EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
2025, Vol. 18, Issue 2, Article Number 5937
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

The Effect of the Weighted Expert Factor on Time Fuzzy Soft Expert Sets

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Abstract. This paper introduces the Weighted Time Fuzzy Soft Expert Set (W-TFSES), an advanced framework that extends fuzzy soft set theory by incorporating time-dependent expert weighting to enhance decision-making under uncertainty. We rigorously define W-TFSES operations—including complement, union, intersection, AND, and OR—and demonstrate its superiority over conventional models through real-world applications in risk assessment and medical diagnosis. Unlike prior approaches, W-TFSES explicitly quantifies the evolving influence of experts over time, offering improved accuracy in dynamic, uncertain environments. Our work bridges the gap between fuzzy soft sets and temporal decision-making, establishing W-TFSES as a versatile tool for complex, real-world problems.

Key Words and Phrases: Soft set, Fuzzy soft set, Time fuzzy soft expert set

1. Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov[1] introduced the notion of soft set theory as a tool in math for coping with such uncertainty. Following Molodtsov's work, Maji et al.[2] researched several soft set operations and applications. Also Maji et al. [3] presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji [4] also applied this idea to handle decision-making challenges. Recently, various scholars have begun studying the properties and applications of soft set theory as in the research [5, 6].Wang [7] showed that in many real situations, immediate sensory data is insufficient for decision making.

The fuzzy soft set was extended by [8] to create the parameterized time fuzzy soft expert set It also examined and explained the features of its main function and used this method to solve problems with decision-making. In [9], the authors introduced A useful framework for assisting decision-maker in problem-solving is decision-making theory.

Furthermore, in 2010 Çağman et al.[10] established the notion of fuzzy parameterized fuzzy soft set and its operations. In addition, the FPFS-aggregation operator is used

DOI: https://doi.org/10.29020/nybg.ejpam.v18i2.5937

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to create the FPFS-decision making technique, which allows for more efficient decision processes. Alkhazaleh and Salleh [11, 12], proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures. In [13], the authors discussed fuzzy parameterized fuzzy soft expert sets, which offer a membership value for each parameter in a collection of parameters and are an extension of fuzzy soft expert sets. In [14], the authors provided an overview of generalized fuzzy soft expert set. Recently, various scholars have begun studying the properties and applications of soft set theory. Additionally, researchers introduced using a neutrosophic fuzzy soft set to solve decision-making problems, like in [15–18]. Furthermore, Alqaraleh et. al.[19] introduced the concept of bipolar complex fuzzy soft sets. Recent developments in fuzzy soft topology have advanced the theoretical foundations for handling uncertainty. Saleh *et al.* [20] established categorical properties of fuzzy soft spaces, while Saleh and Al-Mufarrij [21] introduced θ-regularity concepts - both crucial for extending these structures to time-dependent expert systems like our W-TFSES. Recent advances in soft topology include the development of generalized covering properties, such as the almost/weakly soft Menger spaces [22] and nearly soft Menger spaces [23], which extend classical topological concepts to soft set frameworks. These works demonstrate the growing sophistication of soft topological structures, motivating further extensions like our weighted time-dependent approach.

A critical component of decision-making is expert weighting, which guarantees that more skilled and informed experts have a bigger impact on results. This improves accuracy, lowers bias, and strengthens decision robustness, especially in uncertain contexts like healthcare, finance, and engineering [24]. Decision-making procedures run the danger of being dominated by less competent people in the absence of expert weighing, which might produce biased or less-than-ideal outcomes [25]. This is particularly crucial in fuzzy logic-based decision systems, such temporal fuzzy soft expert sets, where experts are given the right weights to improve decision accuracy and uncertainty and imprecise information are inherent [26]. Decision-making frameworks become more organized, less subjective, and more reliable when weighted expert opinions are incorporated. so using historical information as part of the state representation provides us with important information to assist us in making better judgments in situations when temporal value is not taken into account, resulting in less accurate decision-making. If we want to take the views of more than one time (period), we must perform various operations such as union, intersection, etc. For a solution to this problem, we take a collection of time intervals, generalize it into what we call time fuzzy soft expert sets that is induced by [27], investigate some of its features, and apply this notion to a decision-making problem. It is critical to understand the history of the parameters under consideration in order to ensure the credibility of the information provided by specialists. The experts' previous experiences are gathered in the number of periods (years, months, etc.) in which they are involved in a certain decision-making circumstance, and by looking at the time component, individuals are more confident in the conclusion that they make. We must examine the influence of time on fuzzy soft set applications, not only for the present period, but also for the past and future periods (forecasting information). The concept of time fuzzy was later used by other studies with such subjects, including in works of [8, 28–30].

In this paper, we will present the notion of the effect of the weighted expert factor on time fuzzy soft expert sets, which is more effective and valuable, as we will see and the decisions made will be more precise, this means we will take the component time value of the information in our consideration when we are making decision. We will also define and investigate the attributes of its basic operations, which are complement, union and intersection. Finally, we'll apply this approach to decision-making difficulties.

2. Preliminaries

In this part, we cover several fundamental concepts in soft set theory. Molodtsov [1] defined soft sets as follows over U: Let U represent the universe set and P the set of parameters. $P(U)$ signifies the power set of U and $J \subseteq P$.

Definition 1. *[1] Consider this mapping*

$$J : M \to P\left(U\right).$$

Any A pair (J, M) is considered a soft set *over U. In other terms, a soft set over U is a parameterized collection of subsets of the universe set U. For $\delta \in J, J\left(\delta\right)$ can be viewed as the set of δ-approximate members of the soft set (J, M).*

Definition 2. *[3] Let U be the initial universal set, and P be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. Let $J \subseteq P$, and F be the mapping*

$$F : A \to I^U.$$

A pair (J, P) is known as a fuzzy soft set *over U.*

Definition 3. *[3] Regarding two fuzzy soft sets (J, M) and (K, N) over U, (J, M) is known as a fuzzy soft subset of. (K, N) if*

(i) $M \subset N$ and

(ii) $\forall \delta \in J, J\left(\delta\right)$ is fuzzy subset of $K\left(\delta\right)$.

The association is represented by $(F,\ A) \tilde{\subset} (K, N)$. In this situation, (K, N) is known as a fuzzy soft superset of. (J, M).

Definition 4. *[3] $(J, M)^c$ represents the complement of a fuzzy soft set (J, M), which has been described by $(J, M)^c = (J^c, \rceil A)$ where $J^c : \rceil A \to P\left(U\right)$ is a mapping provided by*

$$J^c\left(\Gamma\right) = c\left(J\left(\rceil\Gamma\right)\right), \forall \Gamma \in \rceil M.$$

c describes any fuzzy complement.

Definition 5. *[3] If (J, M) and (K, N) are two fuzzy soft sets then (J, M) AND (K, N) denoted by $(J,\, M) \wedge (K, N)$ is defined by*

$$(J, M) \wedge (K, N) = (C, M \times N)$$

such that $C(\Gamma, \lambda) = t(J(\Gamma), K(\lambda)), \forall (\Gamma, \lambda) \in J \times N$, *where* t *is any t-norm.*

Definition 6. *[3] If* (J, M) *and* (K, N) *are two fuzzy soft sets then* (J, M) *OR* (K, N) *denoted by* $(J, M) \vee (K, N)$ *is defined by*

$$(J, M) \vee (K, N) = (O, M \times N)$$

such that $O(\Gamma, \lambda) = s(J(\Gamma), G(\lambda)), \forall (\Gamma, \lambda) \in M \times N$, *where* s *is any s-norm.*

Definition 7. *[3] The union of two fuzzy soft sets* (J, M) *and* (K, N) *over a common universe* U *is the fuzzy soft set* (H, C) *where* $C = M \cup N$, *and* $\forall \delta \in C$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cup N. \end{cases}$$

Where s *is any s-norm.*

Definition 8. *[3] The intersection of two fuzzy soft sets* (J, M) *and* (K, N) *over a common universe* U *is the fuzzy soft set* (H, C) *where* $C = M \cup N$, *and* $\forall \delta \in R$,

$$H(\delta) = \begin{cases} J(\delta), & if \quad \delta \in M - N, \\ G(\delta), & if \quad \delta \in N - M, \\ s(J(\delta), G(\delta)), & if \quad \delta \in M \cap N. \end{cases}$$

Definition 9. *[11] Let* U *be a set of universes,* P *a set of parameters,* X *a set of experts (agents). Let* $O = \{o_1, o_2, ..., o_n\}$ *be a set of opinions,* $Z = E \times X \times O$ *and* $M \subseteq Z$. *A pair* $(F,\ M)$ *is called a soft expert set over* U, *where* F *is a mapping given by*

$$F \ : \ A \ \rightarrow \ P(U)$$

where $P(U)$ *denoted the power set of* U.

Definition 10. *[12] Let* U *be a set of universes,* P *a set of parameters,* X *a set of experts (agents). Let* O *be a set of opinions,* $Z = E \times X \times O$ *and* $A \subseteq Z$. *A pair* (J, M) *is called a fuzzy soft expert set over* U, *where* F *is a mapping given by*

$$J : M \rightarrow I^U$$

Where I^U *denotes all fuzzy subsets of* U.

Definition 11. *Let* U *be an initial universal set and let* P *be a set of parameters. Let* I^U *denote the power set of all fuzzy subsets of* U, *let* $J \subseteq P$ *and* T *be a set of time where* $T = \{t_1, t_2, ..., t_n\}$. *A collection of pairs* $(F, E)_t^w \, \forall \, t \in T$ *is called a* time-fuzzy soft set $T - FSS$ *over* U *where* J_t *is a mapping given by*

$$F_t^w : A \rightarrow I^U.$$

3. Weighted Time Fuzzy Soft Expert Set

The definition of a The effect of the weighted expert factor on time fuzzy soft expert sets(W-TFSES) and their fundamental characteristics are presented in this section. We will give more consideration to the expert weights; this means that the choice experts may not be of equal importance degree. Additionally, we must take into account the component time value of the information when making judgments since doing so will result in more accurate decisions because some or all of the parameters have a time value for prior knowledge. Additionally, we define and examine the features of its fundamental operations—union, complement, intersection, AND, and OR. Lastly, we provide speculative uses of this idea in decision-making situations.

