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Quasi $\theta(\tau_1, \tau_2)$-continuous Functions

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Abstract. Our main purpose is to introduce the concept of quasi $\theta(\tau_1, \tau_2)$-continuous functions. Furthermore, several characterizations and some properties concerning quasi $\theta(\tau_1, \tau_2)$-continuous functions are investigated.

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1. Introduction

The notion of continuity is an important concept for the study in topological spaces. This concept has been generalized by weaker forms of open sets such as semi-open sets [23], preopen sets [25], α-open sets [27], β-open sets [19] and θ-open sets [42]. Levine [23] introduced and studied the notion of semi-continuous functions. Arya and Bhamini [1] introduced the concept of θ-semi-continuity as a generalization of semi-continuity. Noiri [28] and Jafari and Noiri [20] have further investigated some characterizations of θ-semi-continuous functions. Viriyapong and Boonpok [44] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [9]. Dungthaisong et al. [18] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [17] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, slightly $(\tau_1, \tau_2)s$-continuous

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functions, $\delta(\tau_1, \tau_2)$-continuous functions, faintly (τ_1, τ_2)-continuous functions and rarely (τ_1, τ_2)-continuous functions were presented in [36], [38], [10], [32], [13], [8], [6], [7], [4], [2], [3], [14], [12], [11], [35], [31], [37] and [41], respectively. Marcus [24] introduced and investigated the notion of quasi continuous functions. Popa [29] introduced and studied the notion of almost quasi continuous functions. Neubrunnovaá [26] showed that quasi continuity is equivalent to semi-continuity due to Levine [23]. Popa and Stan [30] introduced and investigated the notion of weakly quasi continuous functions. Weak quasi continuity is implied by quasi continuity and weak continuity [22] which are independent of each other. Kong-ied et al. [21] introduced and investigated the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [16] introduced and studied the notion of weakly quasi (τ_1, τ_2)-continuous functions. In this paper, we introduce the notion of quasi $\theta(\tau_1, \tau_2)$-continuous functions. We also investigate several characterizations of quasi $\theta(\tau_1, \tau_2)$-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [15] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [15] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [15] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [15] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-Cl(A) and $\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Cl$(A)) = \tau_1\tau_2$-Cl(A).

(2) If $A \subseteq B$, then $\tau_1\tau_2$-Cl$(A) \subseteq \tau_1\tau_2$-Cl$(B)$.

(3) $\tau_1\tau_2$-Cl(A) is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-Cl(A).

(5) $\tau_1\tau_2$-Cl$(X - A) = X - \tau_1\tau_2$-Int$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [15] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [43] (resp. $(\tau_1, \tau_2)s$-*open* [5], $(\tau_1, \tau_2)p$-*open* [5], $(\tau_1, \tau_2)\beta$-*open* [5]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))))$. The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*,

(τ_1,τ_2)*p-closed*, $(\tau_1,\tau_2)\beta$*-closed*). A subset A of a bitopological space (X,τ_1,τ_2) is said to be $\alpha(\tau_1,\tau_2)$*-open* [45] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1,\tau_2)$-open set is said to be $\alpha(\tau_1,\tau_2)$*-closed*. Let A be a subset of a bitopological space (X,τ_1,τ_2). A point $x \in X$ is called a $(\tau_1,\tau_2)\theta$*-cluster point* [43] of A if $\tau_1\tau_2\text{-Cl}(U)\cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1,\tau_2)\theta$-cluster points of A is called the $(\tau_1,\tau_2)\theta$*-closure* [43] of A and is denoted by $(\tau_1,\tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X,τ_1,τ_2) is said to be $(\tau_1,\tau_2)\theta$*-closed* [43] if $(\tau_1,\tau_2)\theta\text{-Cl}(A) = A$. The complement of a $(\tau_1,\tau_2)\theta$-closed set is said to be $(\tau_1,\tau_2)\theta$*-open*. The union of all $(\tau_1,\tau_2)\theta$-open sets of X contained in A is called the $(\tau_1,\tau_2)\theta$*-interior* [43] of A and is denoted by $(\tau_1,\tau_2)\theta\text{-Int}(A)$.

