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A Fractional Model of Abalone Growth Using Adomian Decomposition Method

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Abstract. This study is a modification of the McKendrick equation into a growth model with fractional order to predict the abalone length growth. We have shown that the model is a special form of Taylor's series after it was analysed using Adomian decomposition method and Caputo fractional derivative. By simulating the series with some fractional orders, the results indicate that the greater the fractional order of the model, the series values generated are greater as well. Moreover, the series that is close to the real data is the one with a fractional order $\beta = 0.5$. Therefore, the growth model with a fractional order provides more accuracy than a classical integer order.

2020 Mathematics Subject Classifications: 26A33, 34A08, 92D25, 65M70

Key Words and Phrases: Fractional calculus, modified model, McKendrick equation, Adomian decomposition method, Taylor's series

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1. Introduction

Abalone (Haliotis asinina) is one of the key ethno-fauna of West Nusa Tenggara Province and nationally as a marine commodity for export. The presence of this mollusk has played an important role in the coastal community's economy, not only for local consumption or sale in local markets but also for export to several countries in Asia, Europe, and the United States. Abalone harvesting in the wild has been excessive, leading to a drastic decline in its population, which could threaten the sustainability of the species. Therefore, the abalone cultivation is being widely developed on an industrial scale, but the limited availability of seed stock and the slow growth rate of abalone impose

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great challenges[1]. In this study, the growth of abalone is described mathematically since mathematical modelling is an essential tools in understanding the phenomena. Thomas Robert Malthus in 1798 declared that population in the world would increase over time exponentially and exceed its resources, which was expressed mathematically as:

$$\frac{dP}{dt} = rP, \tag{1}$$

where the initial condition $P(0) = P_0$. By solving the equation (1), we obtain

$$P(t) = P_0 e^{rt}, \tag{2}$$

with $P(t)$ representing the number of population at time t, P_0 is the initial number of population, and r denotes the intrinsic growth rate. The model was applied to predict the number of population in Taraba state, and it showed more realistic results than the logistic model demonstrating a higher R^2 value [2]. Exponential growth model also provide approximation the growth of carcinoma and melanoma properly [3]. In addition, the exponential model was used to describe the dynamical cell growth [4] and in estimating electricity demand in Cameroon [5].

Furthermore, in 1926, McKendrick introduced the growth model in partial differential equation with age structure in one population. Let $w(s,t)$ be the density of population with age s at time t, that is

$$\frac{\partial w(s,t)}{\partial t} + \frac{\partial w(s,t)}{\partial s} = -\mu(s,t)w(s,t), \tag{3}$$

subject to the initial condition

$$w(s,0) = v_0(s), \text{ and w(0,t)} = \int_{s_1}^{s_2} m(s,t)w(s,t)ds, \tag{4}$$

where the value of $\mu(s,t)$ indicates the death process and $m(s,t)$ represents fertility, which are time-dependent. The model is well-known as McKendrick equation [6]. The model was developed by Gourley with involving non-linear effects, that is, the model was combined with competition model to study larval competition based on age-structured [7]. In addition, the equation was expanded into McKendrick-Von Foerster equation with higher order numerical scheme for singular mortality cases [8].

On the other hand, the McKendrick equation has been utilized in various fields, including the determination of optimal hiring and retirement ages for employees [9], developing goodwill model for estimating the duration of a product's life cycle [10], and modeling the transmission of Mycobacterium tuberculosis in Japan, influenced by visitor numbers [11]. Similarly, the model was used to modify SIR and SEIR model by involving age structure in each the compartment [12]. The theory of McKendrick also was applied in ecology problems including zooplankton, insect, mosquito, and tick populations [13]-[14]. Hence, the McKendrick model has become a foundation of partial differential equations models. Moreover, Arora (2023) stated that the equation (3) is a pure growth equation by assuming the $\mu = 0$. The equation was analysed using ADM which was obtaining the exponential function as a solution [15].

