**EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS**
2025, Vol. 18, Issue 2, Article Number 5857
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

# Structural Insights into IUP-Algebras via Intuitionistic Neutrosophic Set Theory

Kannirun Suayngam[1], Pongpun Julatha[2], Warud Nakkhasen[3], Aiyared Iampan[1,*]

[1] *Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand*
[2] *Department of Mathematics, Faculty of Science and Technology, Pibulsongkram Rajabhat University, Phitsanulok 65000, Thailand*
[3] *Department of Mathematics, Faculty of Science, Mahasarakham University, Khamriang, Kantarawichai, Maha Sarakham 44150, Thailand*

---

**Abstract.** This paper introduces and explores the concepts of intuitionistic neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals within the framework of IUP-algebras. By leveraging the principles of complement, characteristic and level subsets, we present a detailed analysis of their structural properties and interrelationships. These innovative approaches not only enhance the theoretical foundations of intuitionistic neutrosophic set theory but also provide new insights into managing uncertainty in algebraic structures. The findings contribute significantly to the advancement of algebraic logic, offering novel perspectives for handling indeterminate, ambiguous and incomplete information in mathematical systems. This study lays the groundwork for future research and potential applications of intuitionistic neutrosophic IUP-algebras in areas such as decision-making, computational intelligence and information theory.

**2020 Mathematics Subject Classifications**: 03G25, 03E72, 03B60

**Key Words and Phrases**: IUP-algebra, intuitionistic neutrosophic set, intuitionistic neutrosophic IUP-subalgebra, intuitionistic neutrosophic IUP-ideal, intuitionistic neutrosophic IUP-filter, intuitionistic neutrosophic strong IUP-ideal

---

## 1. Introduction

In 1965, Zadeh [16] introduced the concept of fuzzy set (FS) theory as a means to manage vague and uncertain information, addressing limitations inherent in classical set theory. This groundbreaking framework proved to be both practical and significant, particularly for scenarios involving incomplete or ambiguous data where truth and falsehood cannot be definitively determined. Fuzzy set theory provided a more nuanced approach to representing ambiguity, thereby facilitating more effective modeling and analysis of

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v18i2.5857

*Email addresses:* kannirun.s@gmail.com (K. Suayngam), pongpun.j@psru.ac.th (P. Julatha), warud.n@msu.ac.th (W. Nakkhasen), aiyared.ia@up.ac.th (A. Iampan)

 

uncertainty-laden information. Building on this foundation, Atanassov [1] in 1986 proposed the concept of intuitionistic fuzzy sets (IFSs), an extension of FS theory that introduced a non-membership degree alongside the membership degree. These two degrees collectively contribute to the indeterminacy degree, capturing a more comprehensive measure of uncertainty. This enhanced framework proved especially adept at addressing decision-making and analytical challenges in environments characterized by complex uncertainties or ignorance, as it accounts for ambiguity arising from both membership and non-membership. Later, in 1995, Smarandache [11] advanced the field further with the introduction of neutrosophic sets (NSs), a powerful generalization of fuzzy set theory. NSs incorporate three independent components: degrees of truth, degrees of falsehood and degrees of indeterminacy, offering a flexible and robust tool for analyzing and making decisions under highly uncertain conditions. By explicitly modeling these three dimensions, the NS framework provides unparalleled utility in addressing complex, ambiguous and incomplete data scenarios.

In 2022, Iampan [9] introduced a novel algebraic framework termed IUP-algebra, representing a sophisticated and abstract extension of algebraic logic. This framework encompasses four foundational subsets: IUP-subalgebras, IUP-filters, IUP-ideals and strong IUP-ideals, along with the concept of homomorphism in IUP-algebras. The flexibility of IUP-algebras allows for integration with diverse mathematical concepts, fostering the generation of new theoretical insights and practical applications. Subsequent research has significantly expanded the theoretical underpinnings and applications of IUP-algebras. In 2023, Chanmanee et al. [8] introduced the concept of the external direct product, analyzing its implications for special subsets of IUP-algebras. They further developed the notion of weak direct products and established fundamental theorems concerning (anti-)IUP-homomorphisms within this context. Later, Chanmanee et al. [7] extended these ideas by investigating direct products for infinite families of IUP-algebras, thereby formalizing the concept of DIUP-algebras and proposing the innovative framework of weak direct product DIUP-algebras, enhancing the structural depth of the theory. Building on these advancements, Kuntama et al. [10] in 2024 applied FS theory to IUP-algebras, introducing fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals and fuzzy strong IUP-ideals. They examined the intricate relationships between these subsets and characteristic functions while also exploring the concepts of prime subsets and prime fuzzy subsets. Their work introduced upper and lower $t$-level (strong) subsets, providing novel tools for analyzing FSs in IUP-algebras. Suayngam et al. [15] significantly enriched the theoretical framework of IUP-algebras by integrating IFSs. Their work introduced intuitionistic fuzzy IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals alongside an in-depth exploration of their fundamental properties and the intricate relationships involving upper and lower $t$-level subsets. Building on this foundation, they [12] further expanded the framework by incorporating NSs, proposing neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals. These advancements provided comprehensive conditions for NSs to conform to these structures, alongside an analysis of their interactions with level subsets, thereby illustrating the adaptability of IUP-algebras in managing uncertainty. Moreover, they [13] extended their work to include Fermatean fuzzy sets, introducing Fermatean fuzzy IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals. This study advanced the algebraic understanding of Fermatean fuzzy structures within the IUP-algebra framework. Additionally, they [14] explored the application of Pythagorean fuzzy sets to IUP-algebras, presenting Pythagorean fuzzy IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals. The analysis revealed that Pythagorean fuzzy IUP-ideals and subalgebras serve as generalizations of Pythagorean fuzzy strong IUP-ideals within IUP-algebras, with the latter constrained to constant Pythagorean fuzzy sets. This body of research underscores the progressive evolution of IUP-algebras as a versatile mathematical framework, capable of accommodating and addressing complex forms of uncertainty through the integration of diverse fuzzy set theories.

In 2017, the concept of NSs was extended through the integration of IFS theory, giving rise to the intuitionistic neutrosophic set (INS), as introduced by Bhowmik and Pal [2]. This novel framework combines the three fundamental degrees of truth, falsehood and uncertainty from NS theory with the IFS principle, which restricts the sum of these degrees to be less than or equal to 2. Such an approach enhances the ability to model and analyze high degrees of uncertainty and ambiguity, making it particularly valuable for complex decision-making and data analysis scenarios. The significance of INSs has been widely acknowledged, spurring further exploration and application in various domains. For instance, in 2010, Bhowmik and Pal [3] introduced refined definitions for operations such as complement, union and intersection within INSs. They also examined relationships between INSs, identifying four specialized types of relations and studying their properties. Building on this foundation, Broumi et al. [4] in 2013 expanded the application of INSs by incorporating them into soft set theory, thereby defining intuitionistic neutrosophic soft sets (INSS). Their work established operations and definitions specific to INSS, creating a basis for further theoretical developments. Later that year, Broumi and Smarandache [5] proposed additional operations on INSS, demonstrating key interconnections and results that elucidate the properties of these operations. In 2014, Broumi et al. [6] extended INS applications to ring theory, introducing the notion of intuitionistic neutrosophic soft sets over rings. Their study analyzed fundamental properties and defined operations such as intersection, union, AND and OR, as well as the product of two INSS over rings. This body of work highlights the adaptability and utility of INSs in addressing complex algebraic structures and uncertainty-rich environments.

## 2. Preliminaries

Algebraic structures have long been a cornerstone of mathematical theory, offering robust frameworks for solving complex problems and modeling abstract relationships. Among these, the IUP-algebra emerges as a sophisticated and highly versatile system, first introduced as an extension of classical algebraic logic. This innovative structure integrates unique axioms that enable the study of operations and relations under specific constraints, making it a powerful tool for exploring uncertainty, symmetry and interdependence in algebraic contexts.

In this section, we delve into the concept of IUP-algebra by defining its fundamental components, which consist of three primary axioms that govern its structure. Additionally, we identify and elaborate on four special subsets—IUP-subalgebra, IUP-filter, IUP-ideal and strong IUP-ideal—which form the backbone of further theoretical developments. These subsets serve as critical tools for analyzing and categorizing the behaviors and properties of IUP-algebraic systems, setting the stage for advanced research and applications in the subsequent topics. The detailed content is organized as follows:

**Definition 1.** *[9] An algebra $X = (X; \cdot, 0)$ of type $(2, 0)$ is called an IUP-algebra, where $X$ is a nonempty set, $\cdot$ is a binary operation on $X$ and $0$ is a fixed element of $X$ if it satisfies the following axioms:*

$$(\forall x \in X)(0 \cdot x = x), \tag{IUP-1}$$

$$(\forall x \in X)(x \cdot x = 0), \tag{IUP-2}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot (x \cdot z) = y \cdot z). \tag{IUP-3}$$

For the sake of simplicity and clarity, we will denote $X$ as an IUP-algebra, expressed in the form $X = (X; \cdot, 0)$, unless stated otherwise.

**Example 1.** *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 3 | 0 | 5 | 1 | 2 | 4 |
| 2 | 5 | 2 | 0 | 4 | 1 | 3 |
| 3 | 1 | 3 | 4 | 0 | 5 | 2 |
| 4 | 4 | 5 | 3 | 2 | 0 | 1 |
| 5 | 2 | 4 | 1 | 5 | 3 | 0 |

*Then $X = (X; \cdot, 0)$ is an IUP-algebra.*

**Example 2.** *[9] Let $(G; \cdot, \mathbf{e})$ be a group where every element is self-inverse, meaning that for all $x \in G$, $x \cdot x = \mathbf{e}$. Under this condition, $(G; \cdot, \mathbf{e})$ satisfies the axioms required for it to be classified as an IUP-algebra. This structural property highlights the inherent symmetry and unique characteristics of self-inverse elements, which play a fundamental role in defining the algebraic operations and logical relationships within the framework of IUP-algebras.*

**Example 3.** *[9] Let $X$ be a set and $\mathcal{P}(X)$ means the power set of $X$. It follows from Example 2 that $(\mathcal{P}(X); \triangle, \emptyset)$ is an IUP-algebra where the binary operation $\triangle$ is defined as the symmetric difference of any two sets.*

**Example 4.** *[9] Let $(G; \cdot, \mathbf{e})$ be a group with the identity element $\mathbf{e}$. Define a binary operation $\bullet$ on $G$ by:*

$$(\forall x, y \in G)(x \bullet y = y \cdot x^{-1}). \tag{2.1}$$

*Then $(G; \bullet, \mathbf{e})$ is an IUP-algebra.*

**Proposition 1.** *[9] In an IUP-algebra $X = (X; \cdot, 0)$, the following assertions are valid (see [9]).*

$$(\forall x, y \in X)((x \cdot 0) \cdot (x \cdot y) = y), \tag{2.2}$$
$$(\forall x \in X)((x \cdot 0) \cdot (x \cdot 0) = 0), \tag{2.3}$$
$$(\forall x, y \in X)((x \cdot y) \cdot 0 = y \cdot x), \tag{2.4}$$
$$(\forall x \in X)((x \cdot 0) \cdot 0 = x), \tag{2.5}$$
$$(\forall x, y \in X)(x \cdot ((x \cdot 0) \cdot y) = y), \tag{2.6}$$
$$(\forall x, y \in X)(((x \cdot 0) \cdot y) \cdot x = y \cdot 0), \tag{2.7}$$
$$(\forall x, y, z \in X)(x \cdot y = x \cdot z \Leftrightarrow y = z), \tag{2.8}$$
$$(\forall x, y \in X)(x \cdot y = 0 \Leftrightarrow x = y), \tag{2.9}$$
$$(\forall x \in X)(x \cdot 0 = 0 \Leftrightarrow x = 0), \tag{2.10}$$
$$(\forall x, y, z \in X)(y \cdot x = z \cdot x \Leftrightarrow y = z), \tag{2.11}$$
$$(\forall x, y \in X)(x \cdot y = y \Rightarrow x = 0), \tag{2.12}$$
$$(\forall x, y, z \in X)((x \cdot y) \cdot 0 = (z \cdot y) \cdot (z \cdot x)), \tag{2.13}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{2.14}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (x \cdot z) \cdot (y \cdot z) = 0), \tag{2.15}$$
$$\text{the right and the left cancellation laws hold.} \tag{2.16}$$

**Definition 2.** *[9] A nonempty subset $S$ of $X$ is called*

(*i*) *an IUP-subalgebra of $X$ if it satisfies the following condition:*

$$(\forall x, y \in S)(x \cdot y \in S) \tag{2.17}$$

(*ii*) *an IUP-filter of $X$ if it satisfies the following conditions:*

$$\text{the constant } 0 \text{ of } X \text{ is in } S, \tag{2.18}$$
$$(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S) \tag{2.19}$$

(*iii*) *an IUP-ideal of $X$ if it satisfies the condition* (2.18) *and the following condition:*

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S) \tag{2.20}$$

(*iv*) *a strong IUP-ideal of $X$ if it satisfies the following condition:*

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S) \tag{2.21}$$

According to [9], the concept of IUP-filters extends and generalizes the notions of both IUP-ideals and IUP-subalgebras, while IUP-ideals and IUP-subalgebras themselves serve as generalizations of strong IUP-ideals. In an IUP-algebra $X$, it is observed that strong IUP-ideals coincide with $X$ itself. A visual representation of these special subsets and their interrelationships is provided in Figure 1, offering a clear and intuitive understanding of the subset hierarchy within the framework of IUP-algebras.

