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Almost Weakly (τ_1, τ_2)-continuous Functions

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Abstract. This paper is concerned with the concept of weakly (τ_1, τ_2)-continuous functions. Furthermore, several characterizations and some properties of almost weakly (τ_1, τ_2)-continuous functions are investigated.

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1. Introduction

Topology as a field of mathematics is concerned with all questions directly or indirectly related to continuity. Semi-open sets [25], preopen sets [26], α-open sets [27] and β-open sets [20] play an important role in the researching of generalizations of continuity in topological spaces. By using these sets many authors introduced and studied various types of weak forms of continuity for functions. Levine [24] introduced the concept of weakly continuous functions in topological spaces. Husain [21] introduced the concept of almost continuous functions. Viriyapong and Boonpok [36] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [8]. Dungthaisong et al. [19] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [18] introduced and investigated the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were presented in [33], [34],

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[9], [31], [12], [7], [5], [6], [3], [1], [2], [13], [11] and [10], respectively. Kong-ied et al. [23] introduced and investigated the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [16] introduced and studied the notion of weakly quasi (τ_1, τ_2)-continuous functions. Prachanpol et al. [30] introduced and investigated the concepts of almost $\delta(\tau_1, \tau_2)$-continuous functions and weakly $\delta(\tau_1, \tau_2)$-continuous functions. Janković [22] defined almost weakly continuous functions as a generalization of both weakly continuous functions due to Levine [25] and almost continuous functions in the sense of Husain [21]. Noiri and Popa [28, 29] investigated further characterizations of almost weakly continuous functions. In this paper, we introduce the notion of almost weakly (τ_1, τ_2)-continuous functions. We also investigate several characterizations of almost weakly (τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [14] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [14] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [14] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [14] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.

(3) $\tau_1\tau_2\text{-}Cl(A)$ is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2\text{-}Cl(A)$.

(5) $\tau_1\tau_2\text{-}Cl(X - A) = X - \tau_1\tau_2\text{-}Int(A)$.

Lemma 2. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $\tau_1\tau_2\text{-}Cl(A) \cap V \subseteq \tau_1\tau_2\text{-}Cl(A \cap V)$ for every $\tau_1\tau_2$-open set V of X;

(2) $\tau_1\tau_2\text{-}Int(A \cup F) \subseteq \tau_1\tau_2\text{-}Int(A) \cup F$ for every $\tau_1\tau_2$-closed set F of X.

A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$-*open* [35] (resp. $(\tau_1, \tau_2)s$-*open* [4], $(\tau_1, \tau_2)p$-*open* [4], $(\tau_1, \tau_2)\beta$-*open* [4]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [37]. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* of A and is denoted by (τ_1, τ_2)-pCl(A). The union of all $(\tau_1, \tau_2)p$-open sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* of A and is denoted by (τ_1, τ_2)-pInt(A).

Lemma 3. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $(\tau_1, \tau_2)\text{-}pCl(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cup A$ [36];

(2) $(\tau_1, \tau_2)\text{-}pInt(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cap A$.

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called $(\tau_1, \tau_2)\theta$-*cluster point* [35] of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [35] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [35] if $A = (\tau_1, \tau_2)\theta$-Cl$(A)$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [35] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 4. [35] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) If A is $\tau_2\tau_2$-open in X, then $\tau_1\tau_2\text{-}Cl(A) = (\tau_1, \tau_2)\theta\text{-}Cl(A)$.

(2) $(\tau_1, \tau_2)\theta\text{-}Cl(A)$ *is $\tau_1\tau_2$-closed in X.*

3. Almost weakly (τ_1, τ_2)-continuous functions

In this section, we introduce the notion of almost weakly (τ_1, τ_2)-continuous functions. Moreover, several characterizations of almost weakly (τ_1, τ_2)-continuous functions are discussed.

