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Subfamilies of Bi-Univalent Functions Defined by Imaginary Error Functions Subordinate to Horadam Polynomials

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Abstract. Several different subfamilies of the bi-univalent function family Ω were introduced and studied by numerous researchers using special functions. In the present paper, utilizing the imaginary error function, we introduce and study a new subfamily $\mathcal{F}_{\Omega}(s, r, u, y, t, \lambda, \tau)$ of bi-univalent functions in the open unit disk Θ, which are connected to the Horadam polynomials, and determine initial coefficients in the Maclaurin series of functions in this subfamily. Moreover, we determine the Fekete-Szegö inequality for functions in this subfamily. The parameters employed in our major results are specialized, and several fresh outcomes are shown to follow.

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1. Introduction and Preliminaries

Ordinary differential equations that meet model constraints are frequently solved using orthogonal polynomials [19] in mathematical model solving. Orthogonal polynomials are useful in physics and engineering and are significant in modern mathematics. The importance of these polynomials in issues pertaining to approximation theory is well known. They are present in quantum physics, approximation theory, probability theory, interpolation, differential equation theory, and mathematical statistics. They also model and

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analyze complicated systems and data sets in the fields of signal processing, image processing, and data analysis (see [5, 11]).

The pair of polynomials $\mathbb{J}_\epsilon$ and $\mathbb{J}_\varepsilon$, of order ϵ and ε, respectively, are orthogonal if

$$\langle \mathbb{J}_\epsilon,\ \mathbb{J}_\varepsilon \rangle = \int_{\sigma_1}^{\sigma_2} \mathbb{J}_\epsilon(y)\mathbb{J}_\varepsilon(y)r(y)dy = 0, \qquad \text{for} \qquad \epsilon \neq \varepsilon, \tag{1}$$

The integral of all finite order polynomials $\mathbb{J}_\epsilon(y)$ is properly defined since $r(y)$ is a non-negative function in the interval (σ_1, σ_2).

Several families of orthogonal polynomials are well-known, such as the Jacobi, Laguerre, Legendre, Hermite, and Chebyshev families. Orthogonal polynomials have many practical qualities and applications, and each family has its own weight function and interval.

The Recurrence relations define the Horadam polynomials as the family of polynomials that is a generalization of the Fibonacci and Lucas polynomials. Murray S. Klamkin Horadam, an Australian mathematician, is credited with their introduction in 1978, hence its name.

Numerous intriguing characteristics of Horadam polynomials and their relationships to other branches of mathematics, such as algebraic geometry, combinatorics, and number theory.

Horzum and Kocer (2009) examined the Horadam polynomials $h_\alpha(y)$, which are ascertained by the following recurrence relation [21].

$$h_\alpha(y) = ryh_{\alpha-1}(y) + uh_{\alpha-2}(y), \quad \alpha \in \{3, 4, \cdots\}, \tag{2}$$

with

$$h_1(y) = s,\ h_2(y) = ty \text{ and } h_3(y) = rty^2 + su,\ \ s, r, u, t \in \mathbb{R}. \tag{3}$$

The Horadam polynomials $h_\alpha(y)$ have the generator

$$\Upsilon(y, \wp) = \sum_{\alpha=1}^{\infty} h_\alpha(y)\wp^{\alpha-1} = \frac{s + (t - sr)y\wp}{1 - ry\wp - u\wp^2}. \tag{4}$$

Remark 1. *Various polynomials can be obtained from the Horadam polynomials $h_\alpha(y)$ for specific values of s, t, r and u (see [18, 21]). For instance:*

(i) When $s = t = r = u = 1$, we receive the Fibonacci polynomials $F_\alpha(y)$;

(ii) When $s = 2$ and $t = r = u = 1$, we receive the Lucas polynomials $L_\alpha(y)$;

(iii) When $s = t = 1$, $r = 2$ and $u = -1$, we receive the first kind of Chebyshev polynomials $T_\alpha(y)$;

(iv) When $s = 1$, $t = r = 2$ and $u = -1$, we receive the second kind of Chebyshev polynomials $U_\alpha(y)$;

(v) When $s = u = 1$ and $t = r = 2$, we receive the Pell polynomials $PL_\alpha(y)$;

(vi) When $s = t = r = 2$ and $u = 1$, we receive the first kind of Pell-Lucas polynomials $u_\alpha(y)$.

