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Nonstandard Finite Difference Predictor Corrector Method for Quadratic Riccati Differential Equation

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Abstract. Solving nonlinear differential equations reliably and effectively is critical for applications in epidemiology, ecology, and finance. Traditional finite difference approaches frequently exhibit numerical instability and fail to maintain crucial features such as positivity and boundedness. Motivated by these restrictions, we provide an unconditionally stable Nonstandard Finite Difference Predictor-Corrector (NSFD-PC) strategy for solving nonlinear quadratic Riccati differential equations. Particularly when the step size h rises, NSFD-PC typically yields the solution that is closer to the exact solution. Riccati differential equation is 1st-order quadratic ode with several systems and control theory applications. Our technique achieves first-order temporal precision while keeping the continuous model's fundamental qualitative properties. We compare the proposed NSFD-PC scheme to the usual Euler Predictor-Corrector approach and find that it performs much better in accuracy and consistency with exact answers. The results show that our method is a superior and dependable option for solving nonlinear differential equations, making it extremely useful in various applications.

2020 Mathematics Subject Classifications: 65L05, 65L20, 34A34, 93D05

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1. Introduction

Ronald Mickens started creating numerical schemes for addressing physical problems that preserve the physical aspects of the original systems, including stability properties, utilizing NSFD schemes [1, 2]. This has led to the development of numerous discretization of nonlinear equations in various fields [3].
In science and engineering, differential equations are a frequently used tool for problem solving. The differential equation's complexity and nonlinearity make it challenging to solve analytically, despite being used to mimic a wide range of real-world issues. Numerical approaches were developed in response to the difficulties in obtaining analytical solutions for real-world issues. NLODE is essential to many branches of pure and applied mathematics [4], including biology [5, 6], astronomy, engineering, applied mechanics [7],and quantum physics. Numerical and analytical solutions to NLODE have drawn interest in numerous practical scientific domains [8–10], the Riccati differential equation is a type of nonlinear differential equation that has great importance. For example, the Riccati differential equation and a 1D static Schrodinger equation are closely connected. [11, 12] A well-known nonlinear differential equation (NLDE), the Riccati differential equation finds use in a variety of scientific and technical fields, including financial mathematics, network synthesis, resilient stabilization, stochastic realization theory, and optimal control. For simulating a variety of physical scenarios, such as spring mass systems, beam bending, resistor-capacitor-induction circuits, chemical reactions [13], pendulums, the motion of rotating masses around bodies, and more, nonlinear differential equations are crucial tools [14]. Even though Riccati equations are used extensively, traditional numerical techniques, including basic finite difference approaches, usually experience numerical instability, especially when step sizes are larger. Important characteristics like positivity and boundedness, which are essential for preserving the physical integrity of solutions, are frequently lost in these conventional approaches. As a result, there is an increasing demand for more robust and precise numerical systems that may overcome these constraints [15, 16]. Numerical instability and bias are problems with standard finite difference methods for solving nonlinear differential equations that model real-world phenomena. If large step sizes are utilized in the numerical simulation, this unwanted behavior is typical. The NSFD-PC technique, which was suggested and made famous by [17], is used to address these drawbacks. Some qualitative aspects of the continuous-time model, like positivity and boundedness, are preserved by the NSFD-PC approach, which is dynamically consistent due to its well-crafted rules.
An Italian Nobel Laureate, the Jacopo Francessco Riccati introduces a famous differential equation titled as Riccati differential equation. It is 1st order non-linear differential equation. Riccati differential equation has many uses in engineering [18, 19]. Non-linear differential equation had been studied by many researchers using different variational and perturbation approaches (VAPA), such as variational iterative methods, Homotopy perturbation methods (HPM) and Adomian decomposition method (ADM) [20, 21]. Notably, a large number of these studies outline the power series solution for these kinds of issues. [18]. Sometimes, for certain values of step size, positivity of solutions and the bounded-

