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Specific Identities Involving Prime Ideals with Generalized P-derivations

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Abstract. In this article, we will investigate the commutativity of the factor ring $\Re/P$, where P is a prime ideal of any ring $\Re$. This investigation will be carried out using generalized P-derivations $\mho$ and $\amalg$ associated with P-derivations χ and $\propto$, respectively, that satisfy specific functional identities linking $\Re$ to P. Moreover, we will discuss some related results. Finally, to reinforce the importance of our assumption regarding the primeness of P, we will provide some examples.

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1. Introduction

Throughout this article, the symbol $\Re$ denotes an associative ring with center $Z(\Re)$. A ring $\Re$ is said to be a prime ring if for any elements $\upsilon, \wp \in \Re$ the condition $\upsilon\Re\wp = \{0\}$ implies that at least one of the elements υ or $\wp$ must be zero. A prime ideal is a proper ideal P of a ring $\Re$ such that if $\upsilon\Re\wp \subseteq P$, then at least one of the elements υ or $\wp$ must be belong to P. A ring $\Re$ is said to be an integral domain if it is a commutative ring with unity and has no zero divisors. Every integral domain is a prime ring, but the converse is not true in general. For all $\upsilon, \wp \in \Re$, the symbols $[\upsilon, \wp] = \upsilon\wp - \wp\upsilon$ and $(\upsilon \circ \wp) = \upsilon\wp + \wp\upsilon$ denote the commutator and anticommutator, respectively. For a subset Θ of $\Re$, a mapping $\chi : \Theta \longrightarrow \Re$ is said to be centralizing (or commuting) on Θ if $[\chi(\upsilon), \upsilon] \in Z(\Re)$ (or $[\chi(\upsilon), \upsilon] = 0$) for all $\upsilon \in \Theta$.

By definition, a derivation is an additive mapping χ from $\Re$ to itself that satisfies $\chi(\upsilon\wp) = \chi(\upsilon)\wp + \upsilon\chi(\wp)$ for all $\upsilon, \wp \in \Re$. A generalized derivation, on the other hand, is an additive mapping $\mho$ from $\Re$ to itself that satisfies $\mho(\upsilon\wp) = \mho(\upsilon)\wp + \upsilon\chi(\wp)$ for all

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$\upsilon, \wp \in \Re$, where χ is the associated derivation with $\mho$. It is evident that every derivation is a generalized derivation, but the converse is not true in general. Another special case of a generalized derivation occurs when χ is restricted to be zero. This is called a multiplier, Ξ, defined as an additive mapping $\Xi : \Re \longrightarrow \Re$ by the rules $\Xi(\upsilon\wp) = \Xi(\upsilon)\wp$ and $\Xi(\upsilon\wp) = \upsilon\Xi(\wp)$ for all $\upsilon, \wp \in \Re$. These are referred to as left and right multipliers, respectively. If Ξ is both a right and left multiplier, it is simply called a multiplier. Examples and counterexamples of these concepts can be found in the literature.

The mapping $\chi : \Re \longrightarrow \Re$ is called P-additive if it satisfies $\chi(\upsilon + \wp) - \chi(\upsilon) - \chi(\wp) \in P$ for all $\upsilon, \wp \in \Re$. A P-additive mapping χ is called a P-derivation if it satisfies the relation $\chi(\upsilon\wp) - \chi(\upsilon)\wp - \upsilon\chi(\wp) \in P$ for all $\upsilon, \wp \in \Re$. A P-additive mapping $\mho : \Re \longrightarrow \Re$ is called a generalized P-derivation associated with a P-derivation χ if it satisfies $\mho(\upsilon\wp) - \mho(\upsilon)\wp - \upsilon\chi(\wp) \in P$ for all $\upsilon, \wp \in \Re$. Additionally, assuming χ is a P-trivial (i.e., $\chi(\Re) \subseteq P$) in the last relation gives us a P-left multiplier concept, defined as $\Xi(\upsilon\wp) - \Xi(\upsilon)\wp \in P$ for all $\upsilon, \wp \in \Re$. The P-right multiplier is defined as $\Xi(\upsilon\wp) - \upsilon\Xi(\wp) \in P$ for all $\upsilon, \wp \in \Re$. Moreover, Ξ is considered a P-multiplier if it is both a P-left and P-right multiplier. It is clear that every generalized derivation is a generalized P-derivation, and that every left multiplier is also a P-left multiplier, but the converse may not be true in general. For examples and counterexamples regarding the existence of these concepts, refer to [1].

Derivations are a crucial area of study in algebraic structure theory. They have their origins in analytic theory, invariant theory, and Galois theory. Derivations play a vital role in both physics and mathematics. Many researchers have presented derivations of various algebraic structures such as rings, and near rings. Based on derivations in rings, Posner proved in [2] that if a non-zero derivation χ centralizing on a prime ring $\Re$, then $\Re$ becomes commutative. Recently, several authors have proven the commutativity of semiprime and prime rings by utilizing appropriately restricted additive mappings that act on these rings or on suitable subsets of them. These mappings include derivations, automorphisms, generalized derivations, multipliers, and others. For more details, one can refer to [3], [4], and [5].

Inspired by previous studies, the commutativity of rings has been discussed in a more expansive way. For instance, the consideration of whether the ring $\Re$ is prime or semiprime has been omitted, and instead the focus has shifted to analyzing the behavior of a factor ring $\Re/P$, where P is a prime ideal of $\Re$. These studies involve the utilization of additive mappings that satisfy certain identities when acting on appropriate subsets of the ring $\Re$. For further details, please refer to references [6], [7], [8], [9], and [10].