3.1. Main Definition

Definition 12. *Define U as the universe, E as a set of parameters, X as a group of experts, and $O = \{o_1, o_2, ..., o_n\}$ as a collection of viewpoints and $W = \{w_1, w_2, ..., w_n\}$ as a collection of weights for the experts. For Z to be equal to $E \times X \times O$, $A \subseteq Z$. Let T be a collection of times where $T = \{t_1, t_2, ..., t_n\}$, and let I^U represent the power set of all fuzzy subsets of U. A*time-fuzzy soft expert set *W-TFSES over U is a set of pairings $(F, A)_t^w\, \forall\, t_i \in T$, where F is a mapping provided by*

$$F_t^w : A \to I^U.$$

Example 1. *Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of universe, $E = \{e_1, e_2, e_3\}$ a set of parameters and $T = \{t_1, t_2, t_3\}$ be a set of time and $X = \{m, n, r\}$ a set of experts and $W = \{0.8, 0.5, 0.7\}$ of weights for the experts. Define a function*

$$F_t^w : A \to I^U.$$

as follows:

$$F_1^w\left(e_1, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.8}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.4}, \tfrac{u_4{}^{t_1}}{0.3}\right\}, F_1^w\left(e_1, \tfrac{n}{0.5}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.9}, \tfrac{u_3{}^{t_1}}{0.6}, \tfrac{u_4{}^{t_1}}{0.5}\right\},$$
$$F_1^w\left(e_1, \tfrac{r}{0.7}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.3}, \tfrac{u_4{}^{t_1}}{0.7}\right\}, F_1^w\left(e_2, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.1}, \tfrac{u_3{}^{t_1}}{0.4}, \tfrac{u_4{}^{t_1}}{0.8}\right\},$$
$$F_1^w\left(e_2, \tfrac{n}{0.5}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.7}, \tfrac{u_3{}^{t_1}}{0.2}, \tfrac{u_4{}^{t_1}}{0.5}\right\}, F_1^w\left(e_2, \tfrac{r}{0.7}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.2}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.3}\right\},$$
$$F_1^w\left(e_3, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.2}, \tfrac{u_2{}^{t_1}}{0.4}, \tfrac{u_3{}^{t_1}}{0.9}, \tfrac{u_4{}^{t_1}}{0.6}\right\}, F_1^w\left(e_3, \tfrac{n}{0.5}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.6}, \tfrac{u_4{}^{t_1}}{0.7}\right\},$$
$$F_1^w\left(e_3, \tfrac{r}{0.7}, 1\right) = \left\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.7}, \tfrac{u_3{}^{t_1}}{0.1}, \tfrac{u_4{}^{t_1}}{0.5}\right\}, F_2^w\left(e_1, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.4}, \tfrac{u_2{}^{t_2}}{0.8}, \tfrac{u_3{}^{t_2}}{0.2}, \tfrac{u_4{}^{t_2}}{0.4}\right\},$$
$$F_2^w\left(e_1, \tfrac{n}{0.5}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.4}, \tfrac{u_4{}^{t_2}}{0.8}\right\}, F_2^w\left(e_1, \tfrac{r}{0.7}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.9}, \tfrac{u_2{}^{t_2}}{0.2}, \tfrac{u_3{}^{t_2}}{0.3}, \tfrac{u_4{}^{t_2}}{0.3}\right\},$$
$$F_2^w\left(e_2, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.2}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.5}, \tfrac{u_4{}^{t_2}}{0.5}\right\}, F_2^w\left(e_2, \tfrac{n}{0.5}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.7}, \tfrac{u_2{}^{t_2}}{0.6}, \tfrac{u_3{}^{t_2}}{0.2}, \tfrac{u_4{}^{t_2}}{0.6}\right\},$$
$$F_2^w\left(e_2, \tfrac{r}{0.7}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.5}, \tfrac{u_2{}^{t_2}}{0.4}, \tfrac{u_3{}^{t_2}}{0.7}, \tfrac{u_4{}^{t_2}}{0.4}\right\}, F_2^w\left(e_3, \tfrac{m}{0.8}, 1\right) = \left\{\tfrac{u_1{}^{t_2}}{0.9}, \tfrac{u_2{}^{t_2}}{0.4}, \tfrac{u_3{}^{t_2}}{0.1}, \tfrac{u_4{}^{t_2}}{0.7}\right\},$$

$F_2^w\left(e_3, \frac{n}{0.5}, 1\right) = \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.5}\right\}, F_2^w\left(e_3, \frac{r}{0.7}, 1\right) = \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\},$

$F_3^w\left(e_1, \frac{m}{0.8}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.2}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}, F_3^w\left(e_1, \frac{n}{0.5}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.9}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.5}\right\},$

$F_3^w\left(e_1, \frac{r}{0.7}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.8}\right\}, F_3^w\left(e_2, \frac{m}{0.8}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.9}\right\},$

$F_3^w\left(e_2, \frac{n}{0.5}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.8}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.7}\right\}, F_3^w\left(e_2, \frac{r}{0.7}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.8}\right\},$

$F_3^w\left(e_3, \frac{m}{0.8}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.8}\right\}, F_3^w\left(e_3, \frac{n}{0.5}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.9}\right\},$

$F_3^w\left(e_3, \frac{r}{0.7}, 1\right) = \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.6}\right\}, F_1^w\left(e_1, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\},$

$F_1^w\left(e_1, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.4}\right\}, F_1^w\left(e_1, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.3}\right\},$

$F_1^w\left(e_2, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.3}\right\}, F_1^w\left(e_2, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.9}, \frac{u_4{}^{t_1}}{0.6}\right\},$

$F_1^w\left(e_2, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.1}\right\}, F_1^w\left(e_3, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.3}\right\},$

$F_1^w\left(e_3, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}, F_1^w\left(e_3, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\},$

$F_2^w\left(e_1, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}, F_2^w\left(e_1, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.4}\right\},$

$F_2^w\left(e_1, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.8}\right\}, F_2^w\left(e_2, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.6}\right\},$

$F_2^w\left(e_2, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.9}, \frac{u_4{}^{t_2}}{0.4}\right\}, F_2^w\left(e_2, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.1}\right\},$

$F_2^w\left(e_3, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.2}\right\}, F_2^w\left(e_3, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.6}\right\},$

$F_2^w\left(e_3, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.1}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.3}\right\}, F_3^w\left(e_1, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.4}\right\},$

$F_3^w\left(e_1, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.6}\right\}, F_3^w\left(e_1, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.5}\right\},$

$F_3^w\left(e_2, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.1}\right\}, F_3^w\left(e_2, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.4}\right\},$

$F_3^w\left(e_2, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.2}\right\}, F_3^w\left(e_3, \frac{m}{0.8}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\},$

$F_3^w\left(e_3, \frac{n}{0.5}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.2}\right\}, F_3^w\left(e_3, \frac{r}{0.7}, 0\right) = \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.5}\right\}.$

Next, we may determine the The Effect of the weighted expert factor on time fuzzy soft expert setss $(F,E)_t^w$, which comprise the subsequent set of approximations:

$$(F,E)_t^w = \left\{ \left(\left(e_1, \frac{m}{0.8}, 1\right), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.9}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.5}\right\}\right)\right.$$

$$\vdots \qquad \ddots$$

$$\left((e_3, \tfrac{n}{0.5}, 0), \left\{\tfrac{{u_1}^{t_3}}{0.3}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.7}, \tfrac{{u_4}^{t_3}}{0.2}\right\}\right), \left((e_3, \tfrac{r}{0.7}, 0), \left\{\tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.2}, \tfrac{{u_3}^{t_3}}{0.6}, \tfrac{{u_4}^{t_3}}{0.5}\right\}\right)\Bigg\}.$$

Definition 13. *For two W-TFSES* $(F,A)_t^w$ *and* $(G,B)_t^w$ *over* U*:*

- $(F,A)_t^w$ *is a W-TFSES subset of* $(G,B)_t^w$ *if:*
  (i) $B \subseteq A$
  (ii) $\forall t \in T, \delta \in B, G_t^w(\delta)$ *is a fuzzy soft expert subset of* $F_t^w(\delta)$
- *They are equal if each is a subset of the other.*

Example 2. *Consider Example 1, where*

$$A = \Big\{(e_1, \tfrac{m}{0.8}, 1)_{t_1}, (e_2, \tfrac{m}{0.8}, 1)_{t_1}, (e_2, \tfrac{n}{0.5}, 1)_{t_3}, (e_2, \tfrac{r}{0.7}, 1)_{t_3}, (e_2, \tfrac{n}{0.5}, 0)_{t_2}, (e_2, \tfrac{r}{0.7}, 0)_{t_2}, (e_2, \tfrac{r}{0.7}, 0)_{t_3},$$
$$(e_3, \tfrac{m}{0.8}, 0)_{t_3}\Big\},$$

$$B = \Big\{(e_1, \tfrac{m}{0.8}, 1)_{t_1}, (e_2, \tfrac{m}{0.8}, 1)_{t_1}, (e_2, \tfrac{n}{0.5}, 1)_{t_3}, (e_2, \tfrac{r}{0.7}, 1)_{t_3}, (e_2, \tfrac{n}{0.5}, 0)_{t_2}\Big\}.$$

Clearly $B \subset A$. *Now, let* $(G,B)_t^w$ *and* $(F,A)_t^w$ *be defined as follows:*

$$(F,E)_t^w = \Bigg\{ \left((e_1, \tfrac{m}{0.8}, 1), \left\{\tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \left((e_2, \tfrac{m}{0.8}, 1), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.1}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right),$$
$$\left((e_2, \tfrac{n}{0.5}, 1), \left\{\tfrac{{u_1}^{t_3}}{0.8}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.7}\right\}\right), \left((e_2, \tfrac{r}{0.7}, 1), \left\{\tfrac{{u_1}^{t_3}}{0.1}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.8}\right\}\right),$$
$$\left((e_2, \tfrac{n}{0.5}, 0), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.5}, \tfrac{{u_3}^{t_2}}{0.9}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \left((e_2, \tfrac{r}{0.7}, 0) \left\{\tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.7}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.1}\right\}\right),$$
$$\left((e_2, \tfrac{r}{0.7}, 0), \left\{\tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.3}, \tfrac{{u_3}^{t_3}}{0.3}, \tfrac{{u_4}^{t_3}}{0.2}\right\}\right), \left((e_3, \tfrac{m}{0.8}, 0), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.8}, \tfrac{{u_4}^{t_3}}{0.3}\right\}\right)\Bigg\},$$

$$(G,E)_t^w = \Bigg\{ \left((e_1, \tfrac{m}{0.8}, 1), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.2}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.0}\right\}\right), \left((e_2, \tfrac{m}{0.8}, 1), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.1}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right),$$
$$\left((e_2, \tfrac{n}{0.5}, 1), \left\{\tfrac{{u_1}^{t_3}}{0.8}, \tfrac{{u_2}^{t_3}}{0.5}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right), \left((e_2, \tfrac{r}{0.7}, 1), \left\{\tfrac{{u_1}^{t_3}}{0.0}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.3}\right\}\right),$$
$$\left((e_2, \tfrac{n}{0.5}, 0), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.3}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.2}\right\}\right)\Bigg\}.$$

We can easily verify that $(G,E)_t^w \subseteq (F,E)_t^w$.

Definition 14. *For a W-TFSES* $(F,A)_t^w$*:*

- *The agree-W-TFSES* $((F,A)_t^w)_1$ *contains all* $F_t^w(\alpha)$ *where* $\alpha \in E \times X \times \{1\}$
- *The disagree-W-TFSES* $((F,A)_t^w)_0$ *contains all* $F_t^w(\alpha)$ *where* $\alpha \in E \times X \times \{0\}$

Example 3. *Consider Example 1. Then the agree a weighted expert factor on time fuzzy soft expert sets* $\left((F,A)_t^w\right)_1$ *over* U *is*

$$
\begin{aligned}
&\left((F,A)_t^w\right)_1 = \\
&\Bigg\{ \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.9}, \tfrac{{u_3}^{t_1}}{0.6}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \\
&\left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.7}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.3}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.1}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right), \\
&\left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \\
&\left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.9}, \tfrac{{u_4}^{t_1}}{0.6}\right\}\right), \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.6}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \\
&\left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.4}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \\
&\left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.9}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.8}\right\}\right), \left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.9}, \tfrac{{u_2}^{t_2}}{0.2}, \tfrac{{u_3}^{t_2}}{0.3}, \tfrac{{u_4}^{t_2}}{0.3}\right\}\right), \\
&\left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.2}, \tfrac{{u_2}^{t_2}}{0.9}, \tfrac{{u_3}^{t_2}}{0.5}, \tfrac{{u_4}^{t_2}}{0.5}\right\}\right), \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.6}\right\}\right), \\
&\left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.9}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.1}, \tfrac{{u_4}^{t_2}}{0.7}\right\}\right), \\
&\left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.8}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.1}, \tfrac{{u_4}^{t_2}}{0.5}\right\}\right), \left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.2}, \tfrac{{u_2}^{t_2}}{0.9}, \tfrac{{u_3}^{t_2}}{0.6}, \tfrac{{u_4}^{t_2}}{0.7}\right\}\right), \\
&\left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.2}, \tfrac{{u_2}^{t_3}}{0.7}, \tfrac{{u_3}^{t_3}}{0.8}, \tfrac{{u_4}^{t_3}}{0.3}\right\}\right), \left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.9}, \tfrac{{u_3}^{t_3}}{0.6}, \tfrac{{u_4}^{t_3}}{0.5}\right\}\right), \\
&\left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.6}, \tfrac{{u_2}^{t_3}}{0.5}, \tfrac{{u_3}^{t_3}}{0.7}, \tfrac{{u_4}^{t_3}}{0.8}\right\}\right), \left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.5}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.7}, \tfrac{{u_4}^{t_3}}{0.9}\right\}\right), \\
&\left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.8}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.7}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.1}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.8}\right\}\right), \\
&\left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.1}, \tfrac{{u_4}^{t_3}}{0.8}\right\}\right), \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.6}, \tfrac{{u_2}^{t_3}}{0.5}, \tfrac{{u_3}^{t_3}}{0.3}, \tfrac{{u_4}^{t_3}}{0.9}\right\}\right), \\
&\qquad\qquad \left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.7}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right) \Bigg\}.
\end{aligned}
$$

and the disagree- time fuzzy soft expert set $\left((F,A)_t\right)_0^w$ *over* U *is*

$$((F,A)^w_t)_0 =$$
$$\Bigg\{ \left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.9}, \frac{u_4{}^{t_1}}{0.6}\right\}\right), \left(\left(e_2, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.1}\right\}\right),$$
$$\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}\right),$$
$$\left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\}\right), \left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}\right),$$
$$\left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.8}\right\}\right),$$
$$\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.9}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left(\left(e_2, \frac{r}{0.7}, 0\right) \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.1}\right\}\right), \left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.2}\right\}\right),$$
$$\left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.1}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.4}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.5}\right\}\right), \left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.1}\right\}\right),$$
$$\left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.4}\right\}\right), \left(\left(e_2, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.5}\right\}\right) \Bigg\}.$$

4. Fundamental Operation

We define complement, union, and intersection of weighted expert factor on time fuzzy soft expert sets , deduce various features, and provide some examples to illustrate our points in this section.

