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Neutrosophic Hypersoft Topological Framework for Agricultural Decision-Making

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Abstract. This paper presents a novel neutrosophic Agricultural Topology based MCDM approach for solving decision-making problems in agriculture, with uncertainties and infinite variables. Neutrosophic sets are introduced to formalize uncertainties and the neutrosophic sub-base can produce a topology for a dual study of the issue via open sets. The work analyses further properties of the neutrosophic hypersoft(NH) topology; basis, subspace, interior, and closure properties. Two algorithms are proposed: one employing NH sets and the other employing NH topology. Numerical examples derived from real-life agriculture situations prove the usefulness of this approach in increasing the efficiency of choice-making strategies.

2020 Mathematics Subject Classifications: 54A40, 03Exx

Key Words and Phrases: Neutrosophic logic, Neutrosophic sets, Fuzzy hypersoft topology, Neutrosophic hypersoft topology, Multicriteria decision-making.

1. Introduction

The uncertainties in agricultural decision-making processes are due to the numerous and imprecise factors affecting agriculture; for instance, climate, topography, and market trends. Traditionally in decision-making, a number of models fail to capture this characteristic because of the inherent impreciseness and uncertainty of such data. Zadeh (1965) [1] in his paper on pioneer work on fuzzy set theory put light to the modeling of uncertainty where information is represented as degrees of membership. The concept advanced by Chang in 1968 took fuzzy sets to develop fuzzy topological spaces that explored fuzzy conditions for studying continuity compactness and convergence [2]. This research founded a fresh scholarly field applying concepts from topology to fuzzy logic to uncover novel mathematical machinery. Atanassov established intuitionistic

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fuzzy sets (IFS) in 1986 [3] because he identified the requirement for precise uncertainty modeling. Traditional fuzzy set theory admits membership degrees alone but intuitionistic fuzzy sets include membership and non-membership functions which generate an enhanced ability to analyze incomplete information. The extension enabled better solutions in pattern recognition and multi-criteria decision-making. In 1988 Atanassov presented both a complete review of intuitionistic fuzzy set theory together with fresh findings to advance its theoretical structure and uncertainty modeling capabilities. New explorations of intuitionistic fuzzy structures began after researchers examined these advancements. Smarandache established Neutrosophic logic in 1999 as an extension of classical and fuzzy logic which integrates indeterminacy as a fundamental part in reasoning processes [4]. The framework implements a comprehensive mathematical framework for dealing with uncertain data and imprecise and vague content in decision procedures and computational solutions.

Dogan Coker introduced intuitionistic fuzzy topological spaces in 1997 as a means to expand intuitionistic fuzzy set theory into topological environments to probe details about intuitionistic fuzzy continuity and compactness [5]. In 2006 Saadati and Park released additional intelligence about intuitionistic fuzzy topology which improved the capabilities of applying it across different mathematical and engineering problems [6]. A major breakthrough came in 1999 when Smarandache introduced neutrosophic sets, which generalized intuitionistic fuzzy sets by incorporating a third parameter: indeterminacy [7]. Neutrosophic sets contain an independent measure of indeterminacy in addition to membership and non-membership values which provides greater resilience for handling contradictory information. The expanded scale has proven useful for medical diagnosis while serving alongside expert systems and artificial intelligence systems. Using Smarandache's theory of neutrosophic logic in 2002 he established fundamental principles for a generalized logic framework to simultaneously evaluate truth values together with falsity and indeterminacy [8]. Neutrosophic systems became better suited to model actual problems containing uncertainty because of this development. Salama and Alblowi developed neutrosophic topological spaces in 2012 by applying neutrosophic sets to topological structure applications [9]. Research benefits from this extension because it enables researchers to study continuity, compactness and separation axioms in neutrosophic conditions providing increased theory robustness. Research efforts today focus on neutrosophic hypersoft sets because they establish powerful mathematical tools to manage diverse uncertain inputs in sophisticated decision situations. In 2023 the research team of Arshad et al. adopted interval complex single-valued neutrosophic hypersoft sets to create advanced decision-making methods [10]. The researchers showed how these sets enhance both accuracy and flexibility performance in multi-criteria evaluations. Neutrosophic topology has obtained new theoretical developments through recent advancements in its field. Through his research on new topology types and improved neutrosophic topologies, Smarandache developed an organized method to address indeterminate spatial structures [11]. The research of compactness and neutrosophic topological spaces utilizing grills created new approaches for studying mathematical spaces under uncertainty conditions [12]. The new discoveries help deepen knowledge regarding topological concepts present in neutrosophic frameworks. Different classes of neutrosophic topologies are distinguished and related through separation axioms investigated within neutrosophic topological spaces [13]. Research about neutrosophic mathematics theories needs to become formalized because it serves as the basis for theoretical development. Linear programming techniques involving neutrosophics serve agricultural land optimization purposes to assist decision-makers with resource distribution [14]. Neutrosophic models show their flexibility in dealing with optimization problems found in real-world situations through these applications. Saqlain et al. established multi-polar interval-valued neutrosophic hypersoft sets during that same year by implementing machine learning methods to improve decision-making systems [15]. The research demonstrated how artificial intelligence techniques increasingly contribute to neutrosophic analysis thus enhancing its adoption across business intelligence and predictive modeling applications.Alanazi and Alrashdi introduced edge-based anomaly detection utilizing neutrosophic models for smart farming systems within 2023 research [16]. The research demonstrated how neutrosophic models enhance agricultural operations by detecting irregular patterns in farming data to achieve better efficiency along with enhanced productivity. The research team composed of Jayasudha and Raghavi developed new operations for neutrosophic hypersoft matrices during 2024 which enhanced the foundational mathematical framework of decision-making systems [17]. The developed operations created powerful analytical and processing methods to handle data uncertainty with increased accuracy. AlHijjawi introduced possibility neutrosophic hypersoft sets to expand the practical reach of neutrosophic sets for complex multi-attribute thinking under various uncertainty conditions [18]. The sets offer detailed handling of ambiguous information which helps solve problems in artificial intelligence and machine learning systems. Research by[19] examines multi-attribute decision-making techniques which use aggregations and similarity measures of neutrosophic hypersoft sets. Such methods create effective evaluation and ranking tools for uncertain situations which prove essential in medical diagnosis and business intelligence and engineering design fields.

Kaur and Singh conducted research that showed that generalized neutrosophic TOPSIS proved efficient as a decision support system in MCDM applications by deploying it to practical decision situations [20]. Novak along with Singh and Tula implemented the use of neutrosophic hypersoft sets to address decision-making problems that heavily rely on uncertain data.Shumaev et al. [21] emphasize innovative project management procedures, highlighting the strategic planning and resource management requirements of agritourism projects. Roman et al. [22] explore new agritourism approaches in Poland, emphasizing local cultural heritage and technological progress as drivers of rural development. Rauniyar et al. [23] offer a systematic literature review and bibliometric analysis, highlighting important trends and research gaps in agritourism. Mironkina et al. [24] study the Russian agritourism market, suggesting development initiatives to increase rural income generation. Mahmoodi et al. [25] make a comparative study of agritourism characteristics and issues in Iran and Poland, highlighting cultural and infrastructural differences. Grillini et al. [26] evaluate support schemes for the development of agritourism in Italy, the USA, and South Africa, highlighting the policy framework's role in driving growth. Domi and Belletti [27] examine the effect of origin products and networking on agritourism performance in Tuscany, illustrating the worth of local branding and cooperation. Wu et al. [28] utilize a fuzzy multi-criteria decision model to choose best agritourism sites in Vietnam, which illustrates the usefulness of fuzzy logic in solving difficult decision issues. Puska et al. [29] apply an integrative fuzzy logic method to support sustainable agro-touristic products in a developing nation, which illustrates the capability of state-of-the-art computational methods. Rezaeifar et al. [30] develop a model of sustainable development for the agriculture sector during the COVID-19 pandemic through fuzzy logic in response to key situations and uncertainty. Ghaforiyan et al. [31] make antifragile tourism policies a priority in a neutrosophic setting, presenting the use of neutrosophic logic in managing uncertainty. Agritourism has become an important contributor to rural economies, ensuring sustainable farm profitability and environmental care. Baby and Kim [32] discuss the intrinsic motivation of tourists, their environmental conduct, and satisfaction levels in the context of sustainable agritourism, highlighting the customer experience in improving farm profitability. Kousar and Kausar [33] suggest a multi-criteria decision-making framework for sustainable agritourism through an integrated fuzzy-rough approach, which stresses the significance of sound analytical tools. Kar et al. [34] establish an ARIMA-based framework for income forecasting from farm commodities in an automated manner, with a focus on data-driven decision-making. Kumar and Pamucar [35] present a critical review of multi-criteria decision-making techniques across two decades, with a strong grasp of emerging analytical tools for agritourism and agriculture decision-making.

