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The Neutrosophic Poisson Distribution Applied to Horadam Polynomial-Subordinate Bi-Univalent Functions

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Abstract. In the open unit disk, we introduce a new subclass of analytic and bi-univalent functions described by the Neutrosophic Poisson distribution series. The estimates for the Fekete–Szegö type and the Taylor coefficients. Then, using the Horadam polynomials, the inequalities corresponding to the functions that belong to this recently discovered subclass are investigated. Along with some corollaries, we also go over the implications of the findings presented in this essay. Numerous branches of mathematics, science, and technology are anticipated to heavily rely on the neutrophilic Poisson distribution.

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1. Preliminaries

The orthogonality and completeness of Legendre polynomials, two of their defining characteristics, make them well-suited for a wide range of applications. Legendre first introduced these polynomials in 1784 [1]. They are utilised quite frequently in analysis, where they are utilised in polynomial interpolation and approximation, in order to address boundary value problems in analysis, such as the Laplace equation and the Schrödinger equation. In addition, they are utilised in a variety of other contexts.

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In the field of physics, Legendre polynomials are put to use for a broad variety of tasks, including the study of spherical harmonics and the development of gravitational potentials, to name just a couple of examples. When it comes to analysing data, particularly in the fields of statistics and signal processing, they can be used to perform regression and smoothing.

A probabilistic model that handles ambiguous or incomplete data in a more flexible manner is called the neutrosophic Poisson distribution. This model is an extension of the traditional Poisson distribution that was developed many years ago. The neutrosophic logic theory is a generalisation of classical logic that permits for the existence of indeterminate or unknown truth values. This distribution was first presented by Florentin Smarandache in 1998 [2], as a part of the neutrosophic logic theory. There have been a few researchers who have combined the Bi-Univalent distribution with this one; for an example, see the citation provided by [3].

Numerous branches of mathematics, including geometry and complex analysis, use a family of analytical functions called "bi-univalent functions," which are defined in the complex plane. The two-univalent functions are the univalent functions' generalization. They are in charge of assigning a specific region to the complex plane's unit disk.

Research on bi-univalent functions continues to be active and has contributed to the development of a number of important theories and breakthroughs in complex analysis and geometry, few to mention [4–8]. generally.

Assume that the class of functions f of the type is $\mathcal{A}$.

$$f(z) = z + m_2 z^2 + \cdots = z + \sum_{k=2}^{\infty} m_k z^k, \quad (z \in \mathfrak{h}). \tag{1}$$

on the disk $\mathfrak{h} = \{z \in \mathbb{C} : |z| < 1\}$, which are analytical. Additionally, we refer to $\mathcal{S}$ as the subclass of $\mathcal{A}$ composed of univalent functions of Eq. (1) in $\mathfrak{h}$.

The study of geometric function theory benefits greatly from the many strong tools that are made possible by the differential subordination of analytical functions. Additionally, see [9]. Miller and Mocanu introduced the first differential subordination problem in [10]. Miller and Mocanu book [11], which includes publication date references, compiles the most of the field's accomplishments.

The symbol for the inverse of each mathematical function $f \in \mathcal{S}$ is f^{-1}.

$$f^{-1}(f(z)) = z \qquad (z \in \mathfrak{h})$$

and

$$w = f(f^{-1}(w)) \qquad (|w| < r_0(f);\ r_0(f) \geq 0.25)$$

where

$$g(w) = f^{-1}(w) = w - m_2 w^2 + (-m_3 + 2m_2^2)w^3 - (m_4 + 5m_2^3 - 5m_3 m_2)w^4 + \cdots . \tag{2}$$

In the case where both $f(z)$ and $f^{-1}(z)$ are univalent in the set $\mathfrak{h}$, it can be concluded that the function is bi-univalent in the set $\mathfrak{h}$. Both of these functions have the same evaluation value, which is the reason why this is the case.

The class of bi-univalent functions in $\mathfrak{h}$ that are defined by the equation (1) will be denoted as Σ using the notation. Here are a few instances from section Σ:

$$\frac{z}{1-z}, \qquad \log\frac{1}{1-z}.$$

On the other hand, the well-known Koebe function is not included in the Σ. Here are some more instances of standard functions that can be found in $\mathfrak{h}$:

$$\frac{2z-z^2}{2} \text{ and } \frac{z}{1-z^2}$$

also don't belong to Σ. To learn about interesting subclasses of functions in class Σ, see ([12–20]).