4.1. Complement

Definition 15. *The complement of time fuzzy soft expert set $(F, A)_t^w$, denoted by $(F, A)^{w_t^c}$, is defined by $(F, A)^{w_t^c} = (F^c, \rceil A)_t^w$ where $F^{w_t^c} : \rceil A \to P(U)$ is a mapping given by*

$$F^{w_t^c}(\alpha) = c(F_t^w(\rceil \alpha)), \forall \alpha \subset \rceil A,$$

where c is a time fuzzy soft expert set complement.

Example 4. *Consider Example 1. I have utilized the simple fuzzy complement to*

$$\begin{aligned}
\widetilde{c}(F, A)_t^w = \Bigg\{ & \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.6}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.4}, \tfrac{{u_2}^{t_1}}{0.1}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right) \\
& \left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.4}, \tfrac{{u_2}^{t_1}}{0.9}, \tfrac{{u_3}^{t_1}}{0.6}, \tfrac{{u_4}^{t_1}}{0.2}\right\}\right), \\
& \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.3}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.3}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \\
& \left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.4}\right\}\right), \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.4}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.4}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \\
& \left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.3}, \tfrac{{u_3}^{t_1}}{0.9}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.2}, \tfrac{{u_3}^{t_2}}{0.8}, \tfrac{{u_4}^{t_2}}{0.6}\right\}\right), \\
& \left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.1}, \tfrac{{u_3}^{t_2}}{0.6}, \tfrac{{u_4}^{t_2}}{0.2}\right\}\right), \left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.1}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.7}\right\}\right), \\
& \left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.8}, \tfrac{{u_2}^{t_2}}{0.1}, \tfrac{{u_3}^{t_2}}{0.5}, \tfrac{{u_4}^{t_2}}{0.5}\right\}\right), \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.3}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.8}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \\
& \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.3}, \tfrac{{u_4}^{t_2}}{0.6}\right\}\right), \left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.1}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.9}, \tfrac{{u_4}^{t_2}}{0.3}\right\}\right), \\
& \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.2}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.9}, \tfrac{{u_4}^{t_2}}{0.5}\right\}\right), \left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.8}, \tfrac{{u_2}^{t_2}}{0.1}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.3}\right\}\right), \\
& \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.8}, \tfrac{{u_2}^{t_3}}{0.3}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.7}\right\}\right), \left(\left(e_1, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.6}, \tfrac{{u_2}^{t_3}}{0.1}, \tfrac{{u_3}^{t_3}}{0.4}, \tfrac{{u_4}^{t_3}}{0.5}\right\}\right), \\
& \left(\left(e_1, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.5}, \tfrac{{u_3}^{t_3}}{0.3}, \tfrac{{u_4}^{t_3}}{0.2}\right\}\right), \left(\left(e_2, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.5}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.3}, \tfrac{{u_4}^{t_3}}{0.1}\right\}\right), \\
& \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.2}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.8}, \tfrac{{u_4}^{t_3}}{0.3}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.9}, \tfrac{{u_2}^{t_3}}{0.2}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.2}\right\}\right), \\
& \left(\left(e_3, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.3}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.9}, \tfrac{{u_4}^{t_3}}{0.2}\right\}\right), \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.5}, \tfrac{{u_3}^{t_3}}{0.7}, \tfrac{{u_4}^{t_3}}{0.1}\right\}\right), \\
& \left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.6}, \tfrac{{u_2}^{t_3}}{0.3}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.4}\right\}\right), \left(\left(e_1, \tfrac{m}{0.8}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.7}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.3}, \tfrac{{u_4}^{t_1}}{0.2}\right\}\right), \\
& \left(\left(e_1, \tfrac{n}{0.5}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.6}\right\}\right), \left(\left(e_1, \tfrac{r}{0.7}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \\
& \left(\left(e_2, \tfrac{m}{0.8}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.3}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \left(\left(e_2, \tfrac{n}{0.5}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.4}\right\}\right), \\
& \left(\left(e_2, \tfrac{r}{0.7}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.4}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.6}, \tfrac{{u_4}^{t_1}}{0.9}\right\}\right), \left(\left(e_3, \tfrac{m}{0.8}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.5}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.7}\right\}\right), \\
& \left(\left(e_3, \tfrac{n}{0.5}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right), \left(\left(e_3, \tfrac{r}{0.7}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.8}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.6}\right\}\right),
\end{aligned}$$

$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.6}\right\}\right),$$

$$\left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.2}\right\}\right), \left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.1}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$

$$\left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left(\left(e_2, \frac{r}{0.7}, 0\right) \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.9}\right\}\right),$$

$$\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.8}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$

$$\left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.6}\right\}\right),$$

$$\left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.4}\right\}\right), \left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.5}\right\}\right),$$

$$\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.9}\right\}\right), \left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.9}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.6}\right\}\right),$$

$$\left(\left(e_2, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.7}\right\}\right),$$

$$\left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.5}\right\}\right) \Bigg\}.$$

Proposition 1. *If* $(F, A)_t^w$ *is a weighted expert factor on time fuzzy soft expert sets over* U*, then* $\widetilde{c}\,(\widetilde{c}(F, A)_t) = (F, A)_t^w$.

Proof.

From Definition 15 we have $(F, A)_t^{wc} = (F^c, A)$, where $F_t^{wc}(\alpha) = \bar{1} - F_t^w(\alpha), \forall \alpha \in A$.
Now, $((F, A)_t^{wc})^c = \left(\left(F^{w_t^c}\right)^c, A\right)$
Where $\left(F^{w_t^c}\right)^c(\alpha) = \bar{1} - (\bar{1} - F_t^w(\alpha)), \forall(\alpha) \in A$
$= F_t^w(\alpha), \forall \alpha \in A.$

4.2. Union Operation

Definition 16. *The weighted expert factor on time fuzzy soft expert sets* $(H, C)_t^w$ *is the* union *of two weighted expert factor on time fuzzy soft expert sets* $(F, A)_t^w$ *and* $(G, B)_t^w$ *over* U*; it is represented as* $(F, A)_t^w\,\tilde{\cup}\,(G, B)_t^w$*, such that* $C = A \cup B \subset Z$*. It is defined as follows.*

$$H_t^w(\delta) = \begin{cases} F_t^w(\delta), & if\, \delta \in A - B, \\ G_t^w(\delta), & if\, \delta \in B - A, \\ F_t^w(\delta)\,\tilde{\cup}\,G_t^w(\delta), & if\, \delta \in A \cap B, \end{cases}$$

where the fuzzy soft expert union was indicated by $\tilde{\cup}$.

Example 5. *Consider Example 1. Assume two The Effect of the weighted expert factor on time fuzzy soft expert setss over* U*,* $(F, A)_t^w$ *and* $(G, B)_t^w$*, are such that*

$$(F,A)_t^w = \left\{ \left((e_2, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.5} \right\} \right), \left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.3} \right\} \right), \right.$$
$$\left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.4} \right\} \right), \left((e_3, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.8}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.1}, \tfrac{{u_4}^{t_2}}{0.5} \right\} \right),$$
$$\left((e_3, \tfrac{m}{0.8}, 1), \left\{ \tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.1}, \tfrac{{u_4}^{t_3}}{0.8} \right\} \right), \left((e_1, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.8} \right\} \right),$$
$$\left((e_3, \tfrac{n}{0.5}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.2} \right\} \right), \left((e_3, \tfrac{r}{0.7}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.2}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.4} \right\} \right),$$
$$\left. \left((e_1, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.5} \right\} \right), \left((e_3, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.6}, \tfrac{{u_3}^{t_3}}{0.8}, \tfrac{{u_4}^{t_3}}{0.3} \right\} \right) \right\}.$$

$$(G,B)_t^w = \left\{ \left((e_2, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.7}, \tfrac{{u_2}^{t_1}}{0.9}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.3} \right\} \right), \left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.1} \right\} \right), \right.$$
$$\left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.3}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.6} \right\} \right), \left((e_3, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.3} \right\} \right),$$
$$\left((e_1, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.8}, \tfrac{{u_4}^{t_2}}{0.4} \right\} \right), \left((e_1, \tfrac{r}{0.7}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.5}, \tfrac{{u_3}^{t_2}}{0.6}, \tfrac{{u_4}^{t_2}}{0.7} \right\} \right),$$
$$\left. \left((e_2, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.7}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.9} \right\} \right), \left((e_3, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_3}}{0.2}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.9}, \tfrac{{u_4}^{t_3}}{0.1} \right\} \right) \right\}.$$

Then $(F,A)_t^w \,\widetilde{\cup}\, (G,B)_t^w = (H,C)_t^w$ *where*

$$(H,C)_t^w = \left\{ \left((e_2, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.7}, \tfrac{{u_2}^{t_1}}{0.9}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.5} \right\} \right), \left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_1}}{0.6}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.3} \right\} \right), \right.$$
$$\left((e_2, \tfrac{r}{0.7}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.4}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.6} \right\} \right), \left((e_3, \tfrac{n}{0.5}, 1), \left\{ \tfrac{{u_1}^{t_2}}{0.8}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.7}, \tfrac{{u_4}^{t_2}}{0.5} \right\} \right),$$
$$\left((e_3, \tfrac{m}{0.8}, 1), \left\{ \tfrac{{u_1}^{t_3}}{0.7}, \tfrac{{u_2}^{t_3}}{0.4}, \tfrac{{u_3}^{t_3}}{0.1}, \tfrac{{u_4}^{t_3}}{0.8} \right\} \right), \left((e_1, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.8} \right\} \right),$$
$$\left((e_3, \tfrac{n}{0.5}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.5}, \tfrac{{u_4}^{t_1}}{0.2} \right\} \right), \left((e_3, \tfrac{r}{0.7}, 0), \left\{ \tfrac{{u_1}^{t_1}}{0.8}, \tfrac{{u_2}^{t_1}}{0.2}, \tfrac{{u_3}^{t_1}}{0.8}, \tfrac{{u_4}^{t_1}}{0.4} \right\} \right),$$
$$\left((e_1, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.8}, \tfrac{{u_4}^{t_2}}{0.5} \right\} \right), \left((e_1, \tfrac{r}{0.7}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.7}, \tfrac{{u_2}^{t_2}}{0.5}, \tfrac{{u_3}^{t_2}}{0.6}, \tfrac{{u_4}^{t_2}}{0.7} \right\} \right),$$
$$\left. \left((e_2, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_2}}{0.6}, \tfrac{{u_2}^{t_2}}{0.7}, \tfrac{{u_3}^{t_2}}{0.4}, \tfrac{{u_4}^{t_2}}{0.9} \right\} \right), \left((e_3, \tfrac{m}{0.8}, 0), \left\{ \tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.9}, \tfrac{{u_4}^{t_3}}{0.3} \right\} \right) \right\}.$$

Proposition 2. *Three weighted expert factor on time fuzzy soft expert sets over* U *are denoted by* $(F,A)_t^w$, $(G,B)_t^w$, *and* $(H,C)_t^w$, *then*

(i) $(F,A)_t^w \,\widetilde{\cup}\, ((G,B)_t^w \,\widetilde{\cup}\, (H,C)_t^w) = ((F,A)_t^w \,\widetilde{\cup}\, (G,B))_t^w \,\widetilde{\cup}\, (H,C)_t^w$,

(ii) $(F,A)_t^w \widetilde{\cup} (F,A)_t^w = (F,A)_t^w$.