Lemma 2. [43] *For a subset A of a bitopological space (X,τ_1,τ_2), the following properties hold:*

(1) If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2\text{-Cl}(A) = (\tau_1,\tau_2)\theta\text{-Cl}(A)$.

(2) $(\tau_1,\tau_2)\theta\text{-Cl}(A)$ is $\tau_1\tau_2$-closed in X.

Let A be a subset of a bitopological space (X,τ_1,τ_2). A point $x \in X$ is called a $\theta(\tau_1,\tau_2)s$*-cluster point* of A if $(\tau_1,\tau_2)\text{-sCl}(U)\cap A \neq \emptyset$ for every $(\tau_1,\tau_2)s$-open set U containing x. The set of all $\theta(\tau_1,\tau_2)s$-cluster points of A is called the $\theta(\tau_1,\tau_2)s$*-closure* of A and is denoted by $\theta(\tau_1,\tau_2)\text{-sCl}(A)$. A subset A of a bitopological space (X,τ_1,τ_2) is said to be $\theta(\tau_1,\tau_2)s$*-closed* if $\theta(\tau_1,\tau_2)\text{-sCl}(A) = A$. The complement of a $\theta(\tau_1,\tau_2)s$-closed set is said to be $\theta(\tau_1,\tau_2)s$*-open*. The union of all $\theta(\tau_1,\tau_2)s$-open sets of X contained in A is called the $\theta(\tau_1,\tau_2)s$*-interior* of A and is denoted by $\theta(\tau_1,\tau_2)\text{-sInt}(A)$.

3. Quasi $\theta(\tau_1,\tau_2)$-continuous functions

In this section, we introduce the notion of quasi $\theta(\tau_1,\tau_2)$-continuous functions. Moreover, some characterizations of quasi $\theta(\tau_1,\tau_2)$-continuous functions are discussed.

Definition 1. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be quasi $\theta(\tau_1,\tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1,\tau_2)s$-open set U of X containing x such that $f((\tau_1,\tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$.*

Theorem 1. *For a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) f is quasi $\theta(\tau_1,\tau_2)$-continuous;

(2) $\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}((\sigma_1,\sigma_2)\theta\text{-Cl}(B)))) \subseteq f^{-1}((\sigma_1,\sigma_2)\theta\text{-Cl}(B))$ for every subset B of Y;

(3) $\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ for every $\sigma_1\sigma_2$-open set V of Y;

(4) $\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))) \subseteq f^{-1}(K)$ for every $(\sigma_1,\sigma_2)r$-closed set K of Y;

(5) $f^{-1}(V) \subseteq \theta(\tau_1, \tau_2)$*-sInt*$(f^{-1}(\sigma_1\sigma_2$*-Cl*$(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(6) $\theta(\tau_1, \tau_2)$*-sCl*$(f^{-1}(\sigma_1\sigma_2$*-Int*$(K))) \subseteq f^{-1}(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(7) $\theta(\tau_1, \tau_2)$*-sCl*$(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$*-Cl*$(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*.*

Proof. (1) ⇒ (2): Let B be any subset of Y. Suppose that $x \notin f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$. Then, $x \in X - f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$ and $f(x) \in Y - (\sigma_1, \sigma_2)\theta\text{-Cl}(B)$. Since $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\sigma_1\sigma_2$-closed in Y, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(Y - (\sigma_1, \sigma_2)\theta\text{-Cl}(B)) = Y - \sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$. Thus, we have $f((\tau_1, \tau_2)\text{-sCl}(U)) \cap \sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)) = \emptyset$ and

$$(\tau_1, \tau_2)\text{-sCl}(U) \cap f^{-1}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))) = \emptyset.$$

This shows that $x \notin \theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))))$. Thus,

$$\theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) \subseteq f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)).$$

(2) ⇒ (3): This is obvious since $\sigma_1\sigma_2\text{-Cl}(V) = (\sigma_1, \sigma_2)\theta\text{-Cl}(V)$ for every $\sigma_1\sigma_2$-open set V of Y.