Inspired by the groundbreaking work of Srivastava et al. [21], several subclasses of the bi-univalent function class Σ were introduced, [22–30] obtained non-sharp estimates on the first two coefficients $|m_2|$ and $|m_3|$ in the Taylor-Maclaurin series expansion (1).

The Horadam polynomials $h_n(x)$, which are determined by the following recurrence relation, were examined by Horzum and Kocer in 2009. [31].

$$h_n(x) = pxh_{n-1}(x) + qh_{n-2}(x), \quad (n \in \mathbb{N} \setminus \{1, 2\}), \tag{3}$$

with

$$h_1(x) = a,\ h_2(x) = tx \text{ and } h_3(x) = ptx^2 + aq, \tag{4}$$

for some real constant a, t, p and q.

Remark 1. *Instances of the Horadam polynomials that are particularly noteworthy.*

i) If $a = t = p = q = 1$, the Fibonacci polynomials sequence is obtained

$$F_n(x) = xF_{n-1}(x) + F_{n-2}(x); \quad F_1(x) = 1, F_2(x) = x.$$

ii) If $a = 2, \quad t = p = q = 1$, the Lucas polynomials sequence is obtained

$$L_{n-1}(x) = xL_{n-2}(x) + L_{n-3}(x); \quad L_0(x) = 2, \quad L_1(x) = x.$$

iii) $a = 1, t = p = 2, q = -1$, the Chebyshev polynomials of second kind sequence is obtained

$$U_{n-1}(x) = 2xU_{n-2}(x) - U_{n-3}(x); \quad U_0(x) = 1, U_1(x) = 2x.$$

iv) If $a = t = 1, p = 2, q = -1$, the Chebyshev polynomials of first kind sequence is obtained

$$T_{n-1}(x) = 2xT_{n-2}(x) - T_{n-3}(x); \quad T_0(x) = 1, T_1(x) = x.$$

v) If $a = q = 1, \quad t = p = 2$, the Pell polynomials sequence is obtained

$$P_n(x) = 2xP_{n-1}(x) + P_{n-2}(x); \quad P_1(x) = 1, P_2(x) = 2x.$$

vi) If $a = t = p = 2, q = 1$, the Pell-Lucas polynomials sequence is obtained

$$Q_{n-1}(x) = 2xQ_{n-2}(x) + Q_{n-3}(x); \quad Q_0(x) = 2, Q_1(x) = 2x.$$

For more information, (see [32] and [31]).

Numerous disciplines in mathematics, physics, statistics, and engineering make use of the Fibonacci, Lucas, Chebyshev, and families of orthogonal polynomials, as well as their extensions. Numerous academic articles have been written.

The polynomials in question.

The following creates Horadam polynomials, $h_n(x)$:

$$\Omega(x, z) = \sum_{n=1}^{\infty} h_n(x) z^{n-1} = \frac{a + (t - ap)xz}{1 - pxz - qz^2}. \tag{5}$$

In recent years, a significant amount of research has concentrated on various essential areas of geometric function theory. These studies looked at coefficient estimates, relationships between different types, and what is needed to belong to certain groups (see [33–39]). Various distributions, such as the Poisson, Pascal, Borel, and Mittag-Leffler-type Poisson distributions, have been utilized in this research, see [40–43].

Neutrosophic theory was just recently, specifically in 1995, presented by Smarandache. It is a novel area of philosophy that serves as a generalisation of both the fuzzy and the intuitionistic fuzzy logic, see [44]. Even though the neutrosophic Poisson distribution of a discrete variable X is a classic Poisson distribution of X, its parameter l is not precise and can be specified with two or more elements. This type of distribution occurs most frequently when l is an interval. Let

$$NP\left(X = d\right) = \frac{(l)^d}{d!} e^{-l}, \quad d = 0, 1, 2, ..., \tag{6}$$

where the expected value and the variance, denoted by the distribution parameter l

$$NL(X) = NV(X) = l \ ,$$

a neutrosophic statistical number is $N = O + I$; for more information, see [44] and the sources therein.We are going to provide a new power series, and the coefficients of this series will be the probabilities of the Neutrosophic Poisson Distribution

$$\mathbb{B}(l, z) = z + \sum_{k=2}^{\infty} \frac{(l)^{k-1} e^{-l}}{(k-1)!} z^k, \qquad z \in \mathfrak{h}. \tag{7}$$

Consider the linear operator $\mathbb{P}_m : \mathcal{A} \to \mathcal{A}$ defined by the convolution

$$\mathbb{P}_m f(z) = \mathbb{B}(l, z) * f(z) = z + \sum_{k=2}^{\infty} \frac{(l)^{k-1} e^{-l}}{(k-1)!} m_k z^k, \qquad z \in \mathfrak{h}. \tag{8}$$

In this study, we derive bounds for the $|m_2|$ and $|m_3|$ Taylor-Maclaurin coefficients and establish a new subclass involving the neutrosophic Poisson distribution associated with Horadam polynomials. For functions in this new class, we also solve the Fekete-Szego functional problems.