ness loses by these methods. In [22] F. M. Fernndez has stated that for the non-linear differential equations (VAPA) methods are not suitable. In his paper, he stated that these methods converges conditionally that depends upon the step sizes and initial conditions. Samia Riaz *et al* in [23] has mentioned that NSFD-PC scheme is more reliable and accurate as compared to Euler method and RK-4 to solve nonlinear quadratic differential equation. To address these shortcomings, this work proposes a more robust alternative: the NSFD-PC technique. Mickens pioneered NSFD-PC approaches, which are notable for preserving important characteristics of the original differential equations. Our technique builds on this principle by creating an unconditionally stable NSFD-PC scheme for nonlinear quadratic Riccati differential equations. Unlike existing methods, our scheme assures first-order temporal precision, preserves crucial aspects of the precise solution such as positivity and boundedness, and is stable across a wide range of step sizes. It convergences accurately and for the differential equations it preserves important properties of exact solutions. This method shows the numerical stability for different step sizes, whereas for larger step sizes other standard techniques lose stability and do not converges to right solution.
This is the structure of the remainder of the paper: In Section 2, the theoretical background of the suggested NSFD-PC system is explained, along with the numerical method. Section 3 digs into the Predictor component of the NSFD method, outlining its development and execution. Section 4 delves into the Riccati differential equation, exploring its significance and the difficulties involved in numerically solving it. In section 5, we apply the NSFD-PC approach to the Riccati equation, demonstrating how this method effectively tackles nonlinearity. Section 6 contains the results and discussion, which compare the accuracy and stability of the proposed scheme to traditional methods. The concluding remarks are discuss in section 7.

2. Numerical Method

Ronald E. Mickens developed the NSFD-PC Method for the numerical solution of differential equations [18]. Other standard numerical methods occasionally produce unsteadiness for larger step sizes, such as the Euler Predictor Corrector Method [24]. The "NSFD-PC method" is a numerical system that has been developed to prevent these kinds of numerical instabilities. The NSFD-PC scheme's structure is responsible for its stability and positively for big step sizes. The predictor and corrector are the two main steps in the NSFD-PC technique. The predictor phase gives an initial solution estimate, which is then refined by the corrector step, which includes additional terms to improve accuracy and stability. This two-stage procedure enables the method to effectively tackle the complexities of nonlinear differential equations, including bigger step sizes where typical methods fail.
When developing the NSFD-PC scheme, we assume the following:

- The differential equation under discussion is first-order and nonlinear, as is common for Riccati-type problems.

- The solution remains bounded and positive, which are characteristics that our strategy is intended to retain.
- The method's step size h is chosen to ensure unconditional stability, which is a substantial improvement over traditional approaches that demand modest step sizes for stability.

2.1. Construction of the NSFD-PC Method

The NSFD-PC method is built on carefully developed principles that distinguish it from traditional finite difference systems. The discrete model is constructed as follows:

- **Predictor Step:** At this point, we generate an initial approximation $y_{n+1}^{(p)}$ of the solution at the following time step using the conventional NSFD scheme.

$$y_{n+1}^{(p)} = y_n + hf(t_n, y_n),$$

where $f(t_n, y_n)$ represents the right-hand side of the differential equation evaluated at (t_n, y_n), h is the step size, and y_n is the function value.

- **Corrector Step:** Through predictor value refinement, the corrector step increases the correctness of the answer. To guarantee stability, we add one more term:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(t_n, y_n) + \frac{\epsilon^{-1}}{h} y_{n+1}^{(p)}\right],$$

where ϵ is a small, positive parameter that has been selected to control the corrector term's influence. This modification aids in solution stabilization for all step sizes.

The corrector step must contain the term $\frac{\epsilon^{-1}}{h} y_{n+1}^{(p)}$ to guarantee that the NSFD-PC approach retains positivity, boundedness, and stability even for higher h. Compared to traditional finite difference approaches, which frequently lose accuracy and stability as h grows, this property is a major advantage. The NSFD-PC method, which solves nonlinear differential equations like the Riccati equation, is a potent tool because it adheres to a two-step predictor-corrector approach that preserves key elements of the original differential equation while increasing accuracy.