Definition 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost weakly (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$,*

$$x \in \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V)))).$$

Theorem 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost weakly (τ_1, τ_2)-continuous;

(2) $f^{-1}(V) \subseteq \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))))$ *for every $\sigma_1\sigma_2$-open set V of Y;*

(3) $\tau_1\tau_2$-*Cl*($\tau_1\tau_2$-*Int*($f^{-1}(V)$)) $\subseteq f^{-1}(\sigma_1\sigma_2$-*Cl*$(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(4) (τ_1, τ_2)-*pCl*$(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$-*Cl*$(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(5) $f^{-1}(V) \subseteq (\tau_1, \tau_2)$-*pInt*$(f^{-1}(\sigma_1\sigma_2$-*Cl*$(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(6) for each $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $f(x)$*, there exists a* $(\tau_1, \tau_2)p$*-open set* U *of* X *containing* x *such that* $f(U) \subseteq \sigma_1\sigma_2$-*Cl*$(V)$*.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y and $x \in f^{-1}(V)$. Then, we have $f(x) \in V$ and by (1), $x \in \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))))$. Thus,

$$f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))).$$

(2) $\Rightarrow$ (3): Let V be any $\sigma_1\sigma_2$-open set of Y. Since $Y - \sigma_1\sigma_2\text{-Cl}(V)$ is $\sigma_1\sigma_2$-open in Y and by (2), we have

$$\begin{aligned} X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)) &= f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))))) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(Y - V))) \\ &= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(X - f^{-1}(V))) \\ &= X - \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(V))) \end{aligned}$$

and hence $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(V))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$.

(3) $\Rightarrow$ (4): Let V be any $\sigma_1\sigma_2$-open set of Y. By (3) and Lemma 3(1),

$$(\tau_1, \tau_2)\text{-pCl}(f^{-1}(V)) = f^{-1}(V) \cup \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(V))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).$$

(4) $\Rightarrow$ (5): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $Y - \sigma_1\sigma_2\text{-Cl}(V)$ is $\sigma_1\sigma_2$-open and by (4), we have

$$\begin{aligned} X - (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) &= (\tau_1, \tau_2)\text{-pCl}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= (\tau_1, \tau_2)\text{-pCl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq f^{-1}(Y - V) \\ &= X - f^{-1}(V) \end{aligned}$$

and hence $f^{-1}(V) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$.

(5) $\Rightarrow$ (6): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. By (5), $x \in f^{-1}(V) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$ and there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$.

(6) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. By (6), there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$; hence $U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$. Thus,

$$x \in U \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(U)) \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))).$$

This shows that f is almost weakly (τ_1, τ_2)-continuous.

Theorem 2. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) f *is almost weakly* (τ_1, τ_2)*-continuous;*

(2) $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(f^{-1}(\sigma_1\sigma_2\text{-}Int(K)))) \subseteq f^{-1}(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(3) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(K))) \subseteq f^{-1}(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(5) $f^{-1}(\sigma_1\sigma_2\text{-}Int(B)) \subseteq (\tau_1, \tau_2)\text{-}pInt(f^{-1}(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(B))))$ *for every subset* B *of* Y*.*

Proof. (1) ⇒ (2): Let K be any $\sigma_1\sigma_2$-closed set of Y. Then, $Y - K$ is $\sigma_1\sigma_2$-open in Y and by Theorem 1, we have

$$\begin{aligned} X - f^{-1}(K) &= f^{-1}(Y - K) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - K)))) \\ &= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(K)))) \\ &= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(X - f^{-1}(\sigma_1\sigma_2\text{-Int}(K)))) \\ &= X - \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K)))) \end{aligned}$$

and hence $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq f^{-1}(K)$.

(2) ⇒ (3): Let K be any $\sigma_1\sigma_2$-closed set of Y. By Lemma 3(1), we have

$$\begin{aligned} (\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))) &= f^{-1}(\sigma_1\sigma_2\text{-Int}(K)) \cup \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K)))) \\ &\subseteq f^{-1}(K). \end{aligned}$$

(3) ⇒ (4): This is obvious.