The geometric and physical interpretation of the models with classical integer-order have well-defined, making them highly useful for solving practical problems across various scientific disciplines. However, the situation is different for fractional-order integration and differentiation, a field that has been rapidly expanding in both theory and real-world applications. Since the introduction of differentiation and integration of arbitrary (non-integer) order, no widely accepted geometric or physical interpretation of these operations existed for over 30 decades. This problem is widely recognized and identified as an open issue [16].

Therefore, this study proposes a new fractional growth model as a modification of McKendrick equation to describe the body length growth of abalone. The results are not only presented in mathematical form but also graphically, so that the meaning of the fractional order derivative in terms of the growth model can also be understood more clearly. In addition, fractional partial differential equations are widely used to describe real-world problems and have proven to be an effective tool for solving various issues in physics and applied mathematics. There are some popular methods for solving fractional partial differential equations such as variational iteration method, generalized differential transform method, homotopy perturbation method, and Adomian decomposition method [17]. In this study the new model is analysed using ADM to obtain a particular form of Taylor's series for abalone length growth. The method was first introduced by George Adomian who designed the solution to both linear and nonlinear differential equations, including systems of these equations [18]. In addition, the ADM provides an efficient approach for obtaining analytical solutions to a broad and complex set of dynamic systems that represent real-world physical problems [19]. In particular, it offers a well-suited solution for fractional mathematical modelling related to physical issues [20] and [21]. Also, the method successfully provided in solving the growth model of the density of particles [15] and approximated solutions for the fuzzy system of Volterra integro-differential equations [22].

The rest of the structure of this study including Sec.2 recalls the preliminaries of the theory. Sec.3 presents the ADM while Sec.4 analyses a new fractional model using the said method, and Sec.5 utilizes the new model to predict the abalone length growth. Finally, Sec.6 discusses the conclusions.

2. Preliminaries

This section is a brief review of some key concepts including fractional integrals and derivatives as well Ulam-Hyers stability.

Definition 2.1. *[23] Let $g : [0, \infty) \longrightarrow \mathbb{R}$ is a continuous function. The definition of the Riemann-Liouville fractional integral and derivative of order β are given as.*

$$ {}_aI_s^{\beta} g(s) = \frac{1}{\Gamma(\beta)} \int_a^s (s-\xi)^{\beta-1} g(\xi) d\xi, \tag{5}$$

and

$$^{RL}D_s^{\beta}g(s) = \begin{cases} \frac{d^n}{ds^n} \;\; if \; \beta = m \in \mathbb{N}, \\ \\ \frac{d^n}{ds^n}\int_0^s \frac{(s-\xi)^{n-\beta-1}}{\Gamma(n-\beta)}g(\xi)d\xi \;\; if \; m-1<\beta<m, m\in\mathbb{N}. \end{cases} \tag{6}$$

However, the equation (6) is indicating that the fractional derivative of constant function is not equal to zero for $\beta \in (m-1, m)$. It means that the Riemann-Liouville fractional derivative contradicts with the fundamental derivative theories. Therefore, the definition of the Caputo fractional derivative is given as follows.

Definition 2.2. *([24], [25]) Let a real function g is continuous on* $[0, \infty)$ *. The derivative of a function g with order* β *is*

$$^{C}_{a}D_s^{\beta}g(s) = \begin{cases} \frac{d^n g(s)}{ds^n} \;\; if \; \beta = m \in \mathbb{N}, \\ \\ \int_a^s \frac{(s-\xi)^{m-\beta-1}}{\Gamma(m-\beta)}g^{(m)}(\xi)d\xi \;\; if \; m-1<\beta<m, m\in\mathbb{N}. \end{cases} \tag{7}$$

The equation (7) indicates that the Caputo fractional derivative conforms to the rules of the fundamental derivative theories. Furthermore, some basic properties of fractional derivative [26] are provided as below.

