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Total Edge Irregularity Strength of Star Snake Graphs

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Abstract. An edge irregular total k-labeling on simple and undirected graph G(V,E) is a map $f : V \cup E \rightarrow \{1, 2, \ldots, k\}$ such that for any different edge xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f\left(x'\right) + f\left(x'y'\right) + f(y')$ are distinct. The minimum positive integer k for which the graph G has an edge irregular total k-labeling is called the total edge irregularity strength of G and is denoted by tes(G). In different fields in our life, like physics, coding theory and computer science, graph labeling plays a vital role and appears in many applications. A labeling of a graph $M(V, E)$ is a map which assigns each element in G with a positive integer number. An edge irregular total ζ -labeling is a function $\Omega : V(M) \cup E(M) \rightarrow \{1, 2, 3, ..., \zeta\}$ such that $W_\Omega(h) \neq W_\Omega(z)$ where $W_\Omega(h)$ and $W_\Omega(z)$ are weights for any two distinct edges. In this case, M has total edge irregularity strength (TEIS) if ζ is minimum. In our paper, we defined a new type of graphs called a triple star snake graph $PS_{3,n}$and m-star snake graph $PS_{m,n}$. Also, we investigated TEIS for a triple star snake graph $PS_{3,n}$. We then generalized the results for m-star snake graph $PS_{m,n}$.

Key Words and Phrases: Edge labeling, Irregularity strength, Irregular labelling Total edge irregularity strength, Star snake graph

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1. Introduction

In real-world systems, interactions between pairs of entities take place every day. Examples of these systems include human interactions, financial networks, social networks, and biological networks. In the field of graph theory, such pairs of entities are referred to as a network, where the substances represent the vertices and the connections between any two substances are denoted as edges [1, 2]. The use of graph theory in condensed matter physics, pioneered by the work of many chemical and physical g(Harary, 1968; Trinajstić,

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1992), is well stablished and gaining more popularity. Some of the most important areas of application of graph theory in physics include condensed matter physics, statistical physics, quantum electrodynamics, electrical networks and vibrational problems [3–5].

The graph G is the ordered pair $(V(G), E(G))$ where $V(G)$ is the set of elements called vertices and $E(G)$ is a finite set of pairs of distinct elements $V(G)$ called set of edge. Two vertices such as v_1 and v_2 are called adjacent, whenever $v_1, v_2 \in E(G)$. Labeling of graph is a map that carries graph elements to positive integers [6]. The domain of mapping is a vertex set, or an edge set, or a union of vertex and edge sets. If the domain is a vertex set, the labeling is called vertex labeling. If the domain is an edge set, the labeling is called edge labeling. If the domain is a union of vertex and edge sets, the labeling is called total labeling. On progress, several types of labeling that has been studied can be seen on Gallian [7].

Spectral graph theory is a beautiful branch of graph theory that utilizes the eigenvalues and eigenvectors of matrices naturally associated with graphs to study them. Some interesting research has been done in the field of spectral graph theory in the past few years. The random walks of octagonal cell network has been investigated in [8] using the Laplacian spectrum method where the mean first passage time (τ) and Kemeny's constant Ξ between nodes was obtained. The work also provide an explicit expression of Kemeny's constant and mean first passage time for octagonal cell network, by their Laplacian eigenvalues and the correlation among roots of characteristic polynomial. In [1], an explicit closed-form formula of the global meanfirst-passage time (GMFPT) for hexagonal model has been established using the decomposition theorem of Laplacian polynomial and characteristic polynomial. They have also shown that, extensive matrix analysis, obtaining GMFPT via spectrums provides an easy calculation in terms of large networks. In [9], the electric network approach and the combinatorial approach have been used to derive the exact expression for resistance distances between any two vertices of the k_4^n ring model. The mean first passage time and Kemeny constant of k_4^n have also been calculated. A study of mean-first-passage time and Kemeny's constant of a random walk by normalized Laplacian matrices of a penta-chain network has been prformed in [10]. Motivtaed by many applications in computer networks, routing protocols, wireless sensor networks, and also by the normalized Laplacian (NL) matrix, the spectrums of the n copies of (k_5^l) chain graph has been obtained in [11]. The number of spanning trees for k_5^l has also been calculated by utilizing these spectrums.

