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A Two-Warehouse Inventory Model for Green Technology Investment: Deteriorating Items with Selling Price and Carbon Emissions

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Abstract. Managing deteriorating inventory in mechatronics presents numerous opportunities. In today's world, nearly every industry utilizes mechatronic tools and processes to slow deterioration, thereby reducing carbon emissions. Given the complexity of global warming, many countries are investing in various initiatives and promoting eco-friendly business practices to minimize carbon emissions. This study examines a two-warehouse inventory model for deteriorating goods that emit carbon. Our focus is on minimizing carbon-emitting items during transportation. Reducing emissions from deteriorating inventory requires a comprehensive strategy that involves multiple supply chain partners and prioritizes environmental sustainability. By adopting green technologies, companies can effectively lower carbon dioxide emissions. In this model, demand is influenced by selling price, and partial backlogging is also considered. Additionally, incorporating time-dependent holding costs enhances the model's applicability. The primary goal of this study is to optimize overall cycle time and costs associated with green technology investments. By optimizing these factors, businesses can manage deteriorating inventory more efficiently while mitigating environmental impacts. Integrating all these elements, we propose an optimized inventory model for deteriorating goods, factoring in selling price and carbon emissions under green technology investment. The assumptions in this study suggest that the cost function is highly nonlinear, leading to a constrained optimization problem. The model is solved using an algorithm implemented in Mathematica software. A numerical example is provided to illustrate the model's application, followed by a sensitivity analysis. Finally, the optimal solution is visually represented through a graphical illustration.

2020 Mathematics Subject Classifications: 90B05, 90C30, 93A30
Key Words and Phrases: Inventory, Two-warehouses, Selling Price, Carbon emission, Green Technology Investment, Optimization

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1. Introduction

Design and manufacturing processes play a vital role in supply chain inventory management. Perishable goods with short shelf lives must be effectively managed as part of inventory control for deteriorating commodities. Strategies such as First-In-First-Out (FIFO) movement, demand forecasting, and continuous monitoring help minimize waste, reduce holding costs, and maintain product quality. Businesses dealing with perishable items, such as grocery stores and pharmaceutical companies, must adhere to these practices.

Many countries support initiatives aimed at reducing carbon emissions and promoting eco-friendly business policies. Sustainable manufacturing and environmentally responsible supply chains enable industries to maintain financial stability while upholding ethical standards. It is well understood that deteriorating goods contribute to lower carbon emissions. Numerous studies have been conducted to validate these approaches and offer practical insights for companies seeking to mitigate environmental impacts, including solid waste, greenhouse gas emissions, and water pollution.

Benjaafar et al. [1] were the first to incorporate carbon emissions into operational decision-making related to production, inventory management, and procurement. To reduce carbon emissions, Bouchery et al. [2] developed an optimization model that integrated sustainable development with multiple objectives. Dye and Yang [3] were the first to examine how carbon emissions impact decisions on preserving or discarding deteriorating goods under various environmental regulations.

Mechatronics plays a crucial role in inventory control for deteriorating goods, with its applications varying based on industry and environmental conditions. In the food sector, mechatronics can help regulate humidity and temperature, while in industrial settings, it enhances the efficiency of machines and equipment, minimizing degradation.

Autonomous inventory control, data collection and analysis, climate regulation systems, automation and artificial intelligence, barcode and RFID technology, predictive maintenance, inventory management software, quality reliability, inventory monitoring, and tracking are some notable applications [4, 5]. To optimize inventory readiness and overall costs, Kattan and Adi [6] developed an inventory model leveraging advanced technology. Raj et al. [7] proposed a hypothetical scenario utilizing green manufacturing technologies. Bhirud et al. [8] formulated a multi-objective optimization model for machining medium carbon steel. Santhi and Muthuswamy [9] explored the role of Industry 4.0 and Industry 5.0 in inventory management. Mehta et al. [10] introduced an innovative sustainable manufacturing strategy aimed at reducing carbon emissions. Additionally, several researchers have developed inventory models that simulate CO2 emissions, as documented in [11–22].

Various researchers have explored different carbon tax strategies for specific products in inventory models to mitigate emissions. However, product type and manufacturing processes play a more significant role in reducing carbon footprints. Zouadi et al. [23] incorporated carbon emission constraints into models for manufacturing and remanufacturing. Between 2001 and 2013, Wang et al. [24] investigated energy constraints and green technology adoption in "Chinese industrial enterprises" as part of efforts to reduce carbon emissions. Recognizing the impact of such investments, Datta [11] developed a deteriorating inventory model. Mukherjee et al. [25] utilized Convolutional Neural Networks (CNN) to minimize emissions in building materials. Poswal et al. [26] proposed a demand model that accounts for both price fluctuations and stock variations. Kumar et al. [27] employed artificial intelligence and optimization techniques, while both Kumar and Gulati [28] and Kakkar et al. [29] applied different optimization strategies across various domains. Mahata and Debnath [30] optimized a manufacturer screening process using KKT conditions to improve efficiency.

The concept of adding an additional warehouse, referred to as the rental warehouse, was first introduced by Hartley [31]. Pakkala and Achary [32] enhanced the two-warehouse inventory model by incorporating deteriorating products with a limited replenishment cycle. Bhunia and Maiti [33] developed a two-warehouse stock model based on a linear consumption rate. Further research on inventory management has been conducted, as noted in studies [34–39]. Ghare and Schrader [40] were the first to investigate the impact of deterioration on inventory models. Das et al. [41] developed a model that integrates price-sensitive demand with declining inventory levels. Paul et al. [42] proposed an Economic Order Quantity (EOQ) model characterized by price-dependent demand and a time-dependent deterioration rate. In our model, we considered deteriorating products that contribute to carbon emissions during storage and examined green technology investments as a strategy for lowering emissions.

A shortfall in inventory management occurs when demand exceeds supply, directly impacting customer satisfaction and operational efficiency. One approach to addressing this issue is partially backlogging shortages, which helps balance cost efficiency and operational effectiveness. This flexible inventory model, which permits partial backlog shortages, can be applied to various inventory challenges. This study explores optimal green technology investments and their influence on company pricing decisions. To enhance realism, the model incorporates product deterioration and fluctuating holding costs. It also allows shortages, with a dynamic backlogging rate. The optimality of the total cost function is analyzed using an analytical optimization approach. Finally, Mathematica software is used to validate the proposed model through a numerical example.

Here is a brief summary of the major contributions:

- Before the product is received, "n" equal installments are required.
- An advance payment is necessary in a two-warehouse system.
- Partial backlogging and advance payment have been taken into account.
- A consistent rate of partially backlogged shortages.
- The demand for the product depends upon its stock.

- Constant rate of deterioration.

The following is the sequence of the remaining sections: A discussion of assumptions and notations is included in Section 2. Section 3 describes the mathematical formulation of the model. In Section 4, by considering the different parameters, we analyze the model's optimal solution. The paper's conclusion is finally presented in Section 5.