In [10], Mouhssine et al. discuss the behavior of a factor near ring $\aleph/P$ when a near ring $\aleph$ admits an (α, τ)-P-derivation χ that satisfies certain identities, where P is a prime ideal of $\aleph$. In 2023, [11] Oukhtite et al. examined the effect of specific differential identities involving P-multipliers on a factor ring $\Re/P$, where P is a prime ideal of any ring $\Re$. In the same year, Sandhu et al. [1] investigated the commutativity of a factor ring $\Re/P$ by exploring certain identities involving a mixture of a generalized P-derivation and a P-multiplier, where P is a prime ideal in $\Re$.

In this article, we will further investigate the commutativity of a factor ring $\Re/P$. We will accomplish this by assuming that the arbitrary ring $\Re$ admits generalized P-

derivations $(\mho, \chi)$ and $(\amalg, \propto)$ that satisfy any of the following identities for each $\upsilon, \wp \in \Re$: (i) $[\chi(\upsilon), \chi(\wp)] \pm [\wp, \amalg(\upsilon)] \in P$, (ii) $\chi(\upsilon) \circ \chi(\wp) \pm [\wp, \amalg(\upsilon)] \in P$, (iii) $[\mho(\upsilon), \chi(\wp)] \pm \wp \circ \amalg(\upsilon) \in P$, (iv) $[\upsilon, \amalg(\wp)] \pm \mho([\upsilon, \wp]) \in P$, (v) $\mho([\upsilon, \wp]) \pm \mho(\wp) \amalg(\upsilon) \in P$, (vi) $\mho([\upsilon, \wp]) \pm \mho(\upsilon) \amalg(\wp) \in P$, (vii) $[\upsilon, \mho(\wp)] \pm \mho(\wp) \amalg(\upsilon) \in P$. Furthermore, we will present several related consequences and provide examples to illustrate the significance of the assumptions in our theorems.

2. Preliminaries

In this section, we will exhibit some important preliminaries that will be used repeatedly to develop proofs of our main theorems.

Lemma 1. *[12, Lemma 2.4] Let $\Re$ be a ring with a semi-prime ideal P, and let $\Re/P$ be 2-torsion free. If χ is a derivation on $\Re$ such that $[\chi^2(\upsilon), \upsilon] \in P$, then χ is P-commuting on $\Re$.*

Lemma 2. *[1, Lemma 10] Let $\Re$ be a ring that admits a generalized P-derivation $\mho$ associated with a P-derivation χ, where P is a prime ideal of $\Re$.*

(i) If $[\upsilon, \mho(\wp)] \in P$ satisfies for every elements $\upsilon, \wp \in \Re$, then $\Re/P$ is an integral domain or $\mho(\Re) \subset P$.

(ii) If $\upsilon \circ \mho(\wp) \in P$ satisfies for every elements $\upsilon, \wp \in \Re$, then $\Re/P$ is an integral domain with $char(\Re/P) = 2$ or $\mho(\Re) \subset P$.

Lemma 3. *Consider a prime ideal P of an arbitrary ring $\Re$. If $\Re$ admits a generalized P-derivation $\mho$ associated with a P-derivation χ such that $[\upsilon, \mho(\upsilon)] \in P$ for all $\upsilon \in \Re$, then either $\Re/P$ is an integral domain or $\chi(\Re) \subseteq P$.*

Proof. The proof can be easily derived from Lemma 2 (i), so it may be skipped as it would not result in any significant changes.

The following corollary is a special case of the previous lemma when $\mho = \chi$.

Corollary 1. *[1, Lemma 1] Consider a prime ideal P of an arbitrary ring $\Re$. If $\Re$ admits a P-derivation χ such that $[\upsilon, \chi(\upsilon)] \in P$ for all $\upsilon \in \Re$, then either $\Re/P$ is an integral domain or $\chi(\Re) \subseteq P$.*

3. Main Results

For brevity, let $(\mho, \chi)$ and $(\amalg, \propto)$ symbolize two generalized P-derivations associated with P-derivations χ and $\propto$, respectively. The symbol $id_\Re$ designates the identity map $id_\Re : \Re \longrightarrow \Re$ defined by $id_\Re(\upsilon) = \upsilon$ for all $\upsilon \in \Re$. In their work, Sandhu et al. [1, Lemma 10] discussed the behavior of a factor ring $\Re/P$ under the influence of a generalized P-derivation satisfying certain algebraic identities involving a prime ideal P of any ring $\Re$. In the following three theorems, we will expand upon those results under the influence of a pair of generalized P-derivations alternating between commutator and anticommutator.

Theorem 1. *Let $\Re$ be a ring equipped with a P-derivation χ and a generalized P-derivation $(\amalg, \propto)$ such that char$(\Re/P) \neq 2$. Then, $[\chi(\upsilon), \chi(\wp)] \pm [\wp, \amalg(\upsilon)] \in P$ for all $\upsilon, \wp \in \Re$ if and only if one of the following is true:*
(i) $\Re/P$ is an integral domain;
(ii) $\chi(\Re) \subseteq P$, and $\amalg(\Re) \subseteq P$.