For a connected and simple graph $M(V, E)$, an edge irregular total ζ-labeling has been introduced by Baca et al. in [12] as a map $\Omega : V(M) \cup E(M) \to \{1, 2, 3, ..., \zeta\}$ such that $W_\Omega(h) \neq W_\Omega(z)$ where $W_\Omega(h)$ and $W_\Omega(z)$ are weights for any two distinct edges. Also, the inequality of TEIS for a graph, with the maximum degree of vertices ΔG, has been deduced in the form

$$tes(M) \geq max\left\{\frac{E(M)+2}{3}, \frac{\Delta(M)+1}{2}\right\} \tag{1}$$

Since then, many authors have begun to find TEIS for many families of graphs. Ivanĉo and Jendroî in [13] determined TEIS for a tree as

$$tes(T) = max\left\{\frac{k+2}{3}, \frac{\Delta(M)+1}{2}\right\} \quad (2)$$

Ahmad et al. [14–20] have investigated TEIS for zigzag graphs, helm and sun graphs, the categorical product of two cycles, the categorical product of two paths, the generalized Petersen graph, certain families of graphs and some classes of plane graphs. Therefore, TEIS has been determined for hexagonal grid graphs in Al-Mushayt and Ahmad [21], planar graphs in Yang et al. [22], for some classes of plane graphs in Tarawneh et al. [23], for fan, wheel, triangular book, and friendship graphs in Tilukay et al. [24], for subdivision of star in Siddiqui [25], for some Cartesian product graphs in Ramdan and Salman [26], for trees in Amar and Togn [27], for generalized web graphs and related graphs in Indriat et al. [28], for generalized prism in Bača and Siddiqui [29], for complete graph and complete bipartite graphs in Jendroî et al. [30], for the disjoint union of wheel graphs in Jeyanth and Sudhai [31], for dense graphs in Majersk et al. [32], for the grids in Miškuf and Jendroî [33], for disjoint union of isomorphic copies of generalized Petersen graph in Naeem and Siddiqui [34], for large graphs in Pfender [35], for centralized uniform theta graphs in Putra and Susanti [36], for series parallel graphs in Rajasingh et al. [37].

Salama [38],[39],[40],[41],[42] has determined TEIS for the polar grid graph, special families of graphs, heptagonal snake graph, uniform theta snake graphs and quintet snake graph.

In this paper, we define new types of graphs called a triple star snake graph $PS_{3,n}$ and m-star snake graph $PS_{m,n}$. Also, we investigate the TEIS for a triple star snake graph $S_{3,n}$. Then, we generalize the results for m-star snake graph $PS_{m,n}$.

2. Main results

In this section, we define the star snake graph and some related graphs. We also determine the TEIS for these graphs.

Definition 1. *In a path P_n, if we replace every edge with a star S_3 we get a new graph called a triple star snake graph, denoted $PS_{3,n}$ (see Figure 1).*

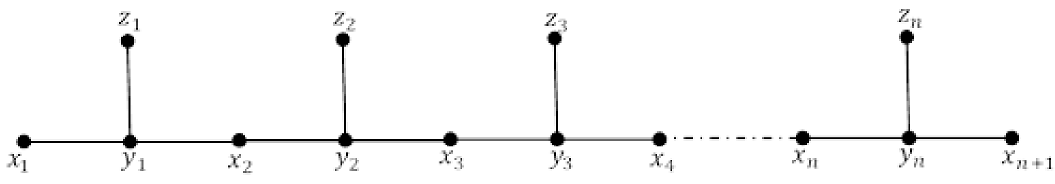

Figure 1: A triple star snake graph $PS_{3,n}$

Theorem 1. *If $PS_{3,n}$ is a triple star snake graph with $3n+1$ vertices, then TEIS is given by : $tes(PS_{3,n}) = n + 1$*