3. Predictor (NSFD)

The following guidelines govern a discrete representation of a system of differential equations using a non-standard finite difference method:

1. In the differential equations, the orders of the discrete derivatives and their corresponding derivatives must be equal.
2. Discrete representations for derivatives such that $\phi(h) = O(h^2)$ can be obtained using a non-trivial denominator function, provided that $h \to 0$, $\phi(h) \to 0$.

3. Nonlinear terms should generally be represented by a non-local discrete representation.

4. The discretized model and its solutions should be subject to the same special criteria that apply to the differential equation or its solutions.

4. Riccati Equation

Let's look at the RE to demonstrate the effectiveness of our suggested NSFD-PC system. [25]:

$$\frac{dy(t)}{dt} = 1 + 2y(t) - y^2(t), \tag{1}$$

with $y(0) = 0$ as the initial condition. The following provides the exact answer to Eq. (1):

$$y = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right). \tag{2}$$

4.1. Qualitative analysis of assumed model

Examine Eq. (1). Without getting explicit solutions, the qualitative analysis is carried out to examine the equilibrium points, their stability, and the system's overall behavior.

4.1.1. Equilibrium points

The equilibrium points y^* are obtained by setting $\frac{dy(t)}{dt} = 0$:

$$1 + 2y - y^2 = 0. \tag{3}$$

After solve the aforementioned equation, the equilibrium points are $y^* = 1 + \sqrt{2}$ and $y^* = 1 - \sqrt{2}$.

4.1.2. Stability Analysis

To assess the stability of the equilibrium points, we differentiate Eq.(3) with respect to y: Evaluating $f'(y)$ at the equilibrium points:

- At $y^* = 1 + \sqrt{2}$:
$$f'(1+\sqrt{2}) = 2 - 2(1+\sqrt{2}) = -2\sqrt{2} < 0,$$
indicating that $y = 1 + \sqrt{2}$ is a **stable equilibrium point**.

- At $y^* = 1 - \sqrt{2}$:
$$f'(1-\sqrt{2}) = 2 - 2(1-\sqrt{2}) = 2\sqrt{2} > 0,$$
which implies that $y = 1 - \sqrt{2}$ is an **unstable equilibrium point**.

4.1.3. Phase Line Analysis

The phase line analysis provides insight into the directional behavior of the solutions around the equilibrium points:

- For $y < 1-\sqrt{2}$, $f(y) = 1 + 2y - y^2 > 0$, indicating that $\frac{dy}{dt} > 0$, and thus $y(t)$ increases.
- For $1-\sqrt{2} < y < 1+\sqrt{2}$, $f(y) < 0$, meaning $\frac{dy}{dt} < 0$, so $y(t)$ decreases.
- For $y > 1+\sqrt{2}$, $f(y) > 0$, which results in $\frac{dy}{dt} > 0$, and $y(t)$ increases.

This analysis indicates that $y = 1+\sqrt{2}$ is an attractor, while $y = 1-\sqrt{2}$ serves as a repeller.