2. Assumptions and Notations

In the process of developing the inventory model, the subsequent assumptions were considered:

- The item's demand is linearly dependent on its price, as it is represented by $D(p) = \phi_1 - \phi_2 p$.
- The market demand is influenced by the selling price.
- There is no item replacement or repair.
- The inventory system has an infinite planning horizon.
- With zero lead time, the replenishment rate is instantaneous.
- The availability of the rented warehouse (RW) is infinite, whereas the own warehouse (OW) has a maximum availability of $\mathcal{W}$ units.
- In comparison to own warehouse (OW), the costs of inventory in rented warehouse (RW) are higher.
- With "n" evenly spaced payments, the business pays a portion "k" of the entire cost of purchasing within the lead time "M". The remaining cost of purchasing is subsequently paid to acquire the lot.
- Shortages are permitted, and a portion "δ" of the demand $D(p) = (\phi_1 - \phi_2 p)$ will be backordered during the stock out time.
- We assume that both warehouses will deteriorate at a steady rate "θ" $(0 < \theta < 1)$
- To lower carbon emissions, in our approach, green technology $(\mathcal{G})$ is under consideration for investment. It is possible to reduce carbon emissions by a certain percentage by using green technology $(\mathcal{G})$, has the formula $\mathcal{F} = \rho\left(1 - e^{-\chi\mathcal{G}}\right)$ where "ρ" is the amount of "CO_2" produced, while "χ" is the possibility of reducing "CO_2" emissions.
- $\mathcal{F} = \rho\left(1 - e^{-\chi\mathcal{G}}\right) \Rightarrow \mathcal{G} = -\frac{1}{\chi}\left[\ln\left(1 - \frac{\mathcal{F}}{\rho}\right)\right]$.

Furthermore, when creating an inventory model, the following notation is utilised.

Notations	Units	Description
$\mathcal{A}$	\$/Order	Ordering Cost
δ	Units	Backlogging unit ($0 < \delta < 1$)
S	Units	Level of total inventory
ϕ_1	Constant	Demand rate's coefficient part ($\phi_1 > 0$)
M	yr	Company's lead time for prepayments
ϕ_2	Constant	Demand rate Constant for price ($\phi_2 > 0$)
θ	Constant	Deterioration rate
n	Constant	Equally distributed prepayments over the lead time
$\mathcal{W}$	Units	Inventory level at OW
c_p	Units	Cost per unit of purchase
c_d	Units	Cost per unit of deterioration
k	Constant	Installment-based payment ($0 < k < 1$)
R	Units	Backlogged units
t_1	yr	$RW's$ inventory level falls to zero at this point.
c_s	Unit	Shortage cost per unit
c_l	Unit	Cost of Opportunity per unit
H	Unit	Holding costs in rupees per unit of OW
F	Unit	Holding costs in rupees per unit of RW
p	Unit	Selling price per unit
$I_o(\mathrm{t})$	Unit	Inventory level at any time "t" in OW
$I_r(\mathrm{t})$	Unit	Inventory level at any time "t" in RW
c_e	Constant	Cost of carbon emission
R_{ce}	Constant	Rate of carbon emissions during deterioration

Decision-making parameters

Notations	Units	Description
t_2	yr	$OW's$ inventory level falls to zero at this point.
$\mathcal{G}$	Unit	Green technology investment
$\mathcal{T}$	yr	The entire length of the cycle of replenishment

3. Problem Definition

In this section, we develop a mathematical model for a two-warehouse inventory system for deteriorating items with selling price and carbon emission under green technology investment.

Based on the following assumptions, let's say a company orders $(S + R)$ units of a given product and pays a percentage "k" of the total price in "n" equal installments within a certain lead time "M". At time $t = 0$, the remaining purchasing cost is paid. As "R" units are used to partially meet backlogged demand, the inventory level becomes "S". Now, "$\mathcal{W}$" units are preserved in "OW", while the remaining fraction "$S-\mathcal{W}$" is saved in "RW". Because "RW" has superior facilities, the holding costs are higher than in "OW", causing "RW" items to be consumed first. As a result of the constant deteriorating rate "θ" and the need to meet customer demand "$D(p)$", "RW"$'s$ inventory level decreases during the time interval $[0, t_1]$. In "RW", it drops to zero at $t = t_1$. As a consequence, inventory level in "OW" decrease due to a constant deteriorating rate "θ" during the interval $[0, t_1]$. Within a short time the inventory in "OW" is depleted due to the customer's demand "$D(p)$" and deteriorating during the interval of time $[t_1, t_2]$. At $t = t_2$, it appears to become zero. Shortages increase at a rate that remains constant "δ" within the interval of time $[t_2, \mathcal{T}]$. The following figure illustrates the inventory level during the complete cycle $[0, \mathcal{T}]$ is as shown below:

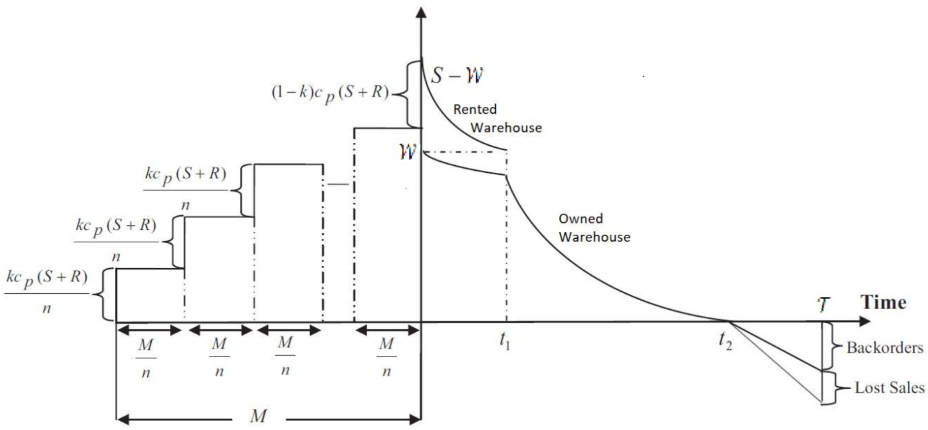

Figure 1: **An illustration of a two-warehouse inventory model for deteriorating items.**

The inventory level at $t = 0$ to $t = \mathcal{T}$ is described in the differential equations as follows:

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -(\phi_1 - \phi_2 p), \qquad 0 \leq t \leq t_1 \tag{1}$$

subject to the conditions:

$$I_r(t) = \begin{cases} S - \mathcal{W}, & at \ t = 0 \\ 0, & at \ t = t_1 \end{cases} \tag{2}$$

On evaluating the differential equations mentioned above:

$$I_r(t) = \frac{\phi_1 - \phi_2 p}{\theta}\{e^{\theta(t_1 - t)} - 1\}, \qquad 0 \leq t \leq t_1. \tag{3}$$

Additionally, at any instant in time "t", inventory level $I_o(\mathrm{t})$ in OW can be represented by the differential equations below:

$$\frac{dI_o(\mathrm{t})}{d\mathrm{t}} + \theta I_o(\mathrm{t}) = 0, \qquad 0 \leq \mathrm{t} \leq \mathrm{t}_1 \tag{4}$$

$$\frac{dI_o(\mathrm{t})}{d\mathrm{t}} + \theta I_o(\mathrm{t}) = -(\phi_1 - \phi_2 p), \qquad \mathrm{t}_1 \leq \mathrm{t} \leq \mathrm{t}_2 \tag{5}$$

$$\frac{dI_o(\mathrm{t})}{d\mathrm{t}} = -\delta(\phi_1 - \phi_2 p), \qquad \mathrm{t}_2 \leq \mathrm{t} \leq \mathcal{T} \tag{6}$$

subject to the conditions:

$$I_o(\mathrm{t}) = \begin{cases} \mathcal{W}, & at \ \mathrm{t} = 0 \\ 0, & at \ \mathrm{t} = \mathrm{t}_2 \\ -R, & at \ \mathrm{t} = \mathcal{T} \end{cases} \tag{7}$$

On evaluating the differential equations mentioned above:

$$I_o(\mathrm{t}) = \mathcal{W} e^{-\theta \mathrm{t}}, \qquad 0 \leq \mathrm{t} \leq \mathrm{t}_1 \tag{8}$$

$$I_o(\mathrm{t}) = \frac{\phi_1 - \phi_2 p}{\theta}\{e^{\theta(\mathrm{t}_2 - \mathrm{t})} - 1\}, \qquad \mathrm{t}_1 \leq \mathrm{t} \leq \mathrm{t}_2 \tag{9}$$

$$I_o(\mathrm{t}) = \delta(\phi_1 - \phi_2 p)(\mathcal{T} - \mathrm{t}) - R, \qquad \mathrm{t}_2 \leq \mathrm{t} \leq \mathcal{T} \tag{10}$$