This paper is organized as: The basic definitions in section 2, The basic definitions and characteristics of exponential fuzzy sets are covered in Section 3, Main results of exponential fuzzy sets are shown in Section 4, application in section 5 and future research possibilities are discussed in Section 6.

1.1. Motivation

The main driving force for this research is to fill crucial gaps in current decision-making methodologies in agriculture, which tend to fail to deal with intricate uncertainties and imprecise information. The research uses Neutrosophic Hypersoft Topology (NHT) to bridge these gaps. The unique gaps being filled are:

- Most classical decision-making models in agriculture tend to miss the natural vagueness and imprecision that actually exist in real-world situations, like soil fertility tests, crop yield forecasting, and pest management plans. $\mathcal{NHT}$ offers a strong mathematical framework for proper handling of these uncertainties.
- Existing models are not capable of dealing with multiple interdependent factors and dynamic environmental conditions. With the extension of neutrosophic hypersoft sets to topological spaces, the present research presents a more flexible and adaptive decision-making framework.
- Crop decision-making consists of varied and intricate datasets like weather pat-

terns, soil quality data, and market trends. Present approaches are inadequate to merge heterogeneous data sources. $\mathcal{NHT}$ supports a richer analysis by reflecting indeterminacy and partial truth.

- Agricultural risk management is vital to reducing the impact of losses from climate change, pest infestation, and market volatility. Traditional models are usually inconsistent in their identification and evaluation of such risks. $\mathcal{NHT}$ facilitates more effective risk evaluation due to its ability to deal with incomplete and inconsistent data.

- The research seeks to illustrate the feasibility of applying neutrosophic hypersoft topology in agricultural decision-making, providing an innovative technique for decision-makers and stakeholders to make more precise and accurate decisions.

1.2. MCDM Problem Based Neutrosophic Agricultural Topology

Choice is essential in arriving at the best solution of a given problem especially when it comes to decision-making involving agriculture. As seen in most real-life problems in agriculture, the time it might be tough to find a good plan of action because of so many variables. In agricultural decision-making, where parameters such as climate variability, soil quality, and resource availability tend to be uncertain, neutrosophic sets offer a sound framework for dealing with intricate data. By building neutrosophic sets for criteria to be evaluated, e.g., soil fertility and water availability, decision-makers can compare possible sites for farms on the basis of the truth degrees, indeterminacy degrees, and falsity degrees. The sets are subsequently employed to build a neutrosophic topology, where open sets are viable solutions. It supports a more variable and precise Multi-Criteria Decision-Making (MCDM) process with the ability to determine the most suitable farm location while considering uncertainties present in agriculture settings. This paper deals with an agricultural decision-making issue by implementing Neutrosophic logic. Firstly, the problem is solved by building Neutrosophic sets to address the uncertainties within and the fuzziness of the attribute values. Subsequently, these Neutrosophic sets are used as a sub-base, to generate a topology. The problem is then reconsidered about the open sets resulting from this topology. This dual approach enables the comparison providing the importance of topology in the context of MCDM for agriculture.

1.3. Novelty

(i) Introduced neutrosophic hypersoft topological spaces, a novel contribution to fuzzy topology.

(ii) Developed key operations such as closure, interior, basis, and sub-space topology within the neutrosophic hypersoft context.

(iii) Proposed a decision-making algorithm using neutrosophic hypersoft topology, a first in the field.

(iv) Demonstrated the practical implementation of neutrosophic hypersoft topology in agricultural decision-making, setting a precedent for future studies.

1.4. Structure of the article

The article is structured as follows: Section 1: The introduction provides an overview of intuitionistic fuzzy hypersoft sets and extends those concepts to Intuitionistic fuzzy hypersoft topological spaces; it also gives the rationale for using these concepts for complex decision-making. Section 2: introduces the concept of neutrosophic hypersoft topological space and studies fundamental aspects of hypersoft topological space including closure, interior, basis, and subspace topology via examples. Section 3: As a result, Algorithm Development suggests a new algorithm for decision-making that considers these topological spaces and the application in agriculture. We discussed the result in Section 4. The paper concludes with Section 5: Conclusion and Future Work.

2. Advanced Neutrosophic Hypersoft Topological Structures

Definition 1. *A neutrosophic set (NS) S in Y is defined as:*

$$S = \{(y, T_S(y), I_S(y), F_S(y)) : y \in Y\}$$

where $T_S : Y \to [0,1]$, $I_S : Y \to [0,1]$, and $F_S : Y \to [0,1]$ represent the degrees of truth-membership, indeterminacy, and falsity-membership of y to S, respectively. These functions must satisfy the following condition for all $y \in Y$:

$$0 \leq T_S(y) + I_S(y) + F_S(y) \leq 3.$$

The set of all neutrosophic sets over Y will be denoted by $NS(Y)$.

Definition 2. *Let Y be the universe,K the set of parameters,$P(Y)$ the power set of Y. A pair (S, K) is called a soft set over Y, where S is a mapping $S : K \to P(Y)$. That is, a soft set is a parameterized family of subsets of set Y.*

Definition 3. *Let Y be an initial universe, and K be a set of parameters. A pair (S, K) is called a neutrosophic soft set over Y, where S is a mapping given by $S : K \to NP(Y)$. On the whole, for every $k \in K$, $S(k)$ is a set of neutrosophic elements of Y, and it is termed as the neutrosophic value set of the parameter k. Clearly, $S(k)$ can be written as a neutrosophic set such that $S(k) = \{(y, T_S(y), I_S(y), F_S(y)) : y \in Y\}$.*

Definition 4. *Let $P(Y)$ denote the power set of Y, where Y is the universal set. A pair*

$$(S : K_1 \times K_2 \times \cdots \times K_n)$$

is said to be a hypersoft set over Y if there exists a mapping

$$S : K_1 \times K_2 \times \cdots \times K_n \to P(Y).$$

Here, $k_1, k_2, k_3, \ldots, k_n$ (for $n \geq 1$) are well-defined attributes, and their corresponding attribute values are the sets $K_1, K_2, \ldots, K_n$. These sets satisfy the condition:

$$K_i \cap K_j = \emptyset, \quad \text{for } i \neq j, \text{ and } i, j \in \{1, 2, \ldots, n\}.$$

Definition 5. *Let $NS(Y)$ be a set of all the neutrosophic sets over the universal set Y. Let $k_1, k_2, k_3, \ldots, k_n$ for certain $n \geq 1$, stand for n well-defined attributes where the attribute values are the sets $K_1, K_2, \ldots K_n$ respectively; $K_i \cap K_j = \emptyset$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Let M_i be the nonempty subset of K_i for each $i = 1, 2, \ldots, n$. A neutrosophic hypersoft set is defined as the pair $(S, M_1 \times M_2 \times \cdots \times M_n)$, where $S : M_1 \times M_2 \times \cdots \times M_n \to NS(Y)$ and $S(M_1 \times M_2 \times \cdots \times M_n)$ $= \left\{ \langle \iota, \frac{y}{T_{S(\iota)}(y), I_{S(\iota)}(y), F_{S(\iota)}(y)} \rangle : y \in Y, \iota \in M_1 \times M_2 \times \cdots \times M_n \subseteq K_1 \times K_2 \times \cdots \times K_n \right\}$, Simplification purposes necessitate the use of the symbols Γ for $M_1 \times M_2 \times \cdots \times M_n$ and Λ for $K_1 \times K_2 \times \cdots \times K_n$.*

Definition 6. *Let $N(Y, \Lambda)$ be the set of all neutrosophic hypersoft subsets over the universe Y and $\grave{\sigma} \subseteq N(Y, \Lambda)$. Then, $\grave{\sigma}$ is called neutrosophic hypersoft topology on Y if the following conditions hold:*

(i) $N_0(Y, \Lambda)$, $N_1(Y, \Lambda)$ *belong to* $\grave{\sigma}$,

(ii) $(S_1, \Gamma_1)\tilde{\cap}(R_2, \Gamma_2) \in \grave{\sigma}$ *implies* $(S_1, \Gamma_1)\tilde{\cap}(R_2, \Gamma_2)$,

(iii) $\{(S_i, \Gamma_i) : i \in I\} \subseteq \grave{\sigma}$ *implies* $\bigcup_{i \in I}(S_i, \Gamma_i) \in \grave{\sigma}$.