(4) ⇒ (5): Let B be any subset of Y. Then by (4),

$$\begin{aligned} X - (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B)))) &= (\tau_1, \tau_2)\text{-pCl}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B)))) \\ &= (\tau_1, \tau_2)\text{-pCl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B)))) \\ &= (\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - B)))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - B)) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(B)). \end{aligned}$$

Thus, $f^{-1}(\sigma_1\sigma_2\text{-Int}(B)) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))$.

(5) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. By (5), we have

$$f^{-1}(V) \subseteq (\tau_1, \tau_2)\text{-pInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$$

and hence f is almost weakly (τ_1, τ_2)-continuous by Theorem 1.

Theorem 3. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost weakly (τ_1, τ_2)-continuous;

(2) $f((\tau_1, \tau_2)\text{-}pCl(A)) \subseteq (\sigma_1, \sigma_2)\theta\text{-}Cl(f(A))$ for every subset A of X;

(3) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(B)) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y;

(4) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int((\sigma_1, \sigma_2)\theta\text{-}Cl(B)))) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ for every subset B of Y;

(5) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(6) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;

(7) $(\tau_1, \tau_2)\text{-}pCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(K))) \subseteq f^{-1}(K)$ for every $(\sigma_1, \sigma_2)r$-closed set K of Y.

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X. Let $x \in (\tau_1, \tau_2)$-pCl(A) and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is almost weakly (τ_1, τ_2)-continuous, by Theorem 1 there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Since $x \in (\tau_1, \tau_2)$-pCl(A), we have $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U \cap A) \subseteq \sigma_1\sigma_2$-Cl$(V) \cap f(A)$. Therefore, $f(x) \in (\sigma_1, \sigma_2)\theta$-Cl$(f(A))$. This shows that

$$f((\tau_1, \tau_2)\text{-pCl}(A)) \subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(f(A)).$$

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then by (2), we have

$$\begin{aligned} f((\tau_1, \tau_2)\text{-pCl}(f^{-1}(B))) &\subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(f(f^{-1}(B))) \\ &\subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(B) \end{aligned}$$

and hence (τ_1, τ_2)-pCl$(f^{-1}(B)) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta$-Cl$(B))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Since $(\sigma_1, \sigma_2)\theta$-Cl$(B)$ is $\sigma_1\sigma_2$-closed in Y and $\sigma_1\sigma_2$-Cl$(V) = (\sigma_1, \sigma_2)\theta$-Cl$(V)$ for every $\sigma_1\sigma_2$-open set V of Y, by (3)

$$\begin{aligned} (\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) &\subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) \\ &= f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))) \\ &= f^{-1}(\sigma_1, \sigma_2)\theta\text{-Cl}(B)). \end{aligned}$$

(4) $\Rightarrow$ (5): This is obvious since $(\sigma_1, \sigma_2)\theta$-Cl$(V) = \sigma_1\sigma_2$-Cl$(V)$ for every $\sigma_1\sigma_2$-open set V of Y.

(5) $\Rightarrow$ (6): This follows from $\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))) = \sigma_1\sigma_2$-Cl(V) for every $(\sigma_1, \sigma_2)p$-open set V of Y.

(6) ⇒ (7): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Then, we have $\sigma_1\sigma_2$-Int(K) is $(\sigma_1, \sigma_2)p$-open in Y. Thus by (6),

$$\begin{aligned}(\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))) &= (\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= f^{-1}(K).\end{aligned}$$

(7) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1, \sigma_2)r$-closed in Y and by (7), we have

$$(\tau_1, \tau_2)\text{-pCl}(f^{-1}(V)) \subseteq (\tau_1, \tau_2)\text{-pCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).$$

It follows from Theorem 1 that f is almost weakly (τ_1, τ_2)-continuous.