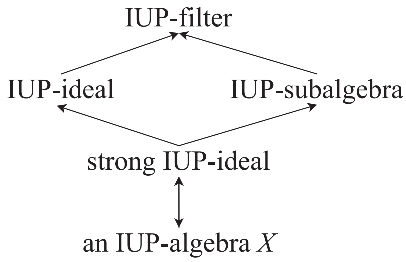


Figure 1: Special subsets of IUP-algebras

## 3. Main results

In this section, we will apply the concept of INSs to IUP-algebras, which will result in the creation of four special subsets. Additionally, we will explore the various properties of these special subsets, which will be further elaborated upon in this section. Before we begin, for all $a, b \in \mathbb{R}$ we will define $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$ for the sake of ease in reading.

**Definition 3.** *[2] An element $x$ of $X$ is called significant with respect to a neutrosophic set (NS) $\mathcal{A}$ of $X$ if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $\mathcal{A}_T$ or $\mathcal{A}_I$ or $\mathcal{A}_F \geq 0.5$. Otherwise, we call it insignificant. Also, for the NS, the truth-membership, indeterminacy-membership and falsity-membership can not be significant.*

*We define an intuitionistic neutrosophic set (INS) by*

$$\psi = \{(x, \psi_T(x), \psi_I(x), \psi_F(x)) \mid x \in X\}, \tag{3.1}$$

*where $\psi_T(x) \wedge \psi_F(x) \leq 0.5, \psi_T(x) \wedge \psi_I(x) \leq 0.5$ and $\psi_F(x) \wedge \psi_I(x) \leq 0.5$ with the condition $0 \leq \psi_T(x) + \psi_I(x) + \psi_F(x) \leq 2$.*

Before we delve into the study complement of an FS $f$, we will introduce the fundamental symbols used in the study complement of $f$ as follows:

$$\overline{f(x)} = 1 - f(x).$$

**Definition 4.** *Let $\psi$ be an INS in a nonempty set $X$. The INS $\overline{\psi}$ is defined by*

$$(\forall x \in X)(\overline{\psi}_T(x) = \overline{\psi_I(x)} \wedge \psi_T(x)), \tag{3.2}$$
$$(\forall x \in X)(\overline{\psi}_I(x) = \psi_I(x)), \tag{3.3}$$
$$(\forall x \in X)(\overline{\psi}_F(x) = \overline{\psi_I(x)} \wedge \psi_F(x)) \tag{3.4}$$

*is called the complement of $\psi$ in $X$.*

**Definition 5.** *An INS $\psi$ in $X$ is called an intuitionistic neutrosophic IUP-subalgebra (INIUP-subalgebra) of $X$ if it satisfies the following conditions:*

$$(\forall x, y \in X)(\psi_T(x \cdot y) \geq (\psi_T(x) \wedge \psi_T(y)) \vee 0.5), \tag{3.5}$$

$$(\forall x, y \in X)(\psi_I(x \cdot y) \leq (\psi_I(x) \vee \psi_I(y)) \wedge 0.5), \tag{3.6}$$
$$(\forall x, y \in X)(\psi_F(x \cdot y) \geq (\psi_F(x) \wedge \psi_F(y)) \vee 0.5). \tag{3.7}$$

**Definition 6.** *An INS $\psi$ in $X$ is called an intuitionistic neutrosophic IUP-ideal (INIUP-ideal) of $X$ if it satisfies the following conditions:*

$$(\forall x \in X)(\psi_T(0) \geq \psi_T(x)), \tag{3.8}$$
$$(\forall x \in X)(\psi_I(0) \leq \psi_I(x)), \tag{3.9}$$
$$(\forall x \in X)(\psi_F(0) \geq \psi_F(x)), \tag{3.10}$$
$$(\forall x, y, z \in X)(\psi_T(x \cdot z) \geq (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5), \tag{3.11}$$
$$(\forall x, y, z \in X)(\psi_I(x \cdot z) \leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5), \tag{3.12}$$
$$(\forall x, y, z \in X)(\psi_F(x \cdot z) \geq (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5). \tag{3.13}$$

**Definition 7.** *An INS $\psi$ in $X$ is called an intuitionistic neutrosophic IUP-filter (INIUP-filter) of $X$ if it satisfies the conditions* (3.8)*,* (3.9) *and* (3.10) *and the following conditions:*

$$(\forall x, y \in X)(\psi_T(y) \geq (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5), \tag{3.14}$$
$$(\forall x, y \in X)(\psi_I(y) \leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5), \tag{3.15}$$
$$(\forall x, y \in X)(\psi_F(y) \geq (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5). \tag{3.16}$$

**Definition 8.** *An INS $\psi$ in $X$ is called an intuitionistic neutrosophic strong IUP-ideal (INSIUP-ideal) of $X$ if it satisfies the following conditions:*

$$(\forall x, y \in X)(\psi_T(x \cdot y) \geq \psi_T(y)), \tag{3.17}$$
$$(\forall x, y \in X)(\psi_I(x \cdot y) \leq \psi_I(y)), \tag{3.18}$$
$$(\forall x, y \in X)(\psi_F(x \cdot y) \geq \psi_F(y)). \tag{3.19}$$

**Lemma 1.** *Every intuitionistic neutrosophic IUP-subalgebra of $X$ satisfies the conditions* (3.8)*,* (3.9) *and* (3.10)*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Let $x \in X$. Then

$$\begin{aligned} \psi_T(0) &= \psi_T(x \cdot x) && \text{(by (IUP-2))} \\ &\geq (\psi_T(x) \wedge \psi_T(x)) \vee 0.5 && \text{(by (3.5))} \\ &= \psi_T(x) \vee 0.5 \\ &\geq \psi_T(x), \end{aligned}$$

$$\begin{aligned} \psi_I(0) &= \psi_I(x \cdot x) && \text{(by (IUP-2))} \\ &\leq (\psi_I(x) \vee \psi_I(x)) \wedge 0.5 && \text{(by (3.6))} \\ &= \psi_I(x) \wedge 0.5 \end{aligned}$$

$$\leq \psi_I(x),$$

$$\begin{aligned}
\psi_F(0) &= \psi_F(x \cdot x) && \text{(by (IUP-2))} \\
&\geq (\psi_F(x) \wedge \psi_F(x)) \vee 0.5 && \text{(by (3.5))} \\
&= \psi_F(x) \vee 0.5 \\
&\geq \psi_F(x).
\end{aligned}$$

Hence, it satisfies the conditions (3.8), (3.9) and (3.10).

**Lemma 2.** *Every intuitionistic neutrosophic strong IUP-ideal of $X$ satisfies the conditions* (3.8)*,* (3.9) *and* (3.10)*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$. Let $x \in X$. Then

$$\begin{aligned}
\psi_T(0) &= \psi_T(x \cdot x) && \text{(by (IUP-2))} \\
&\geq \psi_T(x), && \text{(by (3.17))}
\end{aligned}$$

$$\begin{aligned}
\psi_I(0) &= \psi_I(x \cdot x) && \text{(by (IUP-2))} \\
&\leq \psi_I(x), && \text{(by (3.18))}
\end{aligned}$$

$$\begin{aligned}
\psi_F(0) &= \psi_F(x \cdot x) && \text{(by (IUP-2))} \\
&\geq \psi_F(x). && \text{(by (3.19))}
\end{aligned}$$

Hence, it satisfies the conditions (3.8), (3.9) and (3.10).

**Theorem 1.** *An intuitionistic neutrosophic strong IUP-ideal and constant INS coincide.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$. Let $x \in X$. Then

$$\begin{aligned}
\psi_T(x) &= \psi_T((x \cdot 0) \cdot 0) && \text{(by (2.5))} \\
&\geq \psi_T(0), && \text{(by (3.17))}
\end{aligned}$$

$$\begin{aligned}
\psi_I(x) &= \psi_I((x \cdot 0) \cdot 0) && \text{(by (2.5))} \\
&\leq \psi_I(0), && \text{(by (3.18))}
\end{aligned}$$

$$\begin{aligned}
\psi_F(x) &= \psi_F((x \cdot 0) \cdot 0) && \text{(by (2.5))} \\
&\geq \psi_F(0). && \text{(by (3.19))}
\end{aligned}$$

Hence, $\psi$ is a constant INS of $X$.

Conversely, it is obvious that every constant INS of $X$ is an intuitionistic neutrosophic strong IUP-ideal.

**Example 5.** *Let* $X = \{0, 1, 2, 3, 4, 5\}$ *with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 5 | 0 | 3 | 4 | 2 | 1 |
| 2 | 3 | 2 | 0 | 5 | 1 | 4 |
| 3 | 2 | 4 | 1 | 0 | 5 | 3 |
| 4 | 4 | 3 | 5 | 1 | 0 | 2 |
| 5 | 1 | 5 | 4 | 2 | 3 | 0 |

*Then* $X$ *is an IUP-algebra. We define an INS* $\psi$ *on* $X$ *as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \end{pmatrix}$$

*Then* $\psi$ *is an intuitionistic neutrosophic strong IUP-ideal of* $X$*. Since* $\psi_F(0 \cdot 2) = \psi_F(2) = 0.3 \ngeq 0.5 = 0.3 \vee 0.5 = (0.3 \wedge 0.3) \vee 0.5 = (\psi_F(0) \wedge \psi_F(2)) \vee 0.5$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic IUP-subalgebra of* $X$*.*

**Example 6.** *Let* $X = \{0, 1, 2, 3, 4, 5\}$ *with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 5 | 0 | 4 | 2 | 1 | 3 |
| 2 | 2 | 3 | 0 | 1 | 5 | 4 |
| 3 | 4 | 2 | 5 | 0 | 3 | 1 |
| 4 | 3 | 5 | 1 | 4 | 0 | 2 |
| 5 | 1 | 4 | 3 | 5 | 2 | 0 |

*Then* $X$ *is an IUP-algebra. We define an INS* $\psi$ *on* $X$ *as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.6 & 0.6 & 0.7 & 0.7 & 0.6 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.4 & 0.4 & 0.3 & 0.3 & 0.4 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

*Then* $\psi$ *is an intuitionistic neutrosophic IUP-subalgebra of* $X$*. Since* $\psi_T(5 \cdot 3) = \psi_T(5) = 0.6 \ngeq 0.7 = \psi_T(3)$ *and* $\psi_I(5 \cdot 4) = \psi_I(2) = 0.4 \nleq 0.3 = \psi_I(4)$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic strong IUP-ideal of* $X$*.*

**Example 7.** *Let* $X = \{0, 1, 2, 3, 4, 5\}$ *with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 0 | 4 | 5 | 2 | 3 |
| 2 | 5 | 3 | 0 | 2 | 1 | 4 |
| 3 | 4 | 2 | 5 | 0 | 3 | 1 |
| 4 | 3 | 5 | 1 | 4 | 0 | 2 |
| 5 | 2 | 4 | 3 | 1 | 5 | 0 |

*Then* $X$ *is an IUP-algebra. We define an INS* $\psi$ *on* $X$ *as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.7 & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \end{pmatrix}$$

*Then* $\psi$ *is an intuitionistic neutrosophic strong IUP-ideal of* $X$*. Since* $\psi_I(4 \cdot 3) = \psi_I(4) = 0.7 \nleq 0.5 = 0.7 \wedge 0.5 = (0.7 \vee 0.7) \wedge 0.5 = (\psi_I(0) \vee \psi_I(4)) \wedge 0.5 = (\psi_I(4 \cdot 4) \vee \psi_I(4)) \wedge 0.5 = \psi_I(4 \cdot (4 \cdot 3)) \vee \psi_I(4)) \wedge 0.5$ *and* $\psi_F(0 \cdot 5) = \psi_F(5) = 0.3 \ngeq 0.5 = 0.3 \vee 0.5 = (0.3 \wedge 0.3) \vee 0.5 = (\psi_F(0) \wedge \psi_F(5)) \vee 0.5 = (\psi_F(0 \cdot 0) \wedge \psi_F(5)) \vee 0.5 = \psi_F(0 \cdot (5 \cdot 5)) \wedge \psi_F(5)) \vee 0.5$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic IUP-ideal of* $X$*.*

**Example 8.** *Let* $X = \{0, 1, 2, 3, 4, 5\}$ *with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 0 | 1 | 4 | 5 | 3 |
| 2 | 1 | 2 | 0 | 5 | 3 | 4 |
| 3 | 3 | 5 | 4 | 0 | 2 | 1 |
| 4 | 5 | 4 | 3 | 1 | 0 | 2 |
| 5 | 4 | 3 | 5 | 2 | 1 | 0 |

*Then* $X$ *is an IUP-algebra. We define an INS* $\psi$ *on* $X$ *as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0.21 & 0.21 & 0.4 & 0.4 & 0.4 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0.8 & 0.8 & 0.55 & 0.55 & 0.55 \end{pmatrix}$$

*Then* $\psi$ *is an intuitionistic neutrosophic IUP-ideal of* $X$*. Since* $\psi_I(3 \cdot 1) = \psi_I(5) = 0.4 \nleq 0.21 = \psi_I(1)$ *and* $\psi_F(5 \cdot 0) = \psi_F(4) = 0.55 \ngeq 0.1 = \psi_F(0)$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic strong IUP-ideal of* $X$*.*

**Theorem 2.** *Every intuitionistic neutrosophic IUP-ideal of $X$ is an intuitionistic neutrosophic IUP-filter of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$. By assumption, it satisfies the conditions (3.8), (3.9) and (3.10). Let $x, y \in X$. Then

$$\begin{aligned} \psi_T(y) &= \psi_T(0 \cdot y) && \text{(by (IUP-1))} \\ &\geq (\psi_T(0 \cdot (x \cdot y)) \wedge \psi_T(x)) \vee 0.5 && \text{(by (3.11))} \\ &= (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5, && \text{(by (IUP-1))} \end{aligned}$$

$$\begin{aligned} \psi_I(y) &= \psi_I(0 \cdot y) && \text{(by (IUP-1))} \\ &\leq (\psi_I(0 \cdot (x \cdot y)) \vee \psi_I(x)) \wedge 0.5 && \text{(by (3.12))} \\ &= (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5, && \text{(by (IUP-1))} \end{aligned}$$

$$\begin{aligned} \psi_F(y) &= \psi_F(0 \cdot y) && \text{(by (IUP-1))} \\ &\geq (\psi_F(0 \cdot (x \cdot y)) \wedge \psi_F(x)) \vee 0.5 && \text{(by (3.13))} \\ &= (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5. && \text{(by (IUP-1))} \end{aligned}$$

Hence, $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.

