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Upper and Lower Contra-(τ_1, τ_2)-continuity

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Abstract. This paper presents new classes of multifunctions between bitopological spaces, namely upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations and some properties concerning upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions are investigated.

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1. Introduction

Weaker and stronger forms of open sets in topological spaces such as semi-open sets, preopen sets, α-open sets, β-open sets, δ-open sets and θ-open sets play an important role in the researches of generalizations of continuity. By using these sets many authors introduced and investigated various types of continuity. Viriyapong and Boonpok [1] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [2]. Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [4] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions and slightly $(\tau_1, \tau_2)s$-continuous functions were presented in [5], [6], [7], [8], [9],

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[10], [11], [12], [13], [14], [15], [16], [17], [18] and [19], respectively. Kong-ied at al. [20] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [21] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions. Thongmoon et al. [22] introduced and studied the notion of rarely (τ_1, τ_2)-continuous functions. Srisarakham et al. [23] introduced and investigated the concept of faintly (τ_1, τ_2)-continuous functions. On the other hand, the present authors introduced and studied the notions of $\delta(\tau_1, \tau_2)$-continuous functions [24], quasi $\theta(\tau_1, \tau_2)$-continuous functions [25], almost weakly (τ_1, τ_2)-continuous functions [26] and almost nearly (τ_1, τ_2)-continuous functions [27]. In 1966, Dontchev [28] introduced the notion of contra-continuity in topological spaces. Dontchev and Noiri [29] introduced and studied the concept of RC-continuity between topological spaces which is weaker than contra-continuity. Jafari and Noiri [30] introduced a new class of function called contra-precontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. Ekici [31] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [32], contra-precontinuous functions [30], contra-continuous functions [28], almost s-continuous functions [33] and perfectly continuous functions [34].

In 2008, Ekici et al. [35] extended the notion of contra-continuous functions to the setting of multifunctions. Noiri and Popa [36] introduced the notion of weakly precontinuous multifunctions. Moreover, several characterizations and some properties concerning $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, almost quasi (τ_1, τ_2)-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions, c-quasi (τ_1, τ_2)-continuous multifunctions, s-$(\tau_1, \tau_2)p$-continuous multifunctions, slightly $\alpha(\tau_1, \tau_2)$-continuous multifunctions and slightly $(\tau_1, \tau_2)p$-continuous multifunctions were established in [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64] and [65], respectively. On the other hand, the present authors introduced and investigated the notions of rarely s-$(\tau_1, \tau_2)p$-continuous multifunctions [66], almost nearly (τ_1, τ_2)-continuous multifunctions [67], s-(τ_1, τ_2)-continuous multifunctions [68], quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions [69], almost nearly quasi (τ_1, τ_2)-continuous multifunctions [70], weakly s-(τ_1, τ_2)-continuous multifunctions [71], nearly (τ_1, τ_2)-continuous multifunctions [72] and almost quasi (τ_1, τ_2)-continuous multifunctions [73]. Ekici et al. [74] introduced and studied two new concepts namely contra-precontinuous multifunctions and almost contra-precontinuous multifunctions which are containing the class of contra-continuous multifunctions [35] and contained in the class of weakly precontinuous multifunctions. Ekici et al. [75] introduced and studied a new generalization of contra-continuous multifunctions called almost contra-continuous multifunctions. Recently, the present authors [76] introduced and investigated the notions of upper almost contra-(Λ, sp)-continuous multifunctions and lower almost contra-(Λ, sp)-continuous multifunctions. In this paper, we introduce the concepts of upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions. We also investigate several characterizations of upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [77] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [77] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [77] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [77] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.

(2) If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.

(3) $\tau_1\tau_2\text{-}Cl(A)$ is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2\text{-}Cl(A)$.

(5) $\tau_1\tau_2\text{-}Cl(X - A) = X - \tau_1\tau_2\text{-}Int(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is called $\alpha(\tau_1, \tau_2)$-*open* [78] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is called $\alpha(\tau_1, \tau_2)$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$-*open* [79] (resp. $(\tau_1, \tau_2)s$-*open* [37], $(\tau_1, \tau_2)p$-*open* [37], $(\tau_1, \tau_2)\beta$-*open* [37]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open, $\alpha(\tau_1, \tau_2)$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*, $\alpha(\tau_1, \tau_2)$-*closed*). Let A be a subset of a bitopological space (X, τ_1, τ_2). The set $\cap\{G \mid A \subseteq G \text{ and } G \text{ is } \tau_1\tau_2\text{-open}\}$ is called the $\tau_1\tau_2$-*kernel* [77] of A and is denoted by $\tau_1\tau_2$-$ker(A)$.