Let AU be the family of analytic and univalent functions L in the open disk $\Theta = \{\wp : |\wp| < 1\}$, of the form:

$$L(\wp) = \wp + c_2\wp^2 + c_3\wp^3 + \cdots . \tag{5}$$

For analytic functions L and V , L subordination to V (denoted by $L \prec V$) for all $\wp \in \Theta$, if there exists a function ϖ via $\varpi(0) = 0$ and $|\varpi(\wp)| < 1$, such that

$$L(\wp) = V(\varpi(\wp)).$$

In addition, if V is univalent in Θ, then

$$L(\wp) \prec V(\wp), \text{ iff, } L(0) = V(0)$$

and

$$L(\Theta) \subset V(\Theta).$$

Every function $L \in AU$ has an inverse L^{-1} defined by (see [12, 20]):

$$L^{-1}(L(\wp)) = \wp \qquad (\wp \in \Theta)$$

and

$$\varpi = L(L^{-1}(\varpi)) \qquad (|\varpi| < r_0(L);\ r_0(V) \geq \frac{1}{4}),$$

where

$$V(\varpi) = L^{-1}(\varpi) = \varpi - c_2\varpi^2 + (2c_2^2 - c_3)\varpi^3 - (c_4 + 5c_2^3 - 5c_3c_2)\varpi^4 + \cdots . \tag{6}$$

A function $L \in AU$ is said to be bi-univalent in Θ (the family of bi-univalent functions in Θ denoted by Ω) if both $L(\wp)$ and $L^{-1}(\wp)$ are univalent in Θ (see [15, 22]).

The error function is important in many scientific domains, such as probability, statistics, partial differential equations, and numerous engineering issues. As a result, mathematics has given it a lot of attention. For the error function, a number of noteworthy inequalities and associated subjects were reported; for examples, see [9, 13, 16]. When predicting events that hold with high or low probability, the error function and its approximations are typically utilized.

$$erf(\wp) = \frac{2}{\sqrt{\pi}}\int_0^{\wp} e^{-y^2}dy = \frac{2}{\sqrt{\pi}}\sum_{\nu=0}^{\infty}\frac{(-1)^\nu \wp^{2\nu+1}}{(2\nu+1)\nu!}, \quad \wp \in \mathbb{C}. \tag{7}$$

The imaginary error functions Maclaurin series can be obtained as shown above by wringing the integrand e^{-y^2} as Maclaurin series and integrating term by term; additionally, the imaginary error function, represented by the symbol erfi, has a very similar Maclaurin series, which is explained by (see [2, 10]):

$$erfi(\wp) = \frac{2}{\sqrt{\pi}} \int_0^{\wp} e^{-y^2} dy = \frac{2}{\sqrt{\pi}} \sum_{\nu=0}^{\infty} \frac{\wp^{2\nu+1}}{(2\nu+1)\nu!}, \quad \wp \in \mathbb{C}. \tag{8}$$

Using (7), Ramachandran et al. [8] investigated the normalized analytic error function regarding the form:

$$\mathrm{Erf}(\wp) = \frac{\sqrt{\pi \wp}}{2} erf(\sqrt{\wp}) = \wp + \sum_{\nu=2}^{\infty} \frac{(-1)^{\nu-1} \wp^{\nu}}{(2\nu-1)(\nu-1)!}, \tag{9}$$

and utilizing the convolution product "$*$" defined the following family

$$\mathrm{Erf} * AU = \left\{ R : R(\wp) = (\mathrm{Erf} * L)(\wp) = \wp + \sum_{\nu=2}^{\infty} \frac{(-1)^{\nu-1} c_{\nu}}{(2\nu-1)(\nu-1)!} \wp^{\nu}, \quad L \in AU \right\}. \tag{10}$$