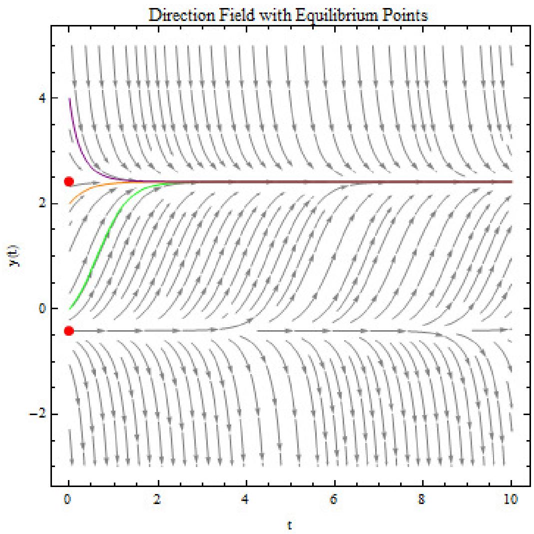

Figure 1: Directional field along with phase portrait

4.1.4. Direction Field and long term behavior

The direction field can be plotted to visualize the trajectories of $y(t)$ over time, which further corroborates the stability characteristics identified. The long-term asymptotic behavior of the solutions reveals that as $t \to \infty$, any trajectory that starts within $y > 1-\sqrt{2}$ will converge to the stable equilibrium $y = 1+\sqrt{2}$. Conversely, initial conditions $y(t_0) < 1-\sqrt{2}$ will diverge away from this region.

5. Application of NSFD-PC Scheme

For the RE, the predictor step of the NSFD-PC method with non-local approximation of nonlinear terms can be built as follows:

$$\frac{y_{n+1} - y_n}{h} = 1 + 2y_n - y_n y_{n+1}, \tag{4}$$

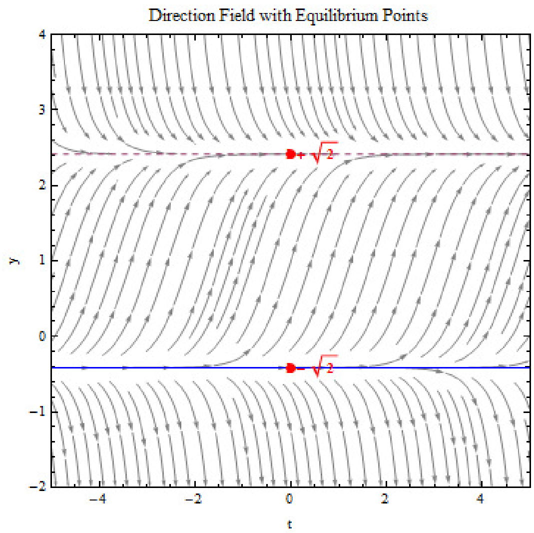

Figure 2: Directional field of stable and unstable points

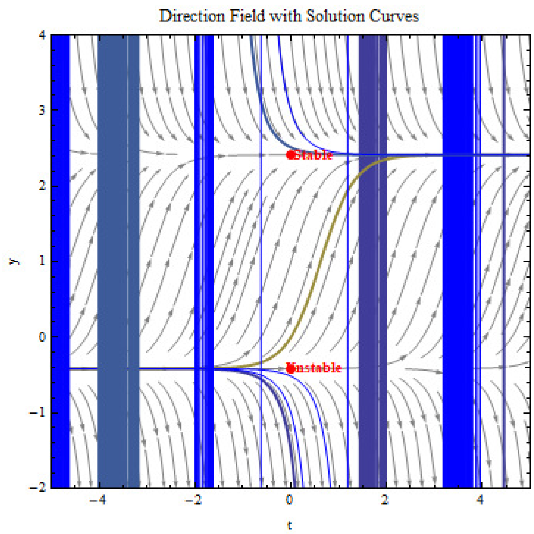

Figure 3: Directional field of stable and unstable points with curves

which provides:

$$y_{n+1} = \frac{y_n + h + 2hy_n}{1 + hy_n}. \tag{5}$$

The non-standard finite difference predictor formula for the given differential equation from Eq.(5) is:

$$y_{n+1}^{(p)} = \frac{y_n^{(p)} + h + 2hy_n^{(p)}}{1 + hy_n^{(p)}}. \tag{6}$$

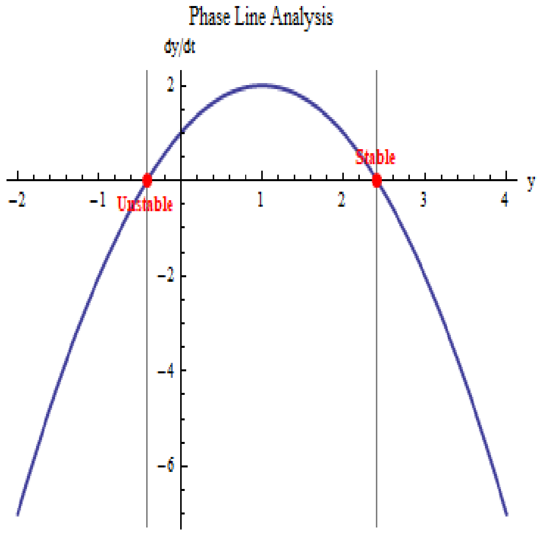

Figure 4: Phase portrait of stable and unstable points

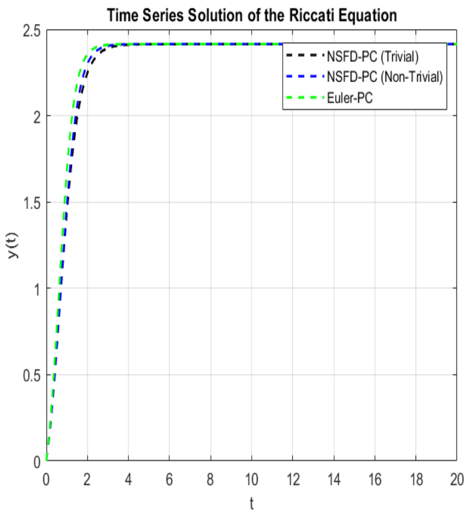

Figure 5: Time series solution

The corrector step can be constructed by adding the term $\frac{\epsilon^{-1}}{h}y_{n+1}$ in the NSFD scheme:

$$\frac{y_{n+1}-y_n}{h} = 1 + 2y_n - y_n y_{n+1} - \frac{\epsilon^{-1}}{h}y_{n+1} + \frac{\epsilon^{-1}}{h}y_{n+1}, \tag{7}$$

which gives:

$$y_{n+1} = \frac{h + y_n + \epsilon^{-1}y_{n+1} + 2hy_n}{1 + hy_n + \epsilon^{-1}}. \tag{8}$$

The non-standard finite difference corrector formula is:

$$y_{n+1}^{(c)} = \frac{h + y_n^{(p)} + \epsilon^{-1} y_{n+1}^{(p)} + 2h y_n^{(p)}}{1 + h y_n^{(p)} + \epsilon^{-1}}. \tag{9}$$

where y_{n+1} indicates that the value of the solution is y_{n+1} at $(n+1)^{th}$ time step and y_n at nth time step, where h is the time step. Only the non-linear term in this formulation is approximated using the non-local method; the time step parameter h is approximated using the conventional method, as in a regular finite difference scheme. If a non-trivial denominator function $\phi(h)$ is used, such that $\phi(h) \to 0$ as $h \to 0$, the scheme is more suitable for large step sizes. For the Riccati problem, the denominator can be approximated as:

$$\varphi(h) = 1 - e^{-h}. \tag{10}$$

This denominator turns our approach into:

$$y_{n+1}^{(p)} = \frac{y_n^{(p)} + 1 - e^{-h} + 2(1 - e^{-h}) y_n^{(p)}}{1 + (1 - e^{-h}) y_n^{(p)}}. \tag{11}$$

The corrector step is:

$$y_{n+1}^{(c)} = \frac{1 - e^{-h} + y_n^{(p)} + \epsilon^{-1} y_{n+1}^{(p)} + 2(1 - e^{-h}) y_n^{(p)}}{1 + (1 - e^{-h}) y_n^{(p)} + \epsilon^{-1}}. \tag{12}$$

We may assess how the non trivial denominator overcomes the NSFD-PC scheme's unstable behavior in the following section.