Lemma 2. [77] *For subsets A, B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $A \subseteq \tau_1\tau_2\text{-}ker(A)$.

(2) If $A \subseteq B$, *then* $\tau_1\tau_2\text{-}ker(A) \subseteq \tau_1\tau_2\text{-}ker(B)$.

(3) If A is $\tau_1\tau_2$-open, then $\tau_1\tau_2\text{-}ker(A) = A$.

(4) $x \in \tau_1\tau_2\text{-}ker(A)$ *if and only if* $A \cap H \neq \emptyset$ *for every $\tau_1\tau_2$-closed set H containing x.*

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and lower contra-(τ_1, τ_2)-continuous multifunctions

In this section, we introduce the concepts of upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions. Furthermore, several characterizations of upper contra-(τ_1, τ_2)-continuous multifunctions and lower contra-(τ_1, τ_2)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-(τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-closed set K of Y such that $x \in F^+(K)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-(τ_1, τ_2)-continuous if F is upper contra-(τ_1, τ_2)-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper contra-(τ_1, τ_2)-continuous;

(2) $F^+(K)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(3) $F^-(V)$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-closed set K of Y containing $F(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that if $y \in U$, then $F(y) \subseteq K$.

Proof. (1) $\Leftrightarrow$ (2): Let K be any $\sigma_1\sigma_2$-closed set of Y and $x \in F^+(K)$. Since F is upper contra-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(K)$. Thus, $F^+(K)$ is $\tau_1\tau_2$-open in X. The converse of the proof is similar.

(2) $\Leftrightarrow$ (3): This follows from the fact that $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(1) $\Leftrightarrow$ (4): Obvious.

Definition 2. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra-(τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-closed set K of Y such that $x \in F^-(K)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^-(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra-(τ_1, τ_2)-continuous if F is lower contra-(τ_1, τ_2)-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower contra-(τ_1, τ_2)-continuous;

(2) $F^-(K)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(3) $F^+(V)$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-closed set K of Y such that $F(x) \cap K \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that if $y \in U$, then $F(y) \cap K \neq \emptyset$.

Proof. The proof is similar to that of Theorem 1.

Theorem 3. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1\tau_2\text{-}Cl(F^-(B)) \subseteq F^-(\sigma_1\sigma_2\text{-}ker(B))$ for every subset B of Y, then F is upper contra-(τ_1, τ_2)-continuous.*

Proof. Suppose that $\tau_1\tau_2\text{-Cl}(F^-(B)) \subseteq F^-(\sigma_1\sigma_2\text{-}ker(B))$ for every subset B of Y. Let V be any $\sigma_1\sigma_2$-open set of Y. By Lemma 2, we have

$$\tau_1\tau_2\text{-Cl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}ker(V)) = F^-(V)$$

and hence $F^-(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 1, F is upper contra-(τ_1, τ_2)-continuous.

Theorem 4. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $F(\tau_1\tau_2\text{-}Cl(A)) \subseteq \sigma_1\sigma_2\text{-}ker(F(A))$ for every subset A of X, then F is lower contra-(τ_1, τ_2)-continuous.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $F(\tau_1\tau_2\text{-Cl}(F^+(V))) \subseteq \sigma_1\sigma_2\text{-}ker(V)$ and hence $\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(V))$. By Lemma 2, we have

$$\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(V)) = F^+(V)$$

and hence $F^+(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 2, F is lower contra-(τ_1, τ_2)-continuous.

Theorem 5. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $\tau_1\tau_2\text{-}Cl(F^+(B)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(B))$ for every subset B of Y, then F is lower contra-(τ_1, τ_2)-continuous.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(V))$ and by Lemma 2, $\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(V)) = F^+(V)$. This implies that $F^+(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 2, F is lower contra-(τ_1, τ_2)-continuous.

Definition 3. [6] *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be upper weakly* (τ_1, τ_2)*-continuous if for each* $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *containing* $F(x)$*, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $F(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$*.*

Theorem 6. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is an upper contra-*(τ_1, τ_2)*-continuous multifunction, then* F *is upper weakly* (τ_1, τ_2)*-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is a $\sigma_1\sigma_2$-closed set Y containing $F(x)$. Since F is upper contra-(τ_1, τ_2)-continuous, by Theorem 1 there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$; hence $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. This shows that F is upper weakly (τ_1, τ_2)-continuous.

The converse of Theorem 6 is not true in general as shown in the following example.