Proof. Since $|E(PS_{3,n})| = 3n$ and $\Delta(PS_{3,n}) = 3$, Then inequality (1) becomes $tes(PS_{3,n}) \geq n+1$.
To complete the proof, we will prove the inverse inequality.
Let $\zeta = n+1$ and $\Omega : V(PS_{3,n}) \cup E(PS_{3,n}) \to \{1, 2, 3, ..., \zeta\}$ is a total ζ-labeling defined as:
$\Omega(x_\delta) = \delta$ for $\delta \in \{1, 2, 3, ..., n+1\}$
$\Omega(x_\delta y_\delta) = \delta$ for $\delta \in \{1, 2, 3, ..., n\}$
$\Omega(y_\delta x_{\delta+1}) = \Omega(y_\delta z_\delta) = \delta + 1$ for $\delta \in \{1, 2, 3, ..., n\}$
The above equations mean that $\zeta = n+1$ is the greatest label of edges and vertices. The weights of edges are given by:
$W_\Omega(x_\delta y_\delta) = 3\delta$ for $\delta \in \{1, 2, 3, ..., n\}$
$W_\Omega(y_\delta x_{\delta+1}) = 3\delta + 2$ for $\delta \in \{1, 2, 3, ..., n\}$
$W_\Omega(y_\delta z_\delta) = 3\delta + 1$ for $\delta \in \{1, 2, 3, ..., n\}$
It is clear that the edges weights are dissimilar. Then, $tes(PS_{3,n}) \geq n+1$ □.

Definition 2. *The m-star snake graph $PS_{m,n}$ is a path P_n in which we replace each edge with a star S_m (see Fig. 2, 3).*

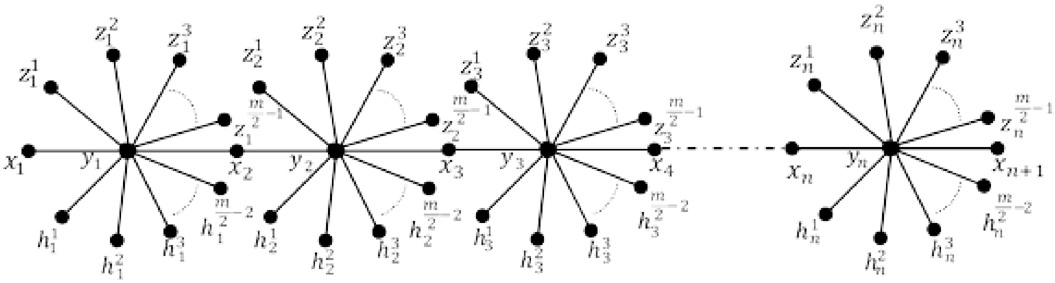

Figure 2: m-star snake graph $PS_{m,n}$, (m is odd)

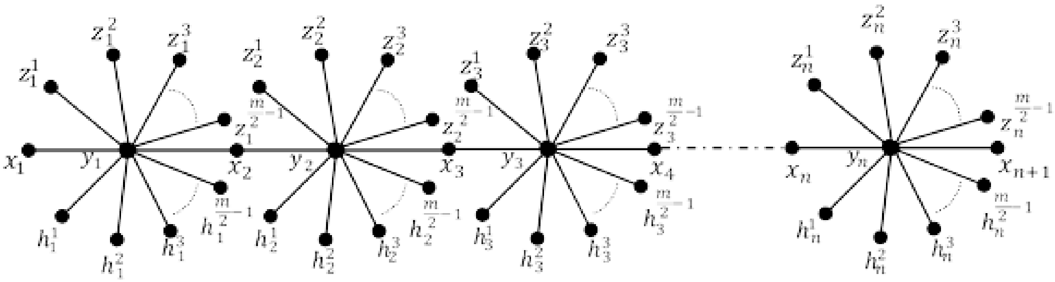

Figure 3: m-star snake graph $PS_{m,n}$, (m is even)