**Example 9.** *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 0 | 5 | 4 | 3 | 2 |
| 2 | 2 | 4 | 0 | 5 | 1 | 3 |
| 3 | 3 | 5 | 4 | 0 | 2 | 1 |
| 4 | 5 | 3 | 1 | 2 | 0 | 4 |
| 5 | 4 | 2 | 3 | 1 | 5 | 0 |

*Then $X$ is an IUP-algebra. We define an INS $\psi$ on $X$ as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.4 & 0.4 & 0.3 & 0.4 & 0.4 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.8 & 0.5 & 0.5 & 0.6 & 0.5 & 0.5 \end{pmatrix}$$

*Then $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$. Since $\psi_I(4 \cdot 5) = \psi_I(4) = 0.4 \not\leq 0.3 = (0.3 \vee 0.3) \wedge 0.5 = (\psi_I(3) \vee \psi_T(3)) \wedge 0.5 = (\psi_I(4 \cdot 1) \vee \psi_I(3)) \wedge 0.5 = (\psi_I(4 \cdot (3 \cdot 5)) \vee \psi_I(3)) \wedge 0.5$ and $\psi_F(4 \cdot 5) = \psi_F(4) = 0.5 \not\geq 0.6 = (0.6 \wedge 0.6) \vee 0.5 = (\psi_F(3) \wedge \psi_F(3)) \vee 0.5 = (\psi_F(4 \cdot 1) \wedge \psi_F(3)) \vee 0.5 = (\psi_F(4 \cdot (3 \cdot 5)) \wedge \psi_F(3)) \vee 0.5$. Hence, $\psi$ is not an intuitionistic neutrosophic IUP-ideal of $X$.*

**Theorem 3.** *Every intuitionistic neutrosophic IUP-subalgebra of $X$ is an intuitionistic neutrosophic IUP-filter of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. By Lemma 1, it satisfies the conditions (3.8), (3.9) and (3.10). Let $x, y \in X$. Then

$$\begin{aligned}
\psi_T(y) &= \psi_T(0 \cdot y) && \text{(by (IUP-1))}\\
&= \psi_T((x \cdot 0) \cdot (x \cdot y)) && \text{(by (IUP-3))}\\
&\geq (\psi_T(x \cdot 0) \wedge \psi_T(x \cdot y)) \vee 0.5 && \text{(by (3.5))}\\
&\geq (((\psi_T(x) \wedge \psi_T(0)) \vee 0.5) \wedge \psi_T(x \cdot y)) \vee 0.5 && \text{(by (3.5))}\\
&= ((\psi_T(x) \vee 0.5) \wedge \psi_T(x \cdot y)) \vee 0.5 && \text{(by (3.8))}\\
&\geq (\psi_T(x) \wedge \psi_T(x \cdot y)) \vee 0.5,
\end{aligned}$$

$$\begin{aligned}
\psi_I(y) &= \psi_I(0 \cdot y) && \text{(by (IUP-1))}\\
&= \psi_I((x \cdot 0) \cdot (x \cdot y)) && \text{(by (IUP-3))}\\
&\leq (\psi_I(x \cdot 0) \vee \psi_I(x \cdot y)) \wedge 0.5 && \text{(by (3.6))}\\
&\leq (((\psi_I(x) \vee \psi_I(x)) \wedge 0.5) \vee \psi_I(x \cdot y)) \wedge 0.5 && \text{(by (3.6))}\\
&= ((\psi_I(x) \wedge 0.5) \vee \psi_I(x \cdot y)) \wedge 0.5 && \text{(by (3.9))}\\
&\leq (\psi_I(x) \vee \psi_I(x \cdot y)) \wedge 0.5,
\end{aligned}$$

$$\begin{aligned}
\psi_F(y) &= \psi_F(0 \cdot y) && \text{(by (IUP-1))}\\
&= \psi_F((x \cdot 0) \cdot (x \cdot y)) && \text{(by (IUP-3))}\\
&\geq (\psi_F(x \cdot 0) \wedge \psi_F(x \cdot y)) \vee 0.5 && \text{(by (3.7))}\\
&\geq (((\psi_F(x) \wedge \psi_F(0)) \vee 0.5) \wedge \psi_F(x \cdot y)) \vee 0.5 && \text{(by (3.7))}\\
&= ((\psi_F(x) \vee 0.5) \wedge \psi_F(x \cdot y)) \vee 0.5 && \text{(by (3.10))}\\
&\geq (\psi_F(x) \wedge \psi_F(x \cdot y)) \vee 0.5.
\end{aligned}$$

Hence, $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.

**Example 10.** *[9] Let $\mathbb{R}^*$ be the set of all nonzero real numbers. Define a binary operation $\cdot$ on $\mathbb{R}^*$ by*

$$(\forall x, y \in \mathbb{R}^*)(x \cdot y = \frac{y}{x}).$$

*Then $(\mathbb{R}^*; \cdot, 1)$ is an IUP-algebra.*

**Example 11.** *From Example 10, let $P = \{x \in \mathbb{R}^* \mid x \geq 1\}$. Then $1 \in P$. Next, let $x, y, z \in \mathbb{R}^*$ be such that $x \cdot (y \cdot z) \geq 1$ and $y \geq 1$. Then $\frac{z}{yx} \geq 1$. Thus, $x \cdot z = \frac{z}{x} = (\frac{z}{yx})y \geq 1$, that is, $x \cdot z \in P$. Hence, $P$ is an IUP-ideal of $\mathbb{R}^*$. Then $P$ is an IUP-filter of $\mathbb{R}^*$. From Theorems 7 and 8, $\psi^G[^{\alpha^+, \beta^-}_{\alpha^-, \beta^+}, 0.5]$ is an intuitionistic neutrosophic IUP-ideal and an* *intuitionistic neutrosophic IUP-filter of* $\mathbb{R}^*$*. Thus,* $\psi$ *is an intuitionistic neutrosophic IUP-ideal and an intuitionistic neutrosophic IUP-filter of* $\mathbb{R}^*$*. Since* $1, 3 \in P$ *but* $3 \cdot 1 = \frac{1}{3} \in P$*, we have* $P$ *is not an IUP-subalgebra of* $\mathbb{R}^*$*. From Theorem 6, we have* $\psi^G[^{\alpha^+,\beta^-}_{\alpha^-,\beta^+}, 0.5]$ *is not an intuitionistic neutrosophic IUP-subalgebra of* $\mathbb{R}^*$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic IUP-subalgebra of* $\mathbb{R}^*$*.*

**Example 12.** *Let* $X = \{0, 1, 2, 3, 4, 5\}$ *with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 0 | 1 | 4 | 5 | 3 |
| 2 | 1 | 2 | 0 | 5 | 3 | 4 |
| 3 | 3 | 4 | 5 | 0 | 1 | 2 |
| 4 | 4 | 5 | 3 | 2 | 0 | 1 |
| 5 | 5 | 3 | 4 | 1 | 2 | 0 |

*Then* $X$ *is an IUP-algebra. We define an INS* $\psi$ *on* $X$ *as follows:*

$$\psi_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.5 & 0.8 \end{pmatrix}$$

$$\psi_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.2 \end{pmatrix}$$

$$\psi_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

*Then* $\psi$ *is an intuitionistic neutrosophic IUP-subalgebra of* $X$*. Since* $\psi_T(2 \cdot 4) = \psi_T(3) = 0.5 \ngeq 0.8 = 0.8 \vee 0.5 = (0.9 \wedge 0.8) \vee 0.5 = (\psi_T(0) \wedge \psi_T(5)) \vee 0.5 = (\psi_T(2 \cdot (5 \cdot 4)) \wedge \psi_T(5)) \vee 0.5$ *and* $\psi_I(2 \cdot 4) = \psi_I(3) = 0.4 \nleq 0.2 = 0.2 \wedge 0.5 = (0 \vee 0.2) \wedge 0.5 = (\psi_I(0) \vee \psi_I(5)) \wedge 0.5 = (\psi_I(2 \cdot (5 \cdot 4)) \vee \psi_I(5)) \wedge 0.5$*. Hence,* $\psi$ *is not an intuitionistic neutrosophic IUP-ideal of* $X$*.*

**Theorem 4.** *If* $\psi$ *is an intuitionistic neutrosophic IUP-subalgebra of* $X$ *satisfying the following condition:*

$$(\forall x, y \in X) \left( x \cdot y \neq 0 \Rightarrow \begin{cases} \psi_T(x) \geq \psi_T(y) \\ \psi_I(x) \leq \psi_I(y) \\ \psi_F(x) \geq \psi_F(y) \end{cases} \right), \tag{3.20}$$

*then* $\psi$ *is an intuitionistic neutrosophic strong IUP-ideal of* $X$*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$ satisfying the condition (3.20). Let $x, y \in X$.

Case 1: Suppose $x \cdot y = 0$. Then

$$\psi_T(x \cdot y) = \psi_T(0)$$

$$\geq \psi_T(y), \qquad \text{(by (3.8))}$$

$$\begin{aligned}\psi_I(x \cdot y) &= \psi_I(0)\\ &\leq \psi_I(y), &\text{(by (3.9))}\end{aligned}$$

$$\begin{aligned}\psi_F(x \cdot y) &= \psi_F(0)\\ &\geq \psi_F(y). &\text{(by (3.10))}\end{aligned}$$

Case 2: Suppose $x \cdot y \neq 0$. Then

$$\begin{aligned}\psi_T(x \cdot y) &\geq \psi_T(x) \wedge \psi_T(y) &\text{(by (3.5))}\\ &= \psi_T(y),\end{aligned}$$

$$\begin{aligned}\psi_I(x \cdot y) &\leq \psi_I(x) \vee \psi_I(y) &\text{(by (3.9))}\\ &= \psi_I(y),\end{aligned}$$

$$\begin{aligned}\psi_F(x \cdot y) &\geq \psi_F(x) \wedge \psi_F(y) &\text{(by (3.10))}\\ &= \psi_F(y).\end{aligned}$$

Hence, $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$.

**Theorem 5.** *If $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$ satisfying the following condition:*

$$(\forall x, y, z \in X) \begin{pmatrix} \psi_T(y \cdot (x \cdot z)) = \psi_T(x \cdot (y \cdot z)) \\ \psi_I(y \cdot (x \cdot z)) = \psi_I(x \cdot (y \cdot z)) \\ \psi_F(y \cdot (x \cdot z)) = \psi_F(x \cdot (y \cdot z)) \end{pmatrix}, \tag{3.21}$$

*then $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$ satisfying the condition (3.21). By assumption, it satisfies the conditions (3.8), (3.9) and (3.10). Let $x, y \in X$. Then

$$\begin{aligned}\psi_T(x \cdot z) &\geq (\psi_T(y \cdot (x \cdot z)) \wedge \psi_T(y)) \vee 0.5 &\text{(by (3.14))}\\ &= (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5,\end{aligned}$$

$$\begin{aligned}\psi_I(x \cdot z) &\leq (\psi_I(y \cdot (x \cdot z)) \vee \psi_I(y)) \wedge 0.5 &\text{(by (3.15))}\\ &= (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5,\end{aligned}$$

$$\psi_F(x \cdot z) \geq (\psi_F(y \cdot (x \cdot z)) \wedge \psi_F(y)) \vee 0.5 \qquad \text{(by (3.16))}$$

$$= (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5.$$

Hence, $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.