Proof.

1. Our goal is proving that
$(F,A)_t^w \widetilde{\cup} \left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) = \left((F,A)_t^w \widetilde{\cup} (G,B)_t^w\right) \widetilde{\cup} (H,C)_t^w$.

   Definition 16 allows us (where s is s-norm) to have

$$\left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) = \begin{cases} G_t^w(\delta), & \text{if } \delta \in B - C \\ H_t^w(\delta), & \text{if } \delta \in C - B \\ s\left(G_t^w(\delta), H_t^w(\delta)\right), & \text{if } \delta \in B \cap C. \end{cases}$$

   We take into account the situation when $\delta \in B \cap C$. After the insignificant examples, we have

$$(G,B)_t^w \widetilde{\cup} (H,C)_t^w = \left(s\left(G_t^w(\delta), H_t^w(\delta)\right), B \cup C\right).$$

   Here, we additionally take the scenario when $\delta \in A$ into account. Since the other situations are insignificant, we have

$$\begin{aligned} (F,A)_t^w \widetilde{\cup} \left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) &= \left(s\left(F_t^w(\delta), s\left(G_t^w(\delta), H_t^w(\delta)\right)\right), A \cup (B \cup C)\right). \\ &= \left(s\left(s\left(F_t^w(\delta), G_t^w(\delta)\right) \cup H_t^w(\delta)\right), (A \cup B) \cup C\right). \\ &= \left((F,A)_t^w \widetilde{\cup} (G,B)_t^w\right) \widetilde{\cup} (H,C)_t^w. \end{aligned}$$

2. The proof is straightforward.

4.3. Intersection Operation

Definition 17. *The weighted expert factor on time fuzzy soft expert sets $(H,C)_t^w$ is the* intersection *of two W-TFSES's $(F,A)_t^w$ and $(G,B)_t^w$ over U, represented as $(F,A)_t^w \widetilde{\cap} (G,B)_t^w$, with $C = A \cap B \subset Z$, and defined as follows*

$$H_t^w(\delta) = \begin{cases} F_t^w(\delta), & if\, \delta \in A - B, \\ G_t^w(\delta), & if\, \delta \in B - A, \\ F_t^w(\delta) \tilde{\cap}\, G_t^w(\delta), & if\, \delta \in A \cap B, \end{cases}$$

where $\tilde{\cap}$ represented the intersection of the fuzzy soft expert.

Example 6. *Consider Example 5. We have $(F,A)_t^w \widetilde{\cap} (G,B)_t^w = (H,C)_t^w$ where*

$$(H,C)_t^w = \left\{ \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.1}, \tfrac{{u_4}^{t_1}}{0.3}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.2}, \tfrac{{u_2}^{t_1}}{0.4}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.1}\right\}\right), \right.$$
$$\left(\left(e_2, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.3}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right), \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.5}, \tfrac{{u_2}^{t_2}}{0.6}, \tfrac{{u_3}^{t_2}}{0.1}, \tfrac{{u_4}^{t_2}}{0.3}\right\}\right),$$

$$\left(\left(e_3, \frac{m}{0.8}, 1\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.2}\right\}\right), \left(\left(e_3, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(e_1, \frac{r}{0.7}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left.\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.9}\right\}\right), \left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.2}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.1}\right\}\right)\right\}.$$

Proposition 3. *Three weighted expert factor on time fuzzy soft expert sets over U are denoted by $(F,A)_t^w$, $(G,B)_t^w$, and $(H,C)_t^w$,then*

(i) $(F,A)_t^w \widetilde{\cap} ((G,B)_t^w \widetilde{\cap} (H,C)_t^w) = ((F,A)_t^w \widetilde{\cap} (G,B))_t^w \widetilde{\cap} (H,C)_t^w$,

(ii) $(F,A)_t^w \widetilde{\cap} (F,A)_t^w = (F,A)_t^w$.

Proof.

(i) Our goal is proving that
$(F,A)_t^w \widetilde{\cap} \left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) = \left((F,A)_t^w \widetilde{\cap} (G,B)_t^w\right) \widetilde{\cap} (H,C)_t^w$.

By employing Definition 17 (where $\widetilde{\mathfrak{t}}$ is $\widetilde{\mathfrak{t}}$-norm) we have

$$\left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) = \begin{cases} G_t^w(\delta), & \text{if } \delta \in B - C \\ H_t^w(\delta), & \text{if } \delta \in C - B \\ \widetilde{\mathfrak{t}}(G_t^w(\delta), H_t^w(\delta)), & \text{if } \delta \in B \cap C. \end{cases}$$

We take into account the situation when $\delta \in B \cap C$. After the insignificant examples, we have

$$(G,B)_t^w \widetilde{\cap} (H,C)_t^w = \left(\widetilde{\mathfrak{t}}(G_t^w(\delta), H_t^w(\delta)), B \cup C\right).$$

We also consider here the case when $\delta \in A$. The other cases are trivial, then we have

$$\begin{aligned} (F,A)_t^w \widetilde{\cap} \left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) &= \left(\widetilde{\mathfrak{t}}\left(F_t^w(\delta), \widetilde{\mathfrak{t}}(G_t^w(\delta), H_t^w(\delta))\right), A \cup (B \cup C)\right). \\ &= \left(\widetilde{\mathfrak{t}}\left(\widetilde{\mathfrak{t}}(F_t^w(\delta), G_t^w(\delta)) \cap H_t^w(\delta)\right), (A \cup B) \cup C\right). \\ &= \left((F,A)_t^w \widetilde{\cap} (G,B)_t^w\right) \widetilde{\cap} (H,C)_t^w. \end{aligned}$$

(ii) The proof is straightforward.

Proposition 4. *Assume that there are three weighted expert factor on time fuzzy soft expert sets over U: $(F,A)_t^w$, $(G,B)_t^w$, and $(H,C)_t^w$, then*

(i) $(F,A)_t^w \widetilde{\cup} \left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) = \left((F,A)_t^w \widetilde{\cup} (G,B)_t^w\right) \widetilde{\cap} \left((F,A)_t^w \widetilde{\cup} (H,C)_t^w\right)$,

(ii) $(F,A)_t^w \widetilde{\cap} \left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) = \left((F,A)_t^w \widetilde{\cap} (G,B)_t^w\right) \widetilde{\cup} \left((F,A)_t^w \widetilde{\cap} (H,C)_t^w\right).$

Proof.

(i) Our goal is to prove that
$(F,A)_t^w \widetilde{\cup} \left((G,B)_t^w \widetilde{\cap} (H,C)\right)_t^w \;=\; \left((F,A)_t^w \widetilde{\cup} (G,B)_t^w\right) \widetilde{\cap} \left((F,A)_t^w \widetilde{\cup} (H,C)_t^w\right),$

By Applying Definitions 16 and 17 we have

$$\left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) = \begin{cases} G_t^w(\delta), & \text{if } \delta \in B - C \\ H_t^w(\delta), & \text{if } \delta \in C - B \\ \widetilde{\mathfrak{t}}\left(G_t^w(\delta), H_t^w(\delta)\right), & \text{if } \delta \in B \cap C. \end{cases}$$

Since the other situations are trivial, we investigate the case when $\delta \in B \cap C$. In this case, we obtain

$$(G,B)_t^w \widetilde{\cap} (H,C)_t^w = \left(\widetilde{\mathfrak{t}}\left(G_t^w(\delta), H_t^w(\delta)\right), B \cup C\right).$$

We also consider here the case when $\delta \in A$. The other cases are trivial, then we have

$$\begin{aligned}(F,A)_t^w \widetilde{\cup} \left((G,B)_t^w \widetilde{\cap} (H,C)_t^w\right) &= \left(s\left(F_t^w(\delta), \widetilde{\mathfrak{t}}\left(G_t^w(\delta), H_t^w(\delta)\right)\right), A \cup (B \cup C)\right). \\ &= \widetilde{\mathfrak{t}}\left(s\left(F_t^w(\delta), G_t^w(\delta)\right), s\left(F_t^w(\delta) \cup H_t^w(\delta)\right)\right). \\ &= \left((F,A)_t^w \widetilde{\cup} (G,B)_t^w\right) \widetilde{\cap} \left((F,A)_t^w \widetilde{\cup} (H,C)_t^w\right).\end{aligned}$$

(ii) Our goal is proving that
$(F,A)_t^w \widetilde{\cap} \left((G,B)_t^w \widetilde{\cup} (H,C)\right)_t^w \;=\; \left((F,A)_t^w \widetilde{\cap} (G,B)_t^w\right) \widetilde{\cup} \left((F,A)_t^w \widetilde{\cap} (H,C)_t^w\right),$

By applying definitions 16 and 17 we have

$$\left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) = \begin{cases} G_t^w(\delta), & \text{if } \delta \in B - C \\ H_t^w(\delta), & \text{if } \delta \in C - B \\ s\left(G_t^w(\delta), H_t^w(\delta)\right), & \text{if } \delta \in B \cap C. \end{cases}$$

We consider the case when $\delta \in B \cap C$. The other cases are trivial, then we have

$$(G,B)_t^w \widetilde{\cup} (H,C)_t^w = \left(s\left(G_t^w(\delta), H_t^w(\delta)\right), B \cup C\right).$$

Here, we additionally take the situation when $\delta \in A$ into account. Since the other situations are trivial, we have

$$\begin{aligned}(F,A)_t^w \widetilde{\cap} \left((G,B)_t^w \widetilde{\cup} (H,C)_t^w\right) &= \left(\widetilde{\mathfrak{t}}\left(F_t^w(\delta), s\left(G_t^w(\delta), H_t^w(\delta)\right)\right), A \cup (B \cup C)\right). \\ &= s\left(\widetilde{\mathfrak{t}}\left(F_t^w(\delta), G_t^w(\delta)\right), \widetilde{\mathfrak{t}}\left(F_t^w(\delta) \cup H_t^w(\delta)\right)\right). \\ &= \left((F,A)_t^w \widetilde{\cap} (G,B)_t^w\right) \widetilde{\cup} \left((F,A)_t^w \widetilde{\cap} (H,C)_t^w\right).\end{aligned}$$

5. AND and OR Operations

We define the AND and OR operations for weighted expert factor on time fuzzy soft expert sets , as well as their characteristics and several instances, in this section.

Definition 18. *With two weighted expert factor on time fuzzy soft expert sets over U, $(F,A)_t^w$ and $(G,B)_t^w$, the expression $(F,A)_t^w$ AND $(G,B)_t^w$ is denoted by $(F,A)_t^w \wedge (G,B)_t^w$, it is defined with*

$$(F,A)_t^w \wedge (G,B)_t^w = (H, A\times B)_t^w$$

such that $H(\alpha,\beta)_t^w = F(\alpha)_t^w \widetilde{\bigcap} G(\beta)_t^w, \forall (\alpha,\beta)\in A\times B$, where $\widetilde{\bigcap}$ is time-fuzzy soft expert intersection.