1. $\left({}_aI_s^{\alpha} \cdot {}_a I_s^{\beta} g\right)(s) = \left({}_aI_s^{\beta} \cdot {}_a I_s^{\alpha} g\right) = \left(I^{\alpha+\beta}g\right)(s)$,
2. ${}_aI_s^{\alpha}(s-a)^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)}(s-a)^{\gamma+\alpha}$,
3. $\left({}_aI_s^{\alpha}\,{}^C_aD_s^{\alpha}g\right)(s) = g(s) - \sum\limits_{k=0}^{m-1} g^{(k)}(a)\frac{(s-a)^k}{k!}$,

with $\alpha, \beta > 0$, $a \geq 0$, $m-1 < \alpha < m$,$m \in \mathbb{N}$, and $\gamma > -1$.

Similarly, the definition of fractional partial derivative is given as follows.

Definition 2.3. *[27] Let* $C, D \subseteq [0, \infty)$ *and a function* $g : C \times D \longrightarrow \mathbb{R}$ *is continuous on* $C \times D$*. The partial derivatives of a function g with order* β *are defined as*

$$\frac{\partial^{\beta}g(s,t)}{\partial s^{\beta}} = \frac{1}{\Gamma(m-\beta)}\int\limits_0^s (s-\xi)^{m-beta-1}\frac{\partial^m g(\xi,t)}{\partial \xi^m}d\xi, \;\; \beta \in (m-1, m) \tag{8}$$

and

$$\frac{\partial^{\beta}g(s,t)}{\partial t^{\beta}} = \frac{1}{\Gamma(m-\beta)}\int\limits_0^t (t-\xi)^{m-\beta-1}\frac{\partial^m g(s,\xi)}{\partial \xi^m}ds, \;\; \beta \in (m-1, m). \tag{9}$$

Thus, the fundamental properties can be generalized similarly. In addition, we provide the definition of the Mittage-Leffler function which involved in solving the fractional differential equations as follows.

Definition 2.4. *[28] The Mittage-Leffler function with parameter α is defined by the series expansion*

$$E_\alpha(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(m\alpha + 1)}, \ \alpha > 0, z \in \mathbb{C} \tag{10}$$

which the series is convergent. This series is a simple generalization of the exponential function.

Similarly, the definition for function with two-parameter is given as follows.

Definition 2.5. *[28] The Mittage-Leffler function of two-parameter is defined as:*

$$E_{\alpha,\beta}(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(m\alpha + \beta)}, \ \alpha, \beta > 0, z \in \mathbb{C} \tag{11}$$

which the series is convergent and where $\Gamma(\cdot)$ is the Gamma function.

Since the result of this study is examined using Ulam-Hyers stability, we provide the following definition.
Let $(\mathbb{R}, \|\cdot\|)$ be a Banach space, $a \in \mathbb{R}$, $b \in \mathbb{R} \cup +\infty$, $D = [a, b) \times \mathbb{R}$, $g : D \to \mathbb{R}$ be a continuous operator. Consider the following fractional partial differential equation:

$$^C D_t^\beta w(x,t) = g(x, t, w(x,t)), \forall (x,y) \in D \tag{12}$$

and the inequality

$$\|^C D_t^\beta w(x,t) - g(x,t,w(x,t))\| < \varepsilon, \forall (x,y) \in D. \tag{13}$$

Definition 2.6. *[29] The equation (12) is Ulam-Hyers stable if there exists a real number $k > 0$ such that for every $\varepsilon > 0$ and for every solution $w \in C^1(D, \mathbb{R})$ of the inequality (13) there exists a solution $v \in C^1(D, \mathbb{R})$ of the equation (12) with*

$$\|w(x,t) - v(x,t)\| < \varepsilon k, \forall (x,y) \in D. \tag{14}$$

3. Adomian Decomposition Method

This section discusses about the brief technique of the ADM. To clarify, we consider the following fractional partial differential equation [30]:

$$\frac{\partial w(s,t)}{\partial t} + \frac{\partial^\beta w(s,t)}{\partial s^\beta} + w(s,t) + (w(x,t))^2 = 0, \tag{15}$$

with the initial condition

$$w(s,0) = w_0(s)$$

where $n - 1 < \beta < n$, $n \in \mathbb{N}$.
The first step of ADM is converting the equation (15) into a linear and non-linear operator form as follows:

$$L_t w + L_s w + N w = 0, \tag{16}$$

where the linear operator of $L_t = \frac{\partial}{\partial t}$, and $L_s = \frac{\partial^\beta}{\partial s^\beta}$ are invertible and $N(w)$ deputizes the non-linear term, that is $N(w) = w(s,t) + (w(s,t))^2$.
Then we obtain

$$w = -L_t^{-1}\left(L_s(w)\right) - L_t^{-1}\left(N(w)\right), \tag{17}$$

In the second step, we assume that w and $N(w)$ are infinite series, which are given as follows:

$$w = \sum_{n=0}^{\infty} w_n \tag{18}$$

and

$$N(w) = \sum_{n=0}^{\infty} A_n, \tag{19}$$

where A_n is called the Adomian polynomial with formula

$$A_n = \frac{1}{n!}\frac{d}{d\lambda^n} N\left(\sum_{i=0}^{n} \lambda^i w_i\right)_{\lambda=0}. \tag{20}$$

In the third step, we determine the iteration formula as below.

$$w(s,t) = \begin{cases} w_0(s,t) = w_0(s), \\ w_{n+1}(s,t) = -L_t^{-1}\left[L_s(w_n)\right] - L_t^{-1}\left(A_n\right) \end{cases}. \tag{21}$$

Therefore, we get the solution of equation (15) as follows:

$$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t) \tag{22}$$

where $w_1, w_2, w_3, ...$ are results of the iteration in equation (21).

4. A Growth Model with Fractional Order

This part we provide a new growth model with fractional order as a modification of the McKendrick equation. The model is constructed for single population species in cultivation condition. Since the species is in cultivation process, we assume that these species have sufficient food. Generally, the growth rate of the species will be slow over the time due to some factors. However, the model does not discuss about the influence factors of species growth in detail. This model is formed to predict the growth of species size by involving intrinsic and extrinsic growth rate of individuals. Let $w(s,t)$ represent the density of population with age s at time t and η denotes the extrinsic growth rate. Since the McKendrick model is based on first integer order, then we propose the growth model with fractional order β for $\beta \in (0,1]$ as follows:

$$\frac{\partial w(s,t)}{\partial t} + \frac{\partial^\beta w(s,t)}{\partial s^\beta} = \eta w(s,t), \tag{23}$$

with the initial condition

$$w(s,0) = Me^{rs},$$

where M is the initial size of individuals and r denotes the intrinsic growth rate. In this case, the equation (23) is analysed using Adomian decomposition method. Then the equation (23) is converted to the linear operator form as follows:

$$L_t w(s,t) + L_s w(s,t) = \eta w(s,t). \tag{24}$$

By setting

$$w(s,t) = \sum_{n=0}^{\infty} w_n(s,t)$$

and $w_0 = Me^{rs}$, we find that

$$w(s,t) = \begin{cases} w_0(s,t) = Me^{rs}, \\ w_{n+1}(s,t) = -L_t^{-1}\left[L_s(w_n)\right] + \eta L^{-1}[w_n] \end{cases}. \tag{25}$$

By applying the equation (25), we find that the sequent terms of the approximated solution as

$$w_1(s,t) = -L_t^{-1}\left[L_s(w_0)\right] + \eta L^{-1}[w_0] = (\eta - r^{\beta})Me^{rs}t, \tag{26}$$

and in the same manner,

$$w_2(s,t) = (\eta - r^{\beta})^2 Me^{rs}\frac{t^2}{2!}, \tag{27}$$

$$w_3(s,t) = (\eta - r^{\beta})^3 Me^{rs}\frac{t^3}{3!}, \tag{28}$$

and the proceeding further the $w_n(s,t)$ is expressed by

$$w_n(s,t) = (\eta - r^{\beta})^n Me^{rs}\frac{t^n}{n!}. \tag{29}$$

Therefore, the solution of the equation (23) is

$$w(s,t) = Me^{rs}\left(1 + (\eta - r^{\beta})t + (\eta - r^{\beta})^2\frac{t^2}{2!} + (\eta - r^{\beta})^3\frac{t^3}{3!} + \cdots\right). \tag{30}$$

The equation (30) is a specific form of Taylor's series for exponential function which can be written as

$$w(s,t) = \lim_{m\to\infty}\sum_{n=0}^{m} w_n(s,t) = Me^{rs}e^{(\eta - r^{\beta})t}. \tag{31}$$

The equation (31) is a continuous function and convergent on the closed interval $[a,c]$ with $a, c \in \mathbb{R}$. Hence, the growth model on the equation (31) is Ulam-Hyers stable. Therefore, the equation (31) will be utilised to predict the abalone growth in the following section.