Then, $(Y, \grave{\sigma}, \Lambda)$ is called a neutrosophic hypersoft topological space over Y. The members of $\grave{\sigma}$ are said to be neutrosophic hypersoft open sets in Y. A neutrosophic hypersoft set (S, Γ) over Y is said to be a neutrosophic hypersoft closed set if its complement $(S, \Gamma)^c$ belongs to $\grave{\sigma}$.

Definition 7. *Let the set of all neutrosophic hypersoft subsets of the universal set Y be represented by $N(Y, \Lambda)$;*

(i) A topology Neutosophic hypersoft indiscrete mode is defined as $\grave{\sigma} = (N_0(Y, \Lambda))$, $(N_1(Y, \Lambda))$. A neutrosophic hypersoft indiscrete topological space over the universe Y is then defined as the triplet $(Y, \grave{\sigma}, \Lambda)$.

(ii) A topology Neutosophic hypersoft discrete topology is given by $\grave{\sigma} = NHS(U, \Delta)$. A neutrosophic hypersoft discrete topological space over the universe Y is then defined as the triplet $(Y, \grave{\sigma}, \Lambda)$.

Example 1. *Let $Y = \{y_1, y_2\}$ be the universe set, and P_1, P_2 be sets of attributes. P_1, P_2 are defined as follows:*

$$P_1 = \{\iota_1, \iota_2, \iota_3\},$$

$$P_2 = \{\kappa_1, \kappa_2\},$$

Suppose that

$$M_1 = \{\iota_1, \iota_2\}, \quad M_2 = \{\kappa_1\},$$

$$N_1 = \{\iota_2, \iota_3\}, \quad N_2 = \{\kappa_1, \kappa_2\},$$

are subsets of P_i for each $i = 1, 2$.

Let $\grave{\sigma} = \{(N_0(Y,\Lambda)), (N_1(Y,\Lambda)), (S_1,\Gamma_1), (S_2,\Gamma_2), (S_3,\Gamma_3), (S_4,\Gamma_4)\}$ *be a subfamily of NHS*(Y,Δ)*, where* $(S_1,\Gamma_1), (S_2,\Gamma_2), (S_3,\Gamma_3), (S_4,\Gamma_4)$ *are neutrosophic hypersoft sets defined as follows:*

$$(S_1,\Gamma_1) = \begin{cases} \big\langle(\iota_1,\kappa_2), \{\frac{y_1}{0.2,0.1,0.7}, \frac{y_2}{0.5,0.1,0.4}\}\big\rangle, \\ \big\langle(\iota_2,\kappa_2), \{\frac{y_1}{0.6,0.1,0.3}, \frac{y_2}{0.8,0.1,0.1}\}\big\rangle. \end{cases}$$

$$(S_2,\Gamma_2) = \begin{cases} \big\langle(\iota_2,\kappa_1), \{\frac{y_1}{0.5,0.2,0.6}, \frac{y_2}{0.7,0.1,0.2}\}\big\rangle, \\ \big\langle(\iota_2,\kappa_2), \{\frac{y_1}{0.4,0.1,0.5}, \frac{y_2}{0.6,0.1,0.4}\}\big\rangle, \\ \big\langle(\iota_3,\kappa_1), \{\frac{y_1}{0.4,0.2,0.4}\}\big\rangle, \\ \big\langle(\iota_3,\kappa_2), \{\frac{y_1}{0.6,0.2,0.2}\}\big\rangle. \end{cases}$$

$$(S_3,\Gamma_3) = \Big\{\big\langle(\iota_3,\kappa_2), \{\frac{y_1}{0.6,0.2,0.2}\big\rangle.$$

Then $\grave{\sigma}$ *is a neutrosophic hypersoft topology, and hence* $(Y,\grave{\sigma},\Lambda)$ *is a neutrosophic hypersoft topological space over the universe* Y*.*

Remark 1. *obvious that each neutrosophic hypersoft topology is a neutrosophic fuzzy soft topology. Consider Example-2.5: When we pick the parameters from a single attribute set, for instance,* P_2

When developing a neutrosophic hypersoft topology, under construction topology appears as a neutrosophic fuzzy soft topology.

This proves that a neutrosophic hypersoft topology is a generalization of a neutrosophic soft topology. Hence, each neutrosophic hypersoft topology is a neutrosophic soft topology. However, the converse is not true; not all neutrosophic soft topology can be extended to neutrosophic hypersoft topology.

Proposition 1. *Let* $(Y,\grave{\sigma}_1,\Lambda)$ *and* $(Y,\grave{\sigma}_2,\Lambda)$ *be two neutrosophic hypersoft topologies over* Y*. Define*

$$\grave{\sigma}_1 \cap \grave{\sigma}_2 = \{(S,\Gamma) \mid (S,\Gamma)) \in \grave{\sigma}_1 \text{ and } (S,\Gamma) \in \grave{\sigma}_2\}.$$

Then $\grave{\sigma}_1 \cap \grave{\sigma}_2$ *is a neutrosophic hypersoft topology on* Y*.*

Proof. Clearly, $(N_0(Y,\Lambda))$ *and* $(N_1(Y,\Lambda)) \in \grave{\sigma}_1 \cap \grave{\sigma}_2$*, where* $\grave{\sigma}_1$ *where* $\grave{\sigma}_1$ *and* $\grave{\sigma}_2$ *are two neutrosophic hypersoft topologies on* Y*. Let* $(S_1,\Gamma_1), (S_2,\Gamma_2) \in \grave{\sigma}_1$ *and* $(S_1,\Gamma_1), (S_2,\Gamma_2) \in \grave{\sigma}_2$*. Then,*

$$(S_1,\Gamma_1) \cap (S_2,\Gamma_2) \in \grave{\sigma}_1 \quad \text{and} \quad (S_1,\Gamma_1) \cap (S_2,\Gamma_2) \in \grave{\sigma}_2.$$

Thus,

$$(S_1,\Gamma_1) \cap (S_2,\Gamma_2) \in \grave{\sigma}_1 \cap \grave{\sigma}_2.$$

Let $\{(S_i,\Gamma_i) : i \in I\} \subseteq \grave{\sigma}_1 \cap \grave{\sigma}_2$*. Then,* $(S_i,\Gamma_i) \in \grave{\sigma}_1$ *and* $(S_i,\Gamma_i) \in \grave{\sigma}_2$ *for any* $i \in I$*. Since* $\grave{\sigma}_1$ *and* $\grave{\sigma}_2$ *are neutrosophic hypersoft topologies on* U*,*

$$\bigcup\{(S_i,\Gamma_i) : i \in I\} \in \grave{\sigma}_1 \quad \text{and} \quad \bigcup\{(S_i,\Gamma_i) : i \in I\} \in \grave{\sigma}_2.$$

Thus,

$$\bigcup\{(S_i,\Gamma_i) : i \in I\} \in \grave{\sigma}_1 \cap \grave{\sigma}_2.$$

Remark 2. *A neutrosophic hypersoft topology doesn't need to be formed by the union of two neutrosophic hypersoft topologies over Y. The following example is used to illustrate this proposition.*

