EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 2, Article Number 5742
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

$\mathfrak{M}-$homomorphisms of Almost Distributive Lattices

N. Rafi[1], Thiti Gaketem[2,*], Ravi Kumar Bandaru[3]

[1] *Department of Mathematics, Bapatla Engineering College, Bapatla, Andhra Pradesh, India*
[2] *Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand*
[3] *Department of Mathematics, School of Advanced Sciences, VIT-AP University, Andhra Pradesh, India*

Abstract. An $\mathfrak{M}-$homomorphism in an Almost Distributive Lattice(ADL) is introduced, with a sufficient condition for it to be an $\mathfrak{M}-$homomorphism. The image and inverse image of an $\mathfrak{M}-$filter under such a homomorphism are shown to be $\mathfrak{M}-$filters. Sufficient conditions for a prime filter to be an $\mathfrak{M}-$filter are established, along with an equivalence between prime $\mathfrak{M}-$filters and minimal prime filters. Finally, any two distinct prime $\mathfrak{M}-$filters in an ADL are shown to be comaximal.

2020 Mathematics Subject Classifications: 06D99, 06D15

Key Words and Phrases: Almost Distributive Lattice(ADL), $\mathfrak{M}-$homomorphism, $\mathfrak{M}-$filter, Co-dense, Co-Kernel, $E-$complemented ADL, Dual dense, $\sqcup-$comaximal

1. Introduction

Swamy and Rao introduced the concept of an Almost Distributive Lattice(ADL) [1], which serves as a common abstraction for numerous ring-theoretic generalizations of Boolean algebra and the class of distributive lattices. In their paper, they defined the notion of an ideal in an ADL, drawing an analogy with ideals in distributive lattices. They observed that the set $PI(\mathcal{R})$ of all principal ideals in an ADL forms a distributive lattice. This observation opened up avenues for extending many existing lattice theory concepts to the class of ADLs. In [2], introduced and study the properties of $\mu-$filters in an ADL. Sambasiva Rao and G.C. Rao introduced the notion of $O-$homomorphisms within the context of Almost Distributive Lattices(ADLs) in [3], exploring their key properties. In [4], author studied the properties of $O-$filters in Lattices. Later, in [5], Rafi established a correspondence between the set of all prime $O-$ideals and the set of minimal prime ideals in an ADL. In that work, they identified a necessary and sufficient condition

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i2.5742

Email addresses: rafimaths@gmail.com (N. Rafi), thiti.ga@up.ac.th (T. Gaketem), ravimaths83@gmail.com (R. Bandaru)

for an $O-$ideal to be prime and demonstrated that distinct prime $O-$ideals in an ADL are always comaximal.

In this paper, we introduce the concept of an $\mathfrak{M}-$homomorphism within the framework of Almost Distributive Lattices and examine its properties. We establish a sufficient condition for a general homomorphism of an ADL to qualify as an $\mathfrak{M}-$homomorphism. Additionally, we demonstrate that both the image and the preimage of an $\mathfrak{M}-$filter under an $\mathfrak{M}-$homomorphism remain $\mathfrak{M}-$filters. We also show that the kernel of a homomorphism is an $\mathfrak{M}-$filter. Furthermore, we derive an equivalence between the class of all prime $\mathfrak{M}-$filters and the class of all minimal prime filters. A necessary and sufficient condition for an $\mathfrak{M}-$filter in an ADL to be a prime filter is also presented. We introduce the notion of an $E-$complemented ADL and investigate its properties. Within an $E-$complemented ADL, we establish several equivalent conditions for an $\mathfrak{M}-$filter to qualify as an annihilator filter. Finally, we prove that any two distinct prime $\mathfrak{M}-$filters in an ADL are comaximal.

2. Preliminaries

The definitions and significant results from [1, 6] are gathered and given in this part; these will be needed during the entire document.

Definition 1. *[1] An algebra* $(\mathcal{R}, \vee, \wedge, 0)$ *of type (2,2,0) satisfying the following specifications is an Almost Distributive Lattice(ADL) with zero :*

(1) $(\theta \vee \vartheta) \wedge \sigma = (\theta \wedge \sigma) \vee (\vartheta \wedge \sigma)$;
(2) $\theta \wedge (\vartheta \vee \sigma) = (\theta \wedge \vartheta) \vee (\theta \wedge \sigma)$;
(3) $(\theta \vee \vartheta) \wedge \vartheta = \vartheta$;
(4) $(\theta \vee \vartheta) \wedge \theta = \theta$;
(5) $\theta \vee (\theta \wedge \vartheta) = \theta$;
(6) $0 \wedge \theta = 0$, *for any* $\theta, \vartheta, \sigma \in \mathcal{R}$.

When $\theta = \theta \wedge \vartheta$, or equivalently, $\theta \vee \vartheta = \vartheta$, occurs for every $\theta, \vartheta \in \mathcal{R}$, then $\theta \leq \vartheta$. This defines a partial $\leq$ on $\mathcal{R}$ in an ADL $(\mathcal{R}, \vee, \wedge, 0)$. As a partial ordering on $\mathcal{R}$, this definition establishes $\leq$. When m in $\mathcal{R}$ holds maximum with respect to the partial ordering $\leq$ on $\mathcal{R}$, it is referred to as *maximal* i.e., for any $m \in \mathcal{R}$, $m \leq \theta \Rightarrow m = \theta$; $\mathcal{M}_{max.elt}(\mathcal{R})$ is the collection of all such maximal elements within $\mathcal{R}$.

In Swamy's work[1], it is noted that an ADL denoted as $\mathcal{R}$ exhibits nearly all features of a distributive lattice[7, 8], apart from the non-commutativity between $\vee$ and $\wedge$ and the right distributivity of $\vee$ over $\wedge$. Either of these properties, if present, would classify $\mathcal{R}$ as a distributive lattice. If, for every $\theta, \vartheta \in \mathcal{I}$ and every $\mu \in \mathcal{R}$, there is a nonempty subset $\mathcal{I}$ of $\mathcal{R}$, then $\theta \vee \vartheta, \theta \wedge \mu \in \mathcal{I}$ (respectively, $\theta \wedge \vartheta, \mu \vee \theta \in \mathcal{F}$) for every $\theta, \vartheta \in \mathcal{I}$. A maximum ideal (filter) contains every appropriate ideal (filter) of $\mathcal{R}$. The smallest ideal that contains $\mathcal{A}$ for each subset $\mathcal{A}$ of $\mathcal{R}$ is $(\mathcal{A}] := \{(\bigvee\limits_{i=1}^{n} \theta_i) \wedge \mu \mid \theta_i \in \mathcal{A}, \mu \in \mathcal{R}$ and $n \in \mathbb{N}\}$. An ideal like $\mathcal{A} = \{\theta\}$ is written as $(\theta]$ rather than $(\mathcal{A}]$; this is known as the principal ideal of $\mathcal{R}$. The same way, for each $\mathcal{A} \subseteq \mathcal{R}$, $[\mathcal{A}) := \{\mu \vee (\bigwedge_{i=1}^{n} \theta_i) \mid \theta_i \in \mathcal{A}, \mu \in \mathcal{R}$ and $n \in \mathbb{N}\}$. A filter like $\mathcal{A} = \{\theta\}$ is written as $[\theta)$ rather than $[\mathcal{A})$; this is known as the principal filter of $\mathcal{R}$. It can be confirmed that $(\theta] \vee (\vartheta] = (\theta \vee \vartheta]$ and $(\theta] \cap (\vartheta] = (\theta \wedge \vartheta]$ hold for any $\theta, \vartheta \in \mathcal{R}$. A sublattice of the distributive lattice $(\mathcal{I}(\mathcal{R}), \vee, \cap)$ of all ideals of $\mathcal{R}$ is thus the set $(\mathcal{PI}(\mathcal{R}), \vee, \cap)$ of all principal ideals of $\mathcal{R}$. For any nonempty subset A of an ADL $\mathcal{R}$, define $\mathcal{A}^+ = \{\mu \in \mathcal{R} \mid \theta \vee \mu$ is maximal, for all $\theta \in \mathcal{A}\}$. Here $\mathcal{A}^+$ is called the dual annihilator of $\mathcal{A}$ in $\mathcal{R}$.

For any $\theta \in \mathcal{R}$, we have $\{\theta\}^+ = [\theta)^+$, where $[\theta)$ is the principal filter generated by θ. An element θ of an ADL $\mathcal{R}$ is called dual dense element if $[\theta)^+ = \mathcal{M}_{max.elt}(\mathcal{R})$ and the set E of all dual dense elements in an ADL $\mathcal{R}$ is an ideal if E is non-empty. In this paper, $\mathcal{R}$ and $\mathcal{R}'$ are used to represent two ADLs with zero elements 0 and $0'$, respectively, while $\mathcal{M}_{max.elt}(\mathcal{R})$ and $\mathcal{M}_{max.elt}(\mathcal{R}')$ denote the sets of all maximal elements in $\mathcal{R}$ and $\mathcal{R}'$, respectively.