5. Applications

In this section we apply the equation (31) to describe the growth of abalone length. In this case, the parameter values which are substituted to the model based on the following data [31].

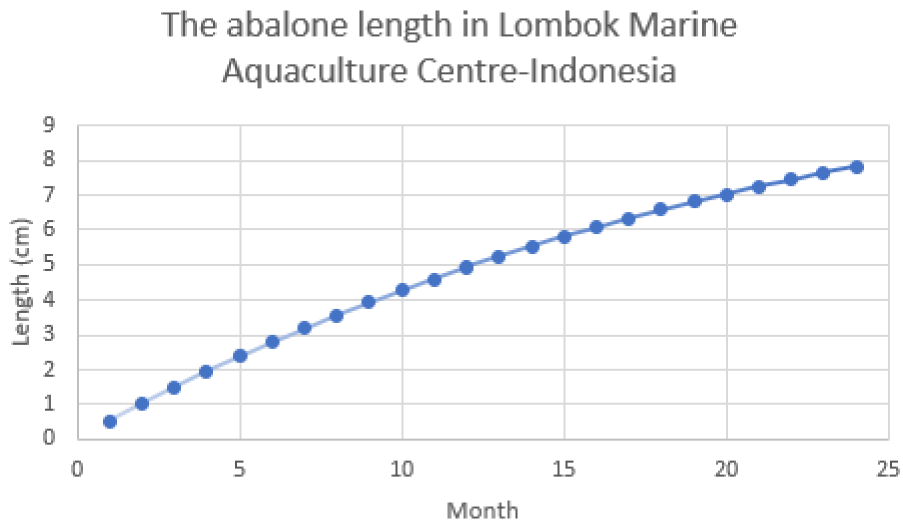

Figure 1: Abalone length for 24 months of observation.

From the data we determine the growth rate of abalone length using formula $\eta = \Delta h/\Delta t$. Meanwhile, the intrinsic growth rate $r = 0.04305$ was presented in the previous study [32]. Let $h(s,t)$ is denoting the body length of abalone. Based on the equation (23), the model for abalone length growth is given as

$$\frac{\partial h(s,t)}{\partial t} + \frac{\partial^{\beta} h(s,t)}{\partial s^{\beta}} = \eta h(s,t), \tag{32}$$

with the initial condition

$$w(s,0) = Ae^{rs},$$

and the solution is

$$h(s,t) = Ae^{rs}e^{(\eta - r^{\beta})t}, \tag{33}$$

where A is the initial body length of abalone. By substituting parameter values and running iteration on the equation (33), the results are given in the following Table 1.