2. Class boundaries $\mathfrak{G}_{\Sigma}^{t}(x, p, q, u, \gamma)$

This section starts by defining the new subclass of Bell distribution series, $\mathfrak{G}_{\Sigma}^{t}(x, p, q, u, \gamma)$.

Definition 1. *A function $f \in \Sigma$ given by (1) is considered to belong to the class $\mathfrak{G}_{\Sigma}^{t}(x, p, q, u, \gamma)$ if the following subordinations are satisfied:*

$$1 + \frac{1}{u}\left[(P_m f(z))' + \gamma z (P_m f(z))'' - 1\right] \prec \Omega(x, z) + 1 - a \tag{9}$$

and

$$1 + \frac{1}{u}\left[(P_m f(w))' + \gamma z (P_m f(w))'' - 1\right] \prec \Omega(x, w) + 1 - a, \tag{10}$$

where $\gamma \geq 0, u \geq 1$, $x \in \mathbb{R}$, and the function $g = f^{-1}$ is given by (2).

Example 1. *The following equation is obtained when $\gamma = 0$ and $u = 1$,The expression $\mathfrak{G}_{\Sigma}^{t}(x, p, q, 1, 0)$, where $\mathfrak{G}_{\Sigma}^{t}(x, p, q)$ denotes the class of functions $f \in \Sigma$ that are provided by (1) and satisfy the subsequent condition:The expression $\mathfrak{G}_{\Sigma}^{t}(x, p, q, 1, 0)$.*

$$(P_m f(z))' \prec \Omega(x, z) + 1 - a \tag{11}$$

and

$$(P_m f(w))' \prec \Omega(x, w) + 1 - a, \tag{12}$$

where $x \in \mathbb{R}$, and the function $g = f^{-1}$ is given by (2).

First, we will provide some estimates for the coefficients that belong to the class $\mathfrak{G}_{\Sigma}^{t}(x, p, q, u, \gamma)$ which may be found in Definition 1.

Theorem 1. *Consider $f \in \Sigma$ to be a member of the class $\mathfrak{G}^t_\Sigma(x,p,q,u,\gamma)$ as indicated by (1). After that,*

$$|m_2| \leq \frac{tx\sqrt{2txu}}{\sqrt{\left[\left|3u\left(1+2\gamma\right)(tx)^2 - 8\left(1+\gamma\right)^2\left(ptx^2+aq\right)e^{-l}\right|\right] l^2 e^{-l}}},$$

and

$$|m_3| \leq \frac{t^2x^2u^2}{4\left(1+\gamma\right)^2 l^2 e^{-2l}} + \frac{2txu}{3\left(1+2\gamma\right) l^2 e^{-l}}.$$

Proof. Suppose that $f \in \mathfrak{G}^t_\Sigma(x,p,q,u,\gamma)$. From 1 Definition, we can write

$$1 + \frac{1}{u}\left[(P_m f(z))' + \gamma z (P_m f(z))'' - 1\right] = \Omega(x, \varkappa(z)) + 1 - a \tag{13}$$

and

$$1 + \frac{1}{u}\left[(P_m f(w))' + \gamma z (P_m f(w))'' - 1\right] = \Omega(x, \tau(w)) + 1 - a, \tag{14}$$

where $\varkappa$ and τ, the analytical functions, have the form

$$\varkappa(z) = c_1 z + c_2 z^2 + c_3 z^3 + \cdots, \quad (z \in \mathfrak{h})$$

and

$$\tau(w) = d_1 w + d_2 w^2 + d_3 w^3 + \cdots, \quad (w \in \mathfrak{h}),$$

such that $\varkappa(0) = \tau(0) = 0$ and $|\varkappa(z)| < 1$, $|\tau(w)| < 1$ for all $z, w \in \mathfrak{h}$.