Lemma 5. *Let (X, τ_1, τ_2) be a bitopological space. If A is $\alpha(\tau_1, \tau_2)$-open in X and B is $(\tau_1, \tau_2)p$-open in X, then $A \cap B$ is $(\tau_1, \tau_2)p$-open in X.*

Proof. Let A be $\alpha(\tau_1, \tau_2)$-open in X and B be $(\tau_1, \tau_2)p$-open in X. Then, we have $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$ and $B \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(B))$. Thus by Lemma 3(1),

$$\begin{aligned}A \cap B &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))) \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(B)) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)) \cap \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(B))) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A) \cap \tau_1\tau_2\text{-Cl}(B))) \\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A \cap B)).\end{aligned}$$

This shows that $A \cap B$ is $(\tau_1, \tau_2)p$-open in X.

Definition 2. [17] *A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-T_2 if for any pair of distinct points x, y in X, there exist disjoint $\tau_1\tau_2$-open sets U and V of X containing x and y, respectively.*

Definition 3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost $\alpha(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists an $\alpha(\tau_1, \tau_2)$-open set U of X such that $f(U) \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$.*

Theorem 4. *Let (Y, σ_1, σ_2) be a (σ_1, σ_2)-T_2 space. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost $\alpha(\tau_1, \tau_2)$-continuous and $g : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost weakly (τ_1, τ_2)-continuous, then the set $\{x \in X \mid f(x) = g(x)\}$ is $(\tau_1, \tau_2)p$-closed in X.*

Proof. Let $A = \{x \in X \mid f(x) = g(x)\}$ and $x \in X - A$. Then, $f(x) \neq g(x)$ and there exist $\sigma_1\sigma_2$-open sets V and V' of Y such that $f(x) \in V$, $g(x) \in V'$ and $V \cap V' = \emptyset$; hence $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)) \cap \sigma_1\sigma_2$-Cl$(V') = \emptyset$. Since f is almost $\alpha(\tau_1, \tau_2)$-continuous, there exists an $\alpha(\tau_1, \tau_2)$-open set U of X such that $f(U) \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$. Since g is almost weakly (τ_1, τ_2)-continuous, by Theorem 1 there exists a $(\tau_1, \tau_2)p$-open set U' of X containing x such that $g(U') \subseteq \sigma_1\sigma_2$-Cl$(V')$. Therefore, $f(U) \cap g(U') = \emptyset$. By Lemma 5, we have $U \cap U'$ is $(\tau_1, \tau_2)p$-open in X. Since $(U \cap U') \cap A = \emptyset$, $x \in X - (\tau_1, \tau_2)$-pCl$(A)$. Thus, A is $(\tau_1, \tau_2)p$-closed in X.

Definition 4. [32] *A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-Urysohn if for each pair of distinct points x and y in X, there exist $\tau_1\tau_2$-open sets U and V such that $x \in U$, $y \in V$ and $\tau_1\tau_2$-Cl$(U) \cap \tau_1\tau_2$-Cl$(V) = \emptyset$.*

Definition 5. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly $\alpha(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists an $\alpha(\tau_1, \tau_2)$-open set U of X such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$.*

Theorem 5. *Let (Y, σ_1, σ_2) be a $\sigma_1\sigma_2$-Urysohn space. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly $\alpha(\tau_1, \tau_2)$-continuous and $g : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost weakly (τ_1, τ_2)-continuous, then the set $\{x \in X \mid f(x) = g(x)\}$ is $(\tau_1, \tau_2)p$-closed in X.*

Proof. The proof is quite similar to that of Theorem 4.