(3) ⇒ (4): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. By (3), we have

$$\begin{aligned}\theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))) &= \theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= f^{-1}(K).\end{aligned}$$

(4) ⇒ (5): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, we have

$$\begin{aligned}X - \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) &= \theta(\tau_1, \tau_2)\text{-sCl}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= \theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V))),\end{aligned}$$

$Y - \sigma_1\sigma_2\text{-Cl}(V) = \sigma_1\sigma_2\text{-Int}(Y - \sigma_1\sigma_2\text{-Cl}(V)) \subseteq \sigma_1\sigma_2\text{-Int}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ and $Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-closed in Y. Thus by (4),

$$\begin{aligned}\theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))) &\subseteq f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq X - f^{-1}(V)\end{aligned}$$

and hence $f^{-1}(V) \subseteq \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$.

(5) ⇒ (6): Let K be any $\sigma_1\sigma_2$-closed set of Y. Then by (5), we have

$$\begin{aligned}X - f^{-1}(K) &= f^{-1}(Y - K) \\ &\subseteq \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - K))) \\ &= \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(K)))\end{aligned}$$

$$= \theta(\tau_1, \tau_2)\text{-sInt}(X - f^{-1}(\sigma_1\sigma_2\text{-Int}(K)))$$
$$= X - \theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))).$$

Thus, $\theta(\tau_1, \tau_2)$-sCl$(f^{-1}(\sigma_1\sigma_2\text{-Int}(K))) \subseteq f^{-1}(K)$.

(6) ⇒ (7): Let V be any $\sigma_1\sigma_2$-closed set of Y. Then, we have $\sigma_1\sigma_2$-Cl(V) is $\sigma_1\sigma_2$-closed in Y and by (6),

$$\theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(V)) \subseteq \theta(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$$
$$\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).$$

(7) ⇒ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, $\sigma_1\sigma_2$-Cl$(Y - \sigma_1\sigma_2\text{-Cl}(V)) \cap f(x) = \emptyset$ and $x \notin f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V)))$. It follows from (7) that $x \notin \theta(\tau_1, \tau_2)$-sCl$(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)))$. Then, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that (τ_1, τ_2)-sCl$(U) \cap f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)) = \emptyset$; hence $f((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2$-Cl$(V)$. This shows that f is quasi $\theta(\tau_1, \tau_2)$-continuous.

Definition 2. [21] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $f(G) \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost quasi (τ_1, τ_2)-continuous if f is almost quasi (τ_1, τ_2)-continuous at each point of X.*

Lemma 3. [21] *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is almost quasi (τ_1, τ_2)-continuous;

(2) for each $x \in X$ and every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$;

(3) $f^{-1}(V)$ is $(\tau_1, \tau_2)s$-open in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;

(4) $f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(5) (τ_1, τ_2)-sCl$(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y;

(6) $f^{-1}(V) \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))))$ for every $\sigma_1\sigma_2$-open set V of Y.

Definition 3. [16] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $f(G) \subseteq \sigma_1\sigma_2$-Cl$(V)$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly quasi (τ_1, τ_2)-continuous if f is weakly quasi (τ_1, τ_2)-continuous at each point of X.*

Lemma 4. [16] *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is weakly quasi (τ_1, τ_2)-continuous;

(2) for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$;

(3) $\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(f^{-1}(\sigma_1\sigma_2\text{-}Int(K)))) \subseteq f^{-1}(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;

(4) $f^{-1}(V) \subseteq (\tau_1, \tau_2)\text{-}sInt(f^{-1}(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(5) $(\tau_1, \tau_2)\text{-}sCl(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.

Theorem 2. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly quasi (τ_1, τ_2)-continuous and almost quasi (τ_1, τ_2)-continuous, then f is quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is weakly quasi (τ_1, τ_2)-continuous, by Lemma 4, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$ and hence $U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$. Since f is almost quasi (τ_1, τ_2)-continuous and $\sigma_1\sigma_2\text{-Cl}(V)$ is a $(\sigma_1, \sigma_2)r$-closed set of Y, by Lemma 3 we have $f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\tau_1, \tau_2)s$-closed in X. Thus, $(\tau_1, \tau_2)\text{-sCl}(U) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ and hence $f((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. This shows that f is quasi $\theta(\tau_1, \tau_2)$-continuous.