3. $\mathfrak{M}-$homomorphisms of ADLs

In this section, we introduce and analyze $\mathfrak{M}-$homomorphisms in Almost Distributive Lattices, establishing conditions for a general homomorphism to be an $\mathfrak{M}-$homomorphism and showing that both images and pre images of $\mathfrak{M}-$filters under these homomorphisms remain $\mathfrak{M}-$filters. We also derive equivalences between minimal prime filters and prime $\mathfrak{M}-$filters, present conditions for $\mathfrak{M}-$filters to be prime, and explore the properties of $E-$complemnted ADLs. We will start this section by presenting the following definition.

Definition 2. *Let m and m' be maximal elements of $\mathcal{R}$ and $\mathcal{R}'$ respectively. Then the mapping $\xi : \mathcal{R} \longrightarrow \mathcal{R}'$ is known as homomorphism if, for all $\theta, \vartheta \in \mathcal{R}$, it satisfies the following:*

(i) $\xi(\theta \vee \vartheta) = \xi(\theta) \vee \xi(\vartheta)$

(ii) $\xi(\theta \wedge \vartheta) = \xi(\theta) \wedge \xi(\vartheta)$

(iii) $\xi(0) = 0'$

(iv) $\xi(m) = m'$.

The Co-kernel of the homomorphism ξ is defined by $Co - Ker\ \xi = \{\theta \in \mathcal{R} \mid \xi(\theta) \in \mathcal{M}_{max.elt}(\mathcal{R}')\}$. Clearly $Co - Ker\ \xi$ is filter in $\mathcal{R}$.

The set of all homomorphism functions from $\mathcal{R}$ to $\mathcal{R}'$ represented as $Hom_{\mathcal{R}}(\mathcal{R}')$.

Lemma 1. *For any $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$, we have:*

(i). for each ideal $\mathcal{F}$ of $\mathcal{R}'$ with $\xi^{-1}(\mathcal{F}) \neq \emptyset$, $\xi^{-1}(\mathcal{F})$ is an ideal of $\mathcal{R}$.

(ii). if ξ is onto then, for each ideal $\mathcal{G}$ of $\mathcal{R}$, $\xi(\mathcal{G})$ is an ideal of $\mathcal{R}'$.

Proof. (i). Consider $\mathcal{F}$ is an ideal of $\mathcal{R}'$ with $\xi^{-1}(\mathcal{F}) \neq \emptyset$. Let $\rho, \sigma \in \xi^{-1}(\mathcal{F})$. Then $\xi(\rho), \xi(\sigma) \in \mathcal{F}$. Since $\mathcal{F}$ is an ideal of $\mathcal{R}'$, we get that $\xi(\rho) \vee \xi(\sigma) \in \mathcal{F}$ and hence $\xi(\rho \vee \sigma) \in \mathcal{F}$. Therefore $\rho \vee \sigma \in \xi^{-1}(\mathcal{F})$. Let $\rho \in \xi^{-1}(\mathcal{F})$ and $\omega \in \mathcal{R}$. Then $\xi(\rho) \in \mathcal{F}$. Now, $\xi(\rho \wedge \omega) = \xi(\rho) \wedge \xi(\omega) \in \mathcal{F}$, since $\xi(\omega) \in \mathcal{R}'$. Therefore $\rho \wedge \omega \in \xi^{-1}(\mathcal{F})$.
(ii). Let ξ be an onto function and $\mathcal{G}$ be any ideal of $\mathcal{R}$. Then $\xi(\mathcal{G})$ is a non-empty set. Let $\delta, \tau \in \xi(\mathcal{G})$. Then $\delta = \xi(\rho)$ and $\tau = \xi(\sigma)$, for some $\rho, \sigma \in \mathcal{G}$. Now $\delta \vee \tau = \xi(\rho) \vee \xi(\sigma) = \xi(\rho \vee \sigma) \in \xi(\mathcal{G})$, since $\rho \vee \sigma \in \mathcal{G}$. Therefore $\delta \vee \tau \in \xi(\mathcal{G})$. Let $\delta \in \xi(\mathcal{G})$ and $\kappa \in \mathcal{R}'$. Then $\delta = \xi(\rho)$, for some $\rho \in \mathcal{G}$. Since ξ is onto and $\kappa \in \mathcal{R}'$, there is $\eta \in \mathcal{R}$ satisfies $\xi(\eta) = \kappa$. Now $\delta \wedge \kappa = \xi(\rho) \wedge \xi(\eta) = \xi(\rho \wedge \eta) \in \xi(\mathcal{G})$, since $\rho \wedge \eta \in \mathcal{G}$. Therefore $\delta \wedge \kappa \in \xi(\mathcal{G})$. Hence $\xi(\mathcal{G})$ is an ideal of $\mathcal{R}'$.

Definition 3 ([9, 10])**.** *For every ideal $\mathcal{F}$ of $\mathcal{R}$, consider the set $\mathfrak{M}(\mathcal{F}) = \{\kappa \in \mathcal{R} | \kappa \vee \rho \in \mathcal{M}_{max.elt}(\mathcal{R}),$ for some $\rho \in \mathcal{F}\}$.*

Lemma 2. *For any $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ and any ideal $\mathcal{F}$ of $\mathcal{R}$, we have $\xi[\mathfrak{M}(\mathcal{F})] \subseteq \mathfrak{M}[\xi(\mathcal{F})]$.*

Proof. Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ and $\mathcal{F}$ be any ideal of $\mathcal{R}$. Let $\kappa \in \xi[\mathfrak{M}(\mathcal{F})]$. Then there is $\rho \in \mathfrak{M}(\mathcal{F})$ satisfying $\kappa = \xi(\rho)$. As $\rho \in \mathfrak{M}(\mathcal{F})$, it is clear that $\rho \vee \sigma \in \mathcal{M}_{max.elt}(\mathcal{R})$, for some $\sigma \in \mathcal{F}$. That implies $\xi(\rho \vee \sigma) \in \mathcal{M}_{max.elt}(\mathcal{R}')$. Let $\eta \in \mathcal{R}'$. Now $[\kappa \vee \xi(\sigma)] \wedge \eta = [\xi(\rho) \vee \xi(\sigma)] \wedge \eta = \xi(\rho \vee \sigma) \wedge \eta = \eta$. Therefore $\kappa \vee \xi(\sigma) \in \mathcal{M}_{max.elt}(\mathcal{R}')$ and hence $\kappa \in \mathfrak{M}[\xi(\mathcal{F})]$. Thus $\xi[\mathfrak{M}(\mathcal{F})] \subseteq \mathfrak{M}[\xi(\mathcal{F})]$.

In general, $\mathfrak{M}[\xi(\mathcal{F})] \nsubseteq \xi[\mathfrak{M}(\mathcal{F})]$ is not true for any ideal $\mathcal{F}$ of $\mathcal{R}$. Consider the subsequent example.

Example 1. *Consider $\mathcal{R} = \{0, \theta, \vartheta, \sigma\}$. Two binary operations $\vee$, $\wedge$ are defined on $\mathcal{R}$ as follows:*

$\vee$	0	θ	ϑ	σ
0	0	θ	ϑ	σ
θ	θ	θ	θ	θ
ϑ	ϑ	ϑ	ϑ	ϑ
σ	σ	θ	ϑ	σ

$\wedge$	0	θ	ϑ	σ
0	0	0	0	0
θ	0	θ	ϑ	σ
ϑ	0	θ	ϑ	σ
σ	0	σ	σ	σ

It is observed easily that $(\mathcal{R}, \vee, \wedge, 0)$ is an ADL with 0.

$$\text{Define } \xi : \mathcal{R} \longrightarrow \mathcal{R} \text{ by } \xi(\kappa) = \begin{cases} 0 & \text{if } \kappa = 0 \\ \sigma & \text{if } \kappa = \sigma \\ \theta & \text{otherwise.} \end{cases}$$

Take an ideal $\mathcal{F} = \{0, \sigma\}$ of an ADL $\mathcal{R}$. Then $\mathfrak{M}(\mathcal{F}) = \{\theta, \vartheta\}$. That implies $\xi[\mathfrak{M}(\mathcal{F})] = \{\theta\}$. Clearly we have that $\xi(\mathcal{F}) = \{0, \sigma\}$. That implies $\mathfrak{M}[\xi(\mathcal{F})] = \{\theta, \vartheta\}$. Therefore $\xi[\mathfrak{M}(\mathcal{F})] \neq \mathfrak{M}[\xi(\mathcal{F})]$. Thus ξ is not an $\mathfrak{M}-$homomorphism.

Now we present the idea of an $\mathfrak{M}-$homomorphism.