From (8), the normalized form of the error function Erfi defined by:

$$\mathrm{Erfi}(\wp) = \frac{\sqrt{\pi \wp}}{2} erfi(\sqrt{\wp}) = \wp + \sum_{\nu=2}^{\infty} \frac{\wp^{\nu}}{(2\nu-1)(\nu-1)!}$$

and by convolution product, we define

$$EL(\wp) = (\mathrm{Erfi} * L)(\wp) = \wp + \sum_{\nu=2}^{\infty} \frac{c_{\nu}}{(2\nu-1)(\nu-1)!} \wp^{\nu}.$$

After we introduced the Horadam polynomials and the normalized form of the error function, we will define the following definition.

Definition 1. *A function $L \in \Omega$ given by (5) is said to be in the family $F_{\Omega}(s, r, u, y, t, \lambda, \tau)$ if satisfying the below two conditions*

$$(1-\tau) \frac{EL(\wp)}{\wp} + \tau \left(EL(\wp)\right)' + \lambda \wp \left(EL(\wp)\right)'' \prec \Upsilon(y, \wp) + 1 - s \tag{11}$$

and

$$(1-\tau) \frac{EV(\varpi)}{\varpi} + \tau \left(EV(\varpi)\right)' + \lambda \varpi \left(EV(\varpi)\right)'' \prec \Upsilon(y, \varpi) + 1 - s, \tag{12}$$

where $\wp, \varpi \in \Theta$, $\tau, \lambda \geq 0$, $y \in \mathbb{R}$, *and the function* $V = L^{-1}$ *is given by (6).*

Example 1. *For $\lambda = 0$, we have, $F_{\Omega}(s, r, u, y, t, 0, \tau) = F_{\Omega}(s, r, u, y, t, \tau)$, in which $F_{\Omega}(s, r, u, y, t, \tau)$ the family of functions $L \in \Omega$ and satisfying the below conditions*

$$(1-\tau) \frac{EL(\wp)}{\wp} + \tau \left(EL(\wp)\right)' \prec \Upsilon(y, \wp) + 1 - s$$

and

$$(1-\tau) \frac{EV(\varpi)}{\varpi} + \tau \left(EV(\varpi)\right)' \prec \Upsilon(y, \varpi) + 1 - s,$$

where $\wp, \varpi \in \Theta$, $\tau \geq 0$, $y \in \mathbb{R}$.

Example 2. *For $\lambda = 0$ and $\tau = 1$, we have, $F_\Omega(s,r,u,y,t,1) = F_\Omega(s,r,u,y,t)$, in which $F_\Omega(s,r,u,y,t)$ the family of functions $L \in \Omega$ and satisfying the below conditions and*

$$(EV(\varpi))' \prec \Upsilon(y,\varpi) + 1 - s,$$

where $\wp, \varpi \in \Theta,\ \ y \in \mathbb{R}$.

Example 3. *For $\lambda = 0$ and $\tau = 0$, we have, $F_\Omega(s,r,u,y,t,0,0) = F_\Omega(s,r,u,y,t,0)$, in which $F_\Omega(s,r,u,y,t,0)$ the family of functions $L \in \Omega$ and satisfying the below conditions*

$$\frac{EL(\wp)}{\wp} \prec \Upsilon(y,\wp) + 1 - s$$

and

$$\frac{EV(\varpi)}{\varpi} \prec \Upsilon(y,\varpi) + 1 - s,$$

where $\wp, \varpi \in \Theta$, $y \in \mathbb{R}$.

Example 4. *For $s = u = 1$ and $t = r = 2$, we have, $F_\Omega(1,2,1,y,2,\lambda,\tau)$ the family of functions $L \in \Omega$ and satisfying the below conditions*

$$(1-\tau)\frac{PL_\alpha(\wp)}{\wp} + \tau\left(PL_\alpha(\wp)\right)' + \lambda\wp\left(PL_\alpha(\wp)\right)'' \prec \Upsilon(y,\wp) + 1 - s$$

and

$$(1-\tau)\frac{PV_\alpha(\varpi)}{\varpi} + \tau\left(PV_\alpha(\varpi)\right)' + \lambda\varpi\left(PV_\alpha(\varpi)\right)'' \prec \Upsilon(y,\varpi) + 1 - s,$$

where $\wp, \varpi \in \Theta,\ \ \tau, \lambda \geq 0$, $y \in \mathbb{R}$.