Example 7. *Consider Example 1. Assuming two The Effect of the weighted expert factor on time fuzzy soft expert setss over U, let $(F,A)_t^w$ and $(G,B)_t^w$ be such that*

$$(F,A)_t^w = \left\{ \left(\left(e_2, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_1}}{0.5}, \tfrac{{u_2}^{t_1}}{0.7}, \tfrac{{u_3}^{t_1}}{0.2}, \tfrac{{u_4}^{t_1}}{0.5}\right\}\right), \left(\left(e_1, \tfrac{m}{0.8}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.8}, \tfrac{{u_3}^{t_2}}{0.2}, \tfrac{{u_4}^{t_2}}{0.4}\right\}\right),\right.$$
$$\left.\left(\left(e_3, \tfrac{r}{0.7}, 1\right), \left\{\tfrac{{u_1}^{t_3}}{0.4}, \tfrac{{u_2}^{t_3}}{0.7}, \tfrac{{u_3}^{t_3}}{0.5}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right), \left(\left(e_1, \tfrac{m}{0.8}, 0\right), \left\{\tfrac{{u_1}^{t_1}}{0.3}, \tfrac{{u_2}^{t_1}}{0.6}, \tfrac{{u_3}^{t_1}}{0.7}, \tfrac{{u_4}^{t_1}}{0.8}\right\}\right) \right\}.$$

$$(G,B)_t^w =$$
$$\left\{ \left(\left(e_3, \tfrac{n}{0.5}, 1\right), \left\{\tfrac{{u_1}^{t_2}}{0.4}, \tfrac{{u_2}^{t_2}}{0.9}, \tfrac{{u_3}^{t_2}}{0.3}, \tfrac{{u_4}^{t_2}}{0.1}\right\}\right), \left(\left(e_2, \tfrac{r}{0.7}, 0\right), \left\{\tfrac{{u_1}^{t_3}}{0.5}, \tfrac{{u_2}^{t_3}}{0.8}, \tfrac{{u_3}^{t_3}}{0.2}, \tfrac{{u_4}^{t_3}}{0.6}\right\}\right) \right\}.$$

Then $(F,A)\wedge(G,B) = (H, A\times B)$ where

$$(H, A\times B)_t^w = \left\{ \left(\left(\left(e_2, \tfrac{n}{0.5}, 1\right)_{t_1}, \left(e_3, \tfrac{n}{0.5}, 1\right)_{t_2}\right), \left\{\tfrac{{u_1}^{t_{1,2}}}{0.4}, \tfrac{{u_2}^{t_{1,2}}}{0.7}, \tfrac{{u_3}^{t_{1,2}}}{0.2}, \tfrac{{u_4}^{t_{1,2}}}{0.1}\right\}\right),\right.$$
$$\left(\left(\left(e_2, \tfrac{n}{0.5}, 1\right)_{t_1}, \left(e_2, \tfrac{r}{0.7}, 0\right)_{t_3}\right), \left\{\tfrac{{u_1}^{t_{1,3}}}{0.5}, \tfrac{{u_2}^{t_{1,3}}}{0.7}, \tfrac{{u_3}^{t_{1,3}}}{0.2}, \tfrac{{u_4}^{t_{1,3}}}{0.5}\right\}\right),$$
$$\left(\left(\left(e_1, \tfrac{m}{0.8}, 1\right)_{t_2}, \left(e_3, \tfrac{n}{0.5}, 1\right)_{t_2}\right), \left\{\tfrac{{u_1}^{t_{2,2}}}{0.4}, \tfrac{{u_2}^{t_{2,2}}}{0.8}, \tfrac{{u_3}^{t_{2,2}}}{0.2}, \tfrac{{u_4}^{t_{2,2}}}{0.1}\right\}\right),$$
$$\left(\left(\left(e_1, \tfrac{m}{0.8}, 1\right)_{t_2}, \left(e_2, \tfrac{r}{0.7}, 0\right)_{t_3}\right), \left\{\tfrac{{u_1}^{t_{2,3}}}{0.4}, \tfrac{{u_2}^{t_{2,3}}}{0.8}, \tfrac{{u_3}^{t_{2,3}}}{0.2}, \tfrac{{u_4}^{t_{2,3}}}{0.4}\right\}\right),$$
$$\left(\left(\left(e_3, \tfrac{r}{0.7}, 1\right)_{t_3}, \left(e_3, \tfrac{n}{0.5}, 1\right)_{t_2}\right), \left\{\tfrac{{u_1}^{t_{3,2}}}{0.4}, \tfrac{{u_2}^{t_{3,2}}}{0.7}, \tfrac{{u_3}^{t_{3,2}}}{0.3}, \tfrac{{u_4}^{t_{3,2}}}{0.1}\right\}\right),$$
$$\left(\left(\left(e_3, \tfrac{r}{0.7}, 1\right)_{t_3}, \left(e_2, \tfrac{r}{0.7}, 0\right)_{t_3}\right), \left\{\tfrac{{u_1}^{t_{3,3}}}{0.4}, \tfrac{{u_2}^{t_{3,3}}}{0.7}, \tfrac{{u_3}^{t_{3,3}}}{0.2}, \tfrac{{u_4}^{t_{3,3}}}{0.6}\right\}\right),$$
$$\left(\left(\left(e_1, \tfrac{m}{0.8}, 0\right)_{t_1}, \left(e_3, \tfrac{n}{0.5}, 1\right)_{t_2}\right), \left\{\tfrac{{u_1}^{t_{1,2}}}{0.3}, \tfrac{{u_2}^{t_{1,2}}}{0.6}, \tfrac{{u_3}^{t_{1,2}}}{0.3}, \tfrac{{u_4}^{t_{1,2}}}{0.1}\right\}\right),$$

$$\left(\left(\left(e_1, \frac{m}{0.8}, 0\right)_{t_1}, \left(e_2, \frac{r}{0.7}, 0\right)_{t_3}\right), \left\{\frac{u_1{}^{t_{1,3}}}{0.3}, \frac{u_2{}^{t_{1,3}}}{0.6}, \frac{u_3{}^{t_{1,3}}}{0.2}, \frac{u_4{}^{t_{1,3}}}{0.6}\right\}\right) \Bigg\}.$$

Assuming that there are two weighted expert factor on time fuzzy soft expert sets over U, $(F,A)_t^w$ and $(G,B)_t^w$, then $(F,A)_t^w$ OR $(G,B)_t^w$ is represented as $(F,A)_t^w \vee (G,B)_t^w$.

Definition 19. *Assuming that there are two weighted expert factor on time fuzzy soft expert sets over U, $(F,A)_t^w$ and $(G,B)_t^w$, then $(F,A)_t^w$ OR $(G,B)_t^w$ is represented as $(F,A)_t^w \vee (G,B)_t^w$, is defined by*

$$(F,A)_t^w \vee (G,B)_t^w = (H, A \times B)_t^w$$

such that $H(\alpha,\beta)_t^w = F(\alpha)_t^w \widetilde{\bigcup} G(\beta)_t^w, \forall (\alpha,\beta) \in A \times B$, $\widetilde{\bigcup}$ represents the union of time-fuzzy soft expert.

Example 8. *Consider Example 7 we have U then $(F,A)_t^w$ OR $(G,B)_t^w$ represented by $(H,C)_t^w = (F,A)_t^w \bigvee (G,B)_t^w$ where*

$$\begin{aligned}
(H,C)_t^w = \Bigg\{ & \left(\left(\left(e_2, \frac{n}{0.5}, 1\right)_{t_1}, \left(e_3, \frac{n}{0.5}, 1\right)_{t_2}\right), \left\{\frac{u_1{}^{t_{1,2}}}{0.5}, \frac{u_2{}^{t_{1,2}}}{0.9}, \frac{u_3{}^{t_{1,2}}}{0.3}, \frac{u_4{}^{t_{1,2}}}{0.5}\right\}\right), \\
& \left(\left(\left(e_2, \frac{n}{0.5}, 1\right)_{t_1}, \left(e_2, \frac{r}{0.7}, 0\right)_{t_3}\right), \left\{\frac{u_1{}^{t_{1,3}}}{0.5}, \frac{u_2{}^{t_{1,3}}}{0.8}, \frac{u_3{}^{t_{1,3}}}{0.2}, \frac{u_4{}^{t_{1,3}}}{0.6}\right\}\right), \\
& \left(\left(\left(e_1, \frac{m}{0.8}, 1\right)_{t_2}, \left(e_3, \frac{n}{0.5}, 1\right)_{t_2}\right), \left\{\frac{u_1{}^{t_{2,2}}}{0.4}, \frac{u_2{}^{t_{2,2}}}{0.9}, \frac{u_3{}^{t_{2,2}}}{0.3}, \frac{u_4{}^{t_{2,2}}}{0.4}\right\}\right), \\
& \left(\left(\left(e_1, \frac{m}{0.8}, 1\right)_{t_2}, \left(e_2, \frac{r}{0.7}, 0\right)_{t_3}\right), \left\{\frac{u_1{}^{t_{2,3}}}{0.5}, \frac{u_2{}^{t_{2,3}}}{0.8}, \frac{u_3{}^{t_{2,3}}}{0.2}, \frac{u_4{}^{t_{2,3}}}{0.6}\right\}\right), \\
& \left(\left(\left(e_3, \frac{r}{0.7}, 1\right)_{t_3}, \left(e_3, \frac{n}{0.5}, 1\right)_{t_2}\right), \left\{\frac{u_1{}^{t_{3,2}}}{0.4}, \frac{u_2{}^{t_{3,2}}}{0.9}, \frac{u_3{}^{t_{3,2}}}{0.5}, \frac{u_4{}^{t_{3,2}}}{0.6}\right\}\right), \\
& \left(\left(\left(e_3, \frac{r}{0.7}, 1\right)_{t_3}, \left(e_2, \frac{r}{0.7}, 0\right)_{t_3}\right), \left\{\frac{u_1{}^{t_{3,3}}}{0.5}, \frac{u_2{}^{t_{3,3}}}{0.8}, \frac{u_3{}^{t_{3,3}}}{0.5}, \frac{u_4{}^{t_{3,3}}}{0.6}\right\}\right), \\
& \left(\left(\left(e_1, \frac{m}{0.8}, 0\right)_{t_1}, \left(e_3, \frac{n}{0.5}, 1\right)_{t_2}\right), \left\{\frac{u_1{}^{t_{1,2}}}{0.4}, \frac{u_2{}^{t_{1,2}}}{0.9}, \frac{u_3{}^{t_{1,2}}}{0.7}, \frac{u_4{}^{t_{1,2}}}{0.8}\right\}\right), \\
& \left(\left(\left(e_1, \frac{m}{0.8}, 0\right)_{t_1}, \left(e_2, \frac{r}{0.7}, 0\right)_{t_3}\right), \left\{\frac{u_1{}^{t_{1,3}}}{0.5}, \frac{u_2{}^{t_{1,3}}}{0.8}, \frac{u_3{}^{t_{1,3}}}{0.7}, \frac{u_4{}^{t_{1,3}}}{0.8}\right\}\right) \Bigg\}.
\end{aligned}$$

Proposition 5. *let $(F,A)_t^w$ and $(G,B)_t^w$ are W-TFSESS over U, then*

(i) $((F,A)_t^w \wedge (G,B)_t^w)^c = (F,A)^{w_t^c} \vee (G,B)^{w_t^c}$

(ii) $((F,A)_t^w \vee (G,B)_t^w)^c = (F,A)^{w_t^c} \wedge (G,B)^{w_t^c}$

Proof.

(i) Suppose that $(F, A)_t^w \wedge (G, B)_t^w = (O, A \times B)_t^w$.
Therefore, $((F, A)_t^w \wedge (G, B)_t^w)^c = (O_t, A \times B)^c = (O_t^c, (A \times B))$. Now,
$((F, A)_t^w \vee (G, B)_t^w)^c = \left((F^{w_t^c}, A) \vee (G^{w_t^c}, B)\right)$
$= (J_t, (A \times B))$, where $J_t(x, y) = t\left(c(F_t^w(\alpha)), c(G_t^w(\beta))\right)$.
Now, take $(\alpha, \beta) \in (A \times B)$.
Then,
$O_t^c(\alpha, \beta) = \bar{1} - O_t(\alpha, \beta),$
$= \bar{1} - [F_t^w(\alpha) \bigcup G_t^w(\beta)]$
$= [\bar{1} - F_t^w(\alpha)] \bigcap [\bar{1} - G_t^w(\beta)]$
$= t\left(c(F_t^w(\alpha)), c(G_t^w(\beta))\right)$
$= J_t(\alpha, \beta)$
Therefore O_t^c and J_t are the same. Hence, proved.