For any fixed numbers $\alpha^+, \alpha^- \in [0.5, 1]$ and $\beta^+, \beta^- \in [0, 0.5)$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-$ and a nonempty subset $G$ of $X$, an INS $\psi^G = (X, \psi_T^G[{}^{\alpha^+}_{\alpha^-}], \psi_I^G[{}^{\beta^-}_{\beta^+}], \psi_F^G[0.5])$ in $X$ where $\psi_T^G[{}^{\alpha^+}_{\alpha^-}]$, $\psi_I^G[{}^{\beta^-}_{\beta^+}]$ and $\psi_F^G[0.5]$ are function on $X$ which are given as follows:

$$\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G \\ \alpha^- & \text{otherwise,} \end{cases}$$
$$\psi_I^G[{}^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G \\ \beta^+ & \text{otherwise,} \end{cases}$$
$$\psi_F^G[0.5](x) = \quad 0.5 \quad \text{for all } x \in X.$$

**Lemma 3.** *Let $G$ be a nonempty subset of $X$. Then the constant $0$ of $X$ is in $G$ if and only if the characteristic INS $\psi^G$ satisfies the conditions (3.8), (3.9) and (3.10).*

*Proof.* Assume that $0 \in G$. Then $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+$, $\psi_I^G[{}^{\beta^-}_{\beta^+}](0) = \beta^-$ and $\psi_F^G[0.5](0) = 0.5$. Thus, $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+ \geq \psi_T^G[{}^{\alpha^+}_{\alpha^-}](x)$, $\psi_I^G[{}^{\beta^-}_{\beta^+}](0) = \beta^- \leq \psi_I^G[{}^{\beta^-}_{\beta^+}](x)$ and $\psi_F^G[0.5](0) = 0.5 \geq \psi_F^G[0.5](x)$ for all $x \in X$, that is, $\psi^G$ satisfies the conditions (3.8), (3.9) and (3.10).

Conversely, assume that $\psi^G$ satisfies the conditions (3.8), (3.9) and (3.10). Then $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](0) \geq \psi_T^G[{}^{\alpha^+}_{\alpha^-}](x)$ for all $x \in X$. Since $G$ is a nonempty subset of $X$, we let $a \in G$. Then $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](0) \geq \psi_T^G[{}^{\alpha^+}_{\alpha^-}](a) = \alpha^+$, so $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](0) = \alpha^+$. Hence, $0 \in G$.

**Theorem 6.** *A nonempty subset $G$ is an IUP-subalgebra of $X$ if and only if the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.*

*Proof.* Assume that $G$ is an IUP-subalgebra of $X$. Let $x, y \in X$. Then

Case 1: Suppose $x, y \in G$. Then $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x) = \alpha^+$ and $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Since $G$ be an IUP-subalgebra of $X$, we have $x \cdot y \in G$. Thus, $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ \geq \alpha^+ \vee 0.5 = (\alpha^+ \wedge \alpha^+) \vee 0.5 = (\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x) \wedge \psi_T^G[{}^{\alpha^+}_{\alpha^-}](y)) \vee 0.5$.

Case 2: Suppose $x \notin G$ or $y \notin G$. Then $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x) = \alpha^-$ or $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](y) = \alpha^-$. Thus, $\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \alpha^- \geq \alpha^- \vee 0.5 = (\psi_T^G[{}^{\alpha^+}_{\alpha^-}](x) \wedge \psi_T^G[{}^{\alpha^+}_{\alpha^-}](y)) \vee 0.5$.

Case 1': Suppose $x, y \in G$. Then $\psi_I^G[{}^{\beta^-}_{\beta^+}](x) = \beta^-$ and $\psi_I^G[{}^{\beta^-}_{\beta^+}](y) = \beta^-$. Since $G$ be an IUP-subalgebra of $X$, we have $x \cdot y \in G$. Thus, $\psi_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- \leq \beta^- \wedge 0.5 = (\beta^- \vee \beta^-) \wedge 0.5 = (\psi_I^G[{}^{\beta^-}_{\beta^+}](x) \vee \psi_I^G[{}^{\beta^-}_{\beta^+}](y)) \wedge 0.5$.

Case 2': Suppose $x \notin G$ or $y \notin G$. Then $\psi_I^G[{}^{\beta^-}_{\beta^+}](x) = \beta^+$ or $\psi_I^G[{}^{\beta^-}_{\beta^+}](y) = \beta^+$. Thus, $\psi_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y) \leq \beta^+ \leq \beta^+ \wedge 0.5 = (\psi_I^G[{}^{\beta^-}_{\beta^+}](x) \vee \psi_I^G[{}^{\beta^-}_{\beta^+}](y)) \wedge 0.5$.

It is obvious to prove that $\psi_F^G[0.5]$ satisfied the condition (3.7).

Hence, the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.

Conversely, assume that the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Let $x, y \in G$. Then $\psi_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+$ and $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. By the condition (3.5), we have $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot y) \geq (\psi_T^G[^{\alpha^+}_{\alpha^-}](x) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](y)) \vee 0.5 = (\alpha^+ \wedge \alpha^+) \vee 0.5 = \alpha^+ \vee 0.5 \geq \alpha^+$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot y) = \alpha^+$, that is, $x\cdot y \in G$. Hence, $G$ is an IUP-subalgebra of $X$.

**Theorem 7.** *A nonempty subset $G$ is an IUP-ideal of $X$ if and only if the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-ideal of $X$.*

*Proof.* Assume that $G$ is an IUP-ideal of $X$. Since $0 \in G$, it follows from Lemma 3 that $\psi_T^G[^{\alpha^+}_{\alpha^-}]$, $\psi_I^G[^{\beta^-}_{\beta^+}]$ and $\psi_F^G[0.5]$ satisfy the conditions (3.8), (3.9) and (3.10), respectively. Next, let $x, y, z \in X$.

Case 1: Suppose $x\cdot(y\cdot z) \in G$ and $y \in G$. Since $G$ is an IUP-ideal of $X$, we have $x\cdot z \in G$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) = \alpha^+ \geq \alpha^+ \vee 0.5 = (\alpha^+ \wedge \alpha^+) \vee 0.5 = (\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](y)) \vee 0.5$.

Case 2: Suppose $x\cdot(y\cdot z) \notin G$ or $y \notin G$. Then $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) = \alpha^-$ or $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^-$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) \geq \alpha^- \geq \alpha^- \vee 0.5 = (\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](y)) \vee 0.5$.

Case 1': Suppose $x\cdot(y\cdot z) \in G$ and $y \in G$. Since $G$ is an IUP-ideal of $X$, we have $x\cdot z \in G$. Thus, $\psi_I^G[^{\beta^-}_{\beta^+}](x\cdot z) = \beta^- \leq \beta^- \wedge 0.5 = (\beta^- \vee \beta^-) \wedge 0.5 = (\psi_I^G[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)) \vee \psi_I^G[^{\beta^-}_{\beta^+}](y)) \wedge 0.5$.

Case 2': Suppose $x\cdot(y\cdot z) \notin G$ or $y \notin G$. Then $\psi_I^G[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)) = \beta^+$ or $\psi_I^G[^{\beta^-}_{\beta^+}](y) = \beta^+$. Thus, $\psi_I^G[^{\beta^-}_{\beta^+}](x\cdot z) \leq \beta^+ \leq \beta^+ \wedge 0.5 = (\psi_I^G[^{\beta^-}_{\beta^+}](x\cdot(y\cdot z)) \vee \psi_I^G[^{\beta^-}_{\beta^+}](y)) \wedge 0.5$.

It is obvious to prove that $\psi_F^G[0.5]$ satisfied the condition (3.13).

Hence, $\psi^G$ is an intuitionistic neutrosophic IUP-ideal of $X$.

Conversely, assume that the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-ideal of $X$. Since $\psi_T^G[^{\alpha^+}_{\alpha^-}]$ satisfies the condition (3.8), it follows from Lemma 3 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x\cdot(y\cdot z) \in G$ and $y \in G$. Then $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) = \alpha^+$ and $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus, $\min\{\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)), \psi_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+$. By the condition (3.11), we have $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) \geq (\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot(y\cdot z)) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](y)) \vee 0.5 = (\alpha^+ \wedge \alpha^+) \vee 0.5 = \alpha^+ \vee 0.5 \geq \alpha^+$, that is, $\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot z) = \alpha^+$. Hence, $x\cdot z \in G$, so $G$ is an IUP-ideal of $X$.

**Theorem 8.** *A nonempty subset $G$ is an IUP-filter of $X$ if and only if the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-filter of $X$.*

*Proof.* Assume that $G$ is an IUP-filter of $X$. Since $0 \in G$, it follows from Lemma 3 that $\psi_T^G[^{\alpha^+}_{\alpha^-}]$, $\psi_I^G[^{\beta^-}_{\beta^+}]$ and $\psi_F^G[0.5]$ satisfy the conditions (3.8), (3.9) and (3.10), respectively. Next, let $x, y \in X$.

Case 1: Suppose $x\cdot y \in G$ and $x \in G$. Since $G$ is an IUP-filter of $X$, we have $y \in G$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq \alpha^+ \vee 0.5 = (\alpha^+ \wedge \alpha^+) \vee 0.5 = (\psi_T^G[^{\alpha^+}_{\alpha^-}](x\cdot y) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](x)) \vee 0.5$.

Case 2: Suppose $x \cdot y \notin G$ or $x \notin G$. Then $\psi_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^-$ or $\psi_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^-$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) \geq \alpha^- \geq \alpha^- \vee 0.5 = (\psi_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](x)) \vee 0.5$.

Case 1': Suppose $x \cdot y \in G$ and $x \in G$. Since $G$ is an IUP-filter of $X$, we have $y \in G$. Thus, $\psi_I^G[^{\beta^-}_{\beta^+}](y) = \beta^- \leq \beta^- \wedge 0.5 = (\beta^- \vee \beta^-) \wedge 0.5 = (\psi_I^G[^{\beta^-}_{\beta^+}](x \cdot y) \vee \psi_I^G[^{\beta^-}_{\beta^+}](x)) \wedge 0.5$.

Case 2': Suppose $x \cdot y \notin G$ or $x \notin G$. Then $\psi_I^G[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^+$ or $\psi_I^G[^{\beta^-}_{\beta^+}](x) = \beta^+$. Thus, $\psi_I^G[^{\beta^-}_{\beta^+}](y) \leq \beta^+ \leq \beta^+ \wedge 0.5 = (\psi_I^G[^{\beta^-}_{\beta^+}](x \cdot y) \vee \psi_I^G[^{\beta^-}_{\beta^+}](x)) \wedge 0.5$.

It is obvious to prove that $\psi_F^G[0.5]$ satisfied the condition (3.16).

Hence, $\psi^G$ is an intuitionistic neutrosophic IUP-filter of $X$.

Conversely, assume that the characteristic INS $\psi^G$ is an intuitionistic neutrosophic IUP-filter of $X$. Since $\psi_T^G[^{\alpha^+}_{\alpha^-}]$ satisfies the condition (3.8), it follows from Lemma 3 that $0 \in G$. Next, let $x, y \in G$ be such that $x \cdot y \in G$ and $x \in G$. Then $\psi_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$ and $\psi_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+$. Thus, $\psi_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+$. By the condition (3.14), we have $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = (\psi_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \wedge \psi_T^G[^{\alpha^+}_{\alpha^-}](x)) \vee 0.5 = \alpha^+ \vee 0.5 \geq \alpha^+$, that is, $\psi_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Hence, $y \in G$, so $G$ is an IUP-filter of $X$.

**Theorem 9.** *A nonempty subset $G$ is a strong IUP-ideal of $X$ if and only if the characteristic INS $\psi^G$ is an intuitionistic neutrosophic strong IUP-ideal of $X$.*

*Proof.* It is straightforward by Theorem 1.

**Lemma 4.** *Let $f$ be an FS in $X$. Then the following statements hold:*

$$(\forall x, y \in X)(1 - (f(x) \vee f(y)) = (1 - f(x)) \wedge (1 - f(y))), \tag{3.22}$$

$$(\forall x, y \in X)(1 - (f(x) \wedge f(y)) = (1 - f(x)) \vee (1 - f(y))). \tag{3.23}$$

*Proof.* Let $x, y \in X$. Suppose $f(x) \vee f(y) = f(x)$. Then $f(y) \leq f(x)$, that is, $1 - f(y) \geq 1 - f(x)$. Thus, $1 - (f(x) \vee f(y)) = 1 - f(x) = (1 - f(x)) \wedge (1 - f(y))$. Similarly, suppose $f(x) \vee f(y) = f(y)$. Then $f(x) \leq f(y)$, that is, $1 - f(x) \geq 1 - f(y)$. Thus, $1 - (f(x) \vee f(y)) = 1 - f(y) = (1 - f(x)) \wedge (1 - f(y))$.

Let $x, y \in X$. Suppose $f(x) \wedge f(y) = f(x)$. Then $f(x) \leq f(y)$, that is, $1 - f(x) \geq 1 - f(y)$. Thus, $1 - (f(x) \wedge f(y)) = 1 - f(x) = (1 - f(x)) \vee (1 - f(y))$. Similarly, suppose $f(x) \wedge f(y) = f(y)$. Then $f(y) \leq f(x)$, that is, $1 - f(y) \geq 1 - f(x)$. Thus, $1 - (f(x) \wedge f(y)) = 1 - f(y) = (1 - f(x)) \vee (1 - f(y))$.