Recently, many researchers have examined bi-univalent functions associated with orthogonal polynomials and found non-sharp estimates on Maclaurin coefficients $|c_2|$ and $|c_3|$ (for details, see [4]-[23]).

In [14], Fekete and Szegö validated the following inequality

$$\left|c_3 - \varphi c_2^2\right| \leq 1 + 2e^{\left(\frac{-2\varphi}{1-\varphi}\right)}$$

for all normalized univalent function L and $\varphi \in [0,1]$. This inequality is sharp for each φ (see [17]-[24]).

Recent years have seen a number of studies use many special functions, such as the Borel, Poisson, Rabotnov, Pascal, Wright and Bessel, to examine important aspects of geometric function theory, such as coefficient estimates, inclusion relations, and requirements for belonging to particular families (see, [1]-[3], [6]-[7]).

This article content is arranged as follows. In section 2 we giving bounds for the coefficients $|c_2|$ and $|c_3|$ in the Maclaurin expansions and estimation of Fekete–Szegö inequality for functions in the family $F_\Omega(s,r,u,y,t,\lambda,\tau)$. Section 3 pertinent links between some of the particular cases of the main results are highlighted. Section 4 concludes the study with a few observations.

2. Bounds of the family $F_\Omega(s,r,u,y,t,\lambda,\tau)$

Section 2 begins with bounds for the coefficients $|c_2|$ and $|c_3|$ in the Maclaurin expansions for functions in the family $F_\Omega(s,r,u,y,t,\lambda,\tau)$.

Theorem 1. *Let $L \in \Omega$ given by (5) belongs to the family $F_\Omega(s,r,u,y,t,\lambda,\tau)$. Then*

$$|c_2| \le \frac{ty\sqrt{2ty}}{\sqrt{\left|\frac{1}{5}(6\lambda+2\tau+1)t^2y^2 - \frac{2}{9}(2\lambda+\tau+1)^2(rty^2+su)\right|}}$$

and

$$|c_3| \le \frac{9t^2y^2}{(2\lambda+\tau+1)^2} + \frac{10ty}{6\lambda+2\tau+1}.$$

Proof. Let $L \in F_\Omega(s,r,u,y,t,\lambda,\tau)$. From Definition 1, we can write

$$(1-\tau)\frac{EL(\wp)}{\wp} + \tau EL'(\wp) + \lambda\wp EL''(\wp) = \Upsilon(y,\varkappa(\wp)) + 1 - s \tag{13}$$

and

$$(1-\tau)\frac{EV(\varpi)}{\varpi} + \tau EV'(\varpi) + \lambda\varpi EV''(\varpi) = \Upsilon(y,\tau(\varpi)) + 1 - s, \tag{14}$$

where $\varkappa$ and τ are analytic and have the form:

$$\varkappa(\wp) = j_1\wp + j_2\wp^2 + j_3\wp^3 + \cdots, \quad (\wp \in \Theta)$$

and

$$\tau(\varpi) = d_1\varpi + d_2\varpi^2 + d_3\varpi^3 + \cdots, \quad (\varpi \in \Theta),$$

such that $\varkappa(0) = \tau(0) = 0$ and $|\varkappa(\wp)| < 1$, $|\tau(\varpi)| < 1$ for all $\wp, \varpi \in \Theta$.