Definition 4. *A homomorphism ξ of $Hom_{\mathcal{R}}(\mathcal{R}')$ is said to be an $\mathfrak{M}-$homomorphism if $\xi[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{F})]$.*

Example 2. *Consider $\mathcal{R} = \{0, \theta, \vartheta, \sigma\}$. Two binary operations $\vee$, $\wedge$ are defined on $\mathcal{R}$ as follows:*

$\vee$	0	θ	ϑ	σ
0	0	θ	ϑ	σ
θ	θ	θ	ϑ	ϑ
ϑ	ϑ	ϑ	ϑ	ϑ
σ	σ	ϑ	ϑ	σ

$\wedge$	0	θ	ϑ	σ
0	0	0	0	0
θ	0	θ	θ	0
ϑ	0	θ	ϑ	σ
σ	0	0	σ	σ

It is observed easily that $(\mathcal{R}, \vee, \wedge, 0)$ is an ADL with 0. Let ξ be an identity mapping on $\mathcal{R}$. Then, ξ is homomorphism on $\mathcal{R}$. Take an ideal $\mathcal{F} = \{0, \theta\}$ of $\mathcal{R}$. Now $\mathfrak{M}(\mathcal{F}) = \bigcup_{\mu \in \mathcal{F}} (\mu)^+ = \{\vartheta, \sigma\}$. Therefore $\xi[\mathfrak{M}(\mathcal{F})] = \{\vartheta, \sigma\}$. Obviously, we get $\xi(\mathcal{F}) = \{0, \theta\}$ and hence $\mathfrak{M}[\xi(\mathcal{F})] = \{\vartheta, \sigma\}$. Therefore $\xi[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{F})]$. Thus ξ is an $\mathfrak{M}-$homomorphism.

Theorem 1. *Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ with onto property and $Co - Ker\ \xi = \mathcal{M}_{max.elt}(\mathcal{R})$. Then ξ is an $\mathfrak{M}-$homomorphism.*

Proof. Clearly, we have that $\xi[\mathfrak{M}(\mathcal{F})] \subseteq \mathfrak{M}[\xi(\mathcal{F})]$. Let $\kappa \in \mathfrak{M}[\xi(\mathcal{F})]$. Then $\kappa \vee \rho \in \mathcal{M}_{max.elt}(\mathcal{R}')$, for some $\rho \in \xi(\mathcal{F})$. Since $\rho \in f(\mathcal{F})$, there exists an element $\sigma \in \mathcal{F}$ such that $\xi(\sigma) = \rho$. Since $\kappa \in \mathcal{R}'$ and ξ is onto, there exists an element $\eta \in \mathcal{R}$ such that $\xi(\eta) = \kappa$. Since $\kappa \vee \rho \in \mathcal{M}_{max.elt}(\mathcal{R}')$, we have that $\xi(\eta) \vee \xi(\sigma) \in \mathcal{M}_{max.elt}(\mathcal{R}')$. That implies $\xi(\eta \vee \sigma) \in \mathcal{M}_{max.elt}(\mathcal{R}')$ and hence $\eta \vee \sigma \in \mathcal{M}_{max.elt}(\mathcal{R})$. That implies that $\eta \in \mathfrak{M}(\mathcal{F})$ and hence $\kappa = \xi(\eta) \in \xi[\mathfrak{M}(\mathcal{F})]$. Therefore $\mathfrak{M}[\xi(\mathcal{F})] \subseteq \xi[\mathfrak{M}(\mathcal{F})]$. Thus $\xi[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{F})]$.

Theorem 2. *Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ with onto property and $Co - Ker\ \xi = \mathcal{M}_{max.elt}(\mathcal{R})$. Then $\mathfrak{M}(\mathcal{F}) = \mathfrak{M}(\mathcal{G}) \Leftrightarrow \mathfrak{M}[\xi(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{G})]$, for each two ideals $\mathcal{F}, \mathcal{G}$ of $\mathcal{R}$.*

Proof. By Theorem 1, we have that ξ is an $\mathfrak{M}-$homomorphism. Assume that $\mathfrak{M}(\mathcal{F}) = \mathfrak{M}(\mathcal{G})$. Now $\mathfrak{M}[\xi(\mathcal{F})] = \xi[\mathfrak{M}(\mathcal{F})] = \xi[\mathfrak{M}(\mathcal{G})] = \mathfrak{M}[\xi(\mathcal{G})]$. Therefore $\mathfrak{M}[\xi(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{G})]$. Conversely assume that $\mathfrak{M}[\xi(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{G})]$. Let $\kappa \in \mathfrak{M}(\mathcal{F})$. Then $\xi(\kappa) \in \xi[\mathfrak{M}(\mathcal{F})]$. That implies $\xi(\kappa) \in \mathfrak{M}[\xi(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{G})]$. That implies $\xi(\kappa) \vee \xi(\eta) \in \mathcal{M}_{max.elt}(\mathcal{R}')$, for some $\eta \in \mathcal{G}$. Hence $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. That gives $\kappa \in \mathfrak{M}(\mathcal{G})$. Therefore $\mathfrak{M}(\mathcal{F}) \subseteq \mathfrak{M}(\mathcal{G})$. Similarly, we get that $\mathfrak{M}(\mathcal{G}) \subseteq \mathfrak{M}(\mathcal{F})$. Hence $\mathfrak{M}(\mathcal{F}) = \mathfrak{M}(\mathcal{G})$.

Definition 5. *[9] A filter $\mathcal{S}$ of $\mathcal{R}$ is referred to as an $\mathfrak{M}-$filter if there exists an ideal $\mathcal{F}$ of $\mathcal{R}$ such that $\mathcal{S} = \mathfrak{M}(\mathcal{F})$.*

It is evident that for any $\kappa \in \mathcal{R}$, the filter $(\kappa)^+$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

Theorem 3. *For any $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ and $\mathfrak{M}-$filter $\mathcal{V}$ of $\mathcal{R}$, $\xi(\mathcal{V})$ is an $\mathfrak{M}-$filter of $\mathcal{R}'$.*

Proof. Let $\mathcal{V}$ be an $\mathfrak{M}-$filter of $\mathcal{R}$. Then, there exists an ideal $\mathcal{F}$ of $\mathcal{R}$ such that $\mathcal{V} = \mathfrak{M}(\mathcal{F})$. From Lemma 1, it follows that $\xi(\mathcal{F})$ forms an ideal of $\mathcal{R}'$. Now $\xi(\mathcal{V}) = \xi[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi(\mathcal{F})]$, since ξ is an $\mathfrak{M}-$homomorphism. Therefore $\xi(\mathcal{V})$ is an $\mathfrak{M}-$filter of $\mathcal{R}'$.

Definition 6. *Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ and $\mathcal{S}$ be a filter of $\mathcal{R}'$. A filter $\xi^{-1}(\mathcal{S})$ of $\mathcal{R}$ is refereed as the contraction of $\mathcal{S}$ with respect to ξ.*

Example 3. *Let $\mathcal{R} = \{0, 1, 2, 3\}$ and define $\vee$, $\wedge$ on $\mathcal{R}$ as follows:*

$\wedge$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	3	3	3

$\vee$	0	1	2	3
0	0	1	2	3
1	1	1	1	1
2	2	2	2	2
3	3	1	2	3

Clearly, $(\mathcal{R}, \vee, \wedge, 0)$ is an ADL. Let $A = \{\breve{0}, \breve{1}\}$ and $B = \{\bar{0}, \bar{1}, \bar{2}\}$ be two discrete ADLs. Then $(\mathcal{R}', \vee, \wedge, \tilde{0})$ is an ADL with respect to the point wise operations and $\tilde{0} = (\breve{0}, \bar{0})$. Define a mapping $f : \mathcal{R} \longrightarrow \mathcal{R}'$ as follows: $f(0) = \tilde{0}$, $f(1) = (\breve{1}, \bar{1})$, $f(2) = (\breve{1}, \bar{2})$, $f(3) = (\breve{1}, \bar{0})$. Clearly f is a homomorphism from $\mathcal{R}$ into $\mathcal{R}'$. Now consider the filter $\mathcal{S} = \{(\breve{1}, \bar{0}), (\breve{1}, \bar{1}), (\breve{1}, \bar{2})\}$ and the ideal $\mathcal{T} = \{(\breve{0}, \bar{0}), (\breve{0}, \bar{1}), (\breve{0}, \bar{2})\}$ in $\mathcal{R}'$. Clearly, $\mathfrak{M}(\mathcal{T}) = \mathcal{S}$. Hence $\mathcal{S}$ is an $\mathfrak{M}-$filter of $\mathcal{R}'$. But $f^{-1}(\mathcal{S}) = \{1, 2, 3\}$ is a filter of $\mathcal{R}$ but not an $\mathfrak{M}-$filter, because $3 \in f^{-1}(\mathcal{S})$ and $(3)^+ = \{1, 2\}$.

Next, we establish a sufficient condition under which the contraction of an $\mathfrak{M}-$filter remains an $\mathfrak{M}-$filter.