Proof. According to the given hypothesis, for every $\upsilon, \wp \in \Re$, we have

$$[\chi(\upsilon), \chi(\wp)] \pm [\wp, \amalg(\upsilon)] \in P. \tag{1}$$

Replacing $\wp$ by $\wp\hbar$ in Equation (1) and applying it, we obtain

$$\chi(\wp)[\chi(\upsilon), \hbar] + [\chi(\upsilon), \wp]\chi(\hbar) \in P \quad \text{for all } \upsilon, \wp, \hbar \in \Re. \tag{2}$$

Taking $\upsilon = \wp = \hbar$ in Equation (2), we obtain

$$\chi(\upsilon)[\chi(\upsilon), \upsilon] + [\chi(\upsilon), \upsilon]\chi(\upsilon) = \chi^2(\upsilon)\upsilon - \chi(\upsilon)\upsilon\chi(\upsilon) + \chi(\upsilon)\upsilon\chi(\upsilon) - \upsilon\chi^2(\upsilon) \in P \quad \text{for all } \upsilon \in \Re. \tag{3}$$

From Equation (3), we conclude

$$[\chi^2(\upsilon), \upsilon] \in P \quad \text{for all } \upsilon \in \Re.$$

By using Lemma 1, we obtain $[\chi(\upsilon), \upsilon] \in P$ for all $\upsilon \in \Re$. Therefore, Corollary 1 implies that either $\Re/P$ is an integral domain or $\chi(\Re) \subseteq P$. Assuming that $\chi(\Re) \subseteq P$, then Equation (1) reduces to $[\wp, \amalg(\upsilon)] \in P$ for every $\upsilon, \wp \in \Re$. By using Lemma 2 (i), we can conclude that either $\Re/P$ is an integral domain or $\amalg(\Re) \subseteq P$.

The following corollary directly results from replacing $\amalg$ by $id_{\Re}$ in the previous theorem and following arguments similar to those used.

Corollary 2. *Let $\Re$ be a ring equipped with a P-derivation χ such that char$(\Re/P) \neq 2$. Then, $[\chi(\upsilon), \chi(\wp)] \pm [\wp, \upsilon] \in P$ for all $\upsilon, \wp \in \Re$ if and only if $\Re/P$ is an integral domain.*

Theorem 2. *Let $\Re$ be a ring equipped with a P-derivation χ and a generalized P-derivation $(\amalg, \propto)$ such that char$(\Re/P) \neq 2$. Then, $\chi(\upsilon) \circ \chi(\wp) \pm [\wp, \amalg(\upsilon)] \in P$ for all $\upsilon, \wp \in \Re$ if and only if one of the following is true:*
(i) $\Re/P$ is an integral domain and $\chi(\Re) \subseteq P$.
(ii) $\chi(\Re) \subseteq P$ and $\amalg(\Re) \subseteq P$.

Proof.
For each $\upsilon, \wp \in \Re$, given the hypothesis

$$\chi(\upsilon) \circ \chi(\wp) + [\wp, \amalg(\upsilon)] \in P. \tag{4}$$

By substituting $\wp$ with $\wp\hbar$ in Equation (4) and applying it, we obtain

$$[\chi(\upsilon), \wp]\chi(\hbar) - \chi(\wp)[\chi(\upsilon), \hbar] \in P \quad \text{for all } \upsilon, \wp, \hbar \in \Re. \tag{5}$$

Replacing $\wp$ by $\kappa\wp$ in Equation (5) and applying it, we get

$$[\chi(\upsilon),\kappa]\wp\chi(\hbar)-\chi(\kappa)\wp[\chi(\upsilon),\hbar]\in P \quad \text{for all } \upsilon,\wp,\hbar,\kappa\in\Re. \tag{6}$$

Setting $\kappa=\chi(\upsilon)$ in Equation (6), we get

$$\chi^2(\upsilon)\wp[\chi(\upsilon),\hbar]\in P \quad \text{for all } \upsilon,\wp,\hbar\in\Re. \tag{7}$$

Replacing $\wp$ by $\hbar\wp$ in Equation (7) and comparing the result with Equation (7), we obtain

$$[\chi^2(\upsilon),\hbar]\wp[\chi(\upsilon),\hbar]\in P \quad \text{for all } \upsilon,\wp,\hbar\in\Re. \tag{8}$$

This implies that $[\chi^2(\upsilon),\hbar]\Re[\chi(\upsilon),\hbar]\subseteq P$ for all $\upsilon,\hbar\in\Re$. The primeness of P yields either $[\chi^2(\upsilon),\hbar]\in P$ or $[\chi(\upsilon),\hbar]\subseteq P$ for all $\upsilon,\hbar\in\Re$. Suppose $[\chi^2(\upsilon),\hbar]\in P$ for all $\upsilon,\hbar\in\Re$. In particular, we have $[\chi^2(\upsilon),\upsilon]\in P$ for all $\upsilon\in\Re$. Using Lemma 1, we find that $[\chi(\upsilon),\upsilon]\in P$ for all $\upsilon\in\Re$. This leads to either $\Re/P$ is an integral domain or $\chi(\Re)\subseteq P$, by using Corollary 1. If $\chi(\Re)\subseteq P$, Equation (4) reduces to $[\wp,\text{II}(\upsilon)]\in P$ for all $\upsilon,\wp\in\Re$. Thus, Lemma 2 (i) implies that either $\Re/P$ is an integral domain or $\text{II}(\Re)\subseteq P$.
Assuming $\Re/P$ is an integral domain, Equation (4) becomes $2\chi(\upsilon)\chi(\wp)\in P$ for all $\upsilon,\wp\in\Re$. Since $char(\Re/P)\neq 2$, then $\chi(\upsilon)\chi(\wp)\in P$ for all $\upsilon,\wp\in\Re$. By replacing $\wp$ with $\wp\hbar$ in the previous equation and using it, we arrive at $\chi(\upsilon)\Re\chi(\hbar)\subseteq P$ for all $\upsilon,\hbar\in\Re$. By utilizing the primeness of P, we can conclude that $\chi(\Re)\subseteq P$. The proof is complete.

To prove the theorem for the identity $\chi(\upsilon)\circ\chi(\wp)-[\wp,\text{II}(\upsilon)]\in P$ for all $\upsilon,\wp\in\Re$, simply repeat the previous arguments to obtain the desired result.