Example 2. *We consider the attributes from Example 1 Let*

$$\dot{\sigma} = \{(N_0(Y,\Lambda)),(N_1(Y,\Lambda)),(S_1,\Gamma_1)\},$$

$$\dot{\sigma} = \{(N_0(Y,\Lambda)),(N_1(Y,\Lambda)),(R_1,\Gamma_1),(R_2,\Gamma_2)\},$$

where

$$(R_1,\Gamma_1) = \begin{cases} \langle(\iota_3,\kappa_2),\{\frac{y_1}{0.3,0.4,0.3},\frac{y_2}{0.6,0.3,0.4}\}\rangle, \\ \langle(\iota_3,\kappa_1),\{\frac{y_1}{0.7,0.1,0.2},\frac{y_2}{0.6,0.1,0.3}\}\rangle. \end{cases}$$

$$(R_2,\Gamma_2) = \begin{cases} \langle(\iota_1,\kappa_2),\{\frac{y_1}{0.7,0.4,0.2},\frac{y_2}{0.8,0.2,0.3}\}\rangle, \\ \langle(\iota_3,\kappa_2),\{\frac{y_1}{0.4,0.2,0.3},\frac{y_2}{0.7,0.1,0.3}\}\rangle \\ \langle(\iota_2,\kappa_2),\{\frac{y_1}{0.4,0.1,0.5},\frac{y_2}{0.6,0.1,0.4}\}\rangle. \end{cases}$$

It is clear that $\dot{\sigma}_1 \cap \dot{\sigma}_2$ *is an neutrosophichypersoft topology. However,*

$$(S_1,\Gamma_1) \cup (R_1,\Gamma_1) \notin \dot{\sigma}_1 \cup \dot{\sigma}_2,$$

which implies that $\dot{\sigma}_1 \cup \dot{\sigma}_2$ *is not an neutrosophic hypersoft topology over the universe* Y*.*

Proposition 2. *Let* $(Y,\dot{\sigma},\Lambda)$ *be a neutrosophic hypersoft topological space over* U*. Then, for any* $\iota \in \Gamma$*, the set*

$$\sigma = \{S(\iota) : (S,\Gamma) \in \dot{\sigma}\}$$

is a neutrosophic topology on Y*.*

Proof.

(1) $(N_0(Y,\Lambda)),(N_1(Y,\Lambda) \in \dot{\sigma}$.

In neutrosophic sets, the null set $\tilde{0} = (y,(T,I,F)) = (y,(0,1,1))$ and the absolute set $\tilde{1} = (y,(T,I,F)) = (y,(0,1,1))$.

It is clear that the values of the null set and absolute set in neutrosophic sets are equal to the values of the null set and absolute set in neutrosophic hypersoft sets.

Therefore, $\tilde{0},\tilde{1} \in \dot{\sigma}$.

(2) Let $R_1,R_2 \in \dot{\sigma}$. Then there exist $(S_1,\Gamma_1),(S_2,\Gamma_2) \in \dot{\sigma}$ such that $R_1 = S_1(\iota_1)$ and $R_2 = S_2(\iota_2)$.

Since $\dot{\sigma}$ is a neutrosophic hypersoft topology on Y, $(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2) \in \dot{\sigma}$.
Let $(S_3,\Gamma_3) = (S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2)$. Then $(S_3,\Gamma_3) \in \dot{\sigma}$.

Note that

$$R_1 \cap R_2 = S_1(\iota_1)\tilde{\cap}S_2(\iota_2) = (S_3,\Gamma_3)$$

and $\sigma = \{S(\iota) : (S,\Gamma) \in \dot{\sigma}\}$. Thus, $R_1 \cap R_2 \in \dot{\sigma}$.

(3) Let $\{R_i : i \in I\} \subseteq \grave{\sigma}$. Then for every $i \in I$, there exists $(S_i, \Gamma_i) \in \grave{\sigma}$ such that $R_i = S_i(\iota_i)$.

Since $\grave{\sigma}$ is a neutrosophic hypersoft topology on Y, $\bigcup\{(S_i, \Gamma_i) : i \in I\} \in \grave{\sigma}$. Let $(S, \Gamma) = \bigcup\{(S_i, \Gamma_i) : i \in I\}$. Then $(S, \Gamma) \in \grave{\sigma}$.

Note that

$$\bigcup_{i\in I} R_i = \bigcup_{i\in I}\{(S_i, \iota_i) : i \in I\} = (S, \Gamma)$$

and $\sigma = \{S(\iota) : (S, \Gamma) \in \grave{\sigma}\}$.

Therefore, $\bigcup_{i\in I} R_i \in \grave{\sigma}$.

Thus, $\sigma = \{S(\iota) : (S, \Gamma) \in \grave{\sigma}\}$ is a neutrosophic topology on Y.

Definition 8. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic hypersoft topological space over Y, and (S, Γ) be a neutrosophic hypersoft set. The neutrosophic hypersoft interior of (S, Γ), denoted by $int_{NH}(S, \Gamma)$, is defined as the neutrosophic hypersoft union of all neutrosophic hypersoft open subsets of (S, Γ).*

Clearly, $int_{NH}(S, \Gamma)$ is the largest neutrosophic hypersoft open set that is contained in (S, Γ).

Theorem 1. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic hypersoft topological space over Y, and $(S_1, \Gamma_1), (S_2, \Gamma_2) \in \mathrm{NHS}(U, \Delta)$. Then, the following properties hold:*

(i) $\mathrm{int}_{\mathrm{NH}}\big(N_0(Y, \Lambda))\big) = (N_0(Y, \Lambda))$ *and* $\mathrm{int}_{\mathrm{NH}}\big((N_1(Y, \Lambda))\big) = (N_1(Y, \Lambda))$,

(ii) $\mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)\tilde{\subseteq}(S_1, \Gamma_1)$,

(iii) (S_1, Γ_1) *is a neutrosophic hypersoft open set if and only if* $\mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1) = (S_1, \Gamma_1)$,

(iv) $\mathrm{int}_{\mathrm{NH}}\big(\mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)\big) = \mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)$,

(v) If $(S_1, \Gamma_1)\tilde{\subseteq}(S_2, \Gamma_2)$, *then* $\mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)\tilde{\subseteq}\mathrm{int}_{\mathrm{NH}}(S_2, \Gamma_2)$,

(vi) $\mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)\tilde{\cap}(S_2, \Gamma_2) \implies \mathrm{int}_{\mathrm{NH}}(S_1, \Gamma_1)\tilde{\cap}\mathrm{int}_{\mathrm{NH}}(S_2, \Gamma_2)$.

Proof. The first two properties are obvious.

(3) Let (S_1, Γ_1) be a neutrosophic hypersoft open set. Since $\mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1)$ is the largest neutrosophic hypersoft open set contained in (S_1, Γ_1), it follows that:

$$\mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1) = (S_1, \Gamma_1).$$

Conversely, suppose that $\mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1) = (S_1, \Gamma_1)$. Since $\mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1)$ is a neutrosophic hypersoft open set, it follows that (S_1, Γ_1) is also a neutrosophic hypersoft open set.

(4) Let $\mathrm{cl}_{\mathrm{NFH}}(S_1, \Gamma_1) = (S_1, \Gamma_1)$. Since (S_2, Γ_2) is a neutrosophic hypersoft open set, we know:

$$\mathrm{int}_{\mathrm{NFH}}(S_2, \Gamma_2) = (S_2, \Gamma_2).$$

Thus, we obtain:

$$\mathrm{int}_{\mathrm{NFH}}\big(\mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1)\big) = \mathrm{int}_{\mathrm{NFH}}(S_1, \Gamma_1).$$

(5) Let $(S_1,\Gamma_1)\tilde{\subseteq}(S_2,\Gamma_2)$. Since:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\subseteq}(S_1,\Gamma_1),$$

it follows that:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\subseteq(S_2,\tilde{\ }\Gamma_2).$$

Also, since $\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)$ is the largest neutrosophic hypersoft open set contained in (S_2,Γ_2), we have:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\subseteq\mathrm{int}_{\mathrm{NFH}}\tilde{(}S_2,\Gamma_2).$$

(6) We have:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\subseteq}(S_1,\Gamma_1)\quad\text{and}\quad\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)\tilde{\subseteq}(S_2,\Gamma_2).$$

Hence:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\cap}\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)\tilde{\subseteq}(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2).$$

Since the largest neutrosophic hypersoft open set contained in $(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2)$ is $\mathrm{int}_{\mathrm{NFH}}[(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2)]$, we have:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\cap}\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)\tilde{\subseteq}\mathrm{int}_{\mathrm{NFH}}[(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2)].$$