**Lemma 5.** *Let $f$ be an FS in $X$. Then the following statements hold:*

$$(\forall x, y, z \in X)(f(z) \geq (f(x) \wedge f(y)) \vee 0.5 \Leftrightarrow \overline{f(z)} \leq (\overline{f(x)} \vee \overline{f(y)}) \wedge 0.5), \tag{3.24}$$

$$(\forall x, y, z \in X)(f(z) \leq (f(x) \vee f(y)) \wedge 0.5 \Leftrightarrow \overline{f(z)} \geq (\overline{f(x)} \wedge \overline{f(y)})) \vee 0.5). \tag{3.25}$$

*Proof.* Let $x, y, z \in X$. Then

$$f(z) \geq (f(x) \wedge f(y)) \vee 0.5 \Leftrightarrow 1 - f(z) \leq 1 - ((f(x) \wedge f(y)) \vee 0.5)$$

$$
\begin{aligned}
&\Leftrightarrow \overline{f(z)} \leq (1-(f(x)\wedge f(y))\wedge(1-0.5) && \text{(by (3.22))}\\
&\Leftrightarrow \overline{f(z)} \leq ((1-f(x))\vee(1-f(y)))\wedge 0.5 && \text{(by (3.23))}\\
&\Leftrightarrow \overline{f(z)} \leq (\overline{f(x)}\vee\overline{f(y)})\wedge 0.5,
\end{aligned}
$$

$$
\begin{aligned}
f(z) \leq ((f(x)\vee f(y))\wedge 0.5) &\Leftrightarrow 1-f(z) \geq 1-((f(x)\vee f(y))\wedge 0.5)\\
&\Leftrightarrow \overline{f(z)} \geq (1-(f(x)\vee f(y)))\vee(1-0.5) && \text{(by (3.23))}\\
&\Leftrightarrow \overline{f(z)} \geq ((1-f(x))\wedge(1-f(y)))\vee 0.5 && \text{(by (3.22))}\\
&\Leftrightarrow \overline{f(z)} \geq (\overline{f(x)}\wedge\overline{f(y)})\vee 0.5.
\end{aligned}
$$

**Theorem 10.** *If $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$, then the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the condition* (3.5) *and the FS $\overline{\psi}_I$ satisfies the condition* (3.6)*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Let $x, y \in X$. Then

$$
\begin{aligned}
\overline{\psi}_T(x\cdot y) &= \overline{\psi_I(x\cdot y)} \wedge \psi_T(x\cdot y)\\
&\geq ((\overline{\psi_I(x)}\wedge\overline{\psi_I(y)})\vee 0.5)\wedge((\psi_T(x)\wedge\psi_T(y))\vee 0.5)\\
&= ((\overline{\psi_I(x)}\wedge\overline{\psi_I(y)})\wedge(\psi_T(x)\wedge\psi_T(y)))\vee 0.5\\
&= ((\overline{\psi_I(x)}\wedge\psi_T(x))\wedge(\overline{\psi_I(y)}\wedge\psi_T(y)))\vee 0.5\\
&= (\overline{\psi}_I(x)\wedge\overline{\psi}_I(y))\vee 0.5,
\end{aligned}
$$

$$
\begin{aligned}
\overline{\psi}_I(x\cdot y) &= \psi_I(x\cdot y)\\
&\leq (\psi_I(x)\vee\psi_I(y))\wedge 0.5\\
&= (\overline{\psi}_I(x)\vee\overline{\psi}_I(y))\wedge 0.5,
\end{aligned}
$$

$$
\begin{aligned}
\overline{\psi}_F(x\cdot y) &= \overline{\psi_I(x\cdot y)} \wedge \psi_F(x\cdot y)\\
&\geq ((\overline{\psi_I(x)}\wedge\overline{\psi_I(y)})\vee 0.5)\wedge((\psi_F(x)\wedge\psi_F(y))\vee 0.5)\\
&= ((\overline{\psi_I(x)}\wedge\overline{\psi_I(y)})\wedge(\psi_F(x)\wedge\psi_F(y)))\vee 0.5\\
&= ((\overline{\psi_I(x)}\wedge\psi_F(x))\wedge(\overline{\psi_I(y)}\wedge\psi_F(y)))\vee 0.5\\
&= (\overline{\psi}_F(x)\wedge\overline{\psi}_F(y))\vee 0.5.
\end{aligned}
$$

Hence, the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the condition (3.5) and the FS $\overline{\psi}_I$ satisfies the condition (3.6).

**Theorem 11.** *If $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$, then the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the conditions* (3.8) *and* (3.11) *and the FS $\overline{\psi}_I$ satisfies the conditions* (3.9) *and* (3.12)*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$. Let $x, y, z \in X$. Then

$$\begin{aligned}\overline{\psi}_T(0) &= \overline{\psi_I(0)} \wedge \psi_T(0)\\ &\geq \overline{\psi_I(x)} \wedge \psi_T(x)\\ &= \overline{\psi}_T(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_I(0) &= \psi_I(0)\\ &\leq \psi_I(x)\\ &= \overline{\psi}_I(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_F(0) &= \overline{\psi_I(0)} \wedge \psi_F(0)\\ &\geq \overline{\psi_I(x)} \wedge \psi_F(x)\\ &= \overline{\psi}_F(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_T(x \cdot z) &= \overline{\psi_I(x \cdot z)} \wedge \psi_T(x \cdot z)\\ &\geq ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \overline{\psi_I(y)}) \vee 0.5) \wedge ((\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5)\\ &= ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \overline{\psi_I(y)}) \wedge (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y))) \vee 0.5\\ &= ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \psi_T(x \cdot (y \cdot z))) \wedge (\overline{\psi_I(y)} \wedge \psi_T(y))) \vee 0.5\\ &= (\overline{\psi}_T(x \cdot (y \cdot z)) \wedge \overline{\psi}_T(y)) \vee 0.5,\end{aligned}$$

$$\begin{aligned}\overline{\psi}_I(x \cdot z) &= \psi_I(x \cdot z)\\ &\leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5\\ &= (\overline{\psi}_I(x \cdot (y \cdot z)) \vee \overline{\psi}_I(y)) \wedge 0.5,\end{aligned}$$

$$\begin{aligned}\overline{\psi}_F(x \cdot z) &= \overline{\psi_I(x \cdot z)} \wedge \psi_F(x \cdot z)\\ &\geq ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \overline{\psi_I(y)}) \vee 0.5) \wedge ((\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5)\\ &= ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \overline{\psi_I(y)}) \wedge (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y))) \vee 0.5\\ &= ((\overline{\psi_I(x \cdot (y \cdot z))} \wedge \psi_F(x \cdot (y \cdot z))) \wedge (\overline{\psi_I(y)} \wedge \psi_F(y))) \vee 0.5\\ &= (\overline{\psi}_F(x \cdot (y \cdot z)) \wedge \overline{\psi}_F(y)) \vee 0.5.\end{aligned}$$

Hence, the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the conditions (3.8) and (3.11) and the FS $\overline{\psi}_I$ satisfies the conditions (3.9) and (3.12).

**Theorem 12.** *If $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$, then the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the conditions* (3.8) *and* (3.14) *and the FS $\overline{\psi}_I$ satisfies the conditions* (3.9) *and* (3.15)*.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$. Let $x, y \in X$. Then

$$\begin{aligned}\overline{\psi}_T(0) &= \overline{\psi_I(0)} \wedge \psi_T(0)\\ &\geq \overline{\psi_I(x)} \wedge \psi_T(x)\\ &= \overline{\psi}_T(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_I(0) &= \psi_I(0)\\ &\leq \psi_I(x)\\ &= \overline{\psi}_I(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_F(0) &= \overline{\psi_I(0)} \wedge \psi_F(0)\\ &\geq \overline{\psi_I(x)} \wedge \psi_F(x)\\ &= \overline{\psi}_F(x),\end{aligned}$$

$$\begin{aligned}\overline{\psi}_T(y) &= \overline{\psi_I(y)} \wedge \psi_T(y)\\ &\geq ((\overline{\psi_I(x \cdot y)} \wedge \overline{\psi_I(x)}) \vee 0.5) \wedge ((\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5)\\ &= ((\overline{\psi_I(x \cdot y)} \wedge \overline{\psi_I(x)}) \wedge (\psi_T(x \cdot y) \wedge \psi_T(x))) \vee 0.5\\ &= ((\overline{\psi_I(x \cdot y)} \wedge \psi_T(x \cdot y)) \wedge (\overline{\psi_I(x)} \wedge \psi_T(x))) \vee 0.5\\ &= (\overline{\psi}_T(x \cdot y) \wedge \overline{\psi}_T(x)) \vee 0.5,\end{aligned}$$

$$\begin{aligned}\overline{\psi}_I(y) &= \psi_I(y)\\ &\leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5\\ &= (\overline{\psi}_I(x \cdot y) \vee \overline{\psi}_I(x)) \wedge 0.5,\end{aligned}$$

$$\begin{aligned}\overline{\psi}_F(y) &= \overline{\psi_I(y)} \wedge \psi_F(y)\\ &\geq ((\overline{\psi_I(x \cdot y)} \wedge \overline{\psi_I(x)}) \vee 0.5) \wedge ((\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5)\\ &= ((\overline{\psi_I(x \cdot y)} \wedge \overline{\psi_I(x)}) \wedge (\psi_F(x \cdot y) \wedge \psi_F(x))) \vee 0.5\\ &= ((\overline{\psi_I(x \cdot y)} \wedge \psi_F(x \cdot y)) \wedge (\overline{\psi_I(x)} \wedge \psi_F(x))) \vee 0.5\\ &= (\overline{\psi}_F(x \cdot y) \wedge \overline{\psi}_F(x)) \vee 0.5.\end{aligned}$$

Hence, the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the conditions (3.8) and (3.14) and the FS $\overline{\psi}_I$ satisfies the conditions (3.9) and (3.15).

**Theorem 13.** *If $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$, then the FSs $\overline{\psi}_T$ and $\overline{\psi}_F$ satisfy the condition* (3.17) *and the FS $\overline{\psi}_I$ satisfies the condition* (3.18).

*Proof.* It is straightforward by Theorem 1.

**Theorem 14.** *An INS $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$ if and only if INS $\Box\psi = (\psi_T, \overline{\psi}_I, \overline{\psi}_T)$ and $\Diamond\psi = (\overline{\psi}_F, \overline{\psi}_I, \psi_F)$ are intuitionistic neutrosophic IUP-subalgebras of $X$.*

*Proof.* It is obvious by Theorem 10.

**Theorem 15.** *An INS $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$ if and only if INS $\Box\psi = (\psi_T, \overline{\psi}_I, \overline{\psi}_T)$ and $\Diamond\psi = (\overline{\psi}_F, \overline{\psi}_I, \psi_F)$ are intuitionistic neutrosophic IUP-ideals of $X$.*

*Proof.* It is obvious by Theorem 11.

**Theorem 16.** *An INS $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$ if and only if INS $\Box\psi = (\psi_T, \overline{\psi}_I, \overline{\psi}_T)$ and $\Diamond\psi = (\overline{\psi}_F, \overline{\psi}_I, \psi_F)$ are intuitionistic neutrosophic IUP-filters of $X$.*

*Proof.* It is obvious by Theorem 12.

**Theorem 17.** *An INS $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$ if and only if INS $\Box\psi = (\psi_T, \overline{\psi}_I, \overline{\psi}_T)$ and $\Diamond\psi = (\overline{\psi}_F, \overline{\psi}_I, \psi_F)$ are intuitionistic neutrosophic strong IUP-ideals of $X$.*

*Proof.* It is obvious by Theorem 13.

**Definition 9.** *Let $f$ be an FS in $X$. For any $t \in [0, 1]$, the sets*

$$U(f; t) = \{x \in X \mid f(x) \geq t\}, \tag{3.26}$$

$$L(f; t) = \{x \in X \mid f(x) \leq t\}, \tag{3.27}$$

$$E(f; t) = \{x \in X \mid f(x) = t\} \tag{3.28}$$

*are called an upper $t$-level subset, a lower $t$-level subset and an equal $t$-level subset of $f$, respectively. The sets*

$$U^{+}(f; t) = \{x \in X \mid f(x) > t\}, \tag{3.29}$$

$$L^{-}(f; t) = \{x \in X \mid f(x) < t\} \tag{3.30}$$

*are called an upper $t$-strong level subset and a lower $t$-strong level subset of $f$, respectively.*

**Theorem 18.** *Let an INS $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Let $\alpha \in [0.5, 1]$ be such that $U(\psi_T; \alpha) \neq \emptyset$. Let $x, y \in U(\psi_T; \alpha)$. Then $\psi_T(x) \geq \alpha$ and $\psi_T(y) \geq \alpha$. Thus, $\psi_T(x) \wedge \psi_T(y) \geq \alpha$. By the condition (3.5), we have $\psi_T(x \cdot y) \geq (\psi_T(x) \wedge \psi_T(y)) \vee 0.5 \geq \alpha \vee 0.5 \geq \alpha$, that is, $\psi_T(x \cdot y) \geq \alpha$. Thus, $x \cdot y \in U(\psi_T; \alpha)$. Hence, $U(\psi_T; \alpha)$ is an IUP-subalgebra of $X$.