Theorem 4. *Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ with onto property and $Co - Ker\ \xi = \mathcal{M}_{max.elt}(\mathcal{R})$. If every ideal of $\mathcal{R}'$ contracts to an ideal of $\mathcal{R}$, then every $\mathfrak{M}-$filter of $\mathcal{R}'$ contracts to an $\mathfrak{M}-$filter of $\mathcal{R}$.*

Proof. Let $\mathcal{S}$ be an $\mathfrak{M}-$filter of $\mathcal{R}'$. Then $\mathcal{S} = \mathfrak{M}(\mathcal{F})$, for some ideal $\mathcal{F}$ of $\mathcal{R}'$. As per our hypothesis, we have that $\xi^{-1}(\mathcal{F})$ is an ideal of $\mathcal{R}$. We derive that $\xi^{-1}[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi^{-1}(\mathcal{F})]$. Let $\kappa \in \mathfrak{M}[\xi^{-1}(\mathcal{F})]$. Then $\kappa \vee \tau \in \mathcal{M}_{max.elt}(\mathcal{R})$, for some $\tau \in \xi^{-1}(\mathcal{F})$. That implies $\xi(\kappa \vee \tau) \in \mathcal{M}_{max.elt}(\mathcal{R}')$ and hence $\xi(\kappa) \vee \xi(\tau) \in \mathcal{M}_{max.elt}(\mathcal{R}')$. That implies $\xi(\kappa) \in \mathfrak{M}(\mathcal{F})$, since $\xi(\tau) \in \mathcal{F}$. That implies $\kappa \in \xi^{-1}[\mathfrak{M}(\mathcal{F})]$. Therefore $\mathfrak{M}[\xi^{-1}(\mathcal{F})] \subseteq \xi^{-1}[\mathfrak{M}(\mathcal{F})]$. Let $\kappa \in \xi^{-1}[\mathfrak{M}(\mathcal{F})]$. Then $\xi(\kappa) \in \mathfrak{M}(\mathcal{F})$. That implies $\xi(\kappa) \vee \omega \in \mathcal{M}_{max.elt}(\mathcal{R}')$, for some $\omega \in \mathcal{F}$. Since ω is an element of an ideal $\mathcal{F}$ of $\mathcal{R}'$ and ξ is an epimorphism, there exists an element $\nu \in \mathcal{R}$ such that $\xi(\nu) = \omega$. That implies $\xi(\kappa) \vee \xi(\nu) \in \mathcal{M}_{max.elt}(\mathcal{R}')$ and hence $\xi(\kappa \vee \nu) \in \mathcal{M}_{max.elt}(\mathcal{R}')$. Therefore $\kappa \vee \nu \in Co - ker\ \xi = \mathcal{M}_{max.elt}(\mathcal{R})$. So that $\kappa \in \mathfrak{M}[\xi^{-1}(\mathcal{F})]$. Hence $\xi^{-1}[\mathfrak{M}(\mathcal{F})] \subseteq \mathfrak{M}[\xi^{-1}(\mathcal{F})]$. Thus $\xi^{-1}[\mathfrak{M}(\mathcal{F})] = \mathfrak{M}[\xi^{-1}(\mathcal{F})]$.

Theorem 5. *Let $\xi \in Hom_{\mathcal{R}}(\mathcal{R}')$ and every $\mathfrak{M}-$filter of $\mathcal{R}'$ contracts to an $\mathfrak{M}-$filter of $\mathcal{R}$. If $\mathcal{R}'$ has dual dense elements, then Co-ker ξ is an $\mathfrak{M}-$filter of $\mathcal{R}$.*

Proof. Let E' be the set of all dual dense elements of $\mathcal{R}'$. Clearly E' is an ideal of $\mathcal{R}'$. Then $\mathfrak{M}(E') = \mathcal{M}_{max.elt}(\mathcal{R}')$. Clearly we have that $\mathcal{M}_{max.elt}(\mathcal{R}')$ is a filter of $\mathcal{R}'$ and hence it is an $\mathfrak{M}-$filter of $\mathcal{R}'$. Clearly, we have that $Co - ker\ \xi = \xi^{-1}(\mathcal{M}_{max.elt}(\mathcal{R}'))$.Therefore $Co - ker\ \xi$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

Definition 7. *For any filter $\mathcal{S}$ of $\mathcal{R}$, define $\mathcal{H}(\mathcal{S}) = \{\kappa \in \mathcal{R} \mid \eta \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R}),$ for some $\eta \in \mathcal{R} \setminus \mathcal{S}\}$*

The following lemma is straightforward to prove, so we omit the proof.

Lemma 3. *For any prime filter $\mathcal{S}$ of $\mathcal{R}$, $\mathcal{H}(\mathcal{S})$ is a filter of $\mathcal{R}$ contained in $\mathcal{S}$.*

Lemma 4. *For any prime filter $\mathcal{S}$ of $\mathcal{R}$, $\mathcal{H}(\mathcal{X}) = \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$.*

Proof. Let $\kappa \in \mathcal{H}(\mathcal{X})$. Then $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$, for some $\eta \in \mathcal{R} \setminus \mathcal{X}$. Since $\mathcal{R} \setminus \mathcal{X}$ is an ideal of $\mathcal{R}$, we get that $\kappa \in \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$ and hence $\mathcal{H}(\mathcal{X}) \subseteq \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$. Similarly, we get that $\mathfrak{M}(\mathcal{R} \setminus \mathcal{X}) \subseteq \mathcal{H}(\mathcal{X})$. Therefore $\mathcal{H}(\mathcal{X}) = \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$.

Theorem 6. *Let $\{\mathcal{T}_\alpha\}_{\alpha \in \triangle}$ be a class of $\mathfrak{M}-$filters of an ADL $\mathcal{R}$. Then $\bigcap\limits_{\alpha \in \triangle} \mathcal{T}_\alpha$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.*

Proof. For each $\alpha \in \triangle$, let $\mathcal{T}_\alpha = \mathfrak{M}(\mathcal{F}_\alpha)$ where $\mathcal{F}_\alpha$ is an ideal of $\mathcal{R}$. Then $\{\mathcal{F}_\alpha\}_{\alpha \in \triangle}$ will be an arbitrary family of ideals in $\mathcal{R}$. For each $\alpha \in \triangle$. Hence $\bigcap\limits_{\alpha \in \triangle} \mathcal{F}_\alpha$ is an ideal of $\mathcal{R}$. Thus we get $\bigcap\limits_{\alpha \in \triangle} \mathfrak{M}(\mathcal{F}_\alpha) = \mathfrak{M}\Big(\bigcap\limits_{\alpha \in \triangle} \mathcal{F}_\alpha\Big)$. Therefore $\bigcap\limits_{\alpha \in \triangle} \mathcal{T}_\alpha$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

Theorem 7. *Let $\mathcal{F}, \mathcal{G}$ be two ideals of an ADL $\mathcal{R}$. Then $\mathfrak{M}(\mathcal{F} \vee \mathcal{G})$ is the smallest $\mathfrak{M}-$filter containing both $\mathfrak{M}(\mathcal{F})$ and $\mathfrak{M}(\mathcal{G})$.*

Proof. Clearly, we get $\mathfrak{M}(\mathcal{F}) \subseteq \mathfrak{M}(\mathcal{F} \vee \mathcal{G})$ and $\mathfrak{M}(\mathcal{G}) \subseteq \mathfrak{M}(\mathcal{F} \vee \mathcal{G})$. Suppose $\mathfrak{M}(\mathcal{F}) \subseteq \mathfrak{M}(\mathcal{V})$ and $\mathfrak{M}(\mathcal{G}) \subseteq \mathfrak{M}(\mathcal{V})$, for some ideal $\mathcal{V}$ of $\mathcal{R}$. Let $\theta \in \mathfrak{M}(\mathcal{F} \vee \mathcal{G})$. Then there exist $\chi \in \mathcal{F}$ and $\upsilon \in \mathcal{G}$ such that $\theta \vee (\chi \vee \upsilon) \in \mathcal{M}_{max.elt}(\mathcal{R})$. Therefore $\theta \vee \chi \in \mathfrak{M}(\mathcal{G}) \subseteq \mathfrak{M}(\mathcal{V})$. There exists $\mu \in \mathcal{V}$ such that $\theta \vee \chi \vee \mu \in \mathcal{M}_{max.elt}(\mathcal{R})$. Since $\mu \vee \pi \in \mathcal{V}$, we get $\theta \in \mathfrak{M}(\mathcal{V})$. Therefore $\mathfrak{M}(\mathcal{F} \vee \mathcal{G})$ is the supremum of $\mathfrak{M}(\mathcal{F})$ and $\mathfrak{M}(\mathcal{G})$.