Definition 6. *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)p$-Hausdorff if for each distinct points $x, y \in X$, there exist $(\tau_1, \tau_2)p$-open sets U and V of X containing x and y, respectively, such that $U \cap V = \emptyset$.*

Theorem 6. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost weakly (τ_1, τ_2)-continuous injection and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, then (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-Hausdorff.*

Proof. Since f is injective, then $f(x) \neq f(y)$ for any distinct points x and y in X. Since (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, there exist σ_1, σ_2-open sets V and V' of Y such that $f(x) \in V$, $f(y) \in V'$ and $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(V') = \emptyset$. Since f is almost weakly (τ_1, τ_2)-continuous, there exist $(\tau_1, \tau_2)p$-open sets U and U' of X containing x and y, respectively, such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$ and $f(U') \subseteq \sigma_1\sigma_2$-Cl$(V')$. This implies that $U \cap U' = \emptyset$. Thus, (X, τ_1, τ_2) is $(\tau_1, \tau_2)p$-Hausdorff.

Definition 7. [15] *A bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-regular if for each $\tau_1\tau_2$-closed set F and each point $x \in X - F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.*

Definition 8. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $(\tau_1, \tau_2)p$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1\tau_2)p$-open set U of X such that $f(U) \subseteq V$.*

Theorem 7. *Let (Y, σ_1, σ_2) be a (σ_1, σ_2)-regular space. Then a function*

$$f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is $(\tau_1, \tau_2)p$-continuous if and only if f is almost weakly (τ_1, τ_2)-continuous.

Proof. We prove only the sufficiency since the necessity is evident. Let $x \in X$ and W be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. By the regularity of (Y, σ_1, σ_2), there exists a $\sigma_1\sigma_2$-open set V of Y such that $f(x) \in V$ and $\sigma_1\sigma_2$-Cl$(V) \subseteq W$. Since f is almost weakly (τ_1, τ_2)-continuous, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. This implies that $f(U) \subseteq W$ and hence f is $(\tau_1, \tau_2)p$-continuous.

Recall that a bitopological space (X, τ_1, τ_2) is said to be *quasi* (τ_1, τ_2)*-*$\mathscr{H}$*-closed* [34] if every $\tau_1\tau_2$-open cover $\{U_\gamma \mid \gamma \in \Gamma\}$, there exists a finite subset Γ_0 of Γ such that $X = \cup\{\tau_1\tau_2\text{-Cl}(U_\gamma) \mid \gamma \in \Gamma_0\}$. A subset K of a bitopological space (X, τ_1, τ_2) is said to be *quasi* (τ_1, τ_2)*-*$\mathscr{H}$*-closed relative to* X if for any cover $\{V_\gamma \mid \gamma \in \Gamma\}$ by $\tau_1\tau_2$-open sets of X, there exists a finite subset Γ_0 of Γ such that $K \subseteq \cup\{\tau_1\tau_2\text{-Cl}(V_\gamma) \mid \gamma \in \Gamma_0\}$. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)*p-compact relative to* X if every cover of A by $(\tau_1, \tau_2)p$-open sets of X has a finite subcover. If $A = X$, then X is said to be (τ_1, τ_2)*p-compact.*

Theorem 8. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is almost weakly* (τ_1, τ_2)*-continuous and* K *is* (τ_1, τ_2)*p-compact relative to* X*, then* $f(K)$ *is quasi* (σ_1, σ_2)*-*$\mathscr{H}$*-closed relative to* Y*.*

Proof. Let $\{V_\gamma \mid \gamma \in \Gamma\}$ be a cover of $f(K)$ by $\sigma_1\sigma_2$-open sets in Y. For each $k \in K$, there exists $\gamma(k) \in \Gamma$ such that $f(k) \in V_{\gamma(k)}$. Since f is almost weakly (τ_1, τ_2)-continuous, by Theorem 1 there exists a $(\tau_1, \tau_2)p$-open set U_k of X containing k such that $f(U_k) \subseteq \sigma_1\sigma_2\text{-Cl}(V_{\gamma(k)})$. Since $\{U_k \mid k \in K\}$ is a cover of K by $(\tau_1, \tau_2)p$-open sets in X, there exists a finite subset K_0 of K such that $K \subseteq \cup\{U_k \mid k \in K_0\}$. Thus, $f(K) \subseteq \cup\{f(U_k) \mid k \in K_0\} \subseteq \cup\{\sigma_1\sigma_2\text{-Cl}(V_{\gamma(k)}) \mid k \in K_0\}$. This shows that $f(K)$ is quasi (σ_1, σ_2)-$\mathscr{H}$-closed relative to Y.