Example 1. *Let* $X = \{a, b, c, d\}$ *with topologies* $\tau_1 = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$*. Let* $Y = \{1, 2, 3, 4\}$ *with topologies*

$$\sigma_1 = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, Y\}$$

and $\sigma_2 = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, Y\}$*. A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is defined as follows:* $F(a) = \{1, 2\}$*,* $F(b) = \{2\}$*,* $F(c) = \{1, 2\}$ *and* $F(d) = \{4\}$*. Then,* F *is upper weakly* (τ_1, τ_2)*-continuous but* F *is not upper contra-*(τ_1, τ_2)*-continuous.*

Definition 4. [6] *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be lower weakly* (τ_1, τ_2)*-continuous if for each* $x \in X$ *and each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$*, there exists a* $\tau_1\tau_2$*-open set* U *of* X *containing* x *such that* $\sigma_1\sigma_2\text{-}Cl(V) \cap F(z) \neq \emptyset$ *for each* $z \in U$*.*

Theorem 7. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a lower contra-*(τ_1, τ_2)*-continuous multifunction, then* F *is lower weakly* (τ_1, τ_2)*-continuous.*

Proof. The proof is similar to that of Theorem 6.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$*-connected* [77] if X cannot be written as the union of two nonempty disjoint $\tau_1\tau_2$-open sets.

Lemma 3. [6] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper weakly* (τ_1, τ_2)*-continuous;*

(2) $F^+(V) \subseteq \tau_1\tau_2\text{-}Int(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(3) $\tau_1\tau_2\text{-}Cl(F^-(\sigma_1\sigma_2\text{-}Int(K))) \subseteq F^-(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $\tau_1\tau_2\text{-}Cl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(5) $F^+(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(F^+(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(B))))$ *for every subset* B *of* Y*;*

(6) $\tau_1\tau_2$-$Cl(F^-(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V)))) \subseteq F^-(\sigma_1\sigma_2$-$Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(7) $\tau_1\tau_2$-$Cl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2$-$Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(8) $\tau_1\tau_2$-$Cl(F^-(\sigma_1\sigma_2$-$Int(K))) \subseteq F^-(K)$ *for every* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*.*

Lemma 4. [6] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower weakly* (τ_1, τ_2)*-continuous;*

(2) $F^-(V) \subseteq \tau_1\tau_2$-$Int(F^-(\sigma_1\sigma_2$-$Cl(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(3) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int(K))) \subseteq F^+(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(B)))) \subseteq F^+(\sigma_1\sigma_2$-$Cl(B))$ *for every subset* B *of* Y*;*

(5) $F^-(\sigma_1\sigma_2$-$Int(B)) \subseteq \tau_1\tau_2$-$Int(F^-(\sigma_1\sigma_2$-$Cl(\sigma_1\sigma_2$-$Int(B))))$ *for every subset* B *of* Y*;*

(6) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V)))) \subseteq F^+(\sigma_1\sigma_2$-$Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(7) $\tau_1\tau_2$-$Cl(F^+(V)) \subseteq F^+(\sigma_1\sigma_2$-$Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(8) $\tau_1\tau_2$-$Cl(F^+(\sigma_1\sigma_2$-$Int(K))) \subseteq F^+(K)$ *for every* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*.*

Theorem 8. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is an upper or lower contra-*(τ_1, τ_2)*-continuous surjective multifunction such that* $F(x)$ *is* $\sigma_1\sigma_2$*-connected for each* $x \in X$ *and* (X, τ_1, τ_2) *is* $\tau_1\tau_2$*-connected, then* (Y, σ_1, σ_2) *is* $\sigma_1\sigma_2$*-connected.*

Proof. Suppose that (Y, σ_1, σ_2) is not $\sigma_1\sigma_2$-connected. There exist nonempty $\sigma_1\sigma_2$-open sets U and V of Y such that $U \cap V = \emptyset$ and $U \cup V = Y$. Since $F(x)$ is $\sigma_1\sigma_2$-connected for each $x \in X$, either $F(x) \subseteq U$ or $F(x) \subseteq V$. If $x \in F^+(U \cup V)$, then $F(x) \subseteq U \cup V$ and hence $x \in F^+(U) \cup F^+(V)$. Moreover, since F is surjective, there exist x and y in X such that $F(x) \subseteq U$ and $F(y) \subseteq V$; hence $x \in F^+(U)$ and $y \in F^+(V)$. Therefore, we obtain the following:

(1) $F^+(U) \cup F^+(V) = F^+(U \cup V) = X$;

(2) $F^+(U) \cap F^+(V) = F^+(U \cap V) = \emptyset$;

(3) $F^+(U) \neq \emptyset$ and $F^+(V) \neq \emptyset$.