From the equalities (13) and (14), we get

$$(1-\tau)\frac{EL(\wp)}{\wp} + \tau EL'(\wp) + \lambda\wp EL''(\wp) = 1 + h_2(y)j_1\wp + \left(h_2(y)j_2 + h_3(y)j_1^2\right)\wp^2 + \cdots \tag{15}$$

and

$$(1-\tau)\frac{EV(\varpi)}{\varpi} + \tau EV'(\varpi) + \lambda\varpi EV''(\varpi) = 1 + h_2(y)d_1\varpi + \left(h_2(y)d_2 + h_3(y)d_1^2\right)\varpi^2 + \cdots. \tag{16}$$

It is common knowledge that if

$$|\varkappa(\wp)| = \left|j_1\wp + j_2\wp^2 + j_3\wp^3 + \cdots\right| < 1, \quad (\wp \in \Theta)$$

and

$$|\tau(\varpi)| = \left|d_1\varpi + d_2\varpi^2 + d_3\varpi^3 + \cdots\right| < 1, \quad \varpi \in \Theta,$$

then

$$|j_i| \le 1 \text{ and } |d_i| \le 1 \text{ for all } i \in \mathbb{N}. \tag{17}$$

Equating the coefficients of both sides in (15) and (16), we get

$$\frac{1}{3}\left(2\lambda+\tau+1\right)c_2 = h_2(y)j_1, \tag{18}$$

$$\frac{1}{10}\left(6\lambda+2\tau+1\right)c_3 = h_2(y)j_2 + h_3(y)j_1^2, \tag{19}$$

$$-\frac{1}{3}\left(2\lambda+\tau+1\right)c_2 = h_2(y)d_1, \tag{20}$$

and

$$\frac{1}{10}\left(6\lambda+2\tau+1\right)\left[2c_2^2 - c_3\right] = h_2(y)d_2 + h_3(y)d_1^2. \tag{21}$$

It follows from (18) and (20) that

$$j_1 = -d_1 \tag{22}$$

and

$$\frac{2}{9}\left(2\lambda+\tau+1\right)^2 c_2^2 = \left[h_2(y)\right]^2\left(j_1^2+d_1^2\right). \tag{23}$$

If we add (19) and (21), we get

$$\frac{1}{5}\left(6\lambda+2\tau+1\right)c_2^2 = h_2(y)\left(j_2+d_2\right) + h_3(y)\left(j_1^2+d_1^2\right). \tag{24}$$

Replacing the value of $\left(c_1^2+d_1^2\right)$ from (23) in the right hand side of (24), we have

$$\left[\frac{1}{5}\left(6\lambda+2\tau+1\right) - \frac{2}{9}\left(2\lambda+\tau+1\right)^2\frac{h_3(y)}{\left[h_2(y)\right]^2}\right]c_2^2 = h_2(y)\left(j_2+d_2\right). \tag{25}$$

Using (3) and (17) in (25), we find that

$$|c_2| \le \frac{ty\sqrt{2ty}}{\sqrt{\left|\frac{1}{5}\left(6\lambda+2\tau+1\right)t^2y^2 - \frac{2}{9}\left(2\lambda+\tau+1\right)^2\left(rty^2+su\right)\right|}}.$$

Also, if we subtract (21) from (19), we obtain

$$\frac{1}{5}\left(6\lambda+2\tau+1\right)\left(c_3-c_2^2\right) = h_2(y)\left(j_2-d_2\right) + h_3(y)\left(j_1^2-d_1^2\right). \tag{26}$$

Then, from (22) and (23), equation (26) becomes

$$c_3 = \frac{9\left[h_2(y)\right]^2}{2\left(2\lambda+\tau+1\right)^2}\left(j_1^2+d_1^2\right) + \frac{5h_2(y)}{6\lambda+2\tau+1}\left(j_2-d_2\right).$$

By applying (3), we conclude that

$$|c_3| \le \frac{9t^2y^2}{\left(2\lambda+\tau+1\right)^2} + \frac{10ty}{6\lambda+2\tau+1}.$$

Using the values of c_2 and c_3, we prove the functional $\left|c_3 - \varphi c_2^2\right|$ for family functions $F_\Omega(s,r,u,y,t,\lambda,\tau)$.