Conversely:

$$\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\cap}\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)\tilde{\cap}\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\quad\text{and}\quad\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\cap}\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)\tilde{\subseteq}\mathrm{int}_{\mathrm{NFH}}(S_2,\Gamma_2)$$

Hence:

$$\mathrm{int}_{\mathrm{NFH}}[(S_1,\Gamma_1)\tilde{\cap}(S_2,\Gamma_2)]\tilde{\subseteq}\mathrm{int}_{\mathrm{NFH}}(S_1,\Gamma_1)\tilde{\cap}\mathrm{cl}_{\mathrm{NFH}}(S_2,\Gamma_2).$$

Example 3. *We consider the attributes from Example 1. Let*

$$\grave{\sigma}=\{(N_0(Y,\Lambda))=(N_1(Y,\Lambda)),(S_1,\Gamma_1),(S_2,\Gamma_2),(S_3,\Gamma_3),(S_4,\Gamma_4)\}$$

is a neutrosophic hypersoft topology on Y. We define

$$(S_5,\Gamma_5)=\begin{cases}\left\langle(\iota_1,\kappa_1),\{\frac{y_1}{0.3,0.1,0.4},\frac{y_2}{0.2,0.2,0.5}\}\right\rangle,\\ \left\langle(\iota_2,\kappa_1),\{\frac{y_1}{0.7,0.2,0.1}\}\right\rangle,\\ \left\langle(\iota_2,\kappa_2),\{\frac{y_2}{0.6,0.2,0.2}\}\right\rangle,\\ \left\langle(\iota_3,\kappa_1),\{\frac{y_1}{0.4,0.1,0.2},\frac{y_2}{0.6,0.2,0.1}\}\right\rangle,\\ \left\langle(\iota_3,\kappa_2),\{\frac{y_1}{0.5,0.1,0.2},\frac{y_2}{0.3,0.2,0.4}\}\right\rangle.\end{cases}$$

Therefore

$$int_{NH}\left(S_5,\Gamma_5\right)=\left(N_0\left(Y,\Lambda\right)\right)\tilde{\cup}\left(S_1,\Gamma_1\right)\tilde{\cup}\left(S_2,\Gamma_2\right)\tilde{\cup}\left(S_3,\Gamma_3\right)\tilde{\cup}\left(S_4,\Gamma_4\right)=\left(S_3,\Gamma_3\right)$$

Definition 9. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic hypersoft topological space over U, and let (S, Γ) be a neutrosophic hypersoft set. The neutrosophic hypersoft closure of (S, Γ), denoted by $\mathrm{cl}_{\mathrm{NFH}}(S, \Gamma)$, is defined as the neutrosophic hypersoft intersection of all neutrosophic hypersoft closed supersets of (S, Γ).*

Clearly, $\mathrm{cl}_{\mathrm{NFH}}(S, \Gamma)$ is the smallest neutrosophic hypersoft closed set that contains (S, Γ).

Proposition 3. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic hypersoft topological space over Y. Then the following properties hold:*

(i) $(N_0(Y, \Lambda))$ *and* $(N_1(Y, \Lambda))$ *are neutrosophic hypersoft closed sets over Y.*

(ii) The intersection of any number of neutrosophic hypersoft closed sets is a neutrosophic hypersoft closed set over Y.

(iii) The union of any two neutrosophic hypersoft closed sets is a neutrosophic hypersoft closed set over Y.

Proof.

(1) For $(N_0(Y, \Lambda)) \in \grave{\sigma}$, it is an NH-open set. Since $(N_0(Y, \Lambda))^c = (N_1(Y, \Lambda))$, it follows that $(N_1(Y, \Lambda))$ is an NH-closed set. Conversely, if $(N_1(Y, \Lambda)) \in \grave{\sigma}$, then $(N_0(Y, \Lambda))$ is an NH-closed set.

(2) If $(S_\iota, \Gamma_\iota)^c \in \grave{\sigma}$ for $\iota \in I$, then $\bigcup_{\iota \in I}(S_\iota, \Gamma_\iota)^c \in \grave{\sigma}$. Thus, $\bigcap_{\iota \in I}(S_\iota, \Gamma_\iota)^c \in \grave{\sigma}$. Hence, $\bigcap_{\iota \in I}(S_\iota, \Gamma_\iota)$ is an NH-closed set over Y.

(3) Let $(S_1, \Gamma_1), (S_2, \Gamma_2) \in \grave{\sigma}^c$. Then $(S_1, \Gamma_1)^c, (S_2, \Gamma_2)^c \in \grave{\sigma}$. Also, we can write $((S_1, \Gamma_1) \cup (S_2, \Gamma_2))^c = (S_1, \Gamma_1)^c \cap (S_2, \Gamma_2)^c$. According to the definition of NH topology, $((S_1, \Gamma_1) \cup (S_2, \Gamma_2))^c \in \grave{\sigma}$, and hence $(S_1, \Gamma_1) \cup (S_2, \Gamma_2) \in \grave{\sigma}^c$.

Theorem 2. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic hypersoft topological space over U, and let $(H_1, \Omega_1), (H_2, \Omega_2) \in NHS(Y, \Delta)$. Then,*

(i) $cl_{NH}(0(\mathrm{UNH})) = 0(\mathrm{UNH})$ *and* $cl_{NH}(1(\mathrm{UNH})) = 1(\mathrm{UNH})$,

(ii) $(H_1, \Omega_1) \subseteq cl_{NH}(H_1, \Omega_1)$,

(iii) (H_1, Ω_1) *is a neutrosophic hypersoft closed set if and only if* $(H_1, \Omega_1) = cl_{NH}(H_1, \Omega_1)$,

(iv) $cl_{NH}(cl_{NH}(H_1, \Omega_1)) = cl_{NH}(H_1, \Omega_1)$,

(v) If $(H_1, \Omega_1) \subseteq (H_2, \Omega_2)$, *then* $cl_{NH}(H_1, \Omega_1) \subseteq cl_{NH}(H_2, \Omega_2)$,

(vi) $cl_{NH}[(H_1, \Omega_1) \cup (H_2, \Omega_2)] = cl_{NH}(H_1, \Omega_1) \cup cl_{NH}(H_2, \Omega_2)$.

Proof. The first two properties are obvious.

(3) Let (S_1, Γ_1) be a neutrosophic hypersoft closed set. By (2), we have $(S_1, \Gamma_1) \subseteq \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$. Since $\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$ is the smallest neutrosophic hypersoft closed set over U which contains (S_1, Γ_1), then $\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \subseteq (S_1, \Gamma_1)$. Hence $(S_1, \Gamma_1) = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$.

Conversely, suppose that $(S_1, \Gamma_1) = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$. Since $\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$ is a neutrosophic hypersoft closed set, then (S_1, Γ_1) is closed.

(4) Let $(S_1, \Gamma_1) = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$. Then, (S_1, Γ_1) is a neutrosophic hypersoft closed set. So, we have

$$\mathrm{cl}_{\mathrm{NH}}(\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)) = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1).$$

(5) If $(S_1, \Gamma_1) \subseteq (S_2, \Gamma_2)$, then

$$(S_2, \Gamma_2) = (S_1, \Gamma_1) \cup (S_1, \Gamma_1) \implies \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2) = \mathrm{cl}_{\mathrm{NH}}[(S_1, \Gamma_1) \cup (S_2, \Gamma_2)] = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \cup \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2).$$

Hence, $\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \subseteq \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2)$.

(6) Since $(S_1, \Gamma_1) \subseteq (S_1, \Gamma_1) \cup (S_2, \Gamma_2)$ and $(S_2, \Gamma_2) \subseteq (S_1, \Gamma_1) \cup (S_2, \Gamma_2)$, from (5),

$$\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1), \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2) \subseteq \mathrm{cl}_{\mathrm{NH}}[(S_1, \Gamma_1) \cup (S_2, \Gamma_2)].$$

Therefore,

$$\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \cup \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2) \subseteq \mathrm{cl}_{\mathrm{NH}}[(S_1, \Gamma_1) \cup (S_2, \Gamma_2)].$$

Conversely, since $(S_1, \Gamma_1) \subseteq \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1)$ and $(S_2, \Gamma_2) \subseteq \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2)$, $\mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \cup \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2)$ is a neutrosophic hypersoft closed set over Y being the union of two neutrosophic hypersoft closed sets. Then,

$$\mathrm{cl}_{\mathrm{NH}}[(S_1, \Gamma_1) \cup (S_2, \Gamma_2)] = \mathrm{cl}_{\mathrm{NH}}(S_1, \Gamma_1) \cup \mathrm{cl}_{\mathrm{NH}}(S_2, \Gamma_2).$$

Theorem 3. *Let $(Y, \grave{\sigma}, \Lambda)$ be a neutrosophic fuzzy hypersoft topological space over U and $(S, \Gamma) \in NHS(Y, \Delta)$. Then:*

(i) $(cl_{NFH}(S, \Gamma))^c = int_{NFH}((S, \Gamma)^c)$,

(ii) $(int_{NFH}(S, \Gamma))^c = cl_{NFH}((S, \Gamma)^c)$.