In order to write the above equation,
we have to consider the continuity at $\mathrm{t} = \mathrm{t}_1$ and $\mathrm{t} = \mathrm{t}_2$:

$$S = \mathcal{W} + \frac{\phi_1 - \phi_2 p}{\theta}\left[e^{\theta \mathrm{t}_1} - 1\right] \tag{11}$$

$$R = \delta\left(\phi_1 - \phi_2 p\right)\left(\mathcal{T} - \mathrm{t}_2\right) \tag{12}$$

$$\mathrm{t}_2 = \mathrm{t}_1 + \frac{1}{\theta}\log\left[1 + \frac{\theta \mathcal{W} e^{-\theta \mathrm{t}_1}}{\phi_1 - \phi_2 p}\right] \tag{13}$$

Presently, the following segments constitute the total cost:

(*a*) Ordering Cost: $\mathcal{A}$

(*b*) Purchase Cost: $c_p(S + R) =$

$$c_p\left(\mathcal{W} + \frac{\phi_1 - \phi_2 p}{\theta}\left(e^{\theta \mathrm{t}_1} - 1\right) + \delta\left(\phi_1 - \phi_2 p\right)\left(\mathcal{T} - \mathrm{t}_2\right)\right)$$

(*c*) Holding Cost:

$$F\int_0^{\mathrm{t}_1} I_r\,(\mathrm{t})d\mathrm{t} + H\int_0^{\mathrm{t}_1} I_o\,(\mathrm{t})d\mathrm{t} + H\int_{\mathrm{t}_1}^{\mathrm{t}_2} I_o\,(\mathrm{t})d\mathrm{t}$$

$$= \frac{F(\phi_1 - \phi_2 p)\left(\theta(-\mathrm{t}_1) + e^{\theta \mathrm{t}_1} - 1\right)}{\theta^2} + \frac{H\left(\mathcal{W} - \mathcal{W}e^{\theta(-\mathrm{t}_1)}\right)}{\theta} + \frac{H(\phi_1 - \phi_2 p)\left(\theta \mathrm{t}_1 + e^{\theta(\mathrm{t}_2 - \mathrm{t}_1)} - \theta \mathrm{t}_2 - 1\right)}{\theta^2}$$

(*d*) Deterioration Cost:

$$c_d\theta \int_0^{\mathrm{t}_1} I_r(\mathrm{t})d\mathrm{t} + c_d\theta \int_0^{\mathrm{t}_1} I_o(\mathrm{t})d\mathrm{t} + c_d\theta \int_{\mathrm{t}_1}^{\mathrm{t}_2} I_o(\mathrm{t})d\mathrm{t}$$

$$= \frac{c_d\theta(\phi_1 - \phi_2 p)\left(\theta \mathrm{t}_1 + e^{\theta(\mathrm{t}_2 - \mathrm{t}_1)} - \theta \mathrm{t}_2 - 1\right)}{\theta^2} + \frac{c_d\theta\left(\mathcal{W} - \mathcal{W}e^{\theta(-\mathrm{t}_1)}\right)}{\theta} + \frac{c_d\theta(\phi_1 - \phi_2 p)\left(\theta(-\mathrm{t}_1) + e^{\theta \mathrm{t}_1} - 1\right)}{\theta^2}$$

(*e*) Shortage Cost:

$$-c_s \int_{\mathrm{t}_2}^{\mathcal{T}} I_o(\mathrm{t})d\mathrm{t} = \frac{1}{2} c_s \delta (\mathcal{T} - \mathrm{t}_2)^2 (\phi_1 - \phi_2 p)$$

(*f*) Opportunity Cost:

$$c_l(1-\delta) \int_{\mathrm{t}_2}^{\mathcal{T}} D d\mathrm{t} = c_l(1-\delta)(\phi_1 - \phi_2 p)(\mathcal{T} - \mathrm{t}_2)$$

(*g*) Capital Cost:

$$I_c \left[\frac{kc_p(S+R)}{n} \frac{M}{n} (1 + 2 + 3 + ... + n)\right]$$

$$= I_c \left(\frac{Mkc_p(n+1)\left((\phi_1 - \phi_2 p)\left(\frac{e^{\theta \mathrm{t}_1}}{\theta} - \frac{1}{\theta}\right) + \delta(\phi_1 - \phi_2 p)(\mathcal{T} - \mathrm{t}_2) + \mathcal{W}\right)}{2n}\right)$$

(*h*) Carbon emission Cost:

$$(c_e.R_{ce}) \int_0^{\mathrm{t}_1} I_r(\mathrm{t})d\mathrm{t} + (c_e.R_{ce}) \int_0^{\mathrm{t}_1} I_o(\mathrm{t})d\mathrm{t} + (c_e.R_{ce}) \int_{\mathrm{t}_1}^{\mathrm{t}_2} I_o(\mathrm{t})d\mathrm{t}$$

$$= (C_e.R_{ce})\left(\frac{(\phi_1-\phi_2 p)\left(\theta(-\mathrm{t}_1)+e^{\theta \mathrm{t}_1}-1\right)}{\theta^2}\right)$$
$$+ (C_e.R_{ce})\left(\frac{(\phi_1-\phi_2 p)\left(\theta \mathrm{t}_1+e^{\theta(\mathrm{t}_2-\mathrm{t}_1)}-\theta \mathrm{t}_2-1\right)}{\theta^2}\right)$$
$$+ (C_e.R_{ce})\left(\frac{\mathcal{W}-\mathcal{W}e^{\theta(-\mathrm{t}_1)}}{\theta}\right)$$

(i) Investment costs for green technology :

$$GTIC = GT$$

(j) Cost reductions from carbon emissions:

$$\left(1-\rho\left(1-e^{-\chi\mathcal{G}}\right)\right)\left[(c_e.R_{ce})\int_0^{\mathrm{t}_1} I_r(\mathrm{t})dt\right.$$
$$\left.+ (c_e.R_{ce})\int_0^{\mathrm{t}_1} I_o(\mathrm{t})d\mathrm{t} + (c_e.R_{ce})\int_{\mathrm{t}_1}^{\mathrm{t}_2} I_o(\mathrm{t})dt\right]$$

$$= \left(1-\rho\left(1-e^{-\chi\mathcal{G}}\right)\right)\left[((C_e.R_{ce})\left(\frac{(\phi_1-\phi_2 p)\left(\theta(-\mathrm{t}_1)+e^{\theta \mathrm{t}_1}-1\right)}{\theta^2}\right)\right.$$
$$+ (C_e.R_{ce})\left(\frac{(\phi_1-\phi_2 p)\left(\theta \mathrm{t}_1+e^{\theta(\mathrm{t}_2-\mathrm{t}_1)}-\theta \mathrm{t}_2-1\right)}{\theta^2}\right)$$
$$\left.+ (C_e.R_{ce})\left(\frac{\mathcal{W}-\mathcal{W}e^{\theta(-\mathrm{t}_1)}}{\theta}\right)\right]$$