The following corollary can be derived immediately from the previous theorem by replacing II by $id_{\Re}$.

Corollary 3. *Let $\Re$ be a ring equipped with a P-derivation χ such that $char(\Re/P)\neq 2$. Then, $\chi(\upsilon)\circ\chi(\wp)\pm[\wp,\upsilon]\in P$ for all $\upsilon,\wp\in\Re$ if and only if $\Re/P$ is an integral domain and $\chi(\Re)\subseteq P$.*

Theorem 3. *Let $\Re$ be a ring equipped with generalized P-derivations $(\mho,\chi)$ and $(\text{II},\propto)$ such that $char(\Re/P)\neq 2$. Then $[\mho(\upsilon),\chi(\wp)]\pm\wp\circ\text{II}(\upsilon)\in P$ for all $\upsilon,\wp\in\Re$ if and only if one of the following satisfies*
(i) $\Re/P$ is an integral domain and $\text{II}(\Re)\subseteq P$;
(ii) $\mho(\Re)\subseteq P$ and $\text{II}(\Re)\subseteq P$;
(iii) $\chi(\Re)\subseteq P$, and $\text{II}(\Re)\subseteq P$.

Proof. From the given hypothesis for each $\upsilon,\wp\in\Re$, we have

$$[\mho(\upsilon),\chi(\wp)]+\wp\circ\text{II}(\upsilon)\in P, \qquad \text{for all } \upsilon,\wp\in\Re. \tag{9}$$

Replacing $\wp$ by $\wp\hbar$ in Equation (9) and applying it, we get

$$\chi(\wp)[\mho(\upsilon),\hbar]+\wp[\mho(\upsilon),\chi(\hbar)]+[\mho(\upsilon),\wp]\chi(\hbar)+\wp[\hbar,\text{II}(\upsilon)]\in P, \quad \text{for all } \upsilon,\wp,\hbar\in\Re. \tag{10}$$

Taking $\wp = \kappa\wp$ in Equation (10) and using it, we get

$$\chi(\kappa)\wp[\mho(\upsilon), \hbar] + [\mho(\upsilon), \kappa]\wp\chi(\hbar) \in P \quad \text{for all } \upsilon, \wp, \hbar, \kappa \in \Re.$$

Letting $\kappa = \hbar$, we obtain $\chi(\hbar)\wp[\mho(\upsilon), \hbar] + [\mho(\upsilon), \hbar]\wp\chi(\hbar) \in P$ for all $\upsilon, \wp, \hbar \in \Re$. Therefore, we can deduce that $\chi(\hbar)\wp[\mho(\upsilon), \hbar] \in P$ and $[\mho(\upsilon), \hbar]\wp\chi(\hbar) \in P$ for all $\upsilon, \wp, \hbar \in \Re$. That is, $\chi(\hbar)\Re[\mho(\upsilon), \hbar] \subseteq P$ for all $\upsilon, \hbar \in \Re$. The primeness of P implies either $\chi(\hbar) \in P$ for all $\hbar \in \Re$ or $[\mho(\upsilon), \hbar] \in P$ for all $\upsilon, \hbar \in \Re$. In the second case, Lemma 2 (i) forces either $\Re/P$ is an integral domain or $\mho(\Re) \subseteq P$. Let's examine the case when $\mho(\Re) \subseteq P$. This reduces Equation (9) to $\wp \circ \amalg(\upsilon) \in P$ for all $\upsilon, \wp \in \Re$. Using the hypothesis that $char(\Re/P) \neq 2$ together with Lemma 2 (ii), we find either $\Re/P$ is an integral domain or $\amalg(\Re) \subseteq P$. Now, assuming $\Re/P$ is an integral domain, Equation (9) reduces to $2\wp\Re\amalg(\upsilon) \subseteq P$ for all $\upsilon, \wp \in \Re$. By using the primeness of P and the fact that $char(\Re/P) \neq 2$, we can conclude that $\amalg(\Re) \subseteq P$.

On the other hand, if $\chi(\hbar) \in P$ for all $\hbar \in \Re$, then Equation (9) becomes $\wp \circ \amalg(\upsilon) \in P$ for all $\upsilon, \wp \in \Re$. By repeating the previous discussion, we arrive at the required conclusion.

To prove the theorem for the identity $[\mho(\upsilon), \chi(\wp)] - \wp \circ \amalg(\upsilon) \in P$ for all $\upsilon, \wp \in \Re$, simply repeat the previous arguments to obtain the desired result.

In [13], Quadri et al. discussed the behavior of a prime ring $\Re$ that admits a generalized derivation $(\mho, \chi)$ satisfying $\mho[\upsilon, \wp] - [\upsilon, \wp] = 0$ for all $\upsilon, \wp \in \Upsilon$, where Υ is a nonzero ideal of $\Re$. In the context of two generalized derivations, Rehman et al. [14] discussed the behavior of a 2-torsion free $*$-prime ring with the identity $[\upsilon, \amalg(\wp)] - \mho([\upsilon, \wp]) = 0$ for all $\upsilon, \wp \in \Lambda$, where Λ is a nonzero square closed $*$-Lie ideal of $\Re$. Bouchannafa et al. [15] studied the relationship between a factor ring $\Re/P$ and a generalized derivation $(\mho, \chi)$ satisfying $\mho[\upsilon, \wp] - [\mho(\upsilon), \wp] \in Z(\Re/P)$ for all $\upsilon, \wp \in \Re$, without imposing primeness on a ring or $char(\Re/P) \neq 2$, where P is a prime ideal of $\Re$. Building on these previous findings, it is natural to inquire about the situation of a factor ring $\Re/P$ when $\Re$ admits generalized P-derivations $(\mho, \chi)$ and $(\amalg, \propto)$ that satisfy the identity $[\upsilon, \amalg(\wp)] \pm \mho([\upsilon, \wp]) \in P$ for all $\upsilon, \wp \in \Re$. To address this question, we will now present the following theorem.