Proof. 1. Using the properties of neutrosophic fuzzy hypersoft closures and interiors:

$$\mathrm{cl}_{\mathrm{NFH}}((S, \Gamma)^c) = \mathrm{cl}_{\mathrm{NFH}}(S, \Gamma) \cap \mathrm{cl}_{\mathrm{NFH}}(S, \Gamma).$$

By the definition of complement and interior:

$$(\mathrm{cl}_{\mathrm{NFH}}(S, \Gamma))^c = \mathrm{int}_{\mathrm{NFH}}((S, \Gamma)^c).$$

2. Similarly, for the interior operator:

$$(\mathrm{int}_{\mathrm{NFH}}(S, \Gamma))^c = \mathrm{cl}_{\mathrm{NFH}}((S, \Gamma)^c).$$

The proof follows directly from the duality of closures and interiors in neutrosophic fuzzy hypersoft topology.

Example 4. *We consider the attributes from Example 1. Let*

$$\grave{\sigma} = \{(N_0(Y,\Lambda)) = (N_1(Y,\Lambda)), (S_1,\Gamma_1), (S_2,\Gamma_2), (S_3,\Gamma_3), (S_4,\Gamma_4)\}$$

is a neutrosophic hypersoft topology on Y. We define

$$(S_5,\Gamma_5) = \begin{cases} \left\langle (\iota_1,\kappa_1), \{\frac{y_1}{0.3,0.1,0.4}, \frac{y_2}{0.2,0.2,0.5}\}\right\rangle, \\ \left\langle (\iota_2,\kappa_1), \{\frac{y_1}{0.7,0.2,0.1}\}\right\rangle, \\ \left\langle (\iota_2,\kappa_2), \{\frac{y_2}{0.6,0.2,0.2}\}\right\rangle, \\ \left\langle (\iota_3,\kappa_1), \{\frac{y_1}{0.4,0.1,0.2}, \frac{y_2}{0.6,0.2,0.1}\}\right\rangle, \\ \left\langle (\iota_3,\kappa_2), \{\frac{y_1}{0.5,0.1,0.2}, \frac{y_2}{0.3,0.2,0.4}\}\right\rangle. \end{cases}$$

Therefore

$$int_{NH}(S_5,\Gamma_5) = (N_0(Y,\Lambda))\,\tilde{\cup}\,(S_1,\Gamma_1)\,\tilde{\cup}\,(S_2,\Gamma_2)\,\tilde{\cup}\,(S_3,\Gamma_3)\,\tilde{\cup}\,(S_4,\Gamma_4) = (S_3,\Gamma_3).$$

Definition 10. *Let $(Y,\grave{\sigma})$ be a neutrosophic fuzzy hypersoft topological space over Y and $\tilde{K} \subseteq \grave{\sigma}$. $\tilde{K}$ is called a neutrosophic fuzzy hypersoft basis for the neutrosophic fuzzy hypersoft topology $\grave{\sigma}$ if every element of $\grave{\sigma}$ can be written as the neutrosophic fuzzy hypersoft union of elements of $\tilde{K}$.*

Proposition 4. *Let $(Y,\grave{\sigma})$ be a neutrosophic fuzzy hypersoft topological space over Y and $\tilde{K}$ be a neutrosophic fuzzy hypersoft basis for $\grave{\sigma}$. Then $\grave{\sigma}$ equals the collection of neutrosophic fuzzy hypersoft unions of elements of $\tilde{K}$.*

Proof. The proof is straightforward from the definition of a neutrosophic fuzzy hypersoft basis. By definition, every element of $\grave{\sigma}$ can be expressed as a neutrosophic fuzzy hypersoft union of elements of $\tilde{K}$.

Example 5. *We consider Example 1 Then,*

$$\tilde{K} = \{(N_0(Y,\Lambda)), (N_1(Y,\Lambda)), (S_1,\Gamma_1), (S_3,\Gamma_3)\}$$

is a neutrosophic hypersoft basis for the neutrosophic hypersoft topology $\grave{\sigma}$.

Theorem 4. *Let $(Y,\grave{\sigma})$ be a neutrosophic hypersoft topological space over Y and (S,Γ) be a neutrosophic hypersoft set over Y. Then the collection*

$$\grave{\sigma}_{(S,\Gamma)} = \{(S,\Gamma)\tilde{\cap}(R_i,\Upsilon_i) : (R_i,\Upsilon_i) \in \grave{\sigma} \text{ for } i \in I\}$$

is a neutrosophic hypersoft topology on the neutrosophic hypersoft subset $(\aleph,\Upsilon)$ relative to the parameter set Υ.

Proof. $N_0(Y,\Lambda)), (N_1(Y,\Lambda) \in \grave{\sigma}_{(S,\Gamma)}$.

Moreover, we have

$$\bigcap_{i\in I}(S,\Gamma)\tilde{\cap}(R_i,\Upsilon_i) = (S,\Gamma)\tilde{\cap}\left(\bigcap_{i\in I}(R_i,\Upsilon_i)\right),$$

and

$$\bigcup_{i\in I}(S,\Gamma)\tilde{\cap}(R_i,\Upsilon_i) = (S,\Gamma)\tilde{\cap}\left(\bigcup_{i\in I}(R_i,\Upsilon_i)\right).$$

Thus, the neutrosophic hypersoft union of any number of neutrosophic hypersoft sets in $\grave{\sigma}_{(S,\Gamma)}$ *belongs to* $\grave{\sigma}_{(S,\Gamma)}$*, and the finite neutrosophic hypersoft intersection of neutrosophic hypersoft sets in* $\grave{\sigma}_{(S,\Gamma)}$ *also belongs to* $\grave{\sigma}_{(S,\Gamma)}$*.*

Hence, $\grave{\sigma}_{(S,\Gamma)}$ *is a neutrosophic hypersoft topology on* (S,Γ)*.*

Definition 11. *Let* $(Y,\grave{\sigma})$ *be a neutrosophic hypersoft topological space over* Y *and* (S,Γ) *be a neutrosophic hypersoft set over* Y*. Then the neutrosophic hypersoft topology*

$$\grave{\sigma}_{(S,\Gamma)} = \{(S,\Gamma)\tilde{\cap}(R_i,\Upsilon_i) : (R_i,\Upsilon_i) \in \grave{\sigma} \text{ for } i \in I\}$$

is called the neutrosophic hypersoft subspace topology, and $((S,\Gamma),\grave{\sigma}_{(S,\Gamma)},\Gamma)$ *is called a neutrosophic hypersoft subspace of* $(Y,\grave{\sigma})$*.*

3. MCDM PROBLEM BASED NEUTROSOPHIC AGRICULTURAL TOPOLOGY

Choice is essential in arriving at the best solution of a given problem especially when it comes to decision-making involving agriculture. As seen in most real-life problems in agriculture the of time it might be tough to find a good plan of action because of so many variables. The Neutrosophic Agricultural Topology-based MCDM approach explained above, offers a good foundation when it comes to these challenges, it adopts the use of neutrosophic logic and topology.

This paper deals with an agricultural decision-making issue by implementing Neutrosophic logic. Firstly, the problem is solved by building Neutrosophic sets to address the uncertainties within and the fuzziness of the attribute values. Subsequently, these Neutrosophic sets are used as a sub-base, to generate a topology. The problem is then reconsidered about the open sets resulting from this topology. This dual approach enables the comparison providing the importance of topology in the context of MCDM for agriculture.