Definition 4. [39] *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)s$-regular if for each $(\tau_1, \tau_2)s$-closed set F of X and each $x \notin F$, there exist disjoint $(\tau_1, \tau_2)s$-open sets V and V such that $x \in U$ and $F \subseteq V$.*

Lemma 5. [39] *A bitopological space (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-regular if and only if for each $x \in X$ and each $(\tau_1, \tau_2)s$-open set U containing x, there exists a $(\tau_1, \tau_2)s$-open set V such that $x \in V \subseteq (\tau_1, \tau_2)\text{-}sCl(V) \subseteq U$.*

Theorem 3. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly quasi (τ_1, τ_2)-continuous and (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-regular, then f is quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Since f is weakly quasi (τ_1, τ_2)-continuous, by Lemma 4, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. By Lemma 5, there exists a $(\tau_1, \tau_2)s$-open set W such that $x \in W \subseteq (\tau_1, \tau_2)\text{-sCl}(W) \subseteq U$. Thus, $f((\tau_1, \tau_2)\text{-sCl}(W) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$ and hence f is quasi $\theta(\tau_1, \tau_2)$-continuous.

Theorem 4. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is quasi $\theta(\tau_1, \tau_2)$-continuous;

(2) $\theta(\tau_1, \tau_2)\text{-}sCl(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-}Cl(V))$ for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;

(3) *$\theta(\tau_1,\tau_2)$-sCl($f^{-1}(\sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl(V)))) $\subseteq f^{-1}(\sigma_1\sigma_2$-Cl($V$)) for every $(\sigma_1,\sigma_2)s$-open set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1,\sigma_2)\beta$-open set of Y. Then,

$$V \subseteq \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$$

and hence $\sigma_1\sigma_2$-Cl$(V) = \sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)))$. Since $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1,\sigma_2)r$-closed in Y, by Theorem 1 we have

$$\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).$$

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, V is $(\sigma_1,\sigma_2)s$-open in Y and by (3), $\theta(\tau_1,\tau_2)$-sCl$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)))) \subseteq f^{-1}(\sigma_1\sigma_2$-Cl$(V))$. Thus by Theorem 1, f is quasi $\theta(\tau_1,\tau_2)$-continuous.

Theorem 5. *For a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) *f is quasi $\theta(\tau_1,\tau_2)$-continuous;*

(2) *$\theta(\tau_1,\tau_2)$-sCl($f^{-1}(\sigma_1\sigma_2$-Int($\sigma_1\sigma_2$-Cl(V)))) $\subseteq f^{-1}(\sigma_1\sigma_2$-Cl($V$)) for every $(\sigma_1,\sigma_2)p$-open set V of Y;*

(3) *$\theta(\tau_1,\tau_2)$-sCl($f^{-1}(V)$) $\subseteq f^{-1}(\sigma_1\sigma_2$-Cl($V$)) for every $(\sigma_1,\sigma_2)p$-open set V of Y;*

(4) *$f^{-1}(V) \subseteq \theta(\tau_1,\tau_2)$-sInt($f^{-1}(\sigma_1\sigma_2$-Cl($V$))) for every $(\sigma_1,\sigma_2)p$-open set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1,\sigma_2)p$-open set of Y. Since $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $\sigma_1\sigma_2$-open set of Y, by Theorem 4 we have

$$\begin{aligned}\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))\\ &= f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).\end{aligned}$$

(2) $\Rightarrow$ (3): Let V be any $(\sigma_1,\sigma_2)p$-open set of Y. Then, $V \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ and by (2),

$$\begin{aligned}\theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(V)) &\subseteq \theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))\\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)).\end{aligned}$$

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1,\sigma_2)p$-open set of Y. Then by (3), we have

$$\begin{aligned}X - \theta(\tau_1,\tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))) &= \theta(\tau_1,\tau_2)\text{-sCl}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))\\ &= \theta(\tau_1,\tau_2)\text{-sCl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(V)))\\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V)))\end{aligned}$$

$$= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$$
$$\subseteq X - f^{-1}(V)$$

and hence $f^{-1}(V) \subseteq \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$.