Theorem 2. *The TEIS of the m-star snake graph $PS_{m,n}$ with $mn+1$ vertices $n > 1$ is given by*

$$tes(PS_{m,n}) = \frac{mn+2}{3} \tag{3}$$

Proof. By substituting with $|E(PS_{4,n})| = mn$ and $\Delta(PS_{m,n}) = m$ in inequality (3), we find

$$tes(PS_{m,n}) \geq \frac{mn+2}{3} \tag{4}$$

An edge irregular total ζ-labeling will be shown assuming that a map $\Omega : E(PS_{m,n}) \cup V(PS_{m,n}) \to \{1, 2, 3, ..., \zeta\}$ is a total ζ-labeling defined in two cases:
Case 1: m is even, Ω is defined as:

$$\Omega(x_\delta) = \begin{cases} \frac{m}{2}(\delta-1)+1 \quad \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}+1\right\} \\ \\ \zeta \quad \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}}+2, ..., n+1\right\} \end{cases}$$

$$\Omega(y_\delta) = \begin{cases} \frac{m}{2}(\delta-1)+1 \quad \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}+1\right\} \\ \\ \zeta \quad \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}}+2, ..., n\right\} \end{cases}$$

$$\Omega(h_\delta^s) = \Omega(z_\delta^s) = \begin{cases} (\frac{m}{2}-1)(\delta-1)+s \quad \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}+1\right\} \\ (\frac{m}{2}-1)(\delta-1)+s \quad \text{for} \quad \begin{cases} \delta = \frac{\zeta-1}{\frac{m}{2}}+2 \\ \\ s = \left\{1, 2, 3, ..., \zeta-(\frac{m}{2}-1)(\delta-1)\right\} \end{cases} \\ \zeta \quad \text{for} \quad \begin{cases} \delta = \frac{\zeta-1}{\frac{m}{2}}+2 \\ \\ s = \left\{\zeta-(\frac{m}{2}-1)(\delta-1)+1, ..., \frac{m}{2}-1\right\} \end{cases} \\ \zeta \quad \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}}+3, ..., n\right\} \end{cases}$$

$$\Omega(x_\delta y_\delta) = \begin{cases} 1 \quad \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}+1\right\} \\ \\ (\delta-1)m-2\zeta+3 \quad \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}}+2, ..., n\right\} \end{cases}$$

$$\Omega(y_\delta x_{\delta+1}) = \begin{cases} \frac{m}{2} \quad \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}\right\} \\ \frac{m}{2}(\delta+1)-\zeta+1 \quad \text{for} \quad \delta = \frac{\zeta-1}{\frac{m}{2}}+1 \\ \delta m-2\zeta+2 \quad \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}}+2, ..., n\right\} \end{cases}$$

$$\Omega(y_\delta z_\delta^s)=\begin{cases}\delta+s \quad \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}+1\right\}\\ \frac{m}{2}(\delta-1)-\zeta+\delta+s+1 \quad \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+2\\ \\ s=\left\{1,2,3,...,\zeta-(\frac{m}{2}-1)(\delta-1)\right\}\end{cases}\\ (\delta-1)m-2\zeta+2s+2 \quad \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+2\\ \\ s=\left\{\zeta-(\frac{m}{2}-1)(\delta-1)+1,...,\frac{m}{2}-1\right\}\end{cases}\\ (\delta-1)m-2\zeta+2s+2 \quad \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+3,...,n\right\}\end{cases}$$

$$\Omega(y_\delta h_\delta^s)=\begin{cases}\delta+s+1 \quad \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}+1\right\}\\ \frac{m}{2}(\delta-1)-\zeta+\delta+s+2 \quad \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+2\\ \\ s=\left\{1,2,3,...,\zeta-(\frac{m}{2}-1)(\delta-1)\right\}\end{cases}\\ (\delta-1)m-2\zeta+2s+3 \quad \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+2\\ \\ s=\left\{\zeta-(\frac{m}{2}-1)(\delta-1)+1,...,\frac{m}{2}-1\right\}\end{cases}\\ (\delta-1)m-2\zeta+2s+3 \quad \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+3,...,n\right\}\end{cases}$$