Corollary 1. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is an almost weakly* (τ_1, τ_2)*-continuous surjection and* (X, τ_1, τ_2) *is* (τ_1, τ_2)*p-compact, then* (Y, σ_1, σ_2) *is quasi* (σ_1, σ_2)*-*$\mathscr{H}$*-closed.*

Definition 9. [14] *A bitopological space* (X, τ_1, τ_2) *is said to be* $\tau_1\tau_2$*-connected if* X *cannot be written as the union of two disjoint nonempty* $\tau_1\tau_2$*-open sets.*

Definition 10. *A bitopological space* (X, τ_1, τ_2) *is said to be* (τ_1, τ_2)*p-connected if* X *cannot be written as the union of two disjoint nonempty* (τ_1, τ_2)*p-open sets.*

Theorem 9. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is an almost weakly* (τ_1, τ_2)*-continuous surjection and* (X, τ_1, τ_2) *is* (τ_1, τ_2)*p-connected, then* (Y, σ_1, σ_2) *is* $\sigma_1\sigma_2$*-connected.*

Proof. Suppose that (Y, σ_1, σ_2) is not $\sigma_1\sigma_2$-connected. Then, there exist nonempty $\sigma_1\sigma_2$-open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. It follows that $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ and $f^{-1}(U) \cup f^{-1}(V) = X$. Since f is surjective and U, V are $\sigma_1\sigma_2$-closed and $\sigma_1\sigma_2$-open, by Theorem 1 the inverse images of U and V are nonempty $(\tau_1, \tau_2)p$-open sets in X. This means that (X, τ_1, τ_2) is not $(\tau_1, \tau_2)p$-connected. This is a contradiction. It follows that (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-connected.

Corollary 2. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a* (τ_1, τ_2)*p-continuous surjection and* (X, τ_1, τ_2) *is* (τ_1, τ_2)*p-connected, then* (Y, σ_1, σ_2) *is* $\sigma_1\sigma_2$*-connected.*

Definition 11. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* (τ_1, τ_2)*p-irresolute if for each* $x \in X$ *and each* (σ_1, σ_2)*p-open set* V *of* Y *containing* $f(x)$*, there exists a* (τ_1, τ_2)*p-open set* U *of* X *such that* $f(U) \subseteq V$*.*

Theorem 10. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)p$-irresolute and*

$$g : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is almost weakly (σ_1, σ_2)-continuous, then the composition $g \circ f : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ is almost weakly (σ_1, σ_2)-continuous.

Proof. Let $x \in X$ and W be any $\rho_1\rho_2$-open set of Z containing $(g \circ f)(x)$. Since g is almost weakly (σ_1, σ_2)-continuous, there exists a $(\sigma_1, \sigma_2)p$-open set V of Y containing $f(x)$ such that $g(V) \subseteq \rho_1\rho_2\text{-Cl}(W)$. Since f is $(\tau_1, \tau_2)p$-irresolute, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $f(U) \subseteq V$. Thus, $(g \circ f)(U) = g(f(U)) \subseteq \rho_1\rho_2\text{-Cl}(W)$. This shows that $g \circ f$ is almost weakly (σ_1, σ_2)-continuous.

4. Conclusion

Stronger and weaker forms of open sets in topological spaces such as semi-open sets, preopen sets, α-open sets, β-open sets, θ-open sets and δ-open sets play an important role in the researching of generalizations of continuity. Using different forms of open sets, many authors have introduced and investigated various types of weak forms of continuity for functions and multifunctions. This work deals with the concept of almost weakly (τ_1, τ_2)-continuous functions. Additionally, several characterizations and some properties concerning almost weakly (τ_1, τ_2)-continuous functions are obtained. The ideas and results of this work may motivate further research.

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