3.1. Definition of the Problem

Farm site selection is an issue that is absolutely critical, yet even at its most basic, can greatly affect the outcome of the crop. The location of a farm determines a number of factors, namely production yields, market access, and access to production requisites such as water and fertilizers. Therefore, the proper location is been of great importance in identifying the premiere area for the establishment of agribusiness centers. In this study, $Y = \{y_1, y_2, y_3, y_4, y_5\}$ represents the universal set of all that contains potential locations for farm development. Let P denote the set of evaluation criteria, defined as follows: $P_1 = \{\iota_1, \iota_2, \iota_3, \iota_4\}$, $P_2 = \{\kappa_1, \kappa_2, \kappa_3\}$ and $P_3 = \{\zeta_1, \zeta_2, \zeta_3\}$.

Suppose M_i and N_i where $i = 1, 2, 3$ are subsets formed by various decision-makers based on these criteria, defined as: $M_1 = \{\iota_1\}$, $M_2 = \{\kappa_3\}$, $M_3 = \{\zeta_2, \zeta_3\}$, $N_1 = \{\iota_1, \iota_2\}$, $N_2 = \{\kappa_3\}$, and $N_3 = \{\zeta_2\}$.

Solving the Problem with NFHSs

Algorithm 1: Steps for Solving the Problem using Neutrosophic Fuzzy Hypersoft Sets

Step-1: Input the neutrosophic hypersoft sets (S_1, Γ_1) and (S_2, Γ_2) over Y.

Step-2: Compute the resultant neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$.

Step-3: Construct the comparison table of the neutrosophic hypersoft set and calculate the row sum (r_i) and column sum (c_i) for all elements y_i.

Step-4: Compute the resulting score O_i for each element y_i using:

$$O_i = r_i - c_i, \quad \forall i.$$

Step-5: The optimal choice is y_j, where:

$$y_j = \arg\max_i \{O_i\}.$$

Suppose that neutrosophic hypersoft sets (S_1, Γ_1) and (S_2, Γ_2) are defined as follows. The tabular representations of (S_1, Γ_1) and (S_2, Γ_2) are shown below. Now, we determine the resultant neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$.

Table 1: Tabular Form of (S_1, Γ_1)

(S_1, Γ_1)	y_1	y_2	y_3	y_4	y_5
$(\iota_1, \kappa_3, \zeta_2) = t_1$	$(0.3, 0.5, 0.2)$	$(0.2, 0.8, 0.1)$	$(0, 1, 0)$	$(0, 1, 0)$	$(0, 1, 0)$
$(\iota_1, \kappa_3, \zeta_3) = t_2$	$(0, 1, 0)$	$(0.6, 0.1, 0.3)$	$(0, 1, 0)$	$(0.7, 0.5, 0.2)$	$(0.3, 0.1, 0.4)$

Table 2: Tabular Form of (S_2, Γ_2)

(S_2, Γ_2)	y_1	y_2	y_3	y_4	y_5
$(\iota_1, \kappa_3, \zeta_2) = u_1$	$(0.5, 0.7, 0.3)$	$(0.4, 0.1, 0.5)$	$(0, 1, 0)$	$(0, 1, 0.7)$	$(0, 1, 0)$
$(\iota_2, \kappa_2, \zeta_2) = u_2$	$(0, 1, 0.8)$	$(0.1, 0.7, 0.2)$	$(0, 1, 0)$	$(0.7, 0.3, 0.5)$	$(0.4, 0.1, 0.6)$

Table 3: Tabular Form of $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$

$(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$	y_1	y_2	y_3	y_4	y_5
$(t_1 \times u_1) = x_1$	$(0.3, 0.5, 0.3)$	$(0.2, 0.1, 0.5)$	$(0, 1, 0)$	$(0, 1, 0.7)$	$(0, 1, 0)$
$(t_1 \times u_2) = x_2$	$(0, 0.5, 0.8)$	$(0.1, 0.7, 0.2)$	$(0, 1, 0)$	$(0, 0.3, 0.5)$	$(0, 0.1, 0.6)$
$(t_2 \times u_1) = x_3$	$(0, 0.7, 0.3)$	$(0.4, 0.1, 0.5)$	$(0, 1, 0)$	$(0, 0.5, 0.7)$	$(0, 0.1, 0.4)$
$(t_2 \times u_2) = x_4$	$(0, 1, 0.8)$	$(0.1, 0.1, 0.3)$	$(0, 1, 0)$	$(0.7, 0.3, 0.5)$	$(0.3, 0.1, 0.6)$

To compare these sets, we construct the comparison table for the neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$ using the algorithm provided by Roy and Maji [36]. The comparison table is a square table where the number of rows equals the number of columns. Both rows and columns are labeled by the object names $y_1, y_2, \ldots, y_n$ from the universe.

The entries x_i, $i = 1, 2, \ldots, n$, are defined as,
$x_i =$ the number of parameters for which the membership value of o_i exceeds or equals the membership value of y_i.

The comparison table is presented below. From this table, we calculate the column sum (c_i) and row sum (r_i). Using these values, the score O_i for each y_i $(i = 1, 2, 3, 4, 5)$ is computed.

According to Table 5, it is evident that the most suitable location is y_2.

In the next section, we will solve the same problem by constructing the neutrosophic hypersoft topology, and the results will be discussed.

Table 4: Comparison Table

$(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$	y_1	y_2	y_3	y_4	y_5
y_1	4	1	4	3	3
y_2	3	4	4	3	3
y_3	3	0	4	3	3
y_4	3	1	4	4	4
y_5	3	1	4	3	4

Table 5: Tabular Form of score-values

Object (y_i)	**Row Sum** (r_i)	**Column Sum** (c_i)	**Score** $(O_i = r_i - c_i)$
y_1	15	16	-1
y_2	17	7	10
y_3	13	20	-7
y_4	16	16	0
y_5	15	17	-2

Solving the Problem with Neutrosophic Hypersoft Topology Algorithm

Algorithm 2

Step-1: Consider a universe of Y.

Step-2: Define a set P of attributes.

Step-3: Construct the neutrosophic hypersoft sets (S_1, Γ_1) and (S_2, Γ_2) over Y.

Step-4: Write the neutrosophic hypersoft topology $\grave{\sigma}$, where (S_1, Γ_1) and (S_2, Γ_2) are open neutrosophic hypersoft sets in $\grave{\sigma}$.

Step-5: Find the resultant neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$ and other open neutrosophic hypersoft sets in $\grave{\sigma}$ using the ÔÇ£ORÔÇØ operation.

Step-6: Construct the comparison table for the neutrosophic hypersoft set and compute the row sum (r_i) and column sum (c_i).

Step-7: Calculate the resulting score O_i for each y_i, $\forall i$, using:

$$O_i = r_i - c_i.$$

Step-8: Determine the optimal choice y_j such that:

$$y_j = \arg\max\{O_i\}.$$

Let us construct the NH topology. We are given (S_1, Γ_1) and (S_2, Γ_2). The goal is to create a topology such that these sets are open sets.

$$\grave{\sigma} = \{(N_0(Y, \Lambda)), (N_1(Y, \Lambda)), (S_1, \Gamma_1), (S_2, \Gamma_2), (S_3, \Gamma_3), (S_4, \Gamma_4)\},$$

where (S_3, Γ_3) and (S_4, Γ_4) are defined as follows:

Table 6: Tabular form of (S_3, Γ_3)

(S_3, Γ_3)	y_1	y_2	y_3	y_4	y_5
$(\iota_1, \kappa_3, \zeta_2) = t_1$	$(0.5, 0.5, 0.3)$	$(0.4, 0.1, 0.5)$	$(0, 1, 0)$	$(0, 1, 0.7)$	$(0, 1, 0)$
$(\iota_1, \kappa_3, \zeta_3) = t_2$	$(0, 1, 0)$	$(0.6, 0.1, 0.3)$	$(0, 1, 0)$	$(0.7, 0.5, 0.2)$	$(0.3, 0.1, 0.4)$
$(\iota_1, \kappa_2, \zeta_2) = t_3$	$(0, 1, 0.8)$	$(0, 1, 0.2)$	$(0, 1, 0)$	$(0.7, 0.3, 0.5)$	$(0.4, 0.1, 0.6)$

Table 7: Tabular form of (S_4, Γ_4)