Consequently, the total inventory cost is

$$TIC = \frac{1}{\mathcal{T}}\left[\begin{array}{l}\langle\text{Ordering Cost}\rangle + \langle\text{Purchase Cost}\rangle + \langle\text{Holding Cost}\rangle\\ + \langle\text{Deterioration Cost}\rangle + \langle\text{Shortage Cost}\rangle\\ + \langle\text{Opportunity Cost}\rangle + \langle\text{Capital Cost}\rangle\\ + \langle\text{Investment costs for green technology }\rangle\\ + \langle\text{Cost reductions from carbon emissions}\rangle\end{array}\right]$$

$$TIC = \frac{1}{\mathcal{T}}\left[(I_c\left(\frac{Mkc_p(n+1)\left((\phi_1-\phi_2 p)\left(\frac{e^{\theta \mathrm{t}_1}}{\theta}-\frac{1}{\theta}\right)+\delta(\phi_1-\phi_2 p)(\mathcal{T}-\mathrm{t}_2)+\mathcal{W}\right)}{2n}\right)\right.$$

$$
\begin{aligned}
&+\left(1-\rho\left(1-e^{-\chi\mathcal{G}}\right)\right)\Bigg[((C_e.R_{ce})\left(\frac{(\phi_1-\phi_2p)\left(\theta(-\mathrm{t}_1)+e^{\theta\mathrm{t}_1}-1\right)}{\theta^2}\right)\\
&\quad+(C_e.R_{ce})\left(\frac{(\phi_1-\phi_2p)\left(\theta\mathrm{t}_1+e^{\theta(\mathrm{t}_2-\mathrm{t}_1)}-\theta\mathrm{t}_2-1\right)}{\theta^2}\right)\\
&\quad+(C_e.R_{ce})\left(\frac{\mathcal{W}-\mathcal{W}e^{\theta(-\mathrm{t}_1)}}{\theta}\right)\Bigg]\\
&\quad+c_p\left(\delta(\mathcal{T}-\mathrm{t}_2)(\phi_1-\phi_2p)+\frac{(\phi_1-\phi_2p)\left(e^{\theta\mathrm{t}_1}-1\right)}{\theta}+\mathcal{W}\right)\\
&\quad+\frac{1}{2}c_s\delta(\mathcal{T}-\mathrm{t}_2)^2(\phi_1-\phi_2p)\\
&\quad+\frac{c_d\theta(\phi_1-\phi_2p)\left(\theta\mathrm{t}_1+e^{\theta(\mathrm{t}_2-\mathrm{t}_1)}-\theta\mathrm{t}_2-1\right)}{\theta^2}\\
&\quad+\frac{c_d\theta(\phi_1-\phi_2p)\left(\theta(-\mathrm{t}_1)+e^{\theta\mathrm{t}_1}-1\right)}{\theta^2}\\
&\quad+c_l(1-\delta)(\phi_1-\phi_2p)(\mathcal{T}-\mathrm{t}_2)\\
&\quad+\frac{F(\phi_1-\phi_2p)\left(\theta(-\mathrm{t}_1)+e^{\theta\mathrm{t}_1}-1\right)}{\theta^2}\\
&\quad+\frac{H(\phi_1-\phi_2p)\left(\theta\mathrm{t}_1+e^{\theta(\mathrm{t}_2-\mathrm{t}_1)}-\theta\mathrm{t}_2-1\right)}{\theta^2}+\mathcal{A}\\
&\quad+\frac{c_d\theta\left(\mathcal{W}-\mathcal{W}e^{\theta(-\mathrm{t}_1)}\right)}{\theta}+\frac{H\left(\mathcal{W}-\mathcal{W}e^{\theta(-\mathrm{t}_1)}\right)}{\theta}+GT\Bigg]
\end{aligned}
\tag{14}
$$

Let $\mathrm{t}_1=\eta\mathrm{t}_2$, $0<\eta<1$, then we get Equ.(15) from Equ.(14)

$$
\begin{aligned}
TIC(\mathrm{t}_2,\mathcal{T},\mathcal{G})=\frac{1}{\mathcal{T}}\Bigg[&\left(I_c\left(\frac{Mkc_p(n+1)\left((\phi_1-\phi_2p)\left(\frac{e^{\theta\eta\mathrm{t}_2}}{\theta}-\frac{1}{\theta}\right)+\delta(\phi_1-\phi_2p)(\mathcal{T}-\mathrm{t}_2)+\mathcal{W}\right)}{2n}\right)\right.\\
&+\left(1-\rho\left(1-e^{-\chi\mathcal{G}}\right)\right)\Bigg[((C_e.R_{ce})\left(\frac{(\phi_1-\phi_2p)\left(\theta(-\eta\mathrm{t}_2)+e^{\theta\eta\mathrm{t}_2}-1\right)}{\theta^2}\right)\\
&\quad+(C_e.R_{ce})\left(\frac{(\phi_1-\phi_2p)\left(\theta\eta\mathrm{t}_2+e^{\theta(\mathrm{t}_2-\eta\mathrm{t}_2)}-\theta\mathrm{t}_2-1\right)}{\theta^2}\right)\\
&\quad+(C_e.R_{ce})\left(\frac{\mathcal{W}-\mathcal{W}e^{\theta(-\eta\mathrm{t}_2)}}{\theta}\right)\Bigg]\\
&\quad+c_p\left(\delta(\mathcal{T}-\mathrm{t}_2)(\phi_1-\phi_2p)+\frac{(\phi_1-\phi_2p)\left(e^{\theta\eta\mathrm{t}_2}-1\right)}{\theta}+\mathcal{W}\right)\\
&\quad+\frac{1}{2}c_s\delta(\mathcal{T}-\mathrm{t}_2)^2(\phi_1-\phi_2p)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{c_d\theta(\phi_1-\phi_2 p)\left(\theta\eta \mathrm{t}_2+e^{\theta(\mathrm{t}_2-\eta \mathrm{t}_2)}-\theta \mathrm{t}_2-1\right)}{\theta^2}\\
&+\frac{c_d\theta(\phi_1-\phi_2 p)\left(\theta(-\eta \mathrm{t}_2)+e^{\theta\eta \mathrm{t}_2}-1\right)}{\theta^2}\\
&+c_l(1-\delta)(\phi_1-\phi_2 p)(\mathcal{T}-\mathrm{t}_2)\\
&+\frac{F(\phi_1-\phi_2 p)\left(\theta\eta(-\mathrm{t}_2)+e^{\theta\eta \mathrm{t}_2}-1\right)}{\theta^2}\\
&+\frac{H(\phi_1-\phi_2 p)\left(\theta\eta \mathrm{t}_2+e^{\theta(\mathrm{t}_2-\eta \mathrm{t}_2)}-\theta \mathrm{t}_2-1\right)}{\theta^2}+\mathcal{A}\\
&\left.+\frac{c_d\theta\left(\mathcal{W}-\mathcal{W}e^{\theta\eta(-\mathrm{t}_2)}\right)}{\theta}+\frac{H\left(\mathcal{W}-\mathcal{W}e^{\theta(-\eta \mathrm{t}_2)}\right)}{\theta}+GT\right]
\end{aligned}
\tag{15}
$$

4. Solution Process

This section discusses the objective function's convexity. In their research work, [38] and [43] also employed the following optimization strategy. To maximize total profit, the following requirements must be met:

$$
\frac{\partial TIC}{\partial \mathrm{t}_2}=0,\quad \frac{\partial TIC}{\partial \mathcal{T}}=0,\quad \frac{\partial TIC}{\partial \mathcal{G}}=0 \tag{16}
$$

Equation (16) yields t_2, $\mathcal{T}$ and $\mathcal{G}$'s optimal values ${\mathrm{t}_2}^*$, $\mathcal{T}^*$ and $\mathcal{G}^*$