Theorem 4. *Let $\Re$ be a ring equipped with generalized P-derivations $(\mho, \chi)$ and $(\amalg, \propto)$ such that $[\upsilon, \amalg(\wp)] \pm \mho([\upsilon, \wp]) \in P$ for all $\upsilon, \wp \in \Re$. Then, $\Re/P$ is an integral domain or $(\amalg \pm \mho)(\Re) \subseteq P$.*

Proof. Our initial hypothesis states:

$$[\upsilon, \amalg(\wp)] \pm \mho([\upsilon, \wp]) \in P, \quad \text{for all } \upsilon, \wp \in \Re. \tag{11}$$

Substituting $\wp$ with $\wp\hbar$ in Equation (11) and applying it, we get

$$\amalg(\wp)[\upsilon, \hbar] + \wp[\upsilon, \propto(\hbar)] + [\upsilon, \wp]\propto(\hbar) \pm \mho(\wp)[\upsilon, \hbar] \pm \wp\chi([\upsilon, \hbar]) \pm [\upsilon, \wp]\chi(\hbar) \in P \quad \text{for all } \upsilon, \wp, \hbar \in \Re. \tag{12}$$

Taking $\upsilon = \hbar$ in Equation (12), we get

$$\wp[\upsilon, \propto(\upsilon)] + [\upsilon, \wp]\propto(\upsilon) \pm [\upsilon, \wp]\chi(\upsilon) \in P \quad \text{for all } \upsilon, \wp \in \Re. \tag{13}$$

By replacing $\wp$ with $\kappa\wp$ in Equation (13) and comparing it with (13), we obtain

$$[\upsilon,\kappa]\Re(\propto(\upsilon)\pm\chi(\upsilon))\subseteq P \quad \text{for all } \upsilon,\kappa\in\Re.$$

Utilizing the primeness of P, we can conclude that either $\Re/P$ is an integral domain or $\propto(\upsilon)\pm\chi(\upsilon)\in P$ for all $\upsilon\in\Re$. Let's examine the case when

$$\propto(\upsilon)\pm\chi(\upsilon)\in P \quad \text{for all } \upsilon\in\Re. \tag{14}$$

This reduces Equation (13) to $\wp[\upsilon,\propto(\upsilon)]\in P$ for all $\upsilon,\wp\in\Re$. The primeness of P, along with Corollary 1, forces either $\Re/P$ is an integral domain or $\propto(\Re)\subseteq P$. If $\propto(\Re)\subseteq P$ for all $\upsilon\in\Re$, then Equation (14) becomes $\chi(\upsilon)\in P$ for all $\upsilon\in\Re$. Hence, Equation (12) reduces to $(\amalg(\wp)\pm\mho(\wp))[\upsilon,\hbar]\in P$ for all $\upsilon,\wp,\hbar\in\Re$. For any $\tau\in\Re$, replacing υ by $\upsilon\tau$ in the last equation and using it, we get $(\amalg(\wp)\pm\mho(\wp))\Re[\tau,\hbar]\subseteq P$ for all $\tau,\wp,\hbar\in\Re$. Again, the primeness of P gives either $\Re/P$ is an integral domain or $\amalg(\wp)\pm\mho(\wp)\in P$ for all $\wp\in\Re$. Therefore, $(\amalg\pm\mho)(\Re)\subseteq P$.

Now we are prepared to gather several corollaries as applications of Theorem 4 as follows:

Corollary 4. *Let $\Re$ be a ring equipped with P-derivations χ and $\propto$ such that $[\upsilon,\propto(\wp)]\pm\chi([\upsilon,\wp])\in P$ for all $\upsilon,\wp\in\Re$. Then $\Re/P$ is an integral domain or $(\propto\pm\chi)(\Re)\subseteq P$.*

Corollary 5. *Let $\Re$ be a ring with char$(\Re/P)\neq 2$. If $\Re$ is equipped with a generalized P-derivation $(\mho,\chi)$ such that $[\upsilon,\mho(\wp)]+\mho([\upsilon,\wp])\in P$ for all $\upsilon,\wp\in\Re$, then $\Re/P$ is an integral domain or $\mho(\Re)\subseteq P$.*

By setting $\amalg = id_{\Re}$, we obtain a generalization of [16, Theorem 1] as shown in the following corollary:

Corollary 6. *Let $\Re$ be a ring equipped with a generalized P-derivation $(\mho,\chi)$ such that $\mho([\upsilon,\wp])\pm([\upsilon,\wp])\in P$ for all $\upsilon,\wp\in\Re$. Then $\Re/P$ is an integral domain or $(\mho\pm id_{\Re})(\Re)\subseteq P$.*

By setting $\mho=\chi$, we get a generalization of [17, Theorem 3] as shown in the following corollary:

Corollary 7. *Let $\Re$ be a ring equipped with a P-derivation χ such that $\chi([\upsilon,\wp])\pm[\upsilon,\wp]\in P$ for all $\upsilon,\wp\in\Re$. Then $\Re/P$ is an integral domain.*

The following example is devoted to explaining the significance of the primeness hypothesis of P in the previous theorems.