(S_4, Γ_4)	y_1	y_2	y_3	y_4	y_5
$(\iota_1, \kappa_3, \zeta_2) = u_1$	$(0.3, 0.7, 0.2)$	$(0.2, 0.8, 0.3)$	$(0, 1, 0)$	$(0, 1, 0)$	$(0, 1, 0)$

Table 8: Tabular form of $(S_3, \Gamma_3) \vee (S_4, \Gamma_4)$

$(S_3, \Gamma_3) \vee (S_4, \Gamma_4)$	y_1	y_2	y_3	y_4	y_5
$(t_1 \times u_1) = v_1$	$(0.3, 0.5, 0.3)$	$(0.2, 0.1, 0.5)$	$(0, 1, 0)$	$(0, 1, 0.7)$	$(0, 1, 0)$
$(t_2 \times u_1) = v_2$	$(0, 0.7, 0.2)$	$(0.2, 0.1, 0.3)$	$(0, 1, 0)$	$(0, 0.5, 0.2)$	$(0.3, 0.1, 0.4)$
$(t_3 \times u_1) = v_3$	$(0, 0.7, 0.2)$	$(0, 0.8, 0.3)$	$(0, 1, 0)$	$(0, 0.3, 0.5)$	$(0, 0.1, 0.6)$

We now calculate $(S_1, \Gamma_1) \vee (S_2, \Gamma_2) \vee (S_3, \Gamma_3) \vee (S_4, \Gamma_4)$.

First, we compute $(S_3, \Gamma_3) \vee (S_4, \Gamma_4)$. Then, we find $(S_1, \Gamma_1) \vee (S_2, \Gamma_2)$. Finally, we calculate the resultant $(S_1, \Gamma_1) \vee (S_2, \Gamma_2) \vee (S_3, \Gamma_3) \vee (S_4, \Gamma_4)$.

Next, we construct the comparison table for the neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2) \vee (S_3, \Gamma_3) \vee (S_4, \Gamma_4)$.

In the comparison table:

- The column sum, c_i, and row sum, r_i, are calculated.
- The score O_i for each y_i $(i = 1, 2, 3, 4, 5)$ is computed as:

$$O_i = r_i - c_i.$$

Table 9: Comparison table of neutrosophic hypersoft set $(S_1, \Gamma_1) \vee (S_2, \Gamma_2) \vee (S_3, \Gamma_3) \vee (S_4, \Gamma_4)$

$(S_1, \Gamma_1) \vee (S_2, \Gamma_2) \vee (S_3, \Gamma_3) \vee (S_4, \Gamma_4)$	y_1	y_2	y_3	y_4	y_5
y_1	12	1	12	11	11
y_2	11	12	12	11	11
y_3	11	0	12	11	11
y_4	11	1	12	12	12
y_5	11	13	12	11	12

Table 10: Tabular Form of score-values

Object (y_i)	**Row Sum** (r_i)	**Column Sum** (c_i)	**Score** $(O_i = r_i - c_i)$
y_1	47	56	-9
y_2	57	15	42
y_3	45	60	-15
y_4	48	56	-8
y_5	47	57	-10

According to Table 10, it is evident that the most suitable location is y_2.

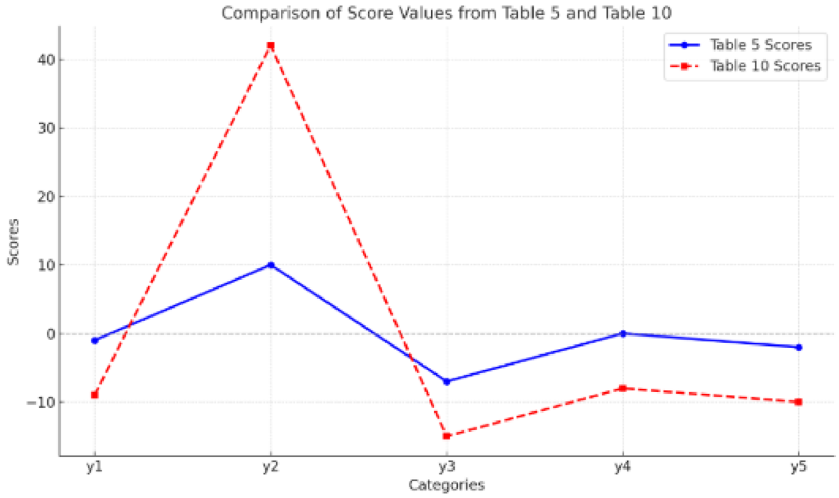

Figure 1: Comparison of score value .

4. Result

Identifying the best land for agriculture stands as the main objective within this problem. The most suitable location according to Algorithm 1 turns out to be u_2. The addition of a topology in Algorithm 2 enables an improved selection refinement method. The introduction of open set treatment to the Algorithm 1 sets expands analytical possibilities in this approach.

The solution space gains tremendous enlargement from the addition of topology components. Algorithm 1 presented at first used four criteria for its selection process. The implementation of topology in Algorithm 2 allows the problem to operate with twelve parameters rather than four. The elongated method deepens organizational decision-making through expanded assessment techniques that merge various factors and their multiple relationships to enhance location comprehension.

A comparison reveals the findings between Algorithm 1 and Algorithm 2 through the provided graph. Algorithm 2 produces the result u_2 as the best land for agriculture. By using both the Algorithm the best land for agriculture is u_2.

5. Conclusion

Analyses of hypersoft topological spaces hold major importance through their establishment of generalized frameworks that integrate parametrized classical topological spaces. We analyze neutrosophic hypersoft topological spaces (NH-TS) in this paper using neutrosophic logic to model topological uncertainty and indeterminacy within the structures. Within NH-TS we examine multiple fundamental characteristics followed by definitions of interior operators and closure functions and basis formulations and subspace topological structures. The paper shows these fundamental concepts through practical examples which display their functional value. The development of decision-making solutions employs concept expansions of neutrosophic hypersoft topology. The study demonstrates practical usefulness by implementing Algorithm 1 with NH set structure to solve a real-world decision problem. The obtained results are analyzed against those produced through Algorithm 2 which used NH topology to solve the same problem. A comparison between the implementations shows crucial findings about the topological structure's effect on decision processes.

The research calls attention to unfinished work that investigates how topological characteristics like continuity connectedness and compactness operate inside NH topological spaces. Research avenues in neutrosophic logic show promising growth potential which extends applicability toward topological investigations and decision-making solutions. The research team plans to develop neutrosophic agricultural topology-based MCDM for extensive agricultural systems alongside predictive analytics through machine learning integration and artificial intelligence. Research should continue to investigate how neutrosophic topology applications handle dynamic as well as temporal agricultural conditions. The construction of a decision support system based on this framework will improve practical applications throughout smart farming and precision agriculture settings.

5.1. Future research

Subsequent studies on neutrosophic hypersoft topological spaces ($\mathcal{NH}-\mathcal{TS}$) can address the application of sophisticated optimization algorithms such as genetic algorithms, particle swarm optimization ($\mathcal{PSO}$), and ant colony optimization ($\mathcal{ACO}$) for improving decision-making efficiency, while creating hybrid models that incorporate NH-TS and machine learning for predictive analysis in precision agriculture. Testing the scalability of the framework with large datasets and complicated agricultural settings, and determining computational complexity and real-time processing for dynamic and temporal scenarios, is critical. Investigating further topological properties like continuity, connectedness, and compactness, and the effects of subspace structures and basis formulations, can enhance decision-making effectiveness. Development of an integrated decision support system ($\mathcal{DSS}$) based on $\mathcal{NH}-\mathcal{TS}$ for smart farming and precision agriculture applications, with predictive analytics modules via artificial intelligence ($\mathcal{AI}$) and machine learning (ML) for yield forecasting, resource planning, and climate adjustment strategies, is crucial. Also, extending the applications of $\mathcal{NH}-\mathcal{TS}$ to healthcare, supply chain management, and environmental monitoring will test its flexibility and efficacy. Theoretical developments in neutrosophic logic and soft set theory, advancements in the creation of new closure functions and interior operators to deal with multi-criteria decision-making ($\mathcal{MCDM}$) in various scenarios, can further improve the $\mathcal{NH}-\mathcal{TS}$ framework's precision, scalability, and efficiency.

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