2)-continuous. Since U and V are $\sigma_1\sigma_2$-clopen in Y, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(U)) = U$ and $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)) = V$. Thus, U and V are $(\sigma_1, \sigma_2)r$-open sets having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. Since F is upper almost nearly (τ_1, τ_2)-continuous, by Theorem 3 $F^+(U)$ and $F^+(V)$ are $\tau_1\tau_2$-open sets. (ii) Let F be lower almost nearly (τ_1, τ_2)-continuous. By Theorem 4, $F^+(U)$ is $\tau_1\tau_2$-closed in X because U is $\sigma_1\sigma_2$-clopen in Y. Therefore, $F^+(V)$ is $\tau_1\tau_2$-open in X. Similarly, we have $F^+(U)$ is $\tau_1\tau_2$-open in X. Thus, (X, τ_1, τ_2) is not $\tau_1\tau_2$-connected.

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