Example 1. *Let $\Re=\mathbb{C}[\upsilon]\times M_2(\Psi)$, where $\mathbb{C}[\upsilon]$ is the polynomial ring of complex numbers with determinant υ, and let Ψ be any ring. Let $P=\{(0,0)\}$. Define $(\mho,\chi),(\amalg,\propto):\Re\longrightarrow\Re$ by*

$$\mho(t(\upsilon),\psi)=\chi(t(\upsilon),\psi)=(t'(\upsilon)<\upsilon^2>,0),$$

and

$$\amalg(t(\upsilon),\psi) = \propto (t(\upsilon),\psi) = (t'(\upsilon) < \upsilon^3 >, 0).$$

It is easy to verify that $\mho$ *and* $\amalg$ *are generalized derivations of* $\Re$ *associated with derivations* χ *and* $\propto$*, respectively. It can also be verified that* $\Re$ *satisfies the identities in Theorems 1, 3 (i), and 4. However, neither* $\Re/P$ *is an integral domain nor* χ*,* $\mho$*,* $\amalg$ *and* $\mho \pm \amalg$ *map* $\Re$ *to* P*. It is important to note that* P *is not a prime ideal of* $\Re$*, since* $(0, \begin{pmatrix} \upsilon & 0 \\ 0 & 0 \end{pmatrix})(0, \begin{pmatrix} 0 & 0 \\ 0 & \wp \end{pmatrix}) \in P$*, but neither* $(0, \begin{pmatrix} \upsilon & 0 \\ 0 & 0 \end{pmatrix}) \in P$ *nor* $(0, \begin{pmatrix} 0 & 0 \\ 0 & \wp \end{pmatrix}) \in P$*. Therefore, the primeness condition in Theorems 1, 3 (i), and 4 is essential.*

Rehman et al. [12] found that either $\chi(\Re) \subseteq P$ or $\Re/P$ is an integral domain when $\Re$ admits generalized derivations $(\mho, \chi)$ and $(\amalg, \propto)$ that satisfy the identities $\mho(\upsilon\wp) + \mho(\upsilon)\amalg(\wp) \in P$ or $\mho(\upsilon)\amalg(\wp) + [\upsilon, \wp] \in P$ for all $\upsilon, \wp \in \Re$, where P is a prime ideal of $\Re$. In the upcoming theorem, we will explore how the identities (i) $\mho([\upsilon, \wp]) \pm \mho(\wp)\amalg(\upsilon) \in P$ and (ii) $\mho([\upsilon, \wp]) \pm \mho(\upsilon)\amalg(\wp) \in P$ for all $\upsilon, \wp \in \Re$, affect the relationship between a factor ring $\Re/P$ and generalized P-derivations.

Theorem 5. *Let* $\Re$ *be a ring equipped with generalized* P*-derivations* $(\mho, \chi)$ *and* $(\amalg, \propto)$ *such that one of the following identities holds for every* $\upsilon, \wp \in \Re$*:*
(i) $\mho([\upsilon, \wp]) \pm \mho(\wp)\amalg(\upsilon) \in P$,
(ii) $\mho([\upsilon, \wp]) \pm \mho(\upsilon)\amalg(\wp) \in P$*. Then either* $\propto(\Re) \subseteq P$ *and* $\Re/P$ *is an integral domain, or* $\mho(\Re)$ *is a subset of* P*.*

Proof. (i) According to the basic assumption, we have

$$\mho([\upsilon, \wp]) \pm \mho(\wp)\amalg(\upsilon) \in P \quad \text{for all } \upsilon, \wp \in \Re. \tag{15}$$

Setting $\upsilon = \upsilon\hbar$ in Equation (15) and using it, we get

$$\mho(\upsilon)[\hbar, \wp] + \upsilon\chi([\hbar, \wp]) + [\upsilon, \wp]\chi(\hbar) \pm \mho(\wp)\upsilon \propto (\hbar) \in P \quad \text{for all } \upsilon, \wp, \hbar \in \Re. \tag{16}$$

By setting $\hbar = \wp$, we have

$$[\upsilon, \wp]\chi(\wp) \pm \mho(\wp)\upsilon \propto (\wp) \in P \quad \text{for all } \upsilon, \wp \in \Re. \tag{17}$$

Replacing υ by $\wp\upsilon$ in Equation (17) and utilizing it, we get $\mho(\wp)\wp\upsilon \propto (\wp) - \wp\mho(\wp)\upsilon \propto (\wp) = [\mho(\wp), \wp]\upsilon \propto (\wp) \in P$ for all $\upsilon, \wp \in \Re$. This implies that $[\mho(\wp), \wp]\Re \propto (\wp) \subseteq P$ for all $\wp \in \Re$. The primeness of P implies either $[\mho(\wp), \wp] \in P$ or $\propto (\wp) \in P$ for all $\wp \in \Re$. To complete the proof, we will discuss the following two cases:

Case (a): If $[\mho(\wp), \wp] \in P$ for all $\wp \in \Re$, this implies that $\Re/P$ is an integral domain or $\chi(\Re) \subseteq P$ by using Lemma 3. If $\Re/P$ is an integral domain, Equation (17) yields $\mho(\wp)\Re \propto (\wp) \subseteq P$ for all $\wp \in \Re$. The primeness of P leads to either $\mho(\Re) \subseteq P$ or $\propto (\Re) \subseteq P$. On the other hand, if $\chi(\Re) \subseteq P$, then Equation (17) gives $\mho(\wp)\Re \propto (\wp) \subseteq P$ for all $\wp \in \Re$. Again, the primeness of P leads to either $\mho(\Re) \subseteq P$ or $\propto (\Re) \subseteq P$. In the scenario where $\propto (\Re) \subseteq P$, Equation (16) becomes $\mho(\upsilon)[\hbar, \wp] \in P$ for all $\upsilon, \wp, \hbar \in \Re$.

Replacing υ by $\upsilon\kappa$ in the last equation yields $\mho(\upsilon)\Re[\hbar,\wp] \subseteq P$ for all $\upsilon,\wp,\hbar \in \Re$. Since P is prime, either $\mho(\Re) \subseteq P$ or $\Re/P$ is an integral domain.