Theorem 2. *Let $L \in \Omega$ given by (5) belongs to the family $F_{\Omega}(s, r, u, y, t, \lambda, \tau)$. Then*

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} \frac{10|ty|}{6\lambda+2\tau+1} & |1-\varphi| \leq \Pi_1, \\ \frac{2t^3y^3|1-\varphi|}{\left|\frac{1}{5}(6\lambda+2\tau+1)t^2y^2 - \frac{2}{9}(2\lambda+\tau+1)^2(rty^2+su)\right|} & |1-\varphi| \geq \Pi_1. \end{cases}$$

where

$$\Pi_1 = 1 - \frac{10\,(2\lambda+\tau+1)^2\,(rty^2+su)}{(6\lambda+2\tau+1)\,t^2y^2}.$$

Proof. From (25) and (26)

$$\begin{aligned} c_3 - \varphi c_2^2 &= \frac{5h_2(y)}{6\lambda+2\tau+1}\,(j_2 - d_2) \\ &+ (1-\varphi)\,\frac{[h_2(y)]^3\,(j_2+d_2)}{\frac{1}{5}\,(6\lambda+2\tau+1)\,[h_2(y)]^2 - \frac{2}{9}\,(2\lambda+\tau+1)^2\,h_3(y)} \\ &= h_2(y)\left[F(\varphi) + \frac{5}{6\lambda+2\tau+1}\right] j_2 + h_2(y)\left[F(\varphi) - \frac{5}{6\lambda+2\tau+1}\right] d_2, \end{aligned}$$

where

$$F(\varphi) = \frac{[h_2(y)]^2\,(1-\varphi)}{\frac{1}{5}\,(6\lambda+2\tau+1)\,[h_2(y)]^2 - \frac{2}{9}\,(2\lambda+\tau+1)^2\,h_3(y)},$$

Then, from (3), we deduce that

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} \frac{10|h_2(y)|}{6\lambda+2\tau+1} & |F(\varphi)| \leq \frac{5}{6\lambda+2\tau+1}, \\ 2\,|h_2(y)|\,|F(\varphi)| & |F(\varphi)| \geq \frac{5}{6\lambda+2\tau+1}. \end{cases}$$

$$\equiv \begin{cases} \frac{10|ty|}{6\lambda+2\tau+1} & |1-\varphi| \leq \Pi_1, \\ \frac{2t^3y^3|1-\varphi|}{\left|\frac{1}{5}(6\lambda+2\tau+1)t^2y^2 - \frac{2}{9}(2\lambda+\tau+1)^2(rty^2+su)\right|} & |1-\varphi| \geq \Pi_1. \end{cases}$$

where

$$\Pi_1 = 1 - \frac{10\,(2\lambda+\tau+1)^2\,(rty^2+su)}{(6\lambda+2\tau+1)\,t^2y^2}$$

3. Particular Cases

The following corollaries are obtained by specializing the parameters λ and τ in the aforementioned theorems in section 2.

Corollary 1. *Let $L \in \Omega$ given by (5) belongs to the family $F_{\Omega}(s,r,u,y,t,\tau)$. Then*

$$|c_2| \leq \frac{ty\sqrt{2ty}}{\sqrt{\left|\left[\frac{1}{5}(2\tau+1)t^2y^2 - \frac{2}{9}(\tau+1)^2(rty^2+su)\right]\right|}},$$

$$|c_3| \leq \frac{9t^2y^2}{(\tau+1)^2} + \frac{10ty}{2\tau+1}$$

and

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} \frac{10|ty|}{2\tau+1} & |1-\varphi| \leq \Pi_2, \\ \frac{2t^3y^3|1-\varphi|}{\left|\frac{1}{5}(2\tau+1)t^2y^2 - \frac{2}{9}(\tau+1)^2(rty^2+su)\right|} & |1-\varphi| \geq \Pi_2. \end{cases}$$

where

$$\Pi_2 = 1 - \frac{10(\tau+1)^2(rty^2+su)}{(2\tau+1)t^2y^2}.$$

Corollary 2. *Let $L \in \Omega$ given by (5) belongs to the family $F_{\Omega}(s,r,u,y,t)$. Then*