(4) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, V is $(\sigma_1, \sigma_2)p$-open in Y and by (4), we have $f^{-1}(V) \subseteq \theta(\tau_1, \tau_2)\text{-sInt}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(V)))$. By Theorem 1, f is quasi $\theta(\tau_1, \tau_2)$-continuous.

Recall that a bitopological space (X, τ_1, τ_2) is said to be *quasi* (τ_1, τ_2)-$\mathscr{H}$*-closed* [40] if every $\tau_1\tau_2$-open cover $\{U_\gamma \mid \gamma \in \Gamma\}$, there exists a finite subset Γ_0 of Γ such that $X = \cup\{\tau_1\tau_2\text{-Cl}(U_\gamma) \mid \gamma \in \Gamma_0\}$. A subset K of a bitopological space (X, τ_1, τ_2) is said to be *quasi* (τ_1, τ_2)-$\mathscr{H}$*-closed relative to* X if for any cover $\{V_\gamma \mid \gamma \in \Gamma\}$ by $\tau_1\tau_2$-open sets of X, there exists a finite subset Γ_0 of Γ such that $K \subseteq \cup\{\tau_1\tau_2\text{-Cl}(V_\gamma) \mid \gamma \in \Gamma_0\}$. A bitopological space (X, τ_1, τ_2) is called s-(τ_1, τ_2)*-closed* [33] if every $(\tau_1, \tau_2)s$-open cover $\{U_\gamma \mid \gamma \in \Gamma\}$, there exists a finite subset Γ_0 of Γ such that $X = \cup\{(\tau_1, \tau_2)\text{-sCl}(U_\gamma) \mid \gamma \in \Gamma_0\}$. A subset K of a bitopological space (X, τ_1, τ_2) is said to be s-(τ_1, τ_2)*-closed relative to* X if for any cover $\{V_\gamma \mid \gamma \in \Gamma\}$ by $(\tau_1, \tau_2)s$-open sets of X, there exists a finite subset Γ_0 of Γ such that $K \subseteq \cup\{(\tau_1, \tau_2)\text{-sCl}(V_\gamma) \mid \gamma \in \Gamma_0\}$.

Theorem 6. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is quasi* $\theta(\tau_1, \tau_2)$*-continuous and* K *is* s-(τ_1, τ_2)*-closed relative to* X*, then* $f(K)$ *is quasi* (σ_1, σ_2)-$\mathscr{H}$*-closed relative to* Y*.*

Proof. Let $\{V_\gamma \mid \gamma \in \Gamma\}$ be a cover of $f(K)$ by $\sigma_1\sigma_2$-open sets in Y. For each $k \in K$, there exists $\gamma(k) \in \Gamma$ such that $f(k) \in V_{\gamma(k)}$. Since f is quasi $\theta(\tau_1, \tau_2)$-continuous, there exists a $(\tau_1, \tau_2)s$-open set U_k of X containing k such that

$$f((\tau_1, \tau_2)\text{-sCl}(U_k)) \subseteq \sigma_1\sigma_2\text{-Cl}(V_{\gamma(k)}).$$

Since $\{U_k \mid k \in K\}$ is a cover of K by $(\tau_1, \tau_2)s$-open sets in X, there exists a finite subset K_0 of K such that $K \subseteq \cup\{U_k \mid k \in K_0\}$. Thus,

$$f(K) \subseteq \cup\{f((\tau_1, \tau_2)\text{-sCl}(U_k)) \mid k \in K_0\}$$
$$\subseteq \cup\{\sigma_1\sigma_2\text{-Cl}(V_{\gamma(k)}) \mid k \in K_0\}.$$

This shows that $f(K)$ is quasi (σ_1, σ_2)-$\mathscr{H}$-closed relative to Y.