Table 1: Approximation of Abalone Length

Month	η	$h_{0.5}$	$h_{0.51}$	$h_{0.6}$	$h_{0.7}$	$h_{0.8}$	$h_{0.9}$	h_1
1	-	0.5322	0.5322	0.5322	0.5322	0.5322	0.5322	0.5322
2	0.4936	0.7370	0.7444	0.7794	0.8119	0.8366	0.8550	0.8687
3	0.4724	1.3924	1.4048	1.4726	1.5341	1.5805	1.6154	1.6413
4	0.4521	1.9934	2.0104	2.1082	2.1962	2.2627	2.3126	2.3496
5	0.4326	2.5435	2.5666	2.6900	2.8023	2.8872	2.9508	2.9981
6	0.4239	3.0768	3.0778	3.2540	3.3898	3.4925	3.5694	3.6267
7	0.3962	3.5397	3.5490	3.7436	3.8998	4.0179	4.1065	4.1723
8	0.0380	3.9611	3.9831	4.1892	4.3641	4.4963	4.5954	4.6691
9	0.3628	4.3558	4.3839	4.6067	4.7989	4.9444	5.0533	5.1344
10	0.3472	4.7240	4.7544	4.9960	5.2045	5.3622	5.4804	5.5683
11	0.3322	5.0643	5.0964	5.3559	5.5794	5.7485	5.8751	5.9694
12	0.3179	5.3797	5.4149	5.6895	5.9269	6.1065	6.2410	6.3412
13	0.3043	5.6726	5.7092	5.9993	6.2497	6.4390	6.5809	6.6865
14	0.2911	5.9436	5.9819	6.2859	6.5482	6.7467	6.8953	7.0059
15	0.2786	6.1961	6.2360	6.5529	6.8264	7.0332	7.1882	7.3035
16	0.2666	6.4308	6.4722	6.8011	7.0849	7.2996	7.4604	7.5801
17	0.2551	6.6491	6.6920	7.0321	7.3255	7.5475	7.7138	7.8375
18	0.2443	6.8540	6.8982	7.2487	7.5512	7.7800	7.9515	8.0790
19	0.2336	7.0429	7.0882	7.4485	7.7593	7.9944	8.1705	8.3016
20	0.2236	7.2206	7.3402	7.6365	7.9551	8.1962	8.3768	8.5111
21	0.2140	7.3866	7.4342	7.8120	8.1380	8.3846	8.5693	8.7068
22	0.2047	7.5410	7.5896	7.9753	8.3081	8.5599	8.7485	8.8888
23	0.1960	7.6869	7.7365	8.1297	8.4689	8.7255	8.9178	9.0608
24	0.1875	7.8225	7.8729	8.2730	8.6182	8.8793	9.0750	9.2205

Consider that h_β represents the abalone length with order β. By comparing the lengths of abalone on the Table 1 with the real data, we find that the mean absolute error of $h_{0,5}, h_{0.6}, h_{0.7}, h_{0.8}, h_{0.9}$, and h_1 are 0.2622; 0.5373; 0.7517; 0.9155; 1.0382; and 1.1294 respectively. Hence, the optimal result is achieved with a fractional order of $\beta = 0.5$. In addition, we use $\Delta\beta = 0.01$ to examine the sensitivity of the model with respect to $\beta = 0.5$ and to the mean value of the difference between $h_{0.5}$ and $h_{0.51}$ which is equal to 0.04. It means that the model is stable. Moreover, it is supported by Ulam-Hyers stability. It can also be shown graphically in Figure 2 below:

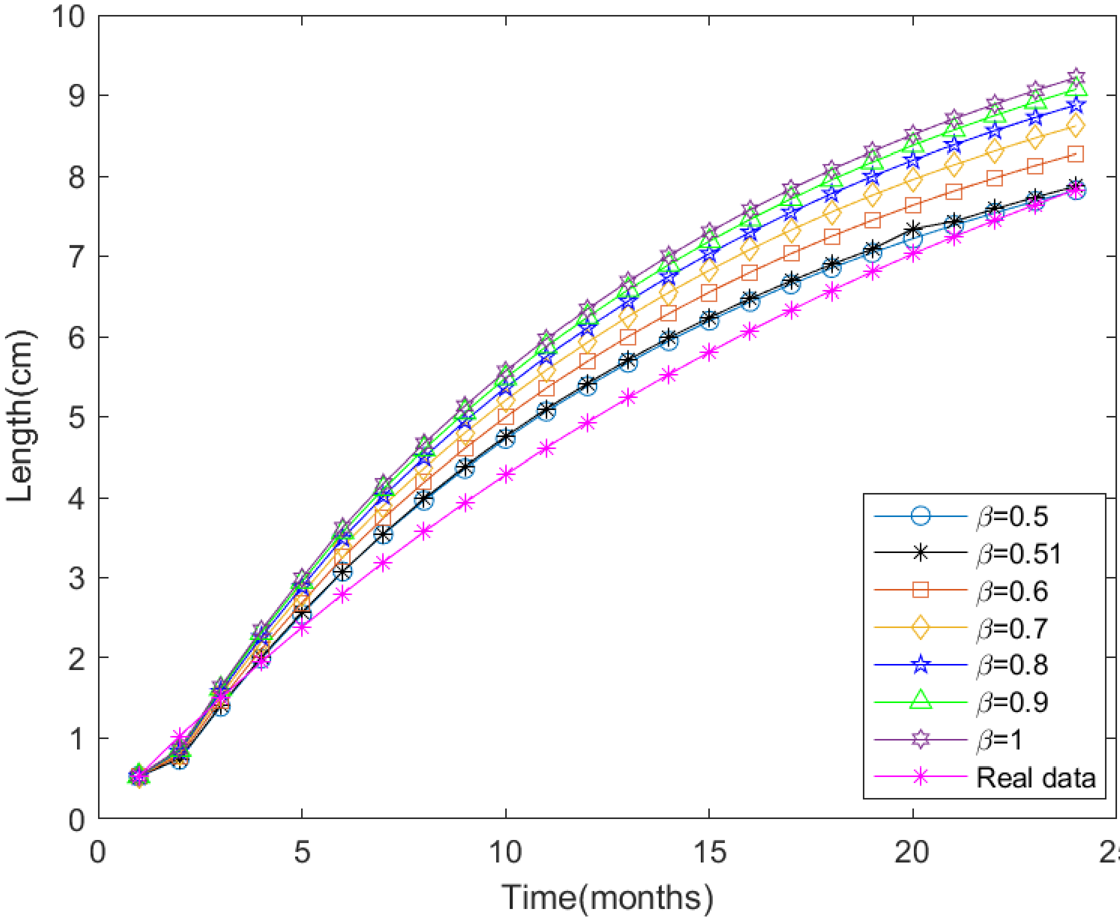

Figure 2: Abalone length comparison for 24 months of observation data.

The Figure 1 shows that as the rate of the abalone length growth and the fractional order decrease, the values of the abalone length approach to the real data. It means that the fractional order in the fractional growth model is the exponent of the gradient of the function. The chart with order $\beta = 0.5$ is the closest to the real data compared with the other charts. Moreover, note that in the 24th month, the abalone length approximation with $\beta = 0.5$ is almost equal to the real data.

On the other side, the following system equations can also be used to express the Equation (33).

$$\frac{\partial h(s,t)}{\partial t} + \frac{\partial^{\beta} h(s,t)}{\partial s^{\beta}} = \eta_1 h(s,t),$$
$$\frac{\partial h(s,t)}{\partial t} + \frac{\partial^{\beta} h(s,t)}{\partial s^{\beta}} = \eta_2 h(s,t),$$
$$\vdots$$
$$\frac{\partial h(s,t)}{\partial t} + \frac{\partial^{\beta} h(s,t)}{\partial s^{\beta}} = \eta_{23} h(s,t)$$

or it can be written as

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{\partial h(s,t)}{\partial t} \\ \dfrac{\partial^{\beta} h(s,t)}{\partial s^{\beta}} \end{bmatrix} = \begin{bmatrix} \eta_1 h(s,t) \\ \eta_2 h(s,t) \\ \vdots \\ \eta_{23} h(s,t) \end{bmatrix}$$

and

$$f(\eta_1, \beta) = h_1, f(\eta_2, \beta) = h_2, \ldots, f(\eta_{23}, \beta) = h_{23}.$$

6. Conclusion

This research showed that modifying the McKendrick model into a new growth model with fractional order, using an exponential function as the initial condition, can accurately predict the abalone length growth. The new model was analysed using the Adomian decomposition method, resulting to a Taylor series. By substituting the parameter values of this series which were obtained from real data and simulated with various fractional orders, the fractional order $\beta = 0.5$ provides results that best reflects the real data, compared to the other orders, including integer orders. For future research, the model can be modified by using Gompertz equation as the initial equation as well involving a parameter.

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