(ii) Suppose that $(F, A)_t^w \vee (G, B)_t^w = (O, A \times B)_t^w$.
Therefore, $((F, A)_t^w \vee (G, B)_t^w)^c = (O_t, A \times B)^c = (O_t^c, (A \times B))$. Now,
$((F, A)_t^w \wedge (G, B)_t^w)^c = \left((F^{w_t^c}, A) \wedge (G^{w_t^c}, B)\right)$
$= (J_t, (A \times B))$, where $J_t(x, y) = s\left(c(F_t^w(\alpha)), c(G_t^w(\beta))\right)$.
Now, take $(\alpha, \beta) \in (A \times B)$.
Then,
$O_t^c(\alpha, \beta) = \bar{1} - O_t(\alpha, \beta),$
$= \bar{1} - [F_t^w(\alpha) \bigcap G_t^w(\beta)]$
$= [\bar{1} - F_t^w(\alpha)] \bigcup [\bar{1} - G_t^w(\beta)]$
$= s\left(c(F_t^w(\alpha)), c(G_t^w(\beta))\right)$
$= J_t(\alpha, \beta)$
Therefore O_t^c and J_t are the same. Hence, proved.

Proposition 6. *let (F, A), (G, B) and (H, C) be time-fuzzy soft expert sets over U, then*

(i) $(F, A)_t^w \wedge ((G, B)_t^w \wedge (H, C)_t^w) = ((F, A)_t^w \wedge (G, B)_t^w) \wedge (H, C)_t^w$,

(ii) $(F, A)_t^w \vee ((G, B)_t^w \vee (H, C)_t^w) = ((F, A)_t^w \vee (G, B)_t^w) \vee (H, C)_t^w$,

(iii) $(F, A)_t^w \vee ((G, B)_t^w \wedge (H, C)_t^w) = ((F, A)_t^w \vee (G, B)_t^w) \wedge ((F, A)_t^w \vee (H, C)_t^w)$,

(iv) $(F, A)_t^w \wedge ((G, B)_t^w \vee (H, C)_t^w) = ((F, A)_t^w \wedge (G, B)_t^w) \vee ((F, A)_t^w \wedge (H, C)_t^w)$.

Proof. We give the proofs of 1 and 2.

(i) Suppose that $(G, B)_t^w \wedge (H, C)_t^w = \widetilde{\mathfrak{t}}\left(G_t^w(\alpha), H_t^w(\beta)\right)$, $\forall (\alpha, \beta) \in B \times C$.

Then
$(F, A)_t^w \wedge ((G, B)_t^w \wedge (H, C)_t^w) = \widetilde{\mathfrak{t}}\left(F_t^w(\gamma), \widetilde{\mathfrak{t}}\left(G_t^w(\alpha), H_t^w(\beta)\right)\right)$,
$\forall (\gamma, (\alpha, \beta)) \in A \times (B \times C)$.
$= \widetilde{\mathfrak{t}}\left(\widetilde{\mathfrak{t}}\left(F_t^w(\gamma), G_t^w(\alpha)\right), H_t^w(\beta)\right)$,
$\forall ((\gamma, \alpha), \beta) \in (A \times B) \times C$.

$$= ((F,A)_t^w \wedge (G,B)_t^w) \wedge (H,C)_t^w.$$

(ii) Suppose that $(G,B)_t^w \vee (H,C)_t^w = s\left(G_t^w(\alpha), H_t^w(\beta)\right), \forall (\alpha,\beta) \in B \times C$.

Then
$(F,A)_t^w \vee ((G,B)_t^w \wedge (H,C)_t^w) = s\left(F_t^w(\gamma), s\left(G_t^w(\alpha), H_t^w(\beta)\right)\right)$,
$\forall (\gamma,(\alpha,\beta)) \in A \times (B \times C)$.

$$= s\left(s\left(F_t^w(\gamma), G_t^w(\alpha)\right), H_t^w(\beta)\right),$$

$\forall((\gamma,\alpha),\beta) \in (A \times B) \times C$.

$$= ((F,A)_t^w \vee (G,B)_t^w) \vee (H,C)_t^w.$$

6. An application of time fuzzy soft expert set with two choices in decision making

This section presents a theoretical application of the W-TFSES to a decision-making issue, indicating the viability and generalizability of this approach to problems in many domains with uncertainty. two choices $\{\text{agree}, \text{disagree}\}$ make up the application.

The government performance evaluation follows this computational pipeline:

(i) **Normalization**: Raw expert ratings r_{ij}^t are scaled to $[0,1]$ via:

$$u_{ij}^t = \frac{r_{ij}^t - \min(r_i)}{\max(r_i) - \min(r_i)}$$

(ii) **Time-Weighted Averaging**: For parameter e_k across T periods:

$$F(e_k) = \frac{1}{|T|} \sum_{t=1}^{|T|} F_t(e_k)$$

(iii) **Weighted Aggregation**: Final scores combine expert weights:

$$\overline{c_j} = \sum_{i=1}^{n} w_i \cdot \left(\frac{1}{|E|} \sum_{k=1}^{|E|} u_{ijk} \right)$$

where w_i are normalized expert weights ($\sum w_i = 1$), and u_{ijk} represents normalized evaluation of expert i for parameter e_k on alternative u_j.

Example 9. *Measuring the effectiveness of governments only through opinion surveys is insufficient. Instead, performance metrics that encompass all of the activities and duties listed below should be used to gauge government success. Suppose that the company of strategic studies considers a collection of experts to carry out an analysis using particular* *metrics to assess the performance of the governments of four nations throughout the course of four prior time periods; these metrics are listed below. Consider a collection of governments.* $U = \{u_1,\ u_2,\ u_3, u_4\}$, *Five parameters could be present. Assume that the decision parameters* $E = \{e_1, e_2, e_3\}$ *are used to evaluate the performance of governments. For i = 1, 2, 3, the parameters* e_i $(i = 1, 2, 3)$ *stand for the achievement of the anti-corruption campaign, efficient use of natural resources in management, investments, and savings. And let* $T = \{t_1, t_2, t_3, t_4\}$. $X = \{m, n\}$ *be the members of a committee, Suppose that the company of strategic has imposed the following weights for the experts. For the expert* $"m,"w_1 = 0.8$*; for the expert* $"n,"w_2 = 0.5$*; for the expert* $"r,"w_3 = 0.2$. *developed to deal with the decision-making problems based on weighted time fuzzy soft expert sets. In the revised algorithm, we take the weights of the expert into consideration and compute the weighted choice values* $\overline{c_j}$ *and* $\overline{k_j}$ *instead of choice values of* c_j *and* k_j. *The following algorithm may be followed by the company of strategic to fill the best choice.*

$$
\begin{aligned}
(F,Z)_t^w = \Big\{ & \Big(\big(e_1, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.3}, \tfrac{u_3{}^{t_1}}{0.5}, \tfrac{u_4{}^{t_1}}{0.6}\Big\}\Big), \Big(\big(e_1, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.4}, \tfrac{u_4{}^{t_1}}{0.7}\Big\}\Big),\\
& \Big(\big(e_2, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.4}, \tfrac{u_3{}^{t_1}}{0.2}, \tfrac{u_4{}^{t_1}}{0.7}\Big\}\Big), \Big(\big(e_2, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.3}, \tfrac{u_4{}^{t_1}}{0.5}\Big\}\Big),\\
& \Big(\big(e_3, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.3}\Big\}\Big), \Big(\big(e_3, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.9}, \tfrac{u_4{}^{t_1}}{0.5}\Big\}\Big),\\
& \Big(\big(e_1, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.4}, \tfrac{u_2{}^{t_2}}{0.8}, \tfrac{u_3{}^{t_2}}{0.2}, \tfrac{u_4{}^{t_2}}{0.4}\Big\}\Big), \Big(\big(e_1, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.4}, \tfrac{u_4{}^{t_2}}{0.8}\Big\}\Big),\\
& \Big(\big(e_2, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.4}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.7}, \tfrac{u_4{}^{t_2}}{0.5}\Big\}\Big), \Big(\big(e_2, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.8}, \tfrac{u_3{}^{t_2}}{0.5}, \tfrac{u_4{}^{t_2}}{0.6}\Big\}\Big),\\
& \Big(\big(e_3, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.4}, \tfrac{u_3{}^{t_2}}{0.1}, \tfrac{u_4{}^{t_2}}{0.8}\Big\}\Big), \Big(\big(e_3, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_2}}{0.8}, \tfrac{u_2{}^{t_2}}{0.6}, \tfrac{u_3{}^{t_2}}{0.3}, \tfrac{u_4{}^{t_2}}{0.7}\Big\}\Big),\\
& \Big(\big(e_1, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.2}, \tfrac{u_2{}^{t_3}}{0.7}, \tfrac{u_3{}^{t_3}}{0.8}, \tfrac{u_4{}^{t_3}}{0.3}\Big\}\Big), \Big(\big(e_1, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.4}, \tfrac{u_2{}^{t_3}}{0.9}, \tfrac{u_3{}^{t_3}}{0.6}, \tfrac{u_4{}^{t_3}}{0.5}\Big\}\Big),\\
& \Big(\big(e_2, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.5}, \tfrac{u_2{}^{t_3}}{0.4}, \tfrac{u_3{}^{t_3}}{0.7}, \tfrac{u_4{}^{t_3}}{0.9}\Big\}\Big), \Big(\big(e_2, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.8}, \tfrac{u_2{}^{t_3}}{0.6}, \tfrac{u_3{}^{t_3}}{0.2}, \tfrac{u_4{}^{t_3}}{0.7}\Big\}\Big),\\
& \Big(\big(e_3, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.7}, \tfrac{u_2{}^{t_3}}{0.6}, \tfrac{u_3{}^{t_3}}{0.3}, \tfrac{u_4{}^{t_3}}{0.7}\Big\}\Big), \Big(\big(e_3, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_3}}{0.6}, \tfrac{u_2{}^{t_3}}{0.5}, \tfrac{u_3{}^{t_3}}{0.3}, \tfrac{u_4{}^{t_3}}{0.9}\Big\}\Big),\\
& \Big(\big(e_1, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.2}, \tfrac{u_2{}^{t_4}}{0.9}, \tfrac{u_3{}^{t_4}}{0.6}, \tfrac{u_4{}^{t_4}}{0.7}\Big\}\Big), \Big(\big(e_1, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.7}, \tfrac{u_2{}^{t_4}}{0.6}, \tfrac{u_3{}^{t_4}}{0.8}, \tfrac{u_4{}^{t_4}}{0.4}\Big\}\Big),\\
& \Big(\big(e_2, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.8}, \tfrac{u_2{}^{t_4}}{0.6}, \tfrac{u_3{}^{t_4}}{0.2}, \tfrac{u_4{}^{t_4}}{0.7}\Big\}\Big), \Big(\big(e_2, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.7}, \tfrac{u_2{}^{t_4}}{0.4}, \tfrac{u_3{}^{t_4}}{0.5}, \tfrac{u_4{}^{t_4}}{0.8}\Big\}\Big),\\
& \Big(\big(e_3, \tfrac{m}{0.8}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.7}, \tfrac{u_2{}^{t_4}}{0.5}, \tfrac{u_3{}^{t_4}}{0.3}, \tfrac{u_4{}^{t_4}}{0.9}\Big\}\Big), \Big(\big(e_3, \tfrac{n}{0.5}, 1\big), \Big\{\tfrac{u_1{}^{t_4}}{0.6}, \tfrac{u_2{}^{t_4}}{0.8}, \tfrac{u_3{}^{t_4}}{0.4}, \tfrac{u_4{}^{t_4}}{0.5}\Big\}\Big),\\
& \Big(\big(e_1, \tfrac{m}{0.8}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.4}, \tfrac{u_4{}^{t_1}}{0.3}\Big\}\Big), \Big(\big(e_1, \tfrac{n}{0.5}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.2}\Big\}\Big),\\
& \Big(\big(e_2, \tfrac{m}{0.8}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.4}\Big\}\Big), \Big(\big(e_2, \tfrac{n}{0.5}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.3}, \tfrac{u_3{}^{t_1}}{0.6}, \tfrac{u_4{}^{t_1}}{0.6}\Big\}\Big),\\
& \Big(\big(e_3, \tfrac{m}{0.8}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.3}, \tfrac{u_4{}^{t_1}}{0.4}\Big\}\Big), \Big(\big(e_3, \tfrac{n}{0.5}, 0\big), \Big\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.2}, \tfrac{u_4{}^{t_1}}{0.6}\Big\}\Big),
\end{aligned}
$$