From the equalities (13) and (14), it is what we get

$$1 + \frac{1}{u}\left[(P_m f(z))' + \gamma z (P_m f(z))'' - 1\right] = 1 + h_2(x) c_1 z + \left[h_2(x) c_2 + h_3(x) c_1^2\right] z^2 + \cdots \tag{15}$$

and

$$1 + \frac{1}{u}\left[(P_m f(w))' + \gamma z (P_m f(w))'' - 1\right] = 1 + h_2(x) d_1 w + \left[h_2(x) d_2 + h_3(x) d_1^2\right] w^2 + \cdots. \tag{16}$$

It is well known that if

$$|\varkappa(z)| = \left|c_1 z + c_2 z^2 + c_3 z^3 + \cdots\right| < 1, \quad (z \in \mathfrak{h})$$

and

$$|\tau(w)| = \left|d_1 w + d_2 w^2 + d_3 w^3 + \cdots\right| < 1, \quad (w \in \mathfrak{h}),$$

then

$$|c_j| \leq 1 \text{ and } |d_j| \leq 1 \text{ for all } j \in \mathbb{N}. \tag{17}$$

The following is the outcome of comparing the pertinent coefficients in (15) and (16)

$$\frac{2(1+\gamma)}{u} l e^{-l} m_2 = h_2(x) c_1, \tag{18}$$

$$\frac{3(1+2\gamma)}{2u} l^2 e^{-l} m_3 = h_2(x) c_2 + h_3(x) c_1^2, \tag{19}$$

$$-\frac{2(1+\gamma)}{u} l e^{-l} m_2 = h_2(x) d_1, \tag{20}$$

and

$$\frac{3(1+2\gamma)}{2u} l^2 e^{-l} \left[2m_2^2 - m_3\right] = h_2(x) d_2 + h_3(x) d_1^2. \tag{21}$$

It follows from (18) and (20) that

$$c_1 = -d_1 \tag{22}$$

and

$$\frac{8(1+\gamma)^2}{u^2} l^2 e^{-2l} m_2^2 = [h_2(x)]^2 \left(c_1^2 + d_1^2\right). \tag{23}$$

If we add (19) and (21), we get

$$\frac{3(1+2\gamma)}{u} l^2 e^{-l} m_2^2 = h_2(x)(c_2 + d_2) + h_3(x)\left(c_1^2 + d_1^2\right). \tag{24}$$

We determine that by replacing the value of $\left(c_1^2 + d_1^2\right)$ from (23) in the right hand side of (24)

$$\left[\frac{3(1+2\gamma)}{u} l^2 e^{-l} - \frac{8(1+\gamma)^2}{u^2} l^2 e^{-2l} \frac{h_3(x)}{[h_2(x)]^2}\right] m_2^2 = h_2(x)(c_2 + d_2). \tag{25}$$

Moreover computations using (4), (17) and (25), we find that

$$|m_2| \leq \frac{tx\sqrt{2txu}}{\sqrt{\left[\left|3u(1+2\gamma)(tx)^2 - 8(1+\gamma)^2\left(ptx^2 + aq\right)e^{-l}\right|\right] l^2 e^{-l}}}.$$

Moreover, if we subtract (21) from (19), we obtain

$$\frac{3(1+2\gamma)}{u} l^2 e^{-l}\left(m_3 - m_2^2\right) = h_2(x)(c_2 - d_2) + h_3(x)\left(c_1^2 - d_1^2\right). \tag{26}$$

Then, in view of (22) and (23), Eq. (26) becomes

$$m_3 = \frac{[h_2(x)]^2 u^2}{8(1+\gamma)^2 l^2 e^{-2l}} \left(c_1^2 + d_1^2\right) + \frac{h_2(x)u}{3(1+2\gamma) l^2 e^{-l}} (c_2 - d_2).$$

Thus applying (4), we conclude that

$$|m_3| \leq \frac{t^2 x^2 u^2}{4(1+\gamma)^2 l^2 e^{-2l}} + \frac{2txu}{3(1+2\gamma) l^2 e^{-l}}.$$

.

1933 [45] was the year when Fekete and Szego succeeded in obtaining a precise constraint for the functional $\left|m_3 - \eta m_2^2\right|$.. This was carried out for a function with a single value known as f and the interval $\eta \in [0,1]$.

We use the values of m_2^2 and m_3 to illustrate that the functional expression for class functions $\mathfrak{G}_{\Sigma}^{t}(x,p,q,u,\gamma)$ is $\left|m_3 - \eta m_2^2\right|$. This is carried out to show that the difference between the two numbers is the key to the answer.