Following are the conditions that need to be met in order to minimize $TIC(\mathrm{t}_2,\mathcal{T},\mathcal{G})$ using the Hessian Matrix, a matrix of partial derivatives of second order:

$$
HM=\begin{bmatrix}
\frac{\partial^2 TIC}{\partial {\mathrm{t}_2}^2} & \frac{\partial^2 TIC}{\partial \mathrm{t}_2\partial\mathcal{T}} & \frac{\partial^2 TIC}{\partial \mathrm{t}_2\partial\mathcal{G}}\\
\frac{\partial^2 TIC}{\partial\mathcal{T}\partial \mathrm{t}_2} & \frac{\partial^2 TIC}{\partial\mathcal{T}^2} & \frac{\partial^2 TIC}{\partial\mathcal{T}\partial\mathcal{G}}\\
\frac{\partial^2 TIC}{\partial\mathcal{G}\partial \mathrm{t}_2} & \frac{\partial^2 TIC}{\partial\mathcal{G}\partial\mathcal{T}} & \frac{\partial^2 TIC}{\partial\mathcal{G}^2}
\end{bmatrix}
$$

$$
\frac{\partial^2 TIC}{\partial {\mathrm{t}_2}^2}>0,\quad
\begin{vmatrix}
\frac{\partial^2 TIC}{\partial {\mathrm{t}_2}^2} & \frac{\partial^2 TIC}{\partial \mathrm{t}_2\partial\mathcal{T}}\\
\frac{\partial^2 TIC}{\partial\mathcal{T}\partial \mathrm{t}_2} & \frac{\partial^2 TIC}{\partial\mathcal{T}^2}
\end{vmatrix}>0,\quad
\begin{vmatrix}
\frac{\partial^2 TIC}{\partial {\mathrm{t}_2}^2} & \frac{\partial^2 TIC}{\partial \mathrm{t}_2\partial\mathcal{T}} & \frac{\partial^2 TIC}{\partial \mathrm{t}_2\partial\mathcal{G}}\\
\frac{\partial^2 TIC}{\partial\mathcal{T}\partial \mathrm{t}_2} & \frac{\partial^2 TIC}{\partial\mathcal{T}^2} & \frac{\partial^2 TIC}{\partial\mathcal{T}\partial\mathcal{G}}\\
\frac{\partial^2 TIC}{\partial\mathcal{G}\partial \mathrm{t}_2} & \frac{\partial^2 TIC}{\partial\mathcal{G}\partial\mathcal{T}} & \frac{\partial^2 TIC}{\partial\mathcal{G}^2}
\end{vmatrix}>0 \tag{17}
$$

After (17), when a matrix is positive definite, we call it a Hessian matrix HM. Based on Mathematica Software, figure 2 illustrates our complete approach for calculating the result of our presented model.

5. Numerical Illustration

This model aims to identify the optimal values $t_2{}^*$, $\mathcal{T}^*$ and $\mathcal{G}^*$ of t_2, $\mathcal{T}$ and $\mathcal{G}$ in order to minimize the total inventory cost function $TIC(t_2, \mathcal{T}, \mathcal{G})$. Due to the complexity of $TIC(t_2, \mathcal{T}, \mathcal{G})$ generated in Equ.(15) it is extremely difficult to analyze it, and to determine the optimum decision variables t_2, $\mathcal{T}$ and $\mathcal{G}$. The following algorithm is implemented to solve this model.

5.1. Algorithm

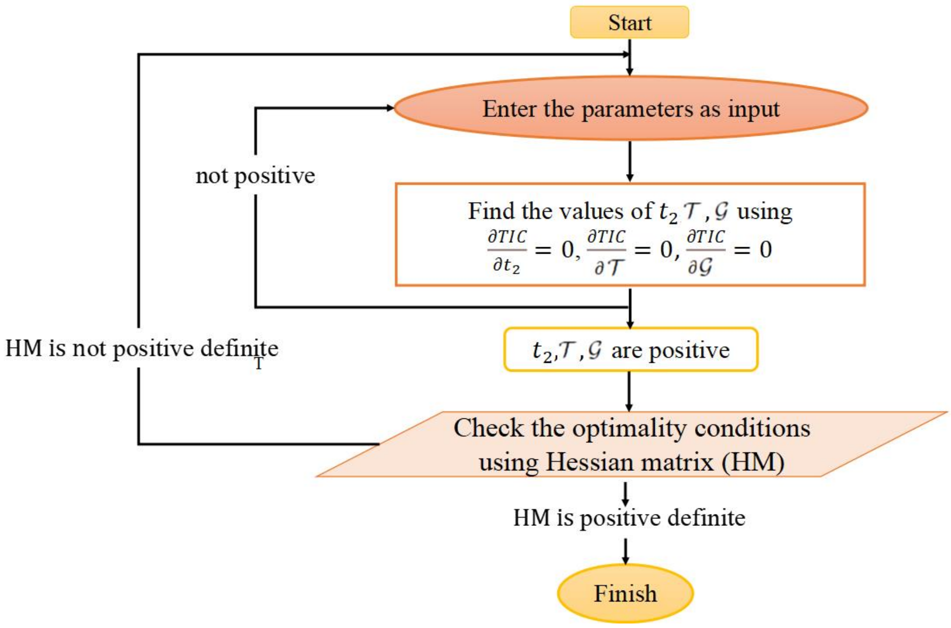

Figure 2: **Flowchart of the solution technique for our defined model.**

- Set the parameters values I_c, M, c_p, n, ϕ_1, ϕ_2, p, θ, η, δ, ρ, χ, C_e, R_{ce}, k, $\mathcal{W}$, c_s, c_d, F, H, $\mathcal{A}$, c_l.
- Construct the function $TIC(t_2, \mathcal{T}, \mathcal{G})$ given by the Equ. (15).
- Minimize $TIC(t_2, \mathcal{T}, \mathcal{G})$ $S.T.C.$ $0 < t_1 < t_2 < \mathcal{T}$ and $0 < c_p < p$.
- Determine the optimal values of $t_2{}^*$, $\mathcal{T}^*$, $\mathcal{G}^*$ and $TIC^*(t_2, \mathcal{T}, \mathcal{G})$

5.2. Case-1 (With an investment in Green Technology)

In this section, a numerical illustration is shown to demonstrate the model's operations. The following values for the variables are entered as follows:
$I_c = 0.25$, $M = 0.25$, $c_p = 10$, $n = 15$, $\phi_1 = 200$, $\phi_2 = 0.5$, $p = 15$, $\theta = 0.3$, $\eta = 0.45$,

$\delta = 0.4$, $\rho = 2.65$, $\chi = 2.8$, $C_e = 1.5$, $R_{ce} = 0.3$, $k = 0.4$, $\mathcal{W} = 100$, $c_s = 1.5$, $c_d = 10$, $F = 3$, $H = 1$, $\mathcal{A} = 500$, $c_l = 1.5$

Here, is an optimal solution as follows:
$t_2{}^* = 0.8044$, $\mathcal{T}^* = 5.5928$, $\mathcal{G}^* = 1.3179$ and $TIC^* = 1507.9032$
In this specific case, investment $\mathcal{G} \neq 0$ in green technology.

5.3. Case-2 (Without an investment in Green Technology)

In this section, a numerical illustration is shown to demonstrate the model's operations. The following values for the variables are entered as follows:
$I_c = 0.25$, $M = 0.25$, $c_p = 10$, $n = 15$, $\phi_1 = 200$, $\phi_2 = 0.5$, $p = 15$, $\theta = 0.3$, $\eta = 0.45$, $\delta = 0.4$, $\rho = 2.65$, $\chi = 2.8$, $C_e = 1.5$, $R_{ce} = 0.3$, $k = 0.4$, $\mathcal{W} = 100$, $c_s = 1.5$, $c_d = 10$, $F = 3$, $H = 1$, $\mathcal{A} = 500$, $c_l = 1.5$, $\mathcal{G} = 0$

Here, is an optimal solution as follows:
$t_2{}^* = 0.6284$, $\mathcal{T}^* = 6.3117$ and $TIC^* = 1524.0395$.
In this specific case, investment $\mathcal{G} = 0$ in green technology.