$$|c_2| \leq \frac{ty\sqrt{2ty}}{\sqrt{\left|\left[\frac{3}{5}t^2y^2 - \frac{8}{9}(rty^2+su)\right]\right|}},$$

$$|c_3| \leq \frac{9t^2y^2}{4} + \frac{10ty}{3}$$

and

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} \frac{10|ty|}{3} & |1-\varphi| \leq \Pi_3, \\ \frac{2t^3y^3|1-\varphi|}{\left|\frac{3}{5}t^2y^2 - \frac{8}{9}(rty^2+su)\right|} & |1-\varphi| \geq \Pi_3. \end{cases}$$

where

$$\Pi_3 = 1 - \frac{40(rty^2+su)}{3t^2y^2}.$$

Corollary 3. *Let $L \in \Omega$ given by (5) belongs to the family $F_{\Omega}(s,r,u,y,t,0)$. Then*

$$|c_2| \leq \frac{ty\sqrt{2ty}}{\sqrt{\left|\left[\frac{1}{5}t^2y^2 - \frac{2}{9}(rty^2+su)\right]\right|}},$$

$$|c_3| \leq 9t^2y^2 + 10ty$$

and

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} 10|ty| & |1-\varphi| \leq 1 - \frac{10(rty^2+su)}{t^2y^2}, \\ \frac{2t^3y^3|1-\varphi|}{\left|\frac{1}{5}t^2y^2 - \frac{2}{9}(rty^2+su)\right|} & |1-\varphi| \geq 1 - \frac{10(rty^2+su)}{t^2y^2}. \end{cases}$$

The following corollary is obtained by specializing the parameters s, r, u and t in the aforementioned theorems in section 2.

Corollary 4. *Let $L \in \Omega$ given by (5) belongs to the family $F_\Omega(1,2,1,y,2,\lambda,\tau)$. Then*

$$|c_2| \leq \frac{4y\sqrt{y}}{\sqrt{\left|\left[\frac{4}{5}\left(6\lambda+2\tau+1\right)y^2 - \frac{2}{9}\left(2\lambda+\tau+1\right)^2\left(4y^2+1\right)\right]\right|}}$$

and

$$|c_3| \leq \frac{36y^2}{\left(2\lambda+\tau+1\right)^2} + \frac{102y}{6\lambda+2\tau+1}.$$

and

$$\left|c_3 - \varphi c_2^2\right| \leq \begin{cases} \frac{20|y|}{6\lambda+2\tau+1} & |1-\varphi| \leq \Pi_4, \\ \frac{16y^3|1-\varphi|}{\left|\frac{1}{5}(6\lambda+2\tau+1)t^2y^2 - \frac{2}{9}(2\lambda+\tau+1)^2(4y^2+1)\right|} & |1-\varphi| \geq \Pi_4. \end{cases}$$

where

$$\Pi_4 = 1 - \frac{10\left(2\lambda+\tau+1\right)^2\left(4y^2+1\right)}{\left(6\lambda+2\tau+1\right)4y^2}.$$

4. Conclusions

In this paper, we defined a comprehensive family of analytic and bi-univalent functions related to imaginary error function and subordinate to Horadam polynomials denoted by $F_\Omega(s,r,u,y,t,\lambda,\tau)$. We estimated for the Maclaurin coefficients $|c_2|$, $|c_3|$ and Fekete-Szegö problems. Furthermore, by specializing the parameters s,r,u,t,λ and τ, one may determine the outcomes for the subfamilies $F_\Omega(s,r,u,y,t,\tau)$, $F_\Omega(s,r,u,y,t)$, $F_\Omega(s,r,u,y,t,0)$ and $F_\Omega(1,2,1,y,2,\lambda,\tau)$ specified in Examples 1, 2, 3 and 4, respectively. Making use of normalized error function could inspire researchers to find the estimates of the coefficients $|c_2|$, $|c_3|$ and Fekete-Szegö problems for functions belonging to new subfamily of bi-univalent functions.

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