Corollary 1. *Let $\{\mathfrak{M}(\mathcal{F}_\alpha)\}_{\alpha \in \triangle}$ be a class of $\mathfrak{M}-$filters of an ADL $\mathcal{R}$, for each $\alpha \in \triangle$. Then $\bigsqcup\limits_{\alpha \in \triangle} \mathfrak{M}(\mathcal{F}_\alpha)$ is the smallest $\mathfrak{M}-$filter containing each $\mathfrak{M}(\mathcal{F}_\alpha)$.*

The set of all $\mathfrak{M}-$filters of $\mathcal{R}$ is denoted by $\mathfrak{F}_{\mathfrak{M}}(\mathcal{R})$

Theorem 8. *Let $\mathfrak{F}_{\mathfrak{M}}(\mathcal{R})$ be a sublattice of $\mathfrak{F}(\mathcal{R})$. If $\{\mathcal{T}_\alpha\}_{\alpha \in \triangle}$ be any class of $\mathfrak{M}-$filters of $\mathcal{R}$, then $\bigvee\limits_{\alpha \in \triangle} \mathcal{T}_\alpha$ is again an $\mathfrak{M}-$filter of $\mathcal{R}$.*

Proof. For each $\alpha \in \triangle$, let $\mathcal{T}_\alpha = \mathfrak{M}(\mathcal{F}_\alpha)$ where $\mathcal{F}_\alpha$ is an ideal of $\mathcal{R}$. Then $\{\mathcal{F}_\alpha\}_{\alpha\in\triangle}$ will be any class family of ideals of $\mathcal{R}$. Since $\mathcal{T}_\alpha = \mathfrak{M}(\mathcal{F}_\alpha) \subseteq \mathfrak{M}(\vee\mathcal{F}_\alpha)$ for each $\alpha \in \triangle$, we get $\bigvee \mathcal{T}_\alpha \subseteq \mathfrak{M}(\bigvee \mathcal{F}_\alpha)$. Let $\theta \in \mathfrak{M}(\bigvee \mathcal{F}_\alpha)$. Then there exists $\chi \in \bigvee \mathcal{F}_\alpha$ such that $\theta \vee \chi \in \mathcal{M}_{max.elt}(\mathcal{R})$. Then there exists a positive integer n such that $\chi = \chi_1 \vee \chi_2 \vee \cdots \vee \chi_n$ where $\chi_i \in \mathcal{F}_{\alpha_i}$. Thus we get $\theta\vee\chi \in \mathcal{M}_{max.elt}(\mathcal{R}) \Rightarrow \theta\vee(\chi_1\vee\chi_2\vee\cdots\vee\chi_n) \in \mathcal{M}_{max.elt}(\mathcal{R}) \Rightarrow$ $(\theta\vee\chi_1)\vee(\theta\vee\chi_2)\vee\cdots\vee(\theta\vee\chi_n) \in \mathcal{M}_{max.elt}(\mathcal{R}) \Rightarrow (\theta\vee\chi_1]\vee(\theta\vee\chi_2]\vee\cdots\vee(\theta\vee\chi_n] = \mathcal{R} \Rightarrow$ $\mathfrak{M}((\theta\vee\chi_1])\vee\mathfrak{M}((\theta\vee\chi_2])\vee\cdots\vee\mathfrak{M}((\theta\vee\chi_n]) = \mathcal{R} \Rightarrow (\theta\vee\chi_1)^+\vee(\theta\vee\chi_2)^+\vee\cdots\vee(\theta\vee\chi_n)^+ = \mathcal{R}$. Since $\theta \in \mathcal{R}$ we get $\theta \in (\theta\vee\chi_1)^+\vee(\theta\vee\chi_2)^+\vee\cdots\vee(\theta\vee\chi_n)^+$. Then there exists $\upsilon_i \in (\theta\vee\chi_i)^+$ for $i = 1, 2, \cdots, n$ such that $\theta = \upsilon_1\wedge\upsilon_2\wedge\cdots\wedge\upsilon_n$. Now, $\theta = \theta\vee\theta = \theta\vee(\upsilon_1\wedge\upsilon_2\wedge\cdots\wedge\upsilon_n) =$ $(\theta\vee\upsilon_1)\wedge(\theta\vee\upsilon_2)\wedge\cdots\wedge(\theta\vee\upsilon_n) \in (\chi_1)^+\vee(\chi_2)^+\vee\cdots\vee(\chi_n)^+ \subseteq \mathfrak{M}(\mathcal{F}_1)\vee\mathfrak{M}(\mathcal{F}_2)\vee\cdots\vee\mathfrak{M}(\mathcal{F}_n) =$ $\mathcal{T}_1 \vee \mathcal{T}_2 \vee \cdots \vee \mathcal{T}_n \subseteq \vee\mathcal{T}_\alpha$. That implies $\mathfrak{M}(\bigvee \mathcal{F}_\alpha) \subseteq \vee\mathcal{T}_\alpha$. Thus $\bigvee \mathcal{T}_\alpha$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

Theorem 9. *Consider the set $\mathfrak{F}_{\mathfrak{M}}(\mathcal{R})$ of all $\mathfrak{M}-$filters of $\mathcal{R}$ is a sublattice of $\mathfrak{F}(\mathcal{R})$. For any filter $\mathcal{T}$, there exists a unique $\mathfrak{M}-$filter contained in $\mathcal{T}$.*

Proof. Let $\mathcal{T}$ be any filter of $\mathcal{R}$. Consider $\mathfrak{M} = \{\mathcal{U} \in \mathfrak{F}_{\mathfrak{M}}(\mathcal{R}) \mid \mathcal{U} \subseteq \mathcal{T}\}$. Since $\mathcal{M}_{max.elt}(\mathcal{R})$ is the $\mathfrak{M}-$filter and $\mathcal{M}_{max.elt}(\mathcal{R}) \subseteq \mathcal{T}$, we get $\mathcal{M}_{max.elt}(\mathcal{R}) \in \mathfrak{M}$. Clearly, $\mathfrak{M}$ satisfies the hypothesis of Zorn's Lemma. Then $\mathfrak{M}$ has a maximal element let it be $\mathcal{N}$. It is enough to show that $\mathcal{N}$ is unique. Let $\mathcal{Y}$ be any maximal element of $\mathfrak{M}$ such that $\mathcal{N} \subseteq \mathcal{Y}$. Clearly, $\mathcal{N} \vee \mathcal{Y} \subseteq \mathcal{T}$. Hence $\mathcal{N} \vee \mathcal{Y} \in \mathfrak{M}$. Therefore $\mathcal{N} = \mathcal{N} \vee \mathcal{Y} = \mathcal{Y}$. Thus $\mathfrak{M}$ has a unique maximal element, which is the required $\mathfrak{M}-$filter contained in $\mathcal{T}$.

Definition 8. *A filter $\mathcal{S}$ of $\mathcal{R}$ is referred as co-dense if $\mathcal{S}^+ = \mathcal{M}_{max.elt}(\mathcal{R})$.*

Example 4. *Let $\mathcal{R} = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and define $\vee$, $\wedge$ on $\mathcal{R}$ as follows:*

$\wedge$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	1	2	3	4	5	6	7
3	0	3	3	3	0	0	3	0
4	0	4	5	0	4	5	7	7
5	0	4	5	0	4	5	7	7
6	0	6	6	3	7	7	6	7
7	0	7	7	0	7	7	7	7

$\vee$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	1	2	3	1	2	6	6
4	4	1	1	1	4	4	1	4
5	5	2	2	2	5	5	2	5
6	6	1	2	6	1	2	6	6
7	7	1	2	6	4	5	6	7

Then $(\mathcal{R}, \vee, \wedge)$ is an ADL. Clearly a filter $\mathcal{S} = \{1, 2, 4, 5, 6, 7\}$ is a co-dense filter of $\mathcal{R}$.

Lemma 5. *Any non co-dense prime filter of an ADL is an $\mathfrak{M}-$filter.*

Proof. Let $\mathcal{S}$ be any non co-dense prime filter of $\mathcal{R}$. Then there exists an element $\kappa \notin \mathcal{M}_{max.elt}(\mathcal{R})$ such that $\kappa \in \mathcal{S}^+$. That implies $[\kappa) \subseteq \mathcal{S}^+$ and hence $\mathcal{S}^{++} \subseteq [\kappa)^+$. That implies $\mathcal{S} \subseteq [\kappa)^+$, since $\mathcal{S} \subseteq \mathcal{S}^{++}$. Let $\tau \in [\kappa)^+$. Then $\tau \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$ and hence $\tau \vee \kappa \in \mathcal{S}$. Since $\mathcal{S}$ is prime, we get that either $\kappa \in \mathcal{S}$ or $\tau \in \mathcal{S}$. Suppose $\kappa \in \mathcal{S}$. Since

$[\kappa) \subseteq \mathcal{S}^+$, we get easily that $[\kappa) \cap \mathcal{S} = \mathcal{M}_{max.elt}(\mathcal{R})$. That implies $\kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$, which is a contradiction to $\kappa \notin \mathcal{M}_{max.elt}(\mathcal{R})$. Which gives $\tau \in \mathcal{S}$ and hence $[\kappa)^+ \subseteq \mathcal{S}$. Thus $\mathcal{S} = [\kappa)^+ = \mathfrak{M}((\kappa])$. Therefore $\mathcal{S}$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

We will now present the definition of an $E-$complemented ADL.