Next, we show that $F^+(U)$ and $F^+(V)$ are $\tau_1\tau_2$-open in X. (i) Let F be upper contra-(τ_1, τ_2)-continuous, by Theorem 6 we have F is upper weakly (τ_1, τ_2)-continuous. By Lemma 3, $F^+(V) \subseteq \tau_1\tau_2$-Int$(F^+(\sigma_1\sigma_2$-Cl$(V))) = \tau_1\tau_2$-Int$(F^+(V))$ since V is $\sigma_1\sigma_2$-clopen. Thus, $F^+(V) = \tau_1\tau_2$-Int$(F^+(V))$ and hence $F^+(V)$ is $\tau_1\tau_2$-open in X. Similarly, we obtain $F^+(U)$ is $\tau_1\tau_2$-open in X. Consequently, this shows that (X, τ_1, τ_2) is not $\tau_1\tau_2$-connected.

(ii) Let F be lower contra-(τ_1, τ_2)-continuous, by Theorem 7 we have F is lower weakly (τ_1, τ_2)-continuous. By Lemma 4, $\tau_1\tau_2\text{-Cl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V)) = F^+(V)$ since V is $\sigma_1\sigma_2$-clopen. Therefore, $F^+(V) = \tau_1\tau_2\text{-Cl}(F^+(V))$ and so $F^+(V)$ is $\tau_1\tau_2$-closed in X. Thus, we have $F^+(U)$ is $\tau_1\tau_2$-open in X. Similarly, we obtain $F^+(V)$ is $\tau_1\tau_2$-open in X. Consequently, this shows that (X, τ_1, τ_2) is not $\tau_1\tau_2$-connected. This completes the proof.

Recall that a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*compact* [77] if every cover of X by $\tau_1\tau_2$-open sets of X has a finite subcover.

Definition 5. [80] *A bitopological space (X, τ_1, τ_2) is said to be strongly S-$\tau_1\tau_2$-closed if every cover of X by $\tau_1\tau_2$-closed sets of X has a finite subcover.*

Theorem 9. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a surjective multifunction and $F(x)$ is strongly S-$\sigma_1\sigma_2$-closed for each $x \in X$. If F is upper contra-(τ_1, τ_2)-continuous and (X, τ_1, τ_2) is $\tau_1\tau_2$-compact, then (Y, σ_1, σ_2) is strongly S-$\sigma_1\sigma_2$-closed.*

Proof. Suppose that (X, τ_1, τ_2) is $\tau_1\tau_2$-compact. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be any cover of Y by $\sigma_1\sigma_2$-closed sets of Y. Since $F(x)$ is strongly S-$\sigma_1\sigma_2$-closed for each $x \in X$, there exists a finite subset $\nabla(x)$ of ∇ such that $F(x) \subseteq \cup\{V_\gamma \mid \gamma \in \nabla(x)\}$. Put $V(x) = \cup\{V_\gamma \mid \gamma \in \nabla(x)\}$. Then, $V(x)$ is $\sigma_1\sigma_2$-closed in Y and $F(x) \subseteq V(x)$. Since F is upper contra-(τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set $U(x)$ of X containing x such that $F(U(x)) \subseteq V(x)$. The family $\{U(x) \mid x \in X\}$ is a $\tau_1\tau_2$-open cover of X. Since (X, τ_1, τ_2) is $\tau_1\tau_2$-compact, there exists a finite number of pints, say, $x_1, x_2, x_3, ..., x_n$ in X such that $X = \cup\{U(x_k) \mid x_k \in X; 1 \leq k \leq n\}$. Thus,

$$Y = F(X) = \cup\{F(U(x_k)) \mid x_k \in X; 1 \leq k \leq n\} \subseteq \cup\{V_{\gamma(x_k)} \mid x_k \in X; 1 \leq k \leq n\}.$$

This shows that (Y, σ_1, σ_2) is strongly S-$\sigma_1\sigma_2$-closed.

Definition 6. [56] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be:*

(1) upper (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq V$;

(2) lower (τ_1, τ_2)-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$.

Lemma 5. [56] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper (τ_1, τ_2)-continuous;

(2) $F^+(V)$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y;

(3) $F^-(K)$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed set K of Y;

(4) $\tau_1\tau_2\text{-}Cl(F^-(B)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(B))$ for every subset B of Y;

(5) $F^+(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(F^+(B))$ *for every subset* B *of* Y.

Theorem 10. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper* (τ_1, τ_2)*-continuous