5.4. Sensitivity Analysis

In this section, the purpose of sensitivity analysis is to determine how parameter values affect optimal values when changing them. It was assumed that one parameter would be changed by ±5% and ±10% at a time, with the other parameters remaining constant throughout the study. Results of the sensitivity analysis are shown in the following table

Based on the observations from the sensitivity analysis, the following insights were drawn.

- Increases in demand parameters ϕ_1 and ϕ_2 will result in higher demand, which will increase the total cost of the inventory.
- Increasing the selling price p, causes decreased demand and related total inventory costs.
- Increase in holding cost results in increase in total inventory costs.
- Increase in deterioration cost θ, results in increase in total inventory costs.
- Investing in green technologies reduces inventory costs when carbon emission costs per unit c_e and rate R_{ce} increases.
- Due to the increase in ordering ($\mathcal{A}$), purchasing (c_p), shortage (c_s), and opportunity (c_l) costs, the total inventory cost increases.

Parameters	**% change**	$t_2{}^*$	$\mathcal{T}^*$	$\mathcal{G}^*$	TIC^*
I_c	-10%	0.7967	6.3897	1.2652	1512.4928
	-5%	0.7961	6.3423	1.2412	1658.3236
	+5%	0.7964	6.3899	1.2642	1799.3813
	+10%	0.7966	6.3936	1.2649	1987.2740
M	-10%	0.7967	6.3897	1.2652	1512.2689
	-5%	0.7831	6.3721	1.2229	1722.3991
	+5%	0.7925	6.3823	1.2034	1833.2162
	+10%	0.7966	6.3936	1.2649	1987.2740
c_p	-10%	0.7967	6.3897	1.2652	1984.7456
	-5%	0.7862	6.3898	1.2881	1956.4609
	+5%	0.7912	6.3877	1.2786	1986.3212
	+10%	0.7966	6.3936	1.2686	1987.2747
n	-10%	0.7966	6.3920	1.2691	1986.1022
	-5%	0.7921	6.3212	1.2421	1985.3991
	+5%	0.7984	6.4121	1.2591	1985.2162
	+10%	0.7943	6.3917	1.2651	1985.9432
ϕ_1	-10%	0.8264	6.7490	1.2462	1813.6896
	-5%	0.8123	6.8291	1.2169	1945.7824
	+5%	0.8612	6.8492	1.2861	2010.3611
	+10%	0.8702	6.9876	1.2821	2156.6885
ϕ_2	-10%	0.7956	6.3795	1.2628	1979.4434
	-5%	0.7911	6.3867	1.2690	1981.0118
	+5%	0.7969	6.3991	1.2860	1988.7939
	+10%	0.7977	6.4116	1.2694	1999.5885
p	-10%	0.7997	6.1530	1.2820	2120.8834
	-5%	0.7999	6.2521	1.3269	1984.8818
	+5%	0.8654	6.4560	1.3486	1881.1123
	+10%	0.7924	6.6226	1.2491	2120.3006

Parameters	**% change**	t_2^*	$\mathcal{T}^*$	$\mathcal{G}^*$	TIC^*
θ	-10%	0.8759	6.4329	1.3135	1981.6955
	-5%	0.8161	6.4516	1.3415	1984.3266
	+5%	0.7999	6.4191'	1.3415	1987.9213
	+10%	0.7279	6.3539	1.2157	1989.6414
η	-10%	0.6031	5.9074	1.0346	2019.5389
	-5%	0.6011	6.0021	1.0244	2006.4111
	+5%	0.6121	6.2931	1.0961	2001.2162
	+10%	0.5563	6.3039	1.1079	2003.4626
δ	-10%	0.6881	6.6473	1.1178	1862.4281
	-5%	0.6923	6.7312	1.0143	1912.2391
	+5%	0.7914	6.7164	1.1986	2001.9481
	+10%	0.8505	6.4046	1.2963	2009.3468
ρ	-10%	0.7768	6.3825	1.2149	1987.1891
	-5%	0.7814	6.3916	1.2629	1986.9191
	+5%	0.8123	6.4112	1.2914	1985.2162
	+10%	0.8170	6.4011	1.3120	1984.8390
$\mathcal{W}$	-10%	0.7801	6.1491	1.2488	1959.8834
	-5%	0.7894	6.2413	1.2694	1976.8818
	+5%	0.7989	6.2871	1.2786	1999.3214
	+10%	0.8119	6.6253	1.2800	2011.2385
$\mathcal{A}$	-10%	0.7849	6.3120	1.2618	1979.8834
	-5%	0.7914	6.3946	1.2769	1984.2169
	+5%	0.8011	6.3969	1.2686	1987.6312
	+10%	0.8082	6.4707	1.2683	1993.7895
C_e	-10%	0.7634	6.4206	1.6420	1512.6434
	-5%	0.7161	6.3123	1.5164	1500.2163
	+5%	0.6919	6.3110	1.5206	1497.6313
	+10%	0.6904	6.3004	1.5314	1400.1685