Definition 9. *An ADL $\mathcal{R}$ is classified as an $E-$complemented ADL if, for every $\kappa \in \mathcal{R}$, there exists $\eta \in \mathcal{R}$ such that both $\kappa \wedge \eta \in E$ and $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$ hold true.*

Example 5. *Consider two discrete ADLs $\mathcal{A} = \{0, \tau\}$ and $\mathcal{B} = \{0, \nu_1, \nu_2\}$. We have $\mathcal{A} \times \mathcal{B} = \{(0,0), (0,\nu_1), (0,\nu_2), (\tau,0), (\tau,\nu_1), (\tau,\nu_2)\}$ let it be $\mathcal{R}$. Define $\vee$ and $\wedge$ on $\mathcal{R}$ under point-wise:*

$\vee$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)
$(0,0)$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)
$(0,\nu_1)$	$(0,\nu_1)$	$(0,\nu_1)$	$(0,\nu_1)$	(τ,ν_1)	(τ,ν_1)	(τ,ν_1)
$(0,\nu_2)$	$(0,\nu_2)$	$(0,\nu_2)$	$(0,\nu_2)$	(τ,ν_2)	(τ,ν_2)	(τ,ν_2)
$(\tau,0)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)
(τ,ν_1)	(τ,ν_1)	(τ,ν_1)	(τ,ν_1)	(τ,ν_1)	(τ,ν_1)	(τ,ν_1)
(τ,ν_2)	(τ,ν_2)	(τ,ν_2)	(τ,ν_2)	(τ,ν_2)	(τ,ν_2)	(τ,ν_2)

$\wedge$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)
$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$(0,\nu_1)$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$
$(0,\nu_2)$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$
$(\tau,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(\tau,0)$	$(\tau,0)$	$(\tau,0)$
(τ,ν_1)	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)
(τ,ν_2)	$(0,0)$	$(0,\nu_1)$	$(0,\nu_2)$	$(\tau,0)$	(τ,ν_1)	(τ,ν_2)

Clearly, $(\mathcal{R}, \vee, \wedge, 0')$ is an ADL, where $0' = (0,0)$.
We have that

(i) $(0,0)^{++} = \{(\tau,\nu_1), (\tau,\nu_2)\}^+ = \mathcal{R} = (\tau,\nu_1)^+$

(ii) $(0,\nu_1)^{++} = \{(\tau,0), (\tau,\nu_1), (\tau,\nu_2)\}^+ = \{(0,\nu_1), (0,\nu_2), (\tau,\nu_1), (\tau,\nu_2)\} = (\tau,0)^+$

(iii) $(0,\nu_2)^{++} = \{(\tau,0), (\tau,\nu_1), (\tau,\nu_2)\}^+ = \{(0,\nu_1), (0,\nu_2), (\tau,\nu_1), (\tau,\nu_2)\} = (\tau,0)^+$

(iv) $(\tau,0)^{++} = \{(0,\nu_1), (0,\nu_2), (\tau,\nu_1), (\tau,\nu_2)\}^+ = \{(\tau,0), (\tau,\nu_1), (\tau,\nu_2)\} = (0,\nu_1)^+$

(v) $(\tau,\nu_1)^{++} = \mathcal{R}^+ = \{(\tau,\nu_1), (\tau,\nu_2)\} = (0,0)^+$

(vi) $(\tau,\nu_2)^{++} = \mathcal{R}^+ = \{(\tau,\nu_1), (\tau,\nu_2)\} = (0,0)^+$

Thus $(\mathcal{R}, \vee, \wedge, 0')$ is an $E-$complemented ADL.

Lemma 6. *In an $E-$complemented ADL $\mathcal{R}$, every prime filter $\mathcal{X}$ with $\mathcal{X} \cap E = \emptyset$ is an $\mathfrak{M}-$filter.*

Proof. Let $\mathcal{X}$ be any prime filter of $\mathcal{R}$ with $\mathcal{X} \cap E = \emptyset$. We have that $\mathfrak{M}(\mathcal{R} \setminus \mathcal{X}) = \mathcal{H}(\mathcal{X}) \subseteq \mathcal{X}$. Let $\kappa \in \mathcal{X}$. As per our hypothesis, there is $\eta \in \mathcal{R}$ such that $\kappa \wedge \eta \in E$ and $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. That implies $\kappa \wedge \eta \notin \mathcal{X}$ and hence $\eta \notin \mathcal{X}$. Since $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$ and $\eta \notin \mathcal{X}$, we get that $\kappa \in \mathcal{H}(\mathcal{X}) = \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$. Which gives $\mathcal{X} \subseteq \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$. Therefore $\mathcal{X} = \mathfrak{M}(\mathcal{R} \setminus \mathcal{X})$. Thus $\mathcal{X}$ is an $\mathfrak{M}-$filter of $\mathcal{R}$.

Theorem 10. *For any proper $\mathfrak{M}-$filter $\mathcal{S}$ of $\mathcal{R}$, $\mathcal{S}$ is prime if and only if $\mathcal{S}$ contains a prime filter.*

Proof. Assume that $\mathcal{S}$ contains a prime filter, say $\mathcal{X}$. Since $\mathcal{S}$ is an $\mathfrak{M}-$filter of $\mathcal{R}$, we have that $\mathcal{S} = \mathfrak{M}(\mathcal{F})$, for some ideal $\mathcal{F}$ of $\mathcal{R}$. We prove that $\mathcal{S}$ is prime filter of $\mathcal{R}$. Let $\kappa, \eta \in \mathcal{R}$ with $\kappa \vee \eta \in \mathcal{S}$. Suppose that $\kappa \notin \mathcal{S}$ and $\eta \notin \mathcal{S}$. Then $\kappa \notin \mathcal{X}$ and $\eta \notin \mathcal{X}$. Since $\mathcal{X}$ is prime, we get $\kappa \vee \eta \notin \mathcal{X}$. That implies $(\kappa \vee \eta)^+ \subseteq \mathcal{X} \subseteq \mathcal{S} = \mathfrak{M}(\mathcal{F})$. Suppose $\kappa \vee \eta \in \mathfrak{M}(\mathcal{F})$. Then $(\kappa \vee \eta) \vee \omega \in \mathcal{M}_{max.elt}(\mathcal{R})$, for some $\omega \in \mathcal{F}$. It gives $\omega \in (\kappa \vee \eta)^+ \subseteq \mathcal{X} \subseteq \mathcal{S} = \mathfrak{M}(\mathcal{F})$. Therefore $\omega \in \mathcal{F} \cap \mathfrak{M}(\mathcal{F})$ and hence $\mathcal{F} \cap \mathfrak{M}(\mathcal{F}) \neq \emptyset$. Thus $\mathcal{S} = \mathcal{F} = \mathfrak{M}(\mathcal{F}) = \mathcal{R}$, which leads a contradiction. Therefore $\kappa \vee \eta \notin \mathfrak{M}(\mathcal{F})$ and hence $\kappa \vee \eta \notin \mathcal{S}$, we get a contradiction. Which leads either $\kappa \in \mathcal{S}$ or $\eta \in \mathcal{S}$. Thus $\mathcal{S}$ is prime.

Theorem 11. *In ADL $\mathcal{R}$, every prime $\mathfrak{M}-$filter is minimal.*

Proof. Let $\mathcal{X}$ be a prime $\mathfrak{M}-$filter of $\mathcal{R}$. Then, there exists an ideal $\mathcal{F}$ of $\mathcal{R}$ such that $\mathcal{X} = \mathfrak{M}(\mathcal{F})$. For an element $\kappa \in \mathcal{X}$, there is an element $\eta \in \mathcal{F}$ satisfying $\kappa \vee \eta \in \mathcal{M}_{\text{max. elt}}(\mathcal{R})$. Assuming that $\eta \in \mathcal{X}$, it follows that $\eta \in \mathfrak{M}(\mathcal{F})$. This leads to the conclusion that $\mathcal{F} \cap \mathfrak{M}(\mathcal{F}) \neq \emptyset$, resulting in the equality $\mathcal{X} = \mathfrak{M}(\mathcal{F}) = \mathcal{F} = \mathcal{R}$, which gives a contradiction. Thus, we conclude that $\eta \notin \mathcal{X}$. Therefore, $\mathcal{X}$ is minimal.

Definition 10. *A filter $\mathcal{S}$ is referred to as annihilator of $\mathcal{R}$ if it satisfies the condition $\mathcal{S} = \mathcal{S}^{++}$.*

Example 6. *From the example 4, consider a filter $\mathcal{T} = \{1, 2, 3, 6\}$. it is clear that $\mathcal{T}$ is an annihilator filter of $\mathcal{R}$, because $\mathcal{T} = \mathcal{T}^{++}$.*

Theorem 12. *In an $E-$complemented ADL $\mathcal{R}$, the statements listed below are equivalent:*

(i) Every $\mathfrak{M}-$filter is an annihilator filter

(ii) Every minimal prime filter is an annihilator filter

(iii) Every prime $\mathfrak{M}-$filter is of the form $(\kappa)^{++}$, for some $\kappa \in \mathcal{R}$

(iv) Every $\mathfrak{M}-$filter is of the form $(\kappa)^{++}$, for some $\kappa \in \mathcal{R}$

(v) Every minimal prime filter is non co-dense.