Theorem 2. *Allow $f \in \Sigma$ to belong to the class $\mathfrak{G}_{\Sigma}^{t}(x,p,q,u,\gamma)$ as given by (1). Then*

$$\left|m_3 - \eta m_2^2\right| \leq \begin{cases} \frac{2|tx|u}{3(1+2\gamma)l^2 e^{-l}}, & |\eta - 1| \leq \wp \\ \frac{2(tx)^3|1-\eta|u^2}{\left[\left|3u(1+2\gamma)[tx]^2 - 8(1+\gamma)^2(ptx^2+aq)e^{-l}\right|\right] l^2 e^{-l}}, & |\eta - 1| \geq \wp, \end{cases}$$

where

$$\wp = 1 - \frac{\left[\left|8(1+\gamma)^2\left(ptx^2+aq\right)\right|\right] e^{-l}}{3u[tx]^2(1+2\gamma)}.$$

Proof. From (25) and (26)

$$\begin{aligned} & m_3 - \eta m_2^2 \\ &= (1-\eta)\frac{[h_2(x)]^3 (c_2+d_2) u^2}{\left[\left|3u(1+2\gamma)[h_2(x)]^2 - 8(1+\gamma)^2 h_3(x) e^{-l}\right|\right] l^2 e^{-l}} \\ &+ \frac{h_2(x)u}{3(1+2\gamma) l^2 e^{-l}}(c_2 - d_2) \\ &= h_2(x)\left[\mathcal{F}(\eta) + \frac{u}{3(1+2\gamma) l^2 e^{-l}}\right] c_2 \\ &+ h_2(x)\left[\mathcal{F}(\eta) - \frac{u}{3(1+2\gamma) l^2 e^{-l}}\right] d_2, \end{aligned}$$

where

$$\mathcal{F}(\eta) = \frac{[h_2(x)]^2 (1-\eta) u^2}{\left[\left|3u(1+2\gamma)[h_2(x)]^2 - 8(1+\gamma)^2 h_3(x) e^{-l}\right|\right] l^2 e^{-l}},$$

Then, in view of (4), we conclude that

$$\left|m_3-\eta m_2^2\right| \leq \begin{cases} \frac{2|h_2(x)|u}{3(1+2\gamma)l^2e^{-l}} & |\mathcal{F}(\eta)| \leq \frac{u}{3(1+2\gamma)l^2e^{-l}}, \\ 2\left|h_2(x)\right|\left|\mathcal{F}(\eta)\right| & |\mathcal{F}(\eta)| \geq \frac{u}{3(1+2\gamma)l^2e^{-l}}. \end{cases}$$

.

3. Corollary

The mentioned theorems 1 and 2 directly generate this corollary.

Corollary 1. *Allow $f \in \Sigma$ to belong to the class $\mathfrak{G}_\Sigma^t(x,p,q,1,0)$ as given by (1). Then*

$$|m_2| \leq \frac{tx\sqrt{2tx}}{\sqrt{\left[\left|3\,(tx)^2-8\left(ptx^2+aq\right)e^{-l}\right|\right]l^2e^{-l}}},$$

$$|m_3| \leq \frac{t^2x^2}{4l^2e^{-2l}}+\frac{2tx}{3l^2e^{-l}}.$$

and

$$\left|m_3-\eta m_2^2\right| \leq \begin{cases} \frac{2|tx|}{3l^2e^{-l}}, & |\eta-1| \leq \Phi \\ \frac{2(tx)^3|1-\eta|}{\left[\left|3[tx]^2-8(ptx^2+aq)e^{-l}\right|\right]l^2e^{-l}}, & |\eta-1| \geq \Phi, \end{cases}$$

where

$$\Phi = \left|1-\frac{8\left(ptx^2+aq\right)e^{-l}}{3\,[tx]^2}\right|.$$

4. Conclusions

The coefficient problems associated with each of the new subclasses of the class of bi-univalent functions have been introduced and investigated in this paper.

$\mathfrak{G}_\Sigma^t(x,p,q,u,\gamma)$ in the open unit disk's configuration $\mathfrak{h}$. Definition 1 thus provides a description of these bi-univalent function classes. For each of these three classes of bi-univalent functions, we have calculated the Taylor–Maclaurin coefficients $|m_2|$ and $|m_3|$ as well as the estimates of the Fekete–Szego functional issues. We discovered that a number of other novel findings would emerge after focussing on the factors that were crucial to our main findings. You can utilise bi-univalent functions in this investigation by using a altered form of the derivative operator proposed by Caputo.

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