Let $\beta \in [0, 0.5)$ be such that $L(\psi_I; \beta) \neq \emptyset$. Let $x, y \in L(\psi_I; \beta)$. Then $\psi_I(x) \leq \beta$ and $\psi_I(y) \leq \beta$. Thus, $\psi_I(x) \vee \psi_I(y) \leq \beta$. By the condition (3.6), we have $\psi_I(x \cdot y) \leq (\psi_I(x) \vee \psi_I(y)) \wedge 0.5 \leq \beta \wedge 0.5 \leq \beta$, that is, $\psi_I(x \cdot y) \leq \beta$. Thus, $x \cdot y \in L(\psi_I; \beta)$. Hence, $L(\psi_I; \beta)$ is an IUP-subalgebra of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U(\psi_F; \gamma) \neq \emptyset$. Let $x, y \in U(\psi_F; \gamma)$. Then $\psi_F(x) \geq \gamma$ and $\psi_F(y) \geq \gamma$. Thus, $\psi_F(x) \wedge \psi_F(y) \geq \gamma$. By the condition (3.7), we have $\psi_F(x \cdot y) \geq (\psi_F(x) \wedge \psi_F(y)) \vee 0.5 \geq \gamma \vee 0.5 \geq \gamma$, that is, $\psi_F(x \cdot y) \geq \gamma$. Thus, $x \cdot y \in U(\psi_F; \gamma)$. Hence, $U(\psi_F; \gamma)$ is an IUP-subalgebra of $X$.

**Theorem 19.** *Let an INS $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$.*

*Proof.* Assume that $\psi$ in $X$ is an intuitionistic neutrosophic IUP-ideal of $X$. Let $\alpha \in [0.5, 1]$ be such that $U(\psi_T; \alpha) \neq \emptyset$. Let $a \in U(\psi_T; \alpha)$. Then $\psi_T(a) \geq \alpha$. By the condition (3.8), we have $\psi_T(0) \geq \psi_T(a) \geq \alpha$. Thus, $0 \in U(\psi_T; \alpha)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\psi_T; \alpha)$ and $y \in U(\psi_T; \alpha)$. Then $\psi_T(x \cdot (y \cdot z)) \geq \alpha$ and $\psi_T(y) \geq \alpha$. Thus, $\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y) \geq \alpha$. By the condition (3.11), we have $\psi_T(x \cdot z) \geq (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5 \geq \alpha \vee 0.5 \geq \alpha$. Thus, $x \cdot z \in U(\psi_T; \alpha)$. Hence, $U(\psi_T; \alpha)$ is an IUP-ideal of $X$.

Let $\beta \in [0, 0.5)$ be such that $L(\psi_I; \beta) \neq \emptyset$. Let $b \in L(\psi_I; \beta)$. Then $\psi_I(b) \leq \beta$. By the condition (3.9), we have $\psi_I(0) \leq \psi_I(b) \leq \beta$. Thus, $0 \in L(\psi_I; \beta)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(\psi_I; \beta)$ and $y \in L(\psi_I; \beta)$. Then $\psi_I(x \cdot (y \cdot z)) \leq \beta$ and $\psi_I(y) \leq \beta$. Thus, $\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y) \leq \beta$. By the condition (3.12), we have $\psi_I(x \cdot z) \leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5 \leq \beta \vee 0.5 \leq \beta$. Thus, $x \cdot z \in L(\psi_I; \beta)$. Hence, $L(\psi_I; \beta)$ is an IUP-ideal of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U(\psi_F; \gamma) \neq \emptyset$. Let $c \in U(\psi_F; \gamma)$. Then $\psi_F(c) \geq \gamma$. By the condition (3.10), we have $\psi_F(0) \geq \psi_F(c) \geq \gamma$. Thus, $0 \in U(\psi_F; \gamma)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\psi_F; \gamma)$ and $y \in U(\psi_F; \gamma)$. Then $\psi_F(x \cdot (y \cdot z)) \geq \gamma$ and $\psi_F(y) \geq \gamma$. Thus, $\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y) \geq \gamma$. By the condition (3.13), we have $\psi_F(x \cdot z) \geq (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5 \geq \gamma \vee 0.5 \geq \gamma$. Thus, $x \cdot z \in U(\psi_F; \gamma)$. Hence, $U(\psi_F; \gamma)$ is an IUP-ideal of $X$.

**Theorem 20.** *Let an INS $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-filters of $X$.*

*Proof.* Assume that $\psi$ in $X$ is an intuitionistic neutrosophic IUP-filter of $X$. Let $\alpha \in [0.5, 1]$ be such that $U(\psi_T; \alpha) \neq \emptyset$. Let $a \in U(\psi_T; \alpha)$. Then $\psi_T(a) \geq \alpha$. By the condition (3.8), we have $\psi_T(0) \geq \psi_T(a) \geq \alpha$. Thus, $0 \in U(\psi_T; \alpha)$. Let $x, y \in X$ be such that $x \cdot y \in U(\psi_T; \alpha)$ and $x \in U(\psi_T; \alpha)$. Then $\psi_T(x \cdot y) \geq \alpha$ and $\psi_T(x) \geq \alpha$. Thus, $\psi_T(x \cdot y) \wedge \psi_T(x) \geq \alpha$. By the condition (3.14), we have $\psi_T(y) \geq (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5 \geq \alpha \vee 0.5 \geq \alpha$. Thus, $y \in U(\psi_T; \alpha)$. Hence, $U(\psi_T; \alpha)$ is an IUP-filter of $X$.

Let $\beta \in [0, 0.5)$ be such that $L(\psi_I; \beta) \neq \emptyset$. Let $b \in L(\psi_I; \beta)$. Then $\psi_I(b) \leq \beta$. By the condition (3.9), we have $\psi_I(0) \leq \psi_I(b) \leq \beta$. Thus, $0 \in L(\psi_I; \beta)$. Let $x, y \in X$ be such that $x \cdot y \in L(\psi_I; \beta)$ and $x \in L(\psi_I; \beta)$. Then $\psi_I(x \cdot y) \leq \beta$ and $\psi_I(x) \leq \beta$. Thus, $\psi_I(x \cdot y) \vee \psi_I(x) \leq \beta$. By the condition (3.15), we have $\psi_I(y) \leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5 \leq \beta \wedge 0.5 \leq \beta$. Thus, $y \in L(\psi_I; \beta)$. Hence, $L(\psi_I; \beta)$ is an IUP-ideal of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U(\psi_F; \gamma) \neq \emptyset$. Let $c \in U(\psi_F; \gamma)$. Then $\psi_F(c) \geq \gamma$. By the condition (3.10), we have $\psi_F(0) \geq \psi_F(c) \geq \gamma$. Thus, $0 \in U(\psi_F; \gamma)$. Let $x, y \in X$ be such that $x \cdot y \in U(\psi_F; \gamma)$ and $x \in U(\psi_F; \gamma)$. Then $\psi_F(x \cdot y) \geq \gamma$ and $\psi_F(x) \geq \gamma$. Thus, $\psi_F(x \cdot y) \wedge \psi_F(x) \geq \gamma$. By the condition (3.16), we have $\psi_F(y) \geq (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5 \geq \gamma \vee 0.5 \geq \gamma$. Thus, $y \in U(\psi_F; \gamma)$. Hence, $U(\psi_F; \gamma)$ is an IUP-filter of $X$.

**Theorem 21.** *Let an INS $\psi$ be an intuitionistic neutrosophic strong IUP-ideal of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or strong IUP-ideals of $X$.*

*Proof.* It is straightforward by Theorem 1.

**Theorem 22.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$. Then $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$. Let $x, y \in X$ be such that $\psi_T(x) \geq 0.5$ and $\psi_T(y) \geq 0.5$. Let $\alpha = \psi_T(x) \wedge \psi_T(y)$. Then $\psi_T(x) \geq \alpha$ and $\psi_T(y) \geq \alpha$. Thus, $x, y \in U(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U(\psi_T; \alpha)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in U(\psi_T; \alpha)$. Thus, $\psi_T(x \cdot y) \geq \alpha = \psi_T(x) \wedge \psi_T(y) \geq (\psi_T(x) \wedge \psi_T(y)) \vee 0.5$.

Let $x, y \in X$ be such that $\psi_I(x) < 0.5$ and $\psi_I(y) < 0.5$. Let $\beta = \psi_I(x) \vee \psi_I(y)$. Then $\psi_I(x) \leq \beta$ and $\psi_I(y) \leq \beta$. Thus, $x, y \in L(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L(\psi_I; \beta)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in L(\psi_I; \beta)$. Thus, $\psi_I(x \cdot y) \leq \beta = \psi_I(x) \vee \psi_I(y) \leq (\psi_I(x) \vee \psi_I(y)) \wedge 0.5$.

Let $x, y \in X$ be such that $\psi_F(x) \geq 0.5$ and $\psi_F(y) \geq 0.5$. Let $\gamma = \psi_F(x) \wedge \psi_F(y)$. Then $\psi_F(x) \geq \gamma$ and $\psi_F(y) \geq \gamma$. Thus, $x, y \in U(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U(\psi_F; \gamma)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in U(\psi_F; \gamma)$. Thus, $\psi_F(x \cdot y) \geq \gamma = \psi_F(x) \wedge \psi_F(y) \geq (\psi_F(x) \wedge \psi_F(y)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.

**Theorem 23.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$. Then $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$. Let $x \in X$. Let $\alpha = \psi_T(x)$. Then $\psi_T(x) \geq \alpha$. Thus, $x \in U(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U(\psi_T; \alpha)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in U(\psi_T; \alpha)$. Then $\psi_T(0) \geq \alpha = \psi_T(x)$. Let $x, y, z \in X$ be such that $\psi_T(x \cdot (y \cdot z)) \geq 0.5$ and $\psi_T(y) \geq 0.5$. Let $\alpha = \psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)$. Then $\psi_T(x \cdot (y \cdot z)) \geq \alpha$ and $\psi_T(y) \geq \alpha$. Thus, $x \cdot (y \cdot z), y \in U(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U(\psi_T; \alpha)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in U(\psi_T; \alpha)$. Thus, $\psi_T(x \cdot z) \geq \alpha = \psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y) \geq (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5$.

Let $x \in X$. Let $\beta = \psi_I(x)$. Then $\psi_I(x) \leq \beta$. Thus, $x \in L(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L(\psi_I; \beta)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in L(\psi_I; \beta)$. Then $\psi_I(0) \leq \beta = \psi_I(x)$. Let $x, y, z \in X$ be such that $\psi_I(x \cdot (y \cdot z)) < 0.5$ and $\psi_I(y) < 0.5$. Let $\beta = \psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)$. Then $\psi_I(x \cdot (y \cdot z)) \leq \beta$ and $\psi_I(y) \leq \beta$. Thus, $x \cdot (y \cdot z), y \in L(\psi_I; \beta) \neq \emptyset$. By assumption, we have $U(\psi_I; \beta)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in L(\psi_I; \beta)$. Thus, $\psi_I(x \cdot z) \leq \beta = \psi_I(x \cdot (y \cdot z)) \vee \psi_I(y) \leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5$.

Let $x \in X$. Let $\gamma = \psi_F(x)$. Then $\psi_F(x) \geq \gamma$. Thus, $x \in U(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U(\psi_F; \gamma)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in U(\psi_F; \gamma)$. Then $\psi_F(0) \geq \gamma = \psi_F(x)$. Let $x, y, z \in X$ be such that $\psi_F(x \cdot (y \cdot z)) \geq 0.5$ and $\psi_F(y) \geq 0.5$. Let $\gamma = \psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)$. Then $\psi_F(x \cdot (y \cdot z)) \geq \gamma$ and $\psi_F(y) \geq \gamma$. Thus, $x \cdot (y \cdot z), y \in U(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U(\psi_F; \gamma)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in U(\psi_F; \gamma)$. Thus, $\psi_F(x \cdot z) \geq \gamma = \psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y) \geq (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.

**Theorem 24.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-filters of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$. Then $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or IUP-filters of $X$. Let $x \in X$. Let $\alpha = \psi_T(x)$. Then $\psi_T(x) \geq \alpha$. Thus, $x \in U(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U(\psi_T; \alpha)$ is an IUP-filter of $X$. By the condition (2.18), we have $0 \in U(\psi_T; \alpha)$. Then $\psi_T(0) \geq \alpha = \psi_T(x)$. Let $x, y \in X$ be such that $\psi_T(x \cdot y) \geq 0.5$ and $\psi_T(x) \geq 0.5$. Let $\alpha = \psi_T(x \cdot y) \wedge \psi_T(x)$. Then $\psi_T(x \cdot y) \geq \alpha$ and $\psi_T(x) \geq \alpha$. Thus, $x \cdot y, x \in U(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U(\psi_T; \alpha)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in U(\psi_T; \alpha)$. Thus, $\psi_T(y) \geq \alpha = \psi_T(x \cdot y) \wedge \psi_T(x) \geq (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5$.

Let $x \in X$. Let $\beta = \psi_I(x)$. Then $\psi_I(x) \leq \beta$. Thus, $x \in L(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L(\psi_I; \beta)$ is an IUP-filter of $X$. By the condition (2.18), we have $0 \in L(\psi_I; \beta)$. Then $\psi_I(0) \leq \beta = \psi_I(x)$. Let $x, y \in X$ be such that $\psi_I(x \cdot y) < 0.5$ and $\psi_I(x) < 0.5$. Let $\beta = \psi_I(x \cdot y) \vee \psi_I(x)$. Then $\psi_I(x \cdot y) \leq \beta$ and $\psi_I(x) \leq \beta$. Thus, $x \cdot y, x \in L(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L(\psi_I; \beta)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in L(\psi_I; \beta)$. Thus, $\psi_I(y) \leq \beta = \psi_I(x \cdot y) \vee \psi_I(x) \leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5$.

Let $x \in X$. Let $\gamma = \psi_F(x)$. Then $\psi_F(x) \geq \gamma$. Thus, $x \in U(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U(\psi_F; \gamma)$ is an IUP-filter of $X$. By the condition (2.18), we have $0 \in U(\psi_F; \gamma)$.