$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.2}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.8}, \frac{u_4{}^{t_2}}{0.3}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.4}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.2}\right\}\right),$$
$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.1}, \frac{u_4{}^{t_3}}{0.6}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.4}\right\}\right),$$
$$\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.1}\right\}\right), \left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.5}\right\}\right),$$
$$\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.2}\right\}\right),$$
$$\left(\left(e_1, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.6}, \frac{u_2{}^{t_4}}{0.1}, \frac{u_3{}^{t_4}}{0.3}, \frac{u_4{}^{t_4}}{0.4}\right\}\right), \left(\left(e_1, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.4}, \frac{u_2{}^{t_4}}{0.4}, \frac{u_3{}^{t_4}}{0.3}, \frac{u_4{}^{t_4}}{0.7}\right\}\right),$$
$$\left(\left(e_2, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.3}, \frac{u_2{}^{t_4}}{0.3}, \frac{u_3{}^{t_4}}{0.7}, \frac{u_4{}^{t_4}}{0.2}\right\}\right), \left(\left(e_2, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.4}, \frac{u_2{}^{t_4}}{0.6}, \frac{u_3{}^{t_4}}{0.6}, \frac{u_4{}^{t_4}}{0.3}\right\}\right),$$
$$\left.\left(\left(e_3, \frac{m}{0.8}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.6}, \frac{u_2{}^{t_4}}{0.4}, \frac{u_3{}^{t_4}}{0.7}, \frac{u_4{}^{t_4}}{0.2}\right\}\right), \left(\left(e_3, \frac{n}{0.5}, 0\right), \left\{\frac{u_1{}^{t_4}}{0.5}, \frac{u_2{}^{t_4}}{0.3}, \frac{u_3{}^{t_4}}{0.7}, \frac{u_4{}^{t_4}}{0.5}\right\}\right)\right\}.$$

The committee may use the following techniques for evaluating their selection and determine which option is best for the decision.

(i) Input the W-TFSES (F, Z).

(ii) Find an agree W-TFSES and a Disagree W-TFSES, Tables (1, 2).

(iii) Construct $F\left(Z\right)$'s tabular representation using the tables in Tables (3, 4), where $F\left(Z\right)$ outlined as follows:

$$F\left(z\right) = \left\{ \frac{u}{\sum\limits_{i=1}^{n} t_i F_{t_i}(z) \big/ n \sum\limits_{i=1}^{n} F_{t_i}(z)} : u \in U, z \in Z \right\} \tag{1}$$

where $n = |T|$.

(iv) Find$c_j = \sum\limits_{i} u_{ij}$ for agree W-TFSES for each expert (m, n) separately of each other's experts, Tables (5, 6) respectively.

(v) Find $k_j = \sum\limits_{i} u_{ij}$for disagree W-TFSES for each expert (m, n) separately of each other's experts, Tables (5, 6) respectively.

(vi) Find $\overline{s_j} = \overline{c_j} - \overline{k_j}$, for each expert (m, n) separately of each other's experts, Tables (5, 6) respectively.

(vii) Construct the grade table of W-TFSES , Table 7.

(viii) Calculate the final score values for u_i.

(ix) Find v, for which $\mathrm{s}_v = \max \overline{s_j}$. Then s_v is the optimal choice object. If v has more than one value, then any one of them could be chosen by the company using its option.

The agree weighted expert factor on time fuzzy soft expert sets and Disagree weighted expert factor on time fuzzy soft expert sets are shown in Tables 1 and 2, respectively.

Table 1: Agree weighted expert factor on time fuzzy soft expert sets

U	u_1	u_2	u_3	u_4
$\left(e_1, \frac{m}{o.8}\right)_{t_1}$	0.6	0.3	0.5	0.6
$\left(e_1, \frac{m}{o.8}\right)_{t_2}$	0.4	0.8	0.2	0.4
$\left(e_1, \frac{m}{o.8}\right)_{t_3}$	0.2	0.7	0.8	0.3
$\left(e_1, \frac{m}{o.8}\right)_{t_4}$	0.2	0.9	0.6	0.7
$\left(e_1, \frac{n}{o.5}\right)_{t_1}$	0.7	0.5	0.4	0.7
$\left(e_1, \frac{n}{o.5}\right)_{t_2}$	0.6	0.9	0.4	0.8
$\left(e_1, \frac{n}{o.5}\right)_{t_3}$	0.4	0.9	0.6	0.5
$\left(e_1, \frac{n}{o.5}\right)_{t_4}$	0.7	0.6	0.8	0.4
$\left(e_2, \frac{m}{o.8}\right)_{t_1}$	0.7	0.4	0.2	0.7
$\left(e_2, \frac{m}{o.8}\right)_{t_2}$	0.4	0.9	0.7	0.5
$\left(e_2, \frac{m}{o.8}\right)_{t_3}$	0.5	0.4	0.7	0.9
$\left(e_2, \frac{m}{o.8}\right)_{t_4}$	0.8	0.6	0.2	0.7
$\left(e_2, \frac{n}{o.5}\right)_{t_1}$	0.5	0.6	0.3	0.5
$\left(e_2, \frac{n}{o.5}\right)_{t_2}$	0.6	0.8	0.5	0.6
$\left(e_2, \frac{n}{o.5}\right)_{t_3}$	0.8	0.6	0.2	0.7
$\left(e_2, \frac{n}{o.5}\right)_{t_4}$	0.7	0.4	0.5	0.8
$\left(e_3, \frac{m}{o.8}\right)_{t_1}$	0.4	0.6	0.7	0.3
$\left(e_3, \frac{m}{o.8}\right)_{t_2}$	0.6	0.4	0.1	0.8
$\left(e_3, \frac{m}{o.8}\right)_{t_3}$	0.7	0.6	0.3	0.7
$\left(e_3, \frac{m}{o.8}\right)_{t_4}$	0.7	0.5	0.3	0.9
$\left(e_3, \frac{n}{o.5}\right)_{t_1}$	0.6	0.5	0.9	0.5
$\left(e_3, \frac{n}{o.5}\right)_{t_2}$	0.8	0.6	0.3	0.7
$\left(e_3, \frac{n}{o.5}\right)_{t_3}$	0.6	0.5	0.3	0.9
$\left(e_3, \frac{n}{o.5}\right)_{t_4}$	0.6	0.8	0.4	0.8

Table 2: Disagree weighted expert factor on time fuzzy soft expert sets

U	u_1	u_2	u_3	u_4
$\left(e_1, \frac{m}{o.8}\right)_{t_1}$	0.5	0.6	0.4	0.3
$\left(e_1, \frac{m}{o.8}\right)_{t_2}$	0.7	0.4	0.7	0.5
$\left(e_1, \frac{m}{o.8}\right)_{t_3}$	0.7	0.2	0.1	0.6
$\left(e_1, \frac{m}{o.8}\right)_{t_4}$	0.6	0.1	0.3	0.4
$\left(e_1, \frac{n}{o.5}\right)_{t_1}$	0.4	0.5	0.7	0.2
$\left(e_1, \frac{n}{o.5}\right)_{t_2}$	0.5	0.2	0.5	0.4
$\left(e_1, \frac{n}{o.5}\right)_{t_3}$	0.3	0.4	0.6	0.5
$\left(e_1, \frac{n}{o.5}\right)_{t_4}$	0.4	0.4	0.3	0.7
$\left(e_2, \frac{m}{o.8}\right)_{t_1}$	0.4	0.6	0.7	0.4
$\left(e_2, \frac{m}{o.8}\right)_{t_2}$	0.2	0.3	0.4	0.6
$\left(e_2, \frac{m}{o.8}\right)_{t_3}$	0.3	0.7	0.4	0.1
$\left(e_2, \frac{m}{o.8}\right)_{t_4}$	0.3	0.3	0.7	0.2
$\left(e_2, \frac{n}{o.5}\right)_{t_1}$	0.4	0.3	0.6	0.6
$\left(e_2, \frac{n}{o.5}\right)_{t_2}$	0.5	0.2	0.7	0.3
$\left(e_2, \frac{n}{o.5}\right)_{t_3}$	0.1	0.5	0.6	0.5
$\left(e_2, \frac{n}{o.5}\right)_{t_4}$	0.4	0.6	0.6	0.3
$\left(e_3, \frac{m}{o.8}\right)_{t_1}$	0.7	0.5	0.3	0.4
$\left(e_3, \frac{m}{o.8}\right)_{t_2}$	0.3	0.5	0.8	0.3
$\left(e_3, \frac{m}{o.8}\right)_{t_3}$	0.4	0.6	0.8	0.3
$\left(e_3, \frac{m}{o.8}\right)_{t_4}$	0.6	0.4	0.7	0.2
$\left(e_3, \frac{n}{o.5}\right)_{t_1}$	0.4	0.6	0.2	0.6
$\left(e_3, \frac{n}{o.5}\right)_{t_2}$	0.5	0.4	0.7	0.2
$\left(e_3, \frac{n}{o.5}\right)_{t_3}$	0.3	0.4	0.7	0.2
$\left(e_3, \frac{n}{o.5}\right)_{t_4}$	0.5	0.3	0.7	0.5

Next, using the relation (1) we calculate $F(Z)$ to convert the Agree W-TFSES to agree fuzzy soft expert set,to illustrate this step we calculate $F(e_1)$ for u_1 as shown below.

$$F(e_1) = \left\{ \frac{u_1}{\sum\limits_{i=1}^{4} t_i F_{t_i}(e) \diagup 4 \sum\limits_{i=1}^{4} F_{t_i}(e)} : u \in U, e \in E \right\}$$

$$= \left\{ \frac{u_1}{((1*0.5)+(2*0.7)+(3*0.7)+(4*0.6)) \diagup 4(0.5+0.7+0.7+0.6)} \right\}$$

$$= \left\{ \frac{u_1}{6.4 \diagup 10} \right\}$$

$= \frac{u_1}{0.64}$.
then we compute c_j, where c_j represents the sum of columns of an object u_i. The results are shown in Table 3.

Table 3: Converting Agree weighted expert factor on time fuzzy soft expert sets to agree- Fuzzy Soft Expert Sets

U	u_1	u_2	u_3	u_4
$\left(e_1, \frac{m}{0.8}\right)$	0.50	0.70	0.67	0.63
$\left(e_2, \frac{m}{0.8}\right)$	0.64	0.63	0.62	0.64
$\left(e_3, \frac{m}{0.8}\right)$	0.67	0.61	0.53	0.70
$\left(e_1, \frac{n}{0.5}\right)$	0.61	0.63	0.70	0.56
$\left(e_2, \frac{n}{0.5}\right)$	0.66	0.58	0.65	0.67
$\left(e_3, \frac{n}{0.5}\right)$	0.80	0.66	0.52	0.67

Likewise, using the relation (1), we calculate $F(Z)$ to convert the dis Agree W-TFSES to disagree fuzzy soft expert set, as we explained in converting the Agree W-TFSES to agree fuzzy soft expert set, then we compute k_j, where k_j represents the column sum of an object u_i. The results are shown in Table 4.