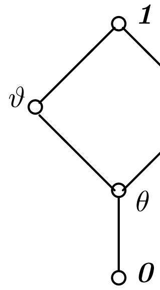
Proof. $1 \Rightarrow 2$: Obvious
$2 \Rightarrow 3$: Assume condition 2. Consider any prime $\mathfrak{M}-$filter $\mathcal{X}$ of $\mathcal{R}$. It follows that $\mathcal{X}$ qualifies as a minimal prime filter of $\mathcal{R}$. By 2, we get $\mathcal{X} = (\kappa)^+$, for some non maximal element κ of $\mathcal{X}^+$. Since $\mathcal{R}$ is an $E-$complemented ADL, there is η of $\mathcal{R}$ satisfying $\kappa \wedge \eta \in E$ and $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. It gives $\eta \in (\kappa)^+ = \mathcal{X}$. Therefore $(\eta)^{++} \subseteq P^{++} = \mathcal{X} = (\kappa)^+$ and it gives $(\eta)^{++} \subseteq (\kappa)^+$. Let $\tau \in (\kappa)^+$. Then $\tau \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$. Which leads $\tau \in (\eta)^{++}$, because $\kappa \in (\eta)^+$. Therefore $(\kappa)^+ \subseteq (\eta)^{++}$. Hence $(\kappa)^+ = (\eta)^{++} = \mathcal{X}$.
$3 \Rightarrow 4$: Assume condition 3. Let $\mathcal{S}$ be any $\mathfrak{M}-$filter of $\mathcal{R}$. Suppose that $\mathcal{S} \neq (\kappa)^{++}$, for all $\kappa \in \mathcal{R}$. Consider $\mathfrak{F} = \{\mathcal{T} \mid \mathcal{T} \text{ is an } \mathfrak{M}-\text{filter and } \mathcal{T} \neq (\kappa)^{++}, \text{ for all } \kappa \in \mathcal{R}\}$. Clearly, $\mathcal{S} \in \mathfrak{F}$ and hence $\mathfrak{F} \neq \emptyset$. By the Zorn's Lemma, $\mathfrak{F}$ has a maximal element say $\mathcal{P}$. Clearly, $\mathcal{P}$ is a prime filter and $M \neq (\kappa)^{++}$, for all $\kappa \in \mathcal{R}$, this leads a contradiction. Hence $\mathcal{S} = (\kappa)^{++}$, for some $\kappa \in \mathcal{R}$.
$4 \Rightarrow 5$: Assume condition 4. Let $\mathcal{X}$ be a minimal prime filter of $\mathcal{R}$. Then $\mathcal{X}$ is an $\mathfrak{M}-$filter of $\mathcal{R}$. By 4, we get that $\mathcal{X} = (\kappa)^{++}$, for some $\kappa \in \mathcal{R}$. Clearly $\kappa \in \mathcal{X}$. As P is minimal and $\kappa \in \mathcal{X}$, there is $\eta \notin \mathcal{X}$ satisfying the property that $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. It gives $\eta \in (\kappa)^+ = \mathcal{X}^+$ and hence $\eta \in \mathcal{X}^+$. We derive that $\mathcal{X}^+ \neq \mathcal{M}_{max.elt}(\mathcal{R})$. If $\mathcal{X}^+ = \mathcal{M}_{max.elt}(\mathcal{R})$ then $\eta \in \mathcal{M}_{max.elt}(\mathcal{R})$, get a contradiction to $\eta \notin \mathcal{X}$. Hence $\mathcal{X}^+ \neq \mathcal{M}_{max.elt}(\mathcal{R})$. Thus $\mathcal{X}$ is non co-dense.
$5 \Rightarrow 1$: Assume condition 5. Let $\mathcal{S}$ be an $\mathfrak{M}-$filter of $\mathcal{R}$. Then $\mathcal{S} = \mathfrak{M}(\mathcal{F})$, for some ideal $\mathcal{F}$ of $\mathcal{R}$. Now we prove that $\mathcal{S} = \mathcal{S}^{++}$. Clearly, we have that $\mathcal{S} \subseteq \mathcal{S}^{++}$. Let $\nu \in \mathcal{S}^{++}$. We show that $(\mathcal{S} \vee \mathcal{S}^+) \cap E \neq \emptyset$. Suppose $(\mathcal{S} \vee \mathcal{S}^+) \cap E = \emptyset$. As E is an ideal of $\mathcal{R}$, there is a prime filter $\mathcal{X}$ of $\mathcal{R}$ satisfies $\mathcal{S} \vee \mathcal{S}^+ \subseteq \mathcal{X}$ and $\mathcal{X} \cap E = \emptyset$. Let $\kappa \in \mathcal{X}$. Since $\mathcal{R}$ is an $E-$complemented ADL, there is $\eta \in \mathcal{R}$ such that $\kappa \wedge \eta \in E$ and $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. Which gives $\kappa \wedge \eta \notin \mathcal{X}$ and hence $\eta \notin \mathcal{X}$. Therefore $\mathcal{X}$ is a minimal prime filter of $\mathcal{R}$. By our assumption we have $\mathcal{X}$ is non co-dense. Since $(\mathcal{S} \vee \mathcal{S}^+) \subseteq \mathcal{X}$, we have that $\mathcal{X}^+ \subseteq (\mathcal{S} \vee \mathcal{S}^+)^+ = \mathcal{S}^+ \cap \mathcal{S}^{++} = \mathcal{M}_{max.elt}(\mathcal{R})$. That implies $\mathcal{X}^+ \subseteq \mathcal{M}_{max.elt}(\mathcal{R})$ and hence $\mathcal{X}^+ = \mathcal{M}_{max.elt}(\mathcal{R})$, it gives a contradiction to $\mathcal{X}$ is a non co-dense. Therefore $(\mathcal{S} \vee \mathcal{S}^+) \cap E \neq \emptyset$. Now choose $\mu \in (\mathcal{S} \vee \mathcal{S}^+) \cap E$. Then $\mu \in \mathcal{S} \vee \mathcal{S}^+$ and $\mu \in E$. That implies $\mu = \delta \wedge \sigma$, for some $\delta \in \mathcal{S}$, $\sigma \in \mathcal{S}^+$ and $(\mu)^+ = \mathcal{M}_{max.elt}(\mathcal{R})$. Now $(\delta)^+ \cap (\sigma)^+ = (\delta \wedge \sigma)^+ = (\mu)^+ = \mathcal{M}_{max.elt}(\mathcal{R})$. Since $\delta \in \mathcal{S} = \mathfrak{M}(\mathcal{F})$, there is $\tau \in \mathcal{F}$ and it satisfying $\delta \vee \tau \in \mathcal{M}_{max.elt}(\mathcal{R})$. Since $\nu \in \mathcal{S}^{++}$ and $\sigma \in \mathcal{S}^+$, we get that $\nu \vee \sigma \in \mathcal{M}_{max.elt}(\mathcal{R})$. Which gives $\nu \in (\sigma)^+$. Since $(\delta)^+ \cap (\sigma)^+ = \mathcal{M}_{max.elt}(\mathcal{R})$, we get that $(\sigma)^+ \subseteq (\delta)^{++} \subseteq (\tau)^+ \subseteq \mathfrak{M}(\mathcal{F}) = \mathcal{S}$. Therefore $\nu \in \mathcal{S}$ and hence $\mathcal{S}^{++} \subseteq \mathcal{S}$. Thus $\mathcal{S} = \mathcal{S}^{++}$. Therefore $\mathcal{S}$ is an annihilator filter of $\mathcal{R}$.

It is straightforward to demonstrate that the set $(\mathfrak{M}_{\mathcal{F}}(\mathcal{R}), \sqcup, \cap)$ of all $\mathfrak{M}-$filters of $\mathcal{R}$ forms a distributive lattice, where $\mathfrak{M}(\mathcal{F}) \cap \mathfrak{M}(\mathcal{G}) = \mathfrak{M}(\mathcal{F} \cap \mathcal{G})$ and $\mathfrak{M}(\mathcal{F}) \sqcup \mathfrak{M}(\mathcal{G}) = \mathfrak{M}(\mathcal{F} \vee \mathcal{G})$, for some ideals $\mathcal{F}$, $\mathcal{G}$ of $\mathcal{R}$. Let us recall that two filters $\mathcal{S}, \mathcal{T}$ of an ADL $\mathcal{R}$ are comaximal if $\mathcal{S} \vee \mathcal{T} = \mathcal{R}$. We now introduce $\sqcup-$comaximality of $\mathfrak{M}-$filters of an ADL.

Definition 11. *Two $\mathfrak{M}-$filters $\mathcal{S}, \mathcal{T}$ of an ADL $\mathcal{R}$ are referred as $\sqcup-$comaximal if $\mathcal{S} \sqcup \mathcal{T} = \mathcal{R}$.*

If two $\mathfrak{M}-$filters are comaximal, then they are necessarily $\sqcup-$comaximal. However, the opposite is not true. Any two comaximal $\mathfrak{M}-$filters are $\sqcup-$comaximal. But the converse is not true. This can be demonstrated with the following example.

Example 7. *Let $\mathcal{R} = \{0, \theta, \vartheta, \sigma, 1\}$ represent a distributive lattice, with its Hasse diagram shown below*



Consider the filters $\mathcal{S} = \{\vartheta, 1\}$ and $\mathcal{T} = \{\sigma, 1\}$. Clearly $\mathcal{F} = \{0, \theta, \vartheta\}$ and $\mathcal{G} = \{0, \theta, \sigma\}$ are ideals in $\mathcal{R}$. It is straightforward to derive that $\mathfrak{M}(\mathcal{F}) = \mathcal{T}$ and $\mathfrak{M}(\mathcal{G}) = \mathcal{S}$. Consequently, $\mathcal{S}$ and $\mathcal{T}$ are two distinct $\mathfrak{M}-$filters of $\mathcal{R}$. Now $\mathcal{S} \sqcup \mathcal{T} = \mathfrak{M}(\mathcal{G}) \sqcup \mathfrak{M}(\mathcal{F}) = \mathfrak{M}(\mathcal{F} \vee \mathcal{G}) = \mathfrak{M}(\mathcal{R}) = \mathcal{R}$. This implies that $\mathcal{S}$ and $\mathcal{T}$ are $\sqcup-$comaximal. However, since $\mathcal{S} \vee \mathcal{T} = \{\theta, \vartheta, \sigma, 1\} \neq \mathcal{R}$, it follows that $\mathcal{S}$ and $\mathcal{T}$ are not comaximal in $\mathcal{R}$.