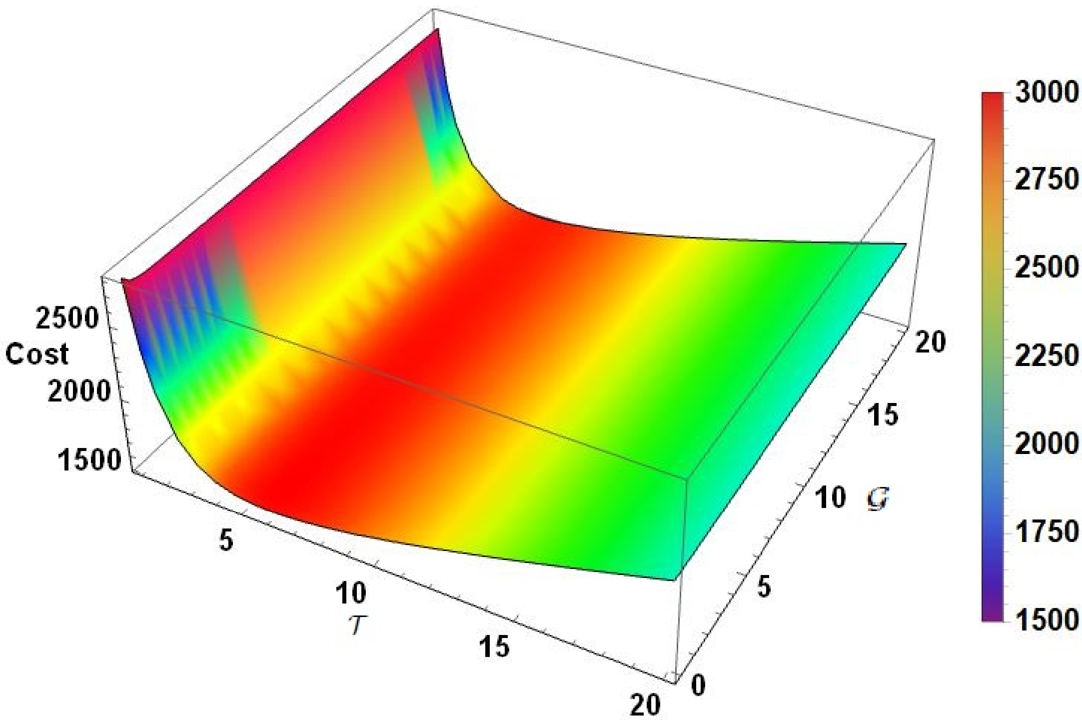

Figure 3: TIC **convexity with respect to** $\mathcal{G}$ **and** $\mathcal{T}$

Parameters	% change	t_2^*	$\mathcal{T}^*$	$\mathcal{G}^*$	TIC^*
R_{ce}	-10%	0.7864	6.2932	1.3624	1514.1223
	-5%	0.7613	6.1244	1.2123	1523.6193
	+5%	0.7214	6.1132‘	1.2615	1458.6012
	+10%	0.6999	6.0022	1.2425	1408.1643
F	-10%	0.7912	6.1324	1.2162	1414.6321
	-5%	0.7014	6.3314	1.3844	1418.7411
	+5%	0.6921	6.3161	1.3012	1506.2342
	+10%	0.7063	6.2141	1.2916	1510.2141
H	-10%	0.7964	6.28741	1.2132	1431.4121
	-5%	0.7162	6.3092	1.1469	1429.2631
	+5%	0.8001	6.2648	1.2013	1506.1234
	+10%	0.8000	6.2962	1.2614	1532.6432
k	-10%	0.7916	6.3241	1.8322	1564.1293
	-5%	0.7994	6.1268	1.9923	1582.1943
	+5%	0.8012	6.2106	1.9968	1614.1023
	+10%	0.7999	6.2004	2.0032	1602.3211
c_s	-10%	0.7964	6.3642	1.2146	1584.2316
	-5%	0.8111	6.2164	1.1163	1599.1106
	+5%	0.8002	6.1146	0.9911	1608.2366
	+10%	0.7999	6.0246	1.0022	1610.1231
c_d	-10%	0.7914	6.2916	1.2783	1442.1662
	-5%	0.7984	6.3100	1.2669	1486.2218
	+5%	0.7916	6.3611	1.2489	1508.1162
	+10%	0.8031	6.3142	1.2013	1510.1064
c_l	-10%	0.7610	6.2642	1.2891	1481.6123
	-5%	0.7811	6.2713	1.2923	1483.1623
	+5%	0.7984	6.3411	1.3216	1506.2842
	+10%	0.7999	6.3942	1.3066	1510.2899

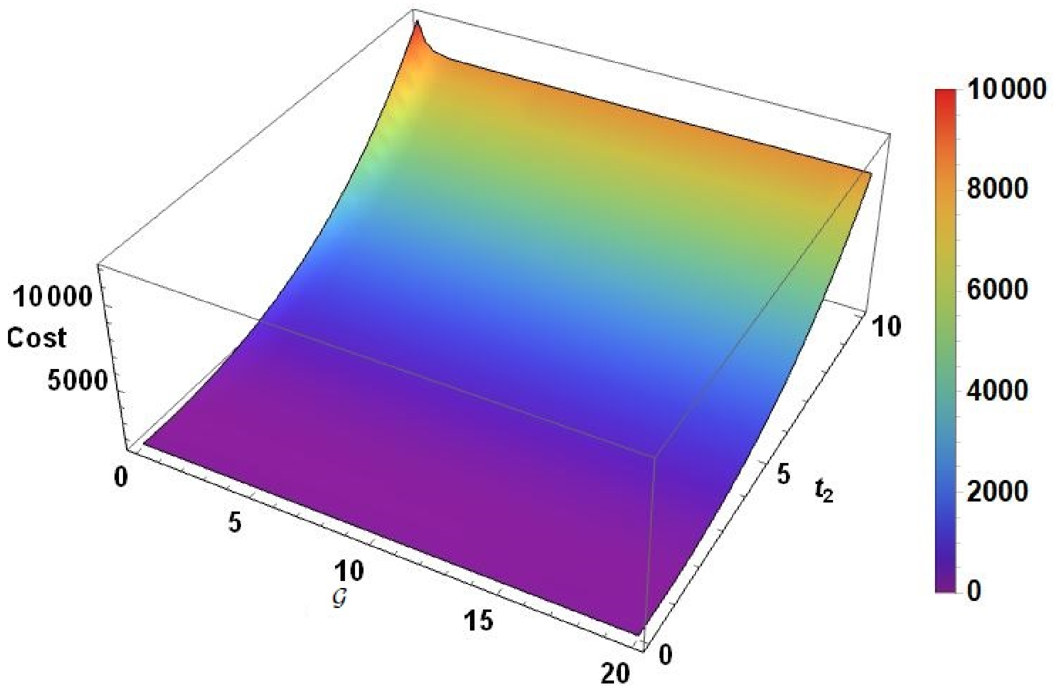

Figure 4: TIC **convexity with respect to** t_2 **and** $\mathcal{G}$

Parameters	**% change**	t_2^*	$\mathcal{T}^*$	$\mathcal{G}^*$	TIC^*
χ	-10%	0.7914	6.3916	1.2436	1584.4216
	-5%	0.7926	6.3412	1.2416	1608.1163
	+5%	0.7999	6.3816‘	1.2812	1610.1823
	+10%	0.8104	6.4102	1.2916	1600.0246

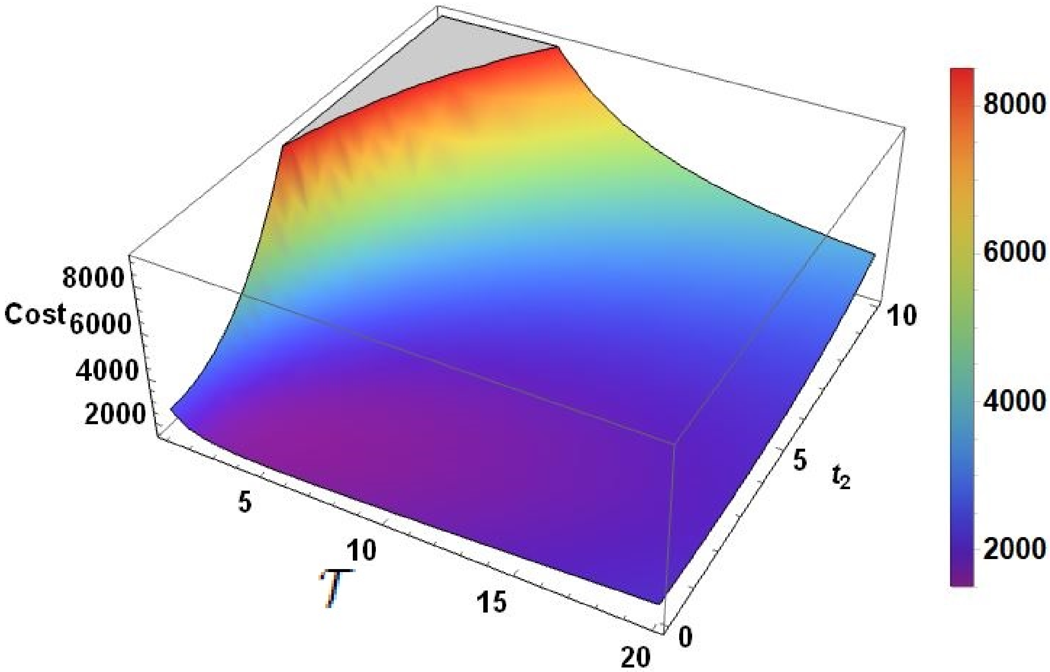

Figure 5: TIC **convexity with respect to** t_2 **and** $\mathcal{T}$**,** $(\mathcal{G} \neq 0)$