Corollary 1. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a quasi* $\theta(\tau_1, \tau_2)$*-continuous surjection and* (X, τ_1, τ_2) *is* s-(τ_1, τ_2)*-closed, then* (Y, σ_1, σ_2) *is quasi* (σ_1, σ_2)-$\mathscr{H}$*-closed.*

Definition 5. [34] *A bitopological space* (X, τ_1, τ_2) *is said to be* $\tau_1\tau_2$*-Urysohn if for each pair of distinct points* x *and* y *in* X*, there exist* $\tau_1\tau_2$*-open sets* U *and* V *such that* $x \in U$*,* $y \in V$ *and* $\tau_1\tau_2$-*Cl*$(U) \cap \tau_1\tau_2$-*Cl*$(V) = \emptyset$*.*

Definition 6. *A bitopological space* (X, τ_1, τ_2) *is called* $(\tau_1, \tau_2)s$*-Hausdorff if for each pair of distinct points* x *and* y *in* X*, there exist* $(\tau_1, \tau_2)s$*-open sets* U *and* V *such that* $x \in U$*,* $y \in V$ *and* $U \cap V = \emptyset$*.*

Lemma 6. *A bitopological space (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-Hausdorff if and only if for each pair of distinct points x and y in X, there exist $(\tau_1, \tau_2)s$-open sets U and V such that $x \in U$, $y \in V$ and (τ_1, τ_2)-sCl$(U) \cap (\tau_1, \tau_2)$-sCl$(V) = \emptyset$.*

Theorem 7. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a quasi $\theta(\tau_1, \tau_2)$-continuous injection and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, then (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-Hausdorff.*

Proof. Since f is injective, then $f(x) \neq f(y)$ for any distinct points x and y in X. Since (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, there exist $\sigma_1\sigma_2$-open sets V and V' of Y such that $f(x) \in V$, $f(y) \in V'$ and $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(V') = \emptyset$. Since f is quasi $\theta(\tau_1, \tau_2)$-continuous, there exist $(\tau_1, \tau_2)s$-open sets U and U' of X containing x and y, respectively, such that $f((\tau_1, \tau_2)$-sCl$(U)) \subseteq \sigma_1\sigma_2$-Cl$(V)$ and $f((\tau_1, \tau_2)$-sCl$(U')) \subseteq \sigma_1\sigma_2$-Cl$(V')$. This implies that (τ_1, τ_2)-sCl$(U) \cap (\tau_1, \tau_2)$-sCl$(U') = \emptyset$. Thus by Lemma 6, (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-Hausdorff.

Definition 7. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the graph*

$$G(f) = \{(x, f(x)) \mid x \in X\}$$

is said to be strong $(\tau_1, \tau_2)s$-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $(\tau_1, \tau_2)s$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $[(\tau_1, \tau_2)$-sCl$(U) \times \sigma_1\sigma_2$-Cl$(V)] \cap G(f) = \emptyset$.

Lemma 7. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ has a strong $(\tau_1, \tau_2)s$-closed graph if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $(\tau_1, \tau_2)s$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $f((\tau_1, \tau_2)$-sCl$(U)) \cap \sigma_1\sigma_2$-Cl$(V) = \emptyset$.*

Theorem 8. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is quasi $\theta(\tau_1, \tau_2)$-continuous and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, then $G(f)$ is strong $(\tau_1, \tau_2)s$-closed.*

Proof. Suppose that $(x, y) \in (X \times Y) - G(f)$. Then, $y \neq f(x)$. Since (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, there there exist $\sigma_1\sigma_2$-open sets V and W of Y containing y and $f(x)$, respectively, such that $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(W) = \emptyset$. Since f is quasi $\theta(\tau_1, \tau_2)$-continuous, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f((\tau_1, \tau_2)$-sCl$(U)) \subseteq \sigma_1\sigma_2$-Cl$(W)$. This implies that $f((\tau_1, \tau_2)$-sCl$(U)) \cap \sigma_1\sigma_2$-Cl$(V) = \emptyset$ and by Lemma 7, $G(f)$ is strong $(\tau_1, \tau_2)s$-closed.

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