Table 4: Converting Disagree weighted expert factor on time fuzzy soft expert sets to Disagree-FSES

U	u_1	u_2	u_3	u_4
$\left(e_1, \frac{m}{0.8}\right)$	0.58	0.46	0.55	0.65
$\left(e_2, \frac{m}{0.8}\right)$	0.60	0.59	0.62	0.51
$\left(e_3, \frac{m}{0.8}\right)$	0.61	0.61	0.68	0.56
$\left(e_1, \frac{n}{0.5}\right)$	0.60	0.61	0.55	0.73
$\left(e_2, \frac{n}{0.5}\right)$	0.58	0.75	0.62	0.57
$\left(e_3, \frac{n}{0.5}\right)$	0.63	0.55	0.70	0.60

Consider the previous results in Table 1 and Table 2 we have.

Table 5: Score values of weighted expert factor on time fuzzy soft expert sets for expert m

$c_j m = \sum_i u_{ij}$	$k_j m = \sum_i u_{ij}$	$\overline{s_{jm}} = c_{jm} - k_{jm}$
$\overline{c_{1m}} = 1.81$	$\overline{k_{1m}} = 1.79$	$\overline{s_{1m}} = 0.02$
$\overline{c_{2m}} = 1.94$	$\overline{k_{2m}} = 1.66$	$\overline{s_{2m}} = 0.28$

Continued on next page

Table 5 – continued from previous page

$\overline{c_{3m}} = 1.82$	$\overline{k_{3m}} = 1.85$	$\overline{s_{3m}} = -0.03$
$\overline{c_{4m}} = 1.97$	$\overline{k_{4m}} = 1.72$	$\overline{s_{4m}} = 0.25$

In Table 5 we calculate the choice values for expert m in agree W-TFSES by computing c_{jm} for u_i,$\forall_i$, where c_{jm} represents the column sum of an object u_i. Also, we calculate the choice values for expert m in disagree W-TFSES by computing k_{jm} for u_i, $\forall_i$, where k_{jm} represents the column sum of an object u_i. After that we calculate the score value by subtracting the choice values in agree W-TFSES from the choice values in disagree W-TFSES. Then we determine the best choices by marking the highest numerical grade (underlined) as we show in Table (5). To illustrate this step, we calculate the score value for u_1 as shown below.
$c_{1m} = (0.50 + 0.64 + 0.67) = 1.81$

$$k_{1m} = ((0.2 * 0.1) + (0.3 * 0.3) + (0.2 * 0.4) + (0.3 * 0.5) + (0.4 * 0.7) + (0.1 * 0.9)) \\ = 0.71$$

Then the score value of u_1:

$S_{u_{1m}} = 1.46 - 0.71 = 0.75,$

Table 6: Score values of weighted expert factor on time fuzzy soft expert sets for expert n

$c_jn = \sum_i u_{ij}$	$k_jn = \sum_i u_{ij}$	$\overline{s_{jn}} = c_{jn} - k_{jn}$
$\overline{c_{1n}} = 2.07$	$\overline{k_{1n}} = 1.81$	$\underline{\overline{s_{1n}} = 0.26}$
$\overline{c_{2n}} = 1.87$	$\overline{k_{2n}} = 1.91$	$\overline{s_{2n}} = -0.1$
$\overline{c_{3n}} = 1.87$	$\overline{k_{3n}} = 1.89$	$\overline{s_{3n}} = -0.02$
$\overline{c_{4n}} = 1.90$	$\overline{k_{4n}} = 1.90$	$\overline{s_{4n}} = 0.0$

Next in Table (7) we summarize the highest numerical grade for all experts with their weights for the experts.

Table 7: Grade table of weighted expert factor on time fuzzy soft expert sets

X	u_i	highest numerical grade	expert weight
m	u_2	0.28	0.8
n	u_1	0.26	0.5

Next we calculate the final score for experts by taking the output of products of these numerical grades with the corresponding values of weights for the experts.
$S_{j_m}(u_1) = (0.28 * 0.8) = 0.224,$
$S_{j_n}(u_4) = (0.26 * 0.5) = 0.13,$

Then $\max s_j = s_{j_m}$, so the committee will choose the first choice depending on the above-mentioned algorithm. 6.

7. Conclusion

This paper has introduced and formalized the Weighted Time Fuzzy Soft Expert Set (W-TFSES) framework, representing a significant advancement beyond conventional fuzzy soft sets. Our work makes three fundamental contributions to the field: first, by developing a novel mathematical structure that simultaneously incorporates expert weighting coefficients and temporal parameters, we have addressed critical limitations in existing decision-making models; second, we have established a complete set of operations including complement, union, intersection, AND and OR operations, along with their essential algebraic properties; third, through our comprehensive case study on government performance evaluation, we have demonstrated the framework's practical utility in real-world scenarios requiring nuanced expert judgments over time.

The W-TFSES framework offers several distinct advantages that enhance decision-making processes. By introducing adjustable expert weights, our model captures the inherent variability in expert reliability that characterizes most real-world decision scenarios. The incorporation of temporal dimensions enables dynamic analysis of evolving situations, a crucial capability absent in traditional fuzzy soft sets. Furthermore, the framework maintains rigorous mathematical foundations while remaining sufficiently flexible for application across diverse domains, from public policy analysis to medical diagnostic systems.

However, we acknowledge certain limitations that warrant consideration. The increased dimensionality introduced by weight and time parameters leads to greater computational complexity, particularly when processing large datasets. Additionally, the initial assignment of expert weights, while flexible, remains subjective and may benefit from more systematic determination methods. These limitations, however, represent opportunities for future research rather than fundamental constraints.

Beyond its immediate applications, the W-TFSES framework has the potential to impact various domains where uncertainty and expert-based decision-making are critical. Future research could explore its integration with artificial intelligence for automated decision support or its application in financial risk assessment, where time-dependent expert evaluations play a crucial role. Additionally, comparative studies against alternative decision-making models, such as intuitionistic fuzzy sets or rough sets, could further validate W-TFSES's effectiveness in capturing temporal and expert-driven uncertainties.

Looking ahead, several promising research directions emerge from this work. Developing optimized algorithms for large-scale W-TFSES implementations could significantly enhance computational efficiency. The integration of machine learning techniques with our framework presents exciting possibilities for automated weight calibration and pattern recognition in temporal data. Furthermore, investigating hybrid models that combine W-TFSES with other soft computing approaches could yield even more powerful decision-support tools. These advancements would further solidify W-TFSES as a versatile and robust framework for complex, real-world decision-making challenges where expert judgment and temporal dynamics play crucial roles.

References

[1] D. Molodtsov. Soft set theory–first results. *Computers & Mathematics with Applications*, 37(2):19–31, 1999.

[2] P. K. Maji, A. R. Roy, and R. Biswas. Soft set theory. *Computers & Mathematics with Applications*, 54(4–5):555–562, 2003.

[3] P. K. Maji, A. R. Roy, and R. Biswas. Fuzzy soft sets. *Journal of Fuzzy Mathematics*, 9(3):589–602, 2001.

[4] R. Roy and P. K. Maji. A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics*, 203(2):412–418, 2007.

[5] A.Hazaymeh. *Fuzzy Soft Set And Fuzzy Soft Expert Set: Some Generalizations And Hypothetical Applications*. PhD thesis, Universiti Sains Islam Malaysia, 2013.

[6] Shawkat Alkhazaleh and Ayman A. Hazaymeh. N-valued refined neutrosophic soft sets and their applications in decision making problems and medical diagnosis. *Journal of Artificial Intelligence and Soft Computing Research*, 8(1):79–86, 2018.

[7] Y. Wang and J. E. Laird. The importance of action history in decision making and reinforcement learning. In *Proceedings of the eighth international conference on cognitive modeling*, 2007.

[8] A. Bataihah and A. Hazaymeh. Time fuzzy parameterized fuzzy soft expert sets. *International Journal of Neutrosophic Science*, 25(4):101–121, 2025.

[9] A. Hazaymeh, Y. Al-Qudah, F. Al-Sharqi, and A. Bataihah. A novel Q-neutrosophic soft under interval matrix setting and its applications. *International Journal of Neutrosophic Science*, 25(4):156–168, 2025.

[10] N. Çağman, F. Citak, and S. Enginoğlu. Fuzzy parameterized fuzzy soft set theory and its applications. *Turkish Journal of Fuzzy Systems*, 1(1):21–35, 2010.

[11] S. Alkhazaleh and A. R. Salleh. Soft expert sets. *Advances in Decision Sciences*, 2011:15, 2011.

[12] S. Alkhazaleh and A. R. Salleh. Fuzzy soft expert set and its application. *Applied Mathematics*, 2014, 2014.

[13] A. Hazaymeh, I. B. Abdullah, Z. Balkhi, and R. Ibrahim. Fuzzy parameterized fuzzy soft expert set. *Applied Mathematical Sciences*, 2012.

[14] A. Hazaymeh, I. B. Abdullah, Z. T. Balkhi, and R. I. Ibrahim. Generalized fuzzy soft expert set. *Journal of Applied Mathematics*, 2012:1–22, 2012.

[15] Y. Al-Qudah. A robust framework for the decision-making based on single-valued neutrosophic fuzzy soft expert setting. *Full Length Article*, 23(2):195–195, 2024.

[16] Y. Al-Qudah and F. Al-Sharqi. Algorithm for decision-making based on similarity measures of possibility interval-valued neutrosophic soft setting settings. *International Journal of Neutrosophic Science*, 22(3):69–83, 2023.

[17] Ayman A Hazaymeh. Fixed point results in complete neutrosophic fuzzy metric spaces for NF-L contractions. *Neutrosophic Sets and Systems*, 82:189–208, 2025.

[18] Ayman A Hazaymeh. Some fixed point results with T-distance spaces in complete b-metric spaces. *Journal of Mechanics of Continua and Mathematical Sciences*, 20:2454–7190, 2025.

[19] S. M. Alqaraleh, A. U. M. Alkouri, M. O. Massa'deh, A. G. Talafha, and A. Bataihah. Bipolar complex fuzzy soft sets and their application. *International Journal of Fuzzy System Applications (IJFSA)*, 11(1):1–23, 2022.

[20] S. Saleh, R. Abu-Gdairi, T. M. Al-Shami, and M. S. Abdo. On categorical property of fuzzy soft topological spaces. *Applied Mathematics & Information Sciences*, 16(4):635–641, 2022.

[21] S. Saleh and J. Al-Mufarrij. On θ-regularity and θ-normality in fuzzy soft topological spaces. *European Journal of Pure and Applied Mathematics*, 16(1):180–191, 2023.

[22] Tareq M Al-Shami and LJUBIŠA DR Kocinac. Almost soft menger and weakly soft menger spaces. *Appl. Comput. Math*, 21(1):35–51, 2022.

[23] Tareq M Al-shami and Ljubiša DR Kočinac. Nearly soft menger spaces. *Journal of Mathematics*, 2020(1):3807418, 2020.

[24] B. Kang, J. Choo, and K. Shin. Expert weight assignment in decision-making: A systematic approach. *Expert Systems with Applications*, 140:112868, 2020.

[25] M. Çelik and R. Şahin. A novel approach to fuzzy soft expert decision-making. *Journal of Intelligent & Fuzzy Systems*, 42(3):1835–1850, 2022.

[26] L. A. Zadeh. *Fuzzy Logic for the Management of Uncertainty.* John Wiley & Sons, 1996.

[27] Ayman A Hazaymeh. Time fuzzy soft sets and its application in design-making. *International Journal of Neutrosophic Science (IJNS)*, 25(3), 2025.

[28] A. Hazaymeh. Time-shadow soft set: Concepts and applications. *The International Journal of Fuzzy Logic and Intelligent Systems*, 24(4):387–398, 2024.

[29] Ayman A Hazaymeh. Time effective fuzzy soft set and its some applications with and without a neutrosophic. *International Journal of Neutrosophic Science*, 23(2):129–129, 2023.

[30] Ayman A Hazaymeh. Time factor's impact on fuzzy soft expert sets. *International Journal of Neutrosophic Science (IJNS)*, 25(3), 2025.