From the previous equations, we can say ζ is the maximum number which labels vertices and edges. The edges' weights of $PS_{m,n}$ are given by:

$$\begin{aligned}W_\Omega(x_\delta y_\delta) &= (\delta-1)m+3\\ W_\Omega(y_\delta x_{\delta+1}) &= m\delta+2\\ W_\Omega(y_\delta z_\delta^s) &= (\delta-1)m+2(s+1)\\ W_\Omega(y_\delta h_\delta^s) &= (\delta-1)m+2s+3\end{aligned}$$

We can say that from the previous equations, the weights are distinct for any two edges. So

$$tes(PS_{m,n})=\frac{mn+2}{3}$$

Case 2: m is odd, Ω is defined as:

$$\Omega(x_\delta)=\begin{cases}\frac{m}{2}(\delta-1)+1 \quad \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}+1\right\}\\ \\ \zeta \quad \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(y_\delta)=\begin{cases}\frac{m}{2}(\delta-1)+1 \quad \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}+1\right\}\\ \\ \zeta \quad \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n+1\right\}\end{cases}$$

$$\Omega(z_\delta^s)=\begin{cases}\frac{m}{2}(\delta-1)+s & \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}\right\}\\ \frac{m}{2}(\delta-1)+s & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{1,2,3,...,\zeta-\frac{m}{2}(\delta-1)\right\}\end{cases}\\ \zeta & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{\zeta-\frac{m}{2}(\delta-1)+1,...,\frac{m}{2}\right\}\end{cases}\\ \zeta & \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(h_\delta^s)=\begin{cases}\frac{m}{2}(\delta-1)+s & \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}\right\}\\ \frac{m}{2}(\delta-1)+s & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{1,2,3,...,\zeta-\frac{m}{2}(\delta-1)\right\}\end{cases}\\ \zeta & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{\zeta-\frac{m}{2}(\delta-1),...,\frac{m}{2}\right\}\end{cases}\\ \zeta & \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(x_\delta y_\delta)=\begin{cases}\delta & \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}+1\right\}\\ (\delta-1)m-2\zeta+3 & \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(y_\delta x_{\delta+1})=\begin{cases}\frac{m}{2}+\delta & \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}\right\}\\ m\delta-\zeta+1-\frac{m}{2}(\delta-1) & \text{for} \quad \delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ \delta m-2\zeta+2 & \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(y_\delta z_\delta^s)=\begin{cases}\delta+s & \text{for} \quad \delta\in\left\{1,2,3,...,\frac{\zeta-1}{\frac{m}{2}}\right\}\\ \delta+s & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{1,2,3,...,\zeta-(\frac{m}{2}-1)(\delta-1)\right\}\end{cases}\\ \frac{m}{2}(\delta-1)-\zeta+2s+\delta & \text{for} \quad \begin{cases}\delta=\frac{\zeta-1}{\frac{m}{2}}+1\\ s=\left\{\zeta-\frac{m}{2}(\delta-1)+1,...,\frac{m}{2}\right\}\end{cases}\\ (\delta-1)m-2\zeta+2s+2 & \text{for} \quad \delta\in\left\{\frac{\zeta-1}{\frac{m}{2}}+2,...,n\right\}\end{cases}$$

$$\Omega(y_\delta h_\delta^s) = \begin{cases} \delta + s + 1 & \text{for} \quad \delta \in \left\{1, 2, 3, ..., \frac{\zeta-1}{\frac{m}{2}}\right\} \\ \delta + s + 1 & \text{for} \quad \begin{cases} \delta = \frac{\zeta-1}{\frac{m}{2}} + 1 \\ s = \left\{1, 2, 3, ..., \zeta - \frac{m}{2}(\delta - 1) - 1\right\} \end{cases} \\ \frac{m}{2}(\delta - 1) - \zeta + 2s + \delta + 1 & \text{for} \quad \begin{cases} \delta = \frac{\zeta-1}{\frac{m}{2}} + 1 \\ s = \left\{\zeta - (\frac{m}{2} - 1)(\delta - 1), ..., \frac{m}{2} - 1\right\} \end{cases} \\ (\delta - 1)m - 2\zeta + 2s + 3 & \text{for} \quad \delta \in \left\{\frac{\zeta-1}{\frac{m}{2}} + 2, ..., n\right\} \end{cases}$$