Then $\psi_F(0) \geq \gamma = \psi_F(x)$. Let $x, y \in X$ be such that $\psi_F(x \cdot y) \geq 0.5$ and $\psi_F(x) \geq 0.5$. Let $\gamma = \psi_F(x \cdot y) \wedge \psi_F(x)$. Then $\psi_F(x \cdot y) \geq \gamma$ and $\psi_F(x) \geq \gamma$. Thus, $x \cdot y, x \in U(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U(\psi_F; \gamma)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in U(\psi_F; \gamma)$. Thus, $\psi_F(y) \geq \gamma = \min\{\psi_F(x \cdot y), \psi_F(x)\} \geq (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.

**Theorem 25.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U(\psi_T; \alpha)$, $L(\psi_I; \beta)$ and $U(\psi_F; \gamma)$ are either empty or strong IUP-ideals of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$. Then $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$.*

*Proof.* It is straightforward by Theorem 1.

**Theorem 26.** *Let $\psi$ be an intuitionistic neutrosophic IUP-subalgebra of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$. Let $\alpha \in [0.5, 1]$ be such that $U^+(\psi_T; \alpha) \neq \emptyset$. Let $x, y \in U^+(\psi_T; \alpha)$. Then $\psi_T(x) > \alpha$ and $\psi_T(y) > \alpha$. Thus, $\psi_T(x) \wedge \psi_T(y) > \alpha$. By the condition (3.5), we have $\psi_T(x \cdot y) \geq (\psi_T(x) \wedge \psi_T(y)) \vee 0.5 > \alpha \vee 0.5 \geq \alpha$. Thus, $x \cdot y \in U^+(\psi_T; \alpha)$. Hence, $U^+(\psi_T; \alpha)$ is an IUP-subalgebra of $X$.

Let $\beta \in [0, 0.5)$ be such that $L^-(\psi_I; \beta) \neq \emptyset$. Let $x, y \in L^-(\psi_I; \beta)$. Then $\psi_I(x) < \beta$ and $\psi_I(y) < \beta$. Thus, $\psi_I(x) \vee \psi_I(y) < \beta$. By the condition (3.6), we have $\psi_I(x \cdot y) \leq (\psi_I(x) \vee \psi_I(y)) \wedge 0.5 < \beta \wedge 0.5 \leq \beta$. Thus, $x \cdot y \in L^-(\psi_I; \beta)$. Hence, $L^-(\psi_I; \beta)$ is an IUP-subalgebra of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U^+(\psi_F; \gamma) \neq \emptyset$. Let $x, y \in U^+(\psi_F; \gamma)$. Then $\psi_F(x) > \gamma$ and $\psi_F(y) > \gamma$. Thus, $\psi_F(x) \wedge \psi_F(y) > \gamma$. By the condition (3.5), we have $\psi_F(x \cdot y) \geq (\psi_F(x) \wedge \psi_F(y)) \vee 0.5 > \gamma \vee 0.5 \geq \gamma$. Thus, $x \cdot y \in U^+(\psi_F; \gamma)$.

Hence, $U^+(\psi_F; \gamma)$ is an IUP-subalgebra of $X$.

**Theorem 27.** *Let $\psi$ be an intuitionistic neutrosophic IUP-ideal of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$.*

*Proof.* Assume that $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$. Let $\alpha \in [0.5, 1]$ be such that $U^+(\psi_T; \alpha) \neq \emptyset$. Let $a \in U^+(\psi_T; \alpha)$. Then $\psi_T(a) > \alpha$. By the condition (3.8), we have $\psi_T(0) \geq \psi_T(a) > \alpha$. Thus, $0 \in U^+(\psi_T; \alpha)$. Let $x, y, z \in U^+(\psi_T; \alpha)$ be such that $x \cdot (y \cdot z), y \in U^+(\psi_T; \alpha)$. Then $\psi_T(x \cdot (y \cdot z)) > \alpha$ and $\psi_T(y) > \alpha$. Thus, $\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y) > \alpha$. By the condition (3.11). we have $\psi_T(x \cdot z) \geq (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5 > \alpha \vee 0.5 \geq \alpha$. Thus, $x \cdot z \in U^+(\psi_T; \alpha)$. Hence, $U^+(\psi_T; \alpha)$ is an IUP-ideal of $X$.

Let $\beta \in [0, 0.5)$ be such that $L^-(\psi_I; \beta) \neq \emptyset$. Let $b \in L^-(\psi_I; \beta)$. Then $\psi_I(b) < \beta$. By the condition (3.9), we have $\psi_I(0) \leq \psi_I(b) < \beta$. Thus, $0 \in L^-(\psi_I; \beta)$. Let $x, y, z \in L^-(\psi_I; \beta)$ be such that $x \cdot (y \cdot z), y \in L^-(\psi_I; \beta)$. Then $\psi_I(x \cdot (y \cdot z)) < \beta$

and $\psi_T(y) < \beta$. Thus, $\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y) < \beta$. By the condition (3.12). we have $\psi_I(x \cdot z) \leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5 < \beta \wedge 0.5 \leq \beta$. Thus, $x \cdot z \in L^-(\psi_I; \beta)$. Hence, $L^-(\psi_I; \beta)$ is an IUP-ideal of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U^+(\psi_F; \gamma) \neq \emptyset$. Let $c \in U^+(\psi_F; \gamma)$. Then $\psi_F(c) > \gamma$. By the condition (3.10), we have $\psi_F(0) \geq \psi_F(c) > \gamma$. Thus, $0 \in U^+(\psi_F; \gamma)$. Let $x, y, z \in U^+(\psi_F; \gamma)$ be such that $x \cdot (y \cdot z), y \in U^+(\psi_F; \gamma)$. Then $\psi_F(x \cdot (y \cdot z)) > \gamma$ and $\psi_F(y) > \gamma$. Thus, $\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y) > \gamma$. By the condition (3.13). we have $\psi_F(x \cdot z) \geq (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5 > \gamma \vee 0.5 \geq \gamma$. Thus, $x \cdot z \in U^+(\psi_F; \gamma)$.

Hence, $U^+(\psi_F; \gamma)$ is an IUP-ideal of $X$.

**Theorem 28.** *Let $\psi$ be an intuitionistic neutrosophic IUP-filter of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-filters of $X$.*

*Proof.* Assume that $\psi$ be an intuitionistic neutrosophic IUP-filter of $X$. Let $\alpha \in [0.5, 1]$ be such that $U^+(\psi_T; \alpha) \neq \emptyset$. Let $a \in U^+(\psi_T; \alpha)$. Then $\psi_T(a) > \alpha$. By the condition (3.8), we have $\psi_T(0) \geq \psi_T(a) > \alpha$. Thus, $0 \in U^+(\psi_T; \alpha)$. Let $x, y \in U^+(\psi_T; \alpha)$ be such that $x \cdot y, x \in U^+(\psi_T; \alpha)$. Then $\psi_T(x \cdot y) > \alpha$ and $\psi_T(x) > \alpha$. Thus, $\psi_T(x \cdot y) \wedge \psi_T(x) > \alpha$. By the condition (3.14). we have $\psi_T(y) \geq (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5 > \alpha \vee 0.5 \geq \alpha$. Thus, $y \in U^+(\psi_T; \alpha)$. Hence, $U^+(\psi_T; \alpha)$ is an IUP-filter of $X$.

Let $\beta \in [0, 0.5)$ be such that $L^-(\psi_I; \beta) \neq \emptyset$. Let $b \in L^-(\psi_I; \beta)$. Then $\psi_I(b) < \beta$. By the condition (3.9), we have $\psi_I(0) \leq \psi_I(b) < \beta$. Thus, $0 \in L^-(\psi_I; \beta)$. Let $x, y \in L^-(\psi_I; \beta)$ be such that $x \cdot y, x \in L^-(\psi_I; \beta)$. Then $\psi_I(x \cdot y) < \beta$ and $\psi_T(x) < \beta$. Thus, $\psi_I(x \cdot y) \vee \psi_I(x) < \beta$. By the condition (3.15). we have $\psi_I(y) \leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5 < \beta \wedge 0.5 \leq \beta$. Thus, $y \in L^-(\psi_I; \beta)$. Hence, $L^-(\psi_I; \beta)$ is an IUP-filter of $X$.

Let $\gamma \in [0.5, 1]$ be such that $U^+(\psi_F; \gamma) \neq \emptyset$. Let $c \in U^+(\psi_F; \gamma)$. Then $\psi_F(c) > \gamma$. By the condition (3.10), we have $\psi_F(0) \geq \psi_F(c) > \gamma$. Thus, $0 \in U^+(\psi_F; \gamma)$. Let $x, y \in U^+(\psi_F; \gamma)$ be such that $x \cdot y, x \in U^+(\psi_F; \gamma)$. Then $\psi_F(x \cdot y) > \gamma$ and $\psi_F(x) > \gamma$. Thus, $\psi_F(x \cdot y) \wedge \psi_F(x) > \gamma$. By the condition (3.16). we have $\psi_F(y) \geq (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5 > \gamma \vee 0.5 \geq \gamma$. Thus, $y \in U^+(\psi_F; \gamma)$. Hence, $U^+(\psi_F; \gamma)$ is an IUP-filter of $X$.

**Theorem 29.** *Let $\psi$ be an intuitionistic neutrosophic strong IUP-ideal of $X$. Then for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or strong IUP-ideals of $X$.*

*Proof.* It is straightforward by Theorem 1.

**Theorem 30.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$ for all $x \in X$, then $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-subalgebras of $X$. Let $x, y \in X$ be such that $\psi_T(x) \geq 0.5$ and $\psi_T(y) \geq 0.5$. Assume that $\psi_T(x \cdot y) < (\psi_T(x) \wedge \psi_T(y)) \vee 0.5$. Let

$\alpha = \psi_T(x \cdot y)$. If $(\psi_T(x) \wedge \psi_T(y)) \vee 0.5 = \psi_T(x) \wedge \psi_T(y)$, then $\psi_T(x) > \alpha$ and $\psi_T(y) > \alpha$. Thus, $x, y \in U^+(\psi_T; \alpha)$. By assumption, we have $U^+(\psi_T; \alpha)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in U^+(\psi_T; \alpha)$. So $\psi_T(x \cdot y) > \alpha = \psi_T(x \cdot y)$, which is a contradiction. If $(\psi_T(x) \wedge \psi_T(y)) \vee 0.5 = 0.5$, then $\alpha = \psi_T(x \cdot y) < 0.5$, which is a contradiction. Thus, $\psi_T(x \cdot y) \geq (\psi_T(x) \wedge \psi_T(y)) \vee 0.5$.

Let $x, y \in X$ be such that $\psi_I(x) < 0.5$ and $\psi_I(y) < 0.5$. Assume that $\psi_I(x \cdot y) > (\psi_I(x) \vee \psi_I(y)) \wedge 0.5$. Let $\beta = \psi_I(x \cdot y)$. If $(\psi_I(x) \vee \psi_I(y)) \wedge 0.5 = \psi_I(x) \vee \psi_I(y)$, then $\psi_I(x) < \beta$ and $\psi_I(y) < \beta$. Thus, $x, y \in L^-(\psi_I; \beta)$. By assumption, we have $L^-(\psi_I; \beta)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in L^-(\psi_I; \beta)$. So $\psi_I(x \cdot y) < \beta = \psi_I(x \cdot y)$, which is a contradiction. If $(\psi_I(x) \vee \psi_I(y)) \wedge 0.5 = 0.5$, then $\beta = \psi_I(x \cdot y) > 0.5$, which is a contradiction. Thus, $\psi_I(x \cdot y) \leq (\psi_I(x) \vee \psi_I(y)) \wedge 0.5$.

Let $x, y \in X$ be such that $\psi_F(x) \geq 0.5$ and $\psi_F(y) \geq 0.5$. Assume that $\psi_F(x \cdot y) < (\psi_F(x) \wedge \psi_F(y)) \vee 0.5$. Let $\gamma = \psi_F(x \cdot y)$. If $(\psi_F(x) \wedge \psi_F(y)) \vee 0.5 = \psi_F(x) \wedge \psi_F(y)$, then $\psi_F(x) > \gamma$ and $\psi_F(y) > \gamma$. Thus, $x, y \in U^+(\psi_F; \gamma)$. By assumption, we have $U^+(\psi_F; \gamma)$ is an IUP-subalgebra of $X$. By the condition (2.17), we have $x \cdot y \in U^+(\psi_F; \gamma)$. So $\psi_F(x \cdot y) > \gamma = \psi_F(x \cdot y)$, which is a contradiction. If $(\psi_F(x) \wedge \psi_F(y)) \vee 0.5 = 0.5$, then $\gamma = \psi_F(x \cdot y) < 0.5$, which is a contradiction. Thus, $\psi_F(x \cdot y) \geq (\psi_F(x) \wedge \psi_F(y)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-subalgebra of $X$.