Case (b): If $\propto(\Re) \subseteq P$, then Equation (17) becomes $[\upsilon,\wp]\chi(\wp) \in P$ for all $\upsilon,\wp \in \Re$. Substituting υ with $\hbar\upsilon$ in the last expression and using it, we obtain $[\hbar,\wp]\Re\chi(\wp) \subseteq P$ for all $\hbar,\wp \in \Re$. The primeness of P implies either $\Re/P$ is an integral domain or $\chi(\Re) \subseteq P$. In the second case, as discussed in **Case** (a), we conclude that either $\Re/P$ is an integral domain or $\mho(\Re) \subseteq P$.

(ii) To prove this part, simply reiterate the arguments used in the proof of part (i) with some minor modifications to achieve the desired result.

Corollary 8. *Let $\Re$ be a ring equipped with a generalized P-derivation $(\mho,\chi)$ such that $\mho([\upsilon,\wp]) \pm \mho(\wp)\mho(\upsilon) \in P$ for all $\upsilon,\wp \in \Re$. Then $\chi(\Re) \subseteq P$ and $\Re/P$ is an integral domain, or $\mho(\Re)$ is a subset of P.*

Corollary 9. *Let $\Re$ be a ring equipped with P-derivations χ and $\propto$ such that $\chi([\upsilon,\wp]) \pm \chi(\wp) \propto(\upsilon) \in P$ for all $\upsilon,\wp \in \Re$. Then $\propto(\Re) \subseteq P$ and $\Re/P$ is an integral domain, or $\chi(\Re)$ is a subset of P.*

Example 2. *Let $\Re = K_{2^2}$ as in [18, Example 2.1], and let $P = \{0\}$. Define $(\mho,\chi) : \Re \longrightarrow \Re$ by*

$$\mho(\upsilon) = \chi(\upsilon) = \begin{cases} 0 & \text{if} \quad \upsilon = 0, c; \\ c & \text{if} \quad \upsilon = a, b. \end{cases}$$

It is easy to verify that $\mho$ is a generalized derivation of $\Re$ associated with derivation χ. It can also be verified that $\Re$ satisfies the identity in Corollary 8. However, neither $\Re/P$ is an integral domain nor $\mho(\Re) \subseteq P$. It is important to note that P is not a prime ideal of $\Re$, since $a\Re c \subseteq P$, but neither $a \in P$ nor $c \in P$. Therefore, the primeness condition in Corollary 8 is essential.

Bouchannafa et al. [9, Theorem 4] investigated that the derivations χ and $\propto$ are subsets of a prime ideal P, or the factor ring $\overline{\Re/P}$ is an integral domain. $\overline{\text{This occurs when}}$ one of the following identities holds: $\overline{\mho(\upsilon)\mho(\wp) \pm \amalg(\upsilon\wp)} \in Z(\Re/P)$, or $\overline{[\mho(\upsilon),\wp] \pm \amalg(\upsilon\wp)} \in Z(\Re/P)$ for all $\upsilon,\wp \in \Upsilon$, where Υ is a non-zero ideal of $\Re$, $(\mho,\chi)$ and $(\amalg,\propto)$ are generalized derivations in $\Re$. In the following theorem, we will examine the structure of a factor ring $\Re/P$ under the influence of a pair of generalized P-derivations that satisfy any of the following algebraic identities for every $\upsilon,\wp \in \Re$: $[\upsilon,\mho(\wp)] \pm \mho(\wp)\amalg(\upsilon) \in P$.

Theorem 6. *Let $\Re$ be a ring equipped with generalized P-derivations $(\mho,\chi)$ and $(\amalg,\propto)$ such that $[\upsilon,\mho(\wp)] \pm \mho(\wp)\amalg(\upsilon) \in P$ for all $\upsilon,\wp \in \Re$. Then one of the following is true:*

(i) $\mho(\Re) \subseteq P$;

(ii) $\chi(\Re) \subseteq P$ and $\propto(\Re) \subseteq P$;

(iii) $\propto(\Re) \subseteq P$ and $\Re/P$ is an integral domain.

Proof. The initial hypothesis states:

$$[\upsilon, \mho(\wp)] \pm \mho(\wp)\,\amalg\,(\upsilon) \in P \qquad \text{for all } \upsilon, \wp \in \Re. \tag{18}$$

Substituting υ by $\upsilon\hbar$ in Equation (18) and using it, we get

$$\upsilon[\hbar, \mho(\wp)] \pm \mho(\wp)\upsilon \propto (\hbar) \in P \qquad \text{for all } \upsilon, \wp, \hbar \in \Re. \tag{19}$$

Replacing υ by $\wp\upsilon$ in Equation (19) and comparing it with (19), we obtain $[\mho(\wp), \wp]\Re \propto (\hbar) \subseteq P$ for all $\wp, \hbar \in \Re$. Primeness of P forces that $[\mho(\wp), \wp] \in P$ or $\propto (\hbar) \in P$ for all $\wp, \hbar \in \Re$.