□

From the previous formulas, we can deduce that ζ is the greatest label of edges and vertices. After calculating the weights of the edges of the graph $PS_{m,n}$ we find:

$$\begin{aligned} W_\Omega(x_\delta y_\delta) &= (\delta - 1)m + 3 \\ W_\Omega(y_\delta x_{\delta+1}) &= m\delta + 2 \\ W_\Omega(y_\delta z_\delta^s) &= (\delta - 1)m + 2(s + 1) \\ W_\Omega(y_\delta h_\delta^s) &= (\delta - 1)m + 2s + 3 \end{aligned}$$

From the equations of weights of edges we see that they are different. So Ω is an edge irregular total ζ-labeling and

$$tes(PS_{m,n}) = \frac{mn + 2}{3}$$

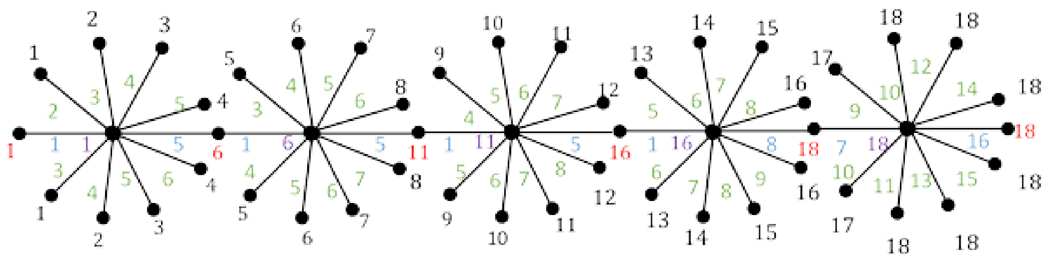

Figure 4: 10-star snake graph $PS_{10,5}$

3. Conclusion

Graph labeling plays an important role in different research areas such as computer science, coding and mathematical physics. The total edge irregularity strength of a graph G and is denoted by $tes(G)$ and is defined as the minimum positive integer k for which the graph G has an edge irregular total k-labeling. The present work aims to study the star snake graph and some related graphs, and determine the TEIS for these graphs. New

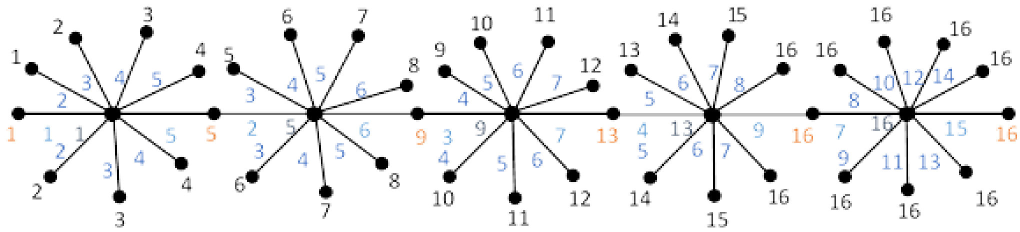

Figure 5: 9-star snake graph $PS_{9,5}$

types of graphs called a triple star snake graph $PS_{m,n}$ and m-star snake graph $PS_{m,n}$ were defined . The following theorem has been proved : If $PS_{m,n}$ is a triple star snake graph with $3n+1$ vertices, then TEIS is : $tes(PS_{3,n}) = n+1$. After that, we have generalized the results for m-star snake graph $PS_{m,n}$as : $tes(PS_{m,n}) = \frac{mn+2}{3}$ where the m-star snake graph $PS_{m,n}$ is defined as a path P_n in which we replace each edge with a star S_m.

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Conflict of interest All authors declare no conflict of interest in this paper.

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