Lemma 7. *For every $\mu, \pi \in \mathcal{R}$ with $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$, $(\kappa)^+$ and $(\eta)^+$ are $\sqcup-$comaximal.*

Proof. Let $\kappa, \eta \in \mathcal{R}$ with $\kappa \vee \eta \in \mathcal{M}_{ma\kappa.elt}(\mathcal{R})$. Clearly, we have that $(\kappa)^+ = \mathfrak{M}((\kappa])$ and $(\eta)^+ = \mathfrak{M}((\eta])$. Now $(\kappa)^+ \sqcup (\eta)^+ = \mathfrak{M}((\kappa]) \sqcup \mathfrak{M}((\eta]) = \mathfrak{M}((\kappa] \vee (\eta]) = \mathfrak{M}((\kappa \vee \eta]) = \mathfrak{M}(\mathcal{R}) = \mathcal{R}$. Therefore $(\kappa)^+$ and $(\eta)^+$ are $\sqcup-$comaximal.

Theorem 13. *Any two distinct prime $\mathfrak{M}-$filters of $\mathcal{R}$ are necessarily $\sqcup-$comaximal.*

Proof. Let $\mathcal{X}$ and $\mathcal{Y}$ be two distinct prime $\mathfrak{M}-$filters of $\mathcal{R}$. Consequently, $\mathcal{X}$ and $\mathcal{Y}$ are minimal. Choose $\sigma \in \mathcal{Y} \setminus \mathcal{X}$ and $\tau \in \mathcal{X} \setminus \mathcal{Y}$. Therefore $\tau \vee \kappa,\ \sigma \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$, for some $\kappa \notin \mathcal{X}$ and $\eta \notin \mathcal{Y}$. Which gives $\sigma \vee \kappa \notin \mathcal{X}$ and $\eta \vee \tau \notin \mathcal{Y}$. Hence $(\sigma \vee \kappa)^+ \subseteq \mathcal{X}$ and $(\eta \vee \tau)^+ \subseteq \mathcal{Y}$. Clearly we have that $(\sigma \vee \kappa) \vee (\tau \vee \eta) \in \mathcal{M}_{max.elt}(\mathcal{R})$. By above result, we get that $(\sigma \vee \kappa)^+ \sqcup (\tau \vee \eta)^+ = \mathcal{R}$ and hence $\mathcal{X} \sqcup \mathcal{Y} = \mathcal{R}$. Thus $\mathcal{X}$ and $\mathcal{Y}$ are $\sqcup-$comaximal.

Definition 12. *For any filter $\mathcal{S}$ of $\mathcal{R}$ and $\kappa \in \mathcal{R}$, define $[\kappa]_{\mathcal{S}} = \{\tau \in \mathcal{S} \mid \tau \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})\}$*

Lemma 8. *For any filter $\mathcal{S}$ of $\mathcal{R}$ and $\kappa \in \mathcal{R}$, we have the the subsequent conditions*

(i) $[\kappa]_{\mathcal{S}} = \mathcal{S} \cap (\kappa)^+$ *is a filter in* $\mathcal{S}$

(ii) *if $\mathcal{S}$ is an $\mathfrak{M}-$filter then $[\kappa]_{\mathcal{S}}$ is an $\mathfrak{M}-$filter.*

Proof. 1. Clearly, $[\kappa]_{\mathcal{S}} \neq \emptyset$. Let $\theta, \vartheta \in [\kappa]_{\mathcal{S}}$. Then $\theta \vee \kappa,\ \vartheta \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$ and hence $(\theta \wedge \vartheta) \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$ Therefore $\theta \wedge \vartheta \in [\kappa]_{\mathcal{S}}$. Let $\theta \in [\kappa]_{\mathcal{S}}$. Then $\theta \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$. For any $\sigma \in \mathcal{S}$, we have $\sigma \vee \theta \vee \kappa \in \mathcal{M}_{max.elt}(\mathcal{R})$. Therefore $\sigma \vee \theta \in [\kappa]_{\mathcal{S}}$. Thus $[\kappa]_{\mathcal{S}}$ is a filter
2. Let $\mathcal{S}$ is an $\mathfrak{M}-$filter of $\mathcal{R}$. There there exists an ideal $\mathcal{F}$ of $\mathcal{R}$ such that $\mathcal{S} = \mathfrak{M}(\mathcal{F})$. Clearly we have $(\kappa)^+ = \mathfrak{M}((\kappa])$. Therefore $(\kappa)^+ \cap \mathcal{S} = \mathfrak{M}(\mathcal{F}) \cap \mathfrak{M}((\kappa]) = \mathfrak{M}(\mathcal{F} \cap (\kappa])$. Hence $[\kappa]_{\mathcal{S}}$ is an $\mathfrak{M}-$filter.

Theorem 14. *Let $\mathcal{S}$ be an $\mathfrak{M}-$filter of an ADL $\mathcal{R}$ with maximal elements. If for any $\kappa, \eta \in \mathcal{R}$ with $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$, then $[\kappa]_{\mathcal{S}}$ and $[\eta]_{\mathcal{S}}$ are $\sqcup-$comaximal in $\mathcal{S}$.*

Proof. Let $\kappa, \eta \in \mathcal{R}$ with $\kappa \vee \eta \in \mathcal{M}_{max.elt}(\mathcal{R})$. By Lemma-7, we have that $(\kappa)^+ \sqcup (\eta)^+ = \mathcal{R}$. Now, $\mathcal{S} = \mathcal{S} \cap \mathcal{R} = \mathcal{S} \cap ((\kappa)^+ \sqcup (\eta)^+) = (\mathcal{S} \cap (\kappa)^+) \sqcup (\mathcal{S} \cap (\eta)^+) = [\kappa]_{\mathcal{S}} \sqcup [\eta]_{\mathcal{S}}$. Therefore $[\kappa]_{\mathcal{S}} \sqcup [\eta]_{\mathcal{S}} = \mathcal{S}$ and hence $[\kappa]_{\mathcal{S}}$ and $[\eta]_{\mathcal{S}}$ are $\sqcup-$comaximal in $\mathcal{S}$.

Conclusions

In this paper, we introduced the notion of $\mathfrak{M}-$homomorphism in the context of an Almost Distributive Lattice(ADL). We have derived a sufficient condition for a homomorphism to become an $\mathfrak{M}-$homomorphism. We have proved that the image and the inverse image of an $\mathfrak{M}-$filter of an ADL under an $\mathfrak{M}-$homomorphism again $\mathfrak{M}-$filters. We have derived some sufficient conditions for a prime filter of an almost distributive lattice to be an $\mathfrak{M}-$filter. We have established a set of equivalent conditions for every $\mathfrak{M}-$filter to be an annihilator filter. We have obtained an equivalency between prime $\mathfrak{M}-$filters and minimal prime filters of an ADL. We have proved that any two distinct prime $\mathfrak{M}-$filters of an ADL are comaximal.

Acknowledgements

The authors wish to thank the anonymous reviewers for their valuable suggestions.

Conflicts of interest or competing interests

The authors declare that they have no conflicts of interest.

Informed Consent

The authors are fully aware and satisfied with the contents of the article.

References

[1] U.M. Swamy and G.C. Rao. Almost distributive lattices. *J. Aust. Math. Soc. (Series A*, 31:77–91, 1981.

[2] N. Rafi and Ravi Kumar Bandaru. μ-filters of almost distributive lattices. *Chamchuri J. Math.*, 10:53–65, 2018.

[3] M. Sambasiva Rao and G.C. Rao. O-homomorphisms of almost distributive lattices. *Chamchuri Journal of Mathematics*, 4:13 – 21, 2012.

[4] M. Sambasiva Rao . O-filters and minimal prime filters of lattices. *Southeast Asian Bulletin of Mathematics*, 39:113 – 121, 2015.

[5] Ravi Kumar Bandaru N. Rafi and G.C. Rao. Prime O-ideals in almost distributive lattices. *Southeast Asian Bulletin of Mathematics*, 39:841–850, 2015.

[6] G.C. Rao. Almost distributive lattices. doctoral thesis, dept. of mathematics, andhra university, visakhapatnam. 1980.

[7] G Birkhoff. *Lattice theory.* Amer. Math. Soc. Collequium Pub, 1967.

[8] G. Gratzer. *General Lattice Theory.* Academic Press, New York, Sanfransisco, 1978.

[9] N. Rafi. A special case of complemented almost distributive lattices. *International Journal of Scientific and Innovative Mathematical Research*, 3(12):17–23, 2015.

[10] N. Rafi. On m-filters of almost distributive lattices. *International Conference on Electrical, Electronics, and Optimization Techniques (ICEEOT) - 2016*, pages 4518–4521, 2016.