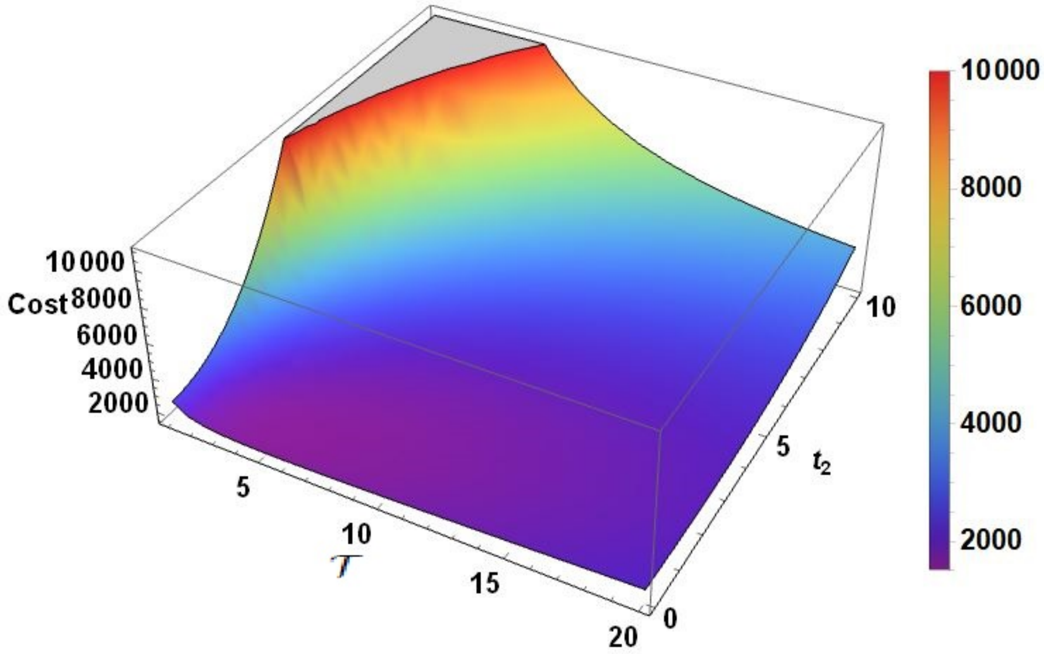

Figure 6: TIC **convexity with respect to** t_2 **and** $\mathcal{T}$**,** $(\mathcal{G} = 0)$

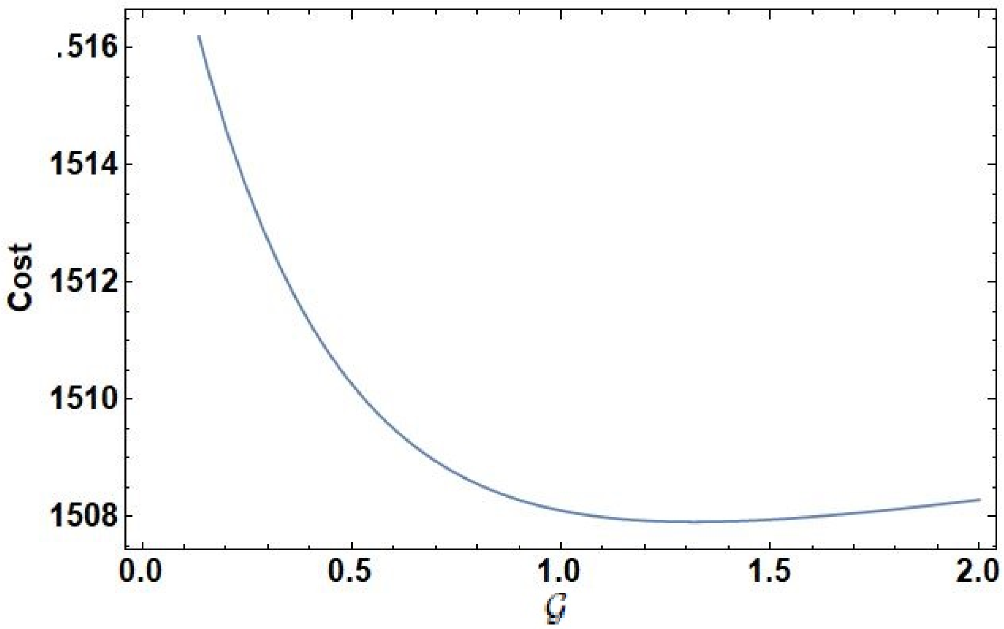

Figure 7: TIC **associated with investment in Green Technology**

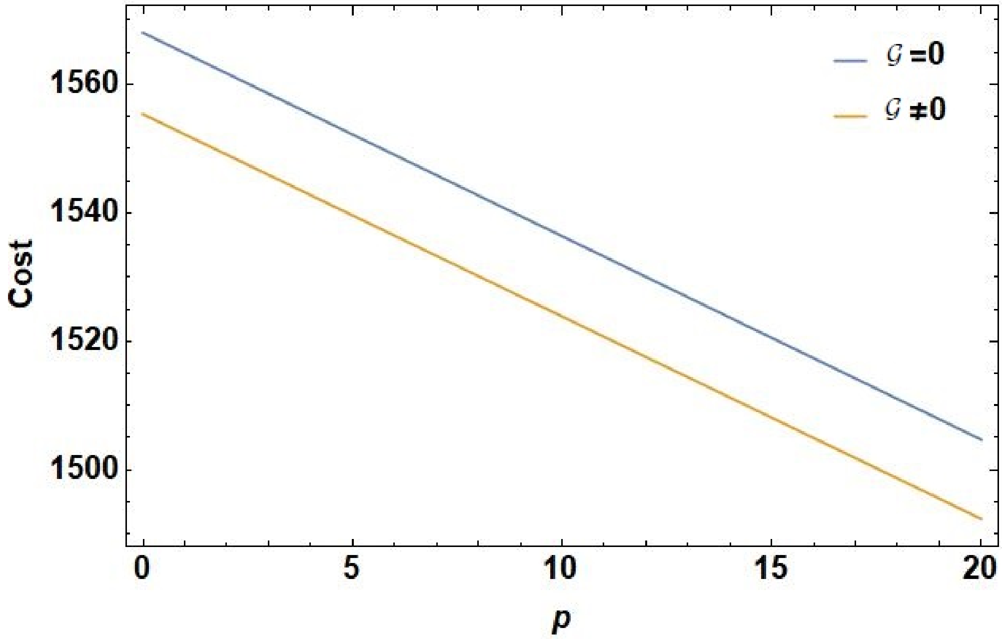

Figure 8: TIC **associated with selling price**

6. Conclusions

We developed an inventory model that is well-suited for high-demand scenarios due to its practical features. These include green technology investments to reduce carbon emissions and a prepayment facility structured in equal installments up to n times. The holding cost parameter is entirely time-dependent. Additionally, the model incorporates a two-warehouse system for storing a larger quantity of deteriorating goods. The demand rate function is influenced by both selling price and time sensitivity.

In this study, we analyzed an inventory system with two storage facilities: an own warehouse (OW) and a rented warehouse (RW). Building on this framework, we optimized a two-warehouse inventory model that integrates green technology investments, deteriorating items with carbon emissions, and selling price considerations. The demand rate in this model depends on selling price, and we also account for partial backlogging.

The primary objective of this approach is to maximize overall cycle time and cost efficiency while investing in green technology. Optimizing these parameters enables businesses to manage deteriorating inventory more effectively while reducing the environmental impact of deterioration. The proposed model is solved using an algorithm implemented in Mathematica software. A numerical example is provided to demonstrate the model's application, followed by a sensitivity analysis. Finally, a graphical illustration of the optimal solution is presented.

This model has significant managerial implications. Business managers must recognize the importance of green marketing and integrate sustainability into their supply chain, products, and services. Implementing green technologies and sustainable policies in production, transportation, and consumption is essential for long-term societal development. Reducing carbon emissions is a key factor in driving investments in green technologies. This scenario is relevant across various industries, including food, chemical, pharmaceutical, automotive, agriculture, horticulture, and retail. The proposed model applies to all decomposing products that align with the given demand pattern and generate carbon emissions during storage and transportation.

This model may not be applicable to all industries or product types, especially those with interval-valued or fuzzy-valued deterioration rates. Future research could explore optimizing the frequency of identical installments to minimize costs while accounting for installment-related expenses. Additionally, this model could be extended by incorporating inventory costs with interval-valued or fuzzy-valued deterioration rates.

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Conflicts of interest or competing interests

The authors declare that they have no conflicts of interest.

Informed Consent

The authors are fully aware and satisfied with the contents of the article.

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