To complete the proof, let's discuss the following two cases:

Case (a): If $[\mho(\wp), \wp] \in P$ for all $\wp \in \Re$, Lemma 3 gives $\chi(\Re) \subseteq P$ or $\Re/P$ is an integral domain. Let's examine $\chi(\Re) \subseteq P$. Then substituting $\wp$ with $\wp\kappa$ in Equation (18), we deduce that $[\upsilon, \mho(\wp)]\kappa + \mho(\wp)[\upsilon, \kappa] \pm \mho(\wp)\kappa\,\amalg\,(\upsilon) \in P$ for all $\upsilon, \wp, \kappa \in \Re$. Right multiplying of Equation (18) by κ and comparing it with the last equation, we get $\mho(\wp)[\upsilon, \kappa] \pm \mho(\wp)[\kappa, \amalg(\upsilon)] \in P$. That is, $\mho(\wp)\Re([\upsilon, \kappa] \mp [\amalg(\upsilon), \kappa]) \subseteq P$ for all $\upsilon, \wp, \kappa \in \Re$. Primeness of P implies that $\mho(\wp) \in P$ or $[\upsilon, \kappa] \mp [\amalg(\upsilon), \kappa] \in P$ for all $\upsilon, \wp, \kappa \in \Re$. From the first scenario, we conclude $\mho(\Re) \subseteq P$. The second scenario with substitution $\kappa = \kappa\upsilon$, yields $[\amalg(\upsilon), \upsilon]\kappa \in P$ for all $\upsilon, \kappa \in \Re$. Again, primeness of P with utilizing Lemma 3, we conclude $\Re/P$ an integral domain or $\propto (\Re) \subseteq P$.

Now, if we consider the scenario when $\Re/P$ is an integral domain, then Equation (19) simplifies to $\mho(\wp)\upsilon \propto (\hbar) \in P$ for all $\upsilon, \wp, \hbar \in \Re$, which implies $\mho(\wp)\Re \propto (\hbar) \subseteq P$ for all $\wp, \hbar \in \Re$. Primeness of P implies $\mho(\Re) \subseteq P$ or $\propto (\Re) \subseteq P$.

Case (b): If $\propto (\hbar) \in P$ for all $\hbar \in \Re$, Equation (19) reduces to $\upsilon[\hbar, \mho(\wp)] \in P$ for all $\upsilon, \wp, \hbar \in \Re$. Primeness of P forces that $[\hbar, \mho(\wp)] \in P$ for all $\wp, \hbar \in \Re$. By using Lemma 2 (i), we conclude $\Re/P$ is an integral domain or $\mho(\Re) \subseteq P$. If $\Re/P$ is an integral domain, then as discussed above the desired result can be obtained.

The following examples aim to emphasize the necessity of the primeness condition of P in Theorems [1–6].

Example 3. *Let* $\Re = \left\{ \begin{pmatrix} 0 & \upsilon & \wp \\ 0 & 0 & 4\hbar \\ 0 & 0 & 0 \end{pmatrix} \mid \upsilon, \wp, \hbar \in \mathbb{Z}_8 \right\}$*, and let* $P = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$*. Defined* $(\mho, \chi); (\amalg, \propto) : \Re \longrightarrow \Re$ *by*

$$\mho \begin{pmatrix} 0 & \upsilon & \wp \\ 0 & 0 & 4\hbar \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2\upsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \textit{with} \qquad \chi \begin{pmatrix} 0 & \upsilon & \wp \\ 0 & 0 & 4\hbar \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \hbar \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$\amalg \begin{pmatrix} 0 & \upsilon & \wp \\ 0 & 0 & 4\hbar \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4\wp & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \textit{with} \qquad \propto \begin{pmatrix} 0 & \upsilon & \wp \\ 0 & 0 & 4\hbar \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \upsilon \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It is easy to verify that $\mho$ *and* $\amalg$ *are generalized derivations of* $\Re$ *associated with derivations* χ *and* $\propto$*, respectively. It can also be verified that* $\Re$ *satisfies the identities in Theorems [1–6]. However, neither* $\Re/P$ *integral domain nor* χ*,* $\propto$*,* $\mho$*,* $\amalg$ *and* $\mho \pm \amalg$ *mapping* $\Re$ *to* P*. It is important to note that* P *is not a prime ideal of* $\Re$*, since* $\begin{pmatrix} 0 & \upsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^2 \in P$*, but* $\begin{pmatrix} 0 & \upsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \notin P$*. Therefore, the primeness condition in Theorems [1–6] is essential.*

Example 4. *Let* $\Re = \Gamma \times \mathbb{H}[x]$*, where* $\Gamma = \{\gamma = \upsilon e_{21} + \wp e_{31} + 2\hbar e_{32} \mid \upsilon, \wp, \hbar \in \mathbb{Z}_4\}$ *and* $\mathbb{Z}[\upsilon]$ *is the polynomial ring of quaternions in determinate* υ*, and let* $P = \{(0,0)\}$*. Define* $(\mho, \chi), (\amalg, \propto) : \Re \longrightarrow \Re$ *by*

$$\mho(\gamma, t(\upsilon)) = (2\upsilon e_{21}, 0) \qquad \textit{with} \qquad \chi(\gamma, t(x\upsilon)) = (-\hbar e_{31}, 0),$$

and

$$\amalg(\gamma, t(\upsilon)) = (2\wp e_{21}, 0) \qquad \textit{with} \qquad \propto(\gamma, t(\upsilon)) = (-\upsilon e_{31}, 0).$$

It is easy to verify that $\mho$ *and* $\amalg$ *are generalized derivations of* $\Re$ *associated with derivations* χ *and* $\propto$*, respectively. It can also be verified that* $\Re$ *satisfies the identities in Theorems [1–6]. However, neither* $\Re/P$ *integral domain nor* χ*,* $\propto$*,* $\mho$*,* $\amalg$ *and* $\mho \pm \amalg$ *mapping* $\Re$ *to* P*. It is important to note that* P *is not a prime ideal of* $\Re$*, since* $(\upsilon e_{21}, 0)\Re(2\wp e_{32}, 0) \in P$*, but neither* $(\upsilon e_{21}, 0) \in P$ *nor* $(2\wp e_{32}, 0) \in P$*. Hence, the primeness condition in Theorems [1–6] is essential.*

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