**Theorem 31.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$ for all $x \in X$, then $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-ideals of $X$. Let $x \in X$. Assume that $\psi_T(0) < \psi_T(x)$. Let $\alpha = \psi_T(0)$. Then $x \in U^+(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U^+(\psi_T; \alpha)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in U^+(\psi_T; \alpha)$. So $\psi_T(0) > \alpha = \psi_T(0)$, which is a contradiction. Thus, $\psi_T(0) \geq \psi_T(x)$. Let $x, y, z \in X$ be such that $\psi_T(x \cdot (y \cdot z)) \geq 0.5$ and $\psi_T(y) \geq 0.5$. Assume that $\psi_T(x \cdot z) < (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5$. Let $\alpha = \psi_T(x \cdot z)$. If $(\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5 = \psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)$, then $x \cdot (y \cdot z), y \in U^+(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U^+(\psi_T; \alpha)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in U^+(\psi_T; \alpha)$. So $\psi_T(x \cdot z) > \alpha = \psi_T(x \cdot z)$, which is a contradiction. If $(\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5 = 0.5$, then $\alpha = \psi_T(x \cdot z) < 0.5$, which is a contradiction. Thus, $\psi_T(x \cdot z) \geq (\psi_T(x \cdot (y \cdot z)) \wedge \psi_T(y)) \vee 0.5$.

Let $x \in X$. Assume that $\psi_I(0) > \psi_I(x)$. Let $\beta = \psi_I(0)$. Then $x \in L^-(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L^-(\psi_I; \beta)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in L^-(\psi_I; \beta)$. So $\psi_I(0) < \beta = \psi_I(0)$, which is a contradiction. Thus, $\psi_I(0) \leq \psi_I(x)$. Let $x, y, z \in X$ be such that $\psi_I(x \cdot (y \cdot z)) < 0.5$ and $\psi_I(y) < 0.5$. Assume that $\psi_I(x \cdot z) > \psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5$. Let $\beta = \psi_I(x \cdot z)$. If $(\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5 = \psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)$, then $x \cdot (y \cdot z), y \in L^-(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L^-(\psi_I; \beta)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in L^-(\psi_I; \beta)$. So $\psi_I(x \cdot z) < \beta = \psi_I(x \cdot z)$, which is a contradiction. If $(\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5 = 0.5$, then $\beta = \psi_I(x \cdot z) \geq 0.5$, which is a contradiction. Thus, $\psi_I(x \cdot z) \leq (\psi_I(x \cdot (y \cdot z)) \vee \psi_I(y)) \wedge 0.5$.

Let $x \in X$. Assume that $\psi_F(0) < \psi_F(x)$. Let $\gamma = \psi_F(0)$. Then $x \in U^+(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U^+(\psi_F; \gamma)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in U^+(\psi_F; \gamma)$. So $\psi_F(0) > \gamma = \psi_F(0)$, which is a contradiction. Thus, $\psi_F(0) \geq \psi_F(x)$. Let $x, y, z \in X$ be such that $\psi_F(x \cdot (y \cdot z)) \geq 0.5$ and $\psi_F(y) \geq 0.5$. Assume that $\psi_F(x \cdot z) < (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5$. Let $\gamma = \psi_F(x \cdot z)$. If $(\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5 = \psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)$, then $x \cdot (y \cdot z), y \in U^+(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U^+(\psi_F; \gamma)$ is an IUP-ideal of $X$. By the condition (2.20), we have $x \cdot z \in U^+(\psi_F; \gamma)$. So $\psi_F(x \cdot z) > \gamma = \psi_F(x \cdot z)$, which is a contradiction. If $(\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5 = 0.5$, then $\gamma = \psi_F(x \cdot z) < 0.5$, which is a contradiction. Thus, $\psi_F(x \cdot z) \geq (\psi_F(x \cdot (y \cdot z)) \wedge \psi_F(y)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-ideal of $X$.

**Theorem 32.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-filters of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$ for all $x \in X$, then $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.*

*Proof.* Assume that for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^+(\psi_T; \alpha)$, $L^-(\psi_I; \beta)$ and $U^+(\psi_F; \gamma)$ are either empty or IUP-filters of $X$. Let $x \in X$. Assume that $\psi_T(0) < \psi_T(x)$. Let $\alpha = \psi_T(0)$. Then $x \in U^+(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U^+(\psi_T; \alpha)$ is an IUP-ideal of $X$. By the condition (2.18), we have $0 \in U^+(\psi_T; \alpha)$. So $\psi_T(0) > \alpha = \psi_T(0)$, which is a contradiction. Thus, $\psi_T(0) \geq \psi_T(x)$. Let $x, y \in X$ be such that $\psi_T(x \cdot y) \geq 0.5$ and $\psi_T(x) \geq 0.5$. Assume that $\psi_T(y) < (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5$. Let $\alpha = \psi_T(y)$. If $(\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5 = \psi_T(x \cdot y) \wedge \psi_T(x)$, then $x \cdot y, x \in U^+(\psi_T; \alpha) \neq \emptyset$. By assumption, we have $U^+(\psi_T; \alpha)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in U^+(\psi_T; \alpha)$. So $\psi_T(y) > \alpha = \psi_T(y)$, which is a contradiction. If $(\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5 = 0.5$, then $\alpha = \psi_T(y) < 0.5$, which is a contradiction. Thus, $\psi_T(y) \geq (\psi_T(x \cdot y) \wedge \psi_T(x)) \vee 0.5$.

Let $x \in X$. Assume that $\psi_I(0) > \psi_I(x)$. Let $\beta = \psi_I(0)$. Then $x \in L^-(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L^-(\psi_I; \beta)$ is an IUP-filter of $X$. By the condition (2.18), we have $0 \in L^-(\psi_I; \beta)$. So $\psi_I(0) < \beta = \psi_I(0)$, which is a contradiction. Thus, $\psi_I(0) \leq \psi_I(x)$. Let $x, y \in X$ be such that $\psi_I(x \cdot y) < 0.5$ and $\psi_I(x) < 0.5$. Assume that $\psi_I(y) > (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5$. Let $\beta = \psi_I(y)$. If $(\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5 = \psi_I(x \cdot y) \vee \psi_I(x)$, Then $x \cdot y, x \in L^-(\psi_I; \beta) \neq \emptyset$. By assumption, we have $L^-(\psi_I; \beta)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in L^-(\psi_I; \beta)$. So $\psi_I(y) < \beta = \psi_I(y)$, which is a contradiction. If $(\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5 = 0.5$, then $\beta = \psi_I(y) > 0.5$, which is a contradiction. Thus, $\psi_I(y) \leq (\psi_I(x \cdot y) \vee \psi_I(x)) \wedge 0.5)$.

Let $x \in X$. Assume that $\psi_F(0) < \psi_F(x)$. Let $\gamma = \psi_F(0)$. Then $x \in U^+(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U^+(\psi_F; \gamma)$ is an IUP-filter of $X$. By the condition (2.18), we have $0 \in U^+(\psi_F; \gamma)$. So $\psi_F(0) > \gamma = \psi_F(0)$, which is a contradiction. Thus, $\psi_F(0) \geq \psi_F(x)$. Let $x, y \in X$ be such that $\psi_F(x \cdot y) \geq 0.5$ and $\psi_F(x) \geq 0.5$. Assume that $\psi_F(y) < (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5$. Let $\gamma = \psi_F(y)$. If $(\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5 = \psi_F(x \cdot y) \wedge \psi_F(x)$, Then $x \cdot y, x \in U^+(\psi_F; \gamma) \neq \emptyset$. By assumption, we have $U^+(\psi_F; \gamma)$ is an IUP-filter of $X$. By the condition (2.19), we have $y \in U^+(\psi_F; \gamma)$. So $\psi_F(y) > \gamma = \psi_F(y)$, which is a contradiction. If $(\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5 = 0.5$, then $\gamma = \psi_F(y) < 0.5$, which is a contradiction. Thus, $\psi_F(y) \geq (\psi_F(x \cdot y) \wedge \psi_F(x)) \vee 0.5$.

Hence, $\psi$ is an intuitionistic neutrosophic IUP-filter of $X$.

**Theorem 33.** *Let for all $\alpha, \gamma \in [0.5, 1]$ and $\beta \in [0, 0.5)$, the sets $U^{+}(\psi_T; \alpha)$, $L^{-}(\psi_I; \beta)$ and $U^{+}(\psi_F; \gamma)$ are either empty or strong IUP-ideals of $X$. If $\psi_T(x) \geq 0.5, \psi_I(x) < 0.5$ and $\psi_F(x) \geq 0.5$ for all $x \in X$, then $\psi$ is an intuitionistic neutrosophic strong IUP-ideal of $X$.*

*Proof.* It is straightforward by Theorem 1.

## 4. Conclusions

This study has introduced the foundational concepts of intuitionistic neutrosophic IUP-subalgebras, intuitionistic neutrosophic IUP-ideals, intuitionistic neutrosophic IUP-filters and intuitionistic neutrosophic strong IUP-ideals within the framework of IUP-algebras. Through rigorous analysis, we explored their intrinsic properties and established their relationships with the notions of characteristic, complement and level subsets. A key contribution of this work is the derivation of a comprehensive diagram, as depicted in Figure 2, which illustrates the hierarchical structure and interactions among these subsets. This visualization provides an intuitive framework for understanding the interconnections within the intuitionistic neutrosophic IUP-algebraic system.

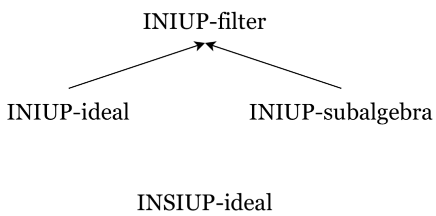

Figure 2: INSs in IUP-algebras

The findings presented herein mark a significant advancement in INS theory as applied to IUP-algebras, offering both theoretical and practical insights. By systematically bridging these concepts, this study lays the groundwork for future explorations, including the potential application of INS theory to other algebraic systems and the extension of these ideas into more generalized frameworks.

In future work, we aim to expand upon this foundation by examining the integration of soft set theory and cubic set theory with neutrosophic IUP-algebras. These efforts will involve investigating neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters and strong IUP-ideals within these alternative frameworks. The hierarchical relationships, as detailed in Figure 2, will serve as a critical reference point for further studies into the algebraic and logical underpinnings of IUP-algebras, ultimately enhancing their utility in modeling and decision-making under conditions of uncertainty.

## Acknowledgements

This research was supported by University of Phayao and Thailand Science Research and Innovation Fund (Fundamental Fund 2025, Grant No. 5027/2567).


## References


[1] K. T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.*, 20(1):87–96, 1986.

[2] M. Bhowmik and M. Pal. Intuitionistic neutrosophic set. *J. Inf. Comput. Inf. Sci. Eng.*, 4(2):142–152, 2009.

[3] M. Bhowmik and M. Pal. Intuitionistic neutrosophic set relations and some of its properties. *J. Inf. Comput. Inf. Sci. Eng.*, 5(3):183–192, 2010.

[4] S. Broumi and F. Smarandache. Intuitionistic neutrosophic soft set. *J. Inf. Comput. Sci.*, 8(2):130–140, 2013.

[5] S. Broumi and F. Smarandache. More on intuitionistic neutrosophic soft sets. *Comp. Sci. Inf. Tech.*, 1(4):257–268, 2013.

[6] S. Broumi, F. Smarandache, and P. K. Maji. Intuitionistic neutrosphic soft set over rings. *Math. Stat.*, 2(3):120–126, 2014.

[7] C. Chanmanee, W. Nakkhasen, R. Prasertpong, P. Julatha, and A. Iampan. Notes on external direct products of dual IUP-algebras. *South East Asian J. Math. Math. Sci.*, 19(3):13–30, 2023.

[8] C. Chanmanee, R. Prasertpong, P. Julatha, N. Lekkoksung, and A. Iampan. On external direct products of IUP-algebras. *Int. J. Innov. Comput. Inf. Control*, 19(3):775–787, 2023.

[9] A. Iampan, P. Julatha, P. Khamrot, and D. A. Romano. Independent UP-algebras. *J. Math. Comput. Sci., JMCS*, 27(1):65–76, 2022.

[10] K. Kuntama, P. Krongchai, R. Prasertpong, P. Julatha, and A. Iampan. Fuzzy set theory applied to IUP-algebras. *J. Math. Comput. Sci., JMCS*, 34(2):128–143, 2024.

[11] F. Smarandache. Neutrosophic set – a generalization of the intuitionistic fuzzy set. *2006 IEEE International Conference on Granular Computing*, pages 38–42, 2006.

[12] K. Suayngam, P. Julatha, R. Prasertpong, and A. Iampan. Neutrosophic sets in IUP-algebras: a new exploration. *Int. J. Neutrosophic Sci.*, 25(3):540–560, 2025.

[13] K. Suayngam, R. Prasertpong, N. Lekkoksung, P. Julatha, and A. Iampan. Fermatean fuzzy set theory applied to IUP-algebras. *European J. Pure App. Math.*, 17(4):3022–3042, 2024.

[14] K. Suayngam, R. Prasertpong, W. Nakkhasen, P. Julatha, and A. Iampan. Pythagorean fuzzy sets: a new perspective on IUP-algebras. *Int. J. Innov. Comput. Inf. Control*, 21(2):accepted, 2025.

[15] K. Suayngam, T. Suwanklang, P. Julatha, R. Prasertpong, and A. Iampan. New results on intuitionistic fuzzy sets in IUP-algebras. *Int. J. Innov. Comput. Inf. Control*, 20(4):1125–1141, 2024.

[16] L. A. Zadeh. Fuzzy sets. *Inf. Cont.*, 8(3):338–353, 1965.