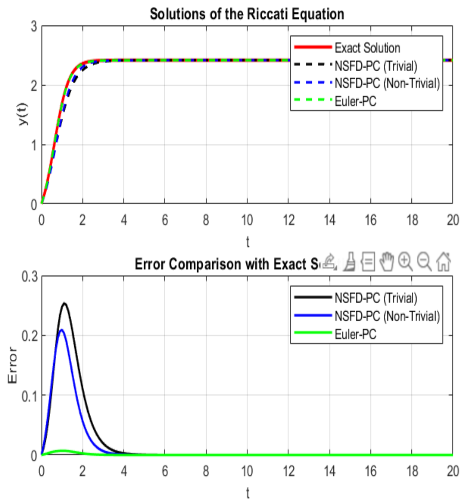

Figure 6: Comparison of NSFD-PC Scheme and Euler-PC with exact solution at h=0.1

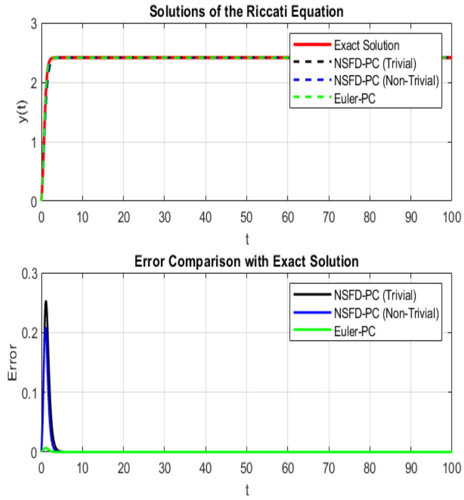

Figure 7: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for step size 0.1

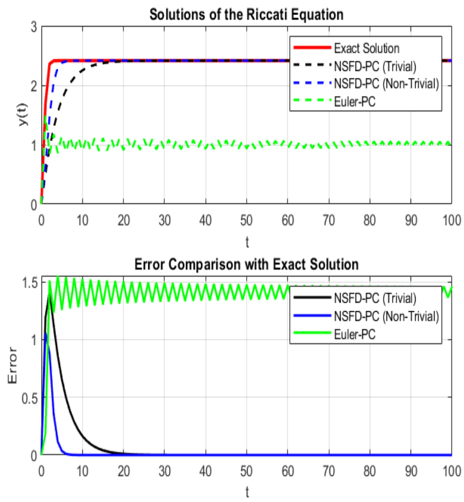

Figure 8: Comparison of NSFD-PC and Euler-PC with exact solution with step size h=1

6. Results and Discussions

In this section, we apply the suggested NSFD-PC algorithms to the Riccati problem in order to assess their performance. We compare the numerical results from the NSFD-PC schemes, as defined in Eq. (1), with the precise solution in order to evaluate the method's accuracy. Initially, we observe that for small step sizes $h = 0.1$, $h = 1$, 1.1, 1.8, the NSFD-PC scheme shows excellent consistency with the exact solution, as illustrated in Figures $7, 8, 9$ and 10. This illustrates the accuracy and consistency of the approach under more precise discretization. Nevertheless, the NSFD-PC technique with a trivial denominator starts to show non-physical oscillations when the step size is raised to h

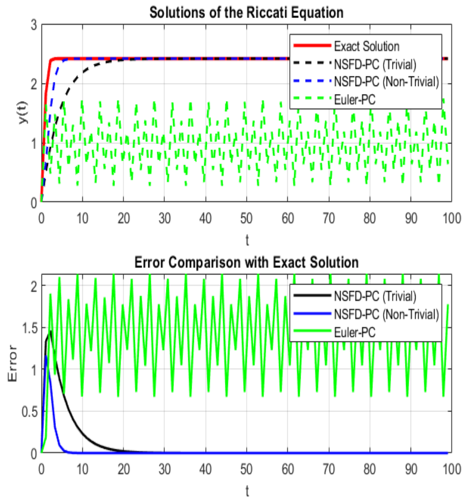

Figure 9: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=1.1

= 5. Although these oscillations are prominent, the solution tends to align with the exact solution after some time, suggesting that the method still maintains a degree of accuracy over extended periods. When the step size is further increased to $h = 100$ and even $h = 200$, the Euler scheme exhibits increasingly large oscillations. These oscillations indicate a divergence from the exact solution, demonstrating the limitations of the method when a trivial denominator is used at large step sizes. Despite this, the NSFD-PC scheme overcomes this issue. For large step sizes, including $h = 1.8$, $h = 5$, and beyond, the NSFD-PC method with a non-trivial denominator remains consistent with the exact solution across all time intervals, highlighting its robustness. The ability of the NSFD-PC method with a non-trivial denominator to handle large step sizes is a significant improvement, as it allows for accurate results even with discretizations, reducing computational cost. This adjustment effectively mitigates the oscillatory behavior observed with the trivial denominator and makes the method reliable for a wider range of step sizes. In contrast, the Euler-PC method, which was also applied to the Riccati equation for comparison, becomes unstable when the step size is increased to $h = 1.8$, $h = 5$, h=100, h=200 as shown in Figures 10, 11, 12, 13 and 14, the Euler-PC method exhibits non-physical oscillations and eventually becomes unbounded over a longer time interval, indicating a significant loss of accuracy. The instability of the Euler-PC method for larger step sizes further emphasizes the superiority of the NSFD-PC method, especially when using a non-trivial denominator. Time series solution of the NSFD-PC and Euler-PC methods are presented in Figure 5, where it becomes evident that the NSFD-PC method significantly outperforms the Euler-PC method in terms of minimizing numerical error. The results highlight the reliability and accuracy of the NSFD-PC method, especially for large step sizes, and suggest its applicability in solving nonlinear differential equations like the Riccati equation across varying time intervals and step sizes. Finally the systems stability, phase line analysis and directional fields are shown in figures 1, 2, 3, and 4. The system's asymptotic stability

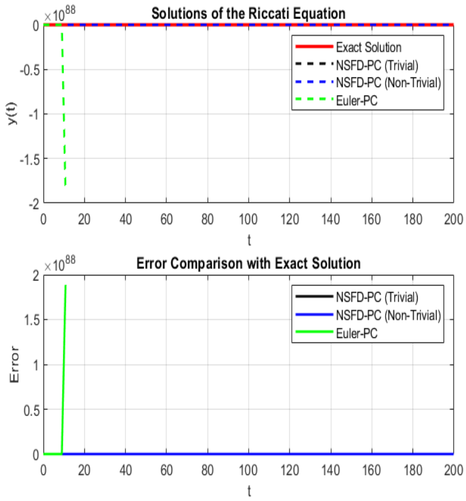

Figure 10: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=1.8

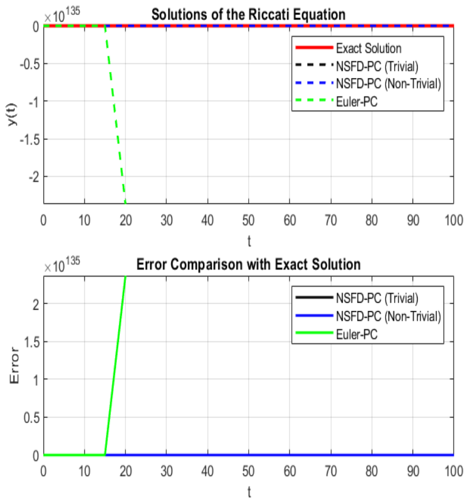

Figure 11: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=5

toward the positive equilibrium is highlighted by this analysis.

7. Conclusion

In this work, we propose an NSFD-PC method for solving the nonlinear quadratic Riccati differential equation that is unconditionally stable. Even with larger step sizes, the NSFD-PC method, which incorporates a non-trivial denominator, has demonstrated significant benefits, especially in terms of stability. A comprehensive examination revealed that the Euler-PC method exhibits needless oscillations or even diverges when the step size grows. In contrast, the NSFD-PC approach is stable and consistent with the exact

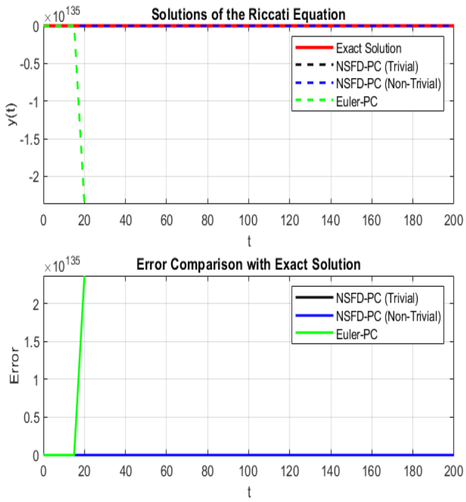

Figure 12: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=5

solution regardless of step size. This durability makes it a great candidate for solving nonlinear and stiff differential equations. The comparison clearly demonstrates the advantages of the NSFD-PC method over the Euler-PC method. The NSFD-PC method not only improves accuracy but also increases stability while solving complex differential equations. These characteristics are especially significant when dealing with stiff systems, where small step sizes are typically necessary to maintain stability using traditional methods. The NSFD-PC method minimizes the overall computational cost for long-term simulations by permitting bigger step sizes while maintaining accuracy, making it an efficient alternative for a wide range of applications. While it is true that the NSFD-PC technique requires slightly more computing power due to its non-trivial structure, the trade-off is negligible when compared to the significant improvements in accuracy and stability. Its ability to create solutions that closely resemble exact solutions while remaining stable across a wide range of step sizes makes it the favored method for numerical analysis. Overall, the NSFD-PC approach stands out as a dependable, precise, and computationally efficient tool for solving complex, nonlinear, and stiff differential equations, highlighting its importance in applied mathematics and computer research.

Authors' contributions

BR participated in the conceptualization, investigation, validation, visualization and writing the original draft. MAB participated in the formal analysis, data curation, investigation, supervision, review and editing of the manuscript. AK participated in the conceptualization, administration, validation, visualization and writing of the manuscript. DKA participated in the review and editing of the manuscript. TA participated in the administration, validation, visualization, review and editing of the manuscript. All authors read and approved the final manuscript.

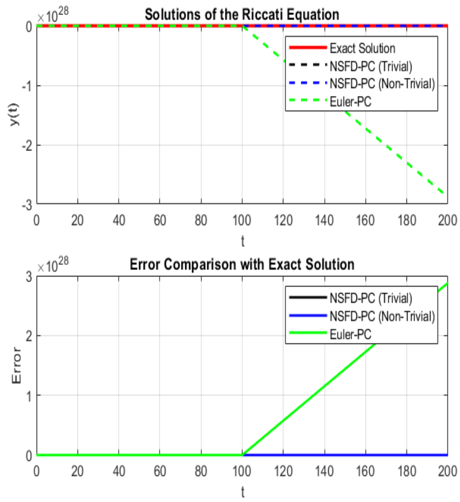

Figure 13: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=100

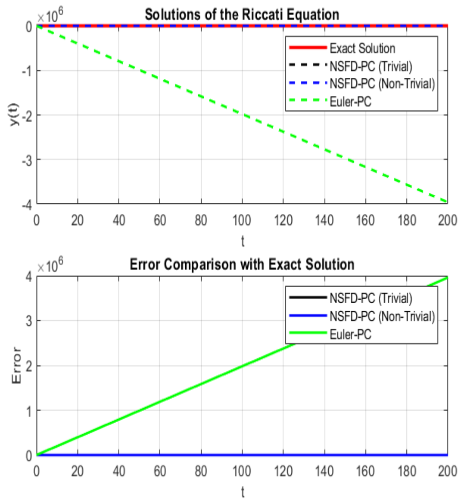

Figure 14: Comparison of NSFD-PC Scheme and Euler-PC with exact solution for h=200

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Availability of Data and Materials

All data that support the findings of this study are included in the article.
Competing interests: The authors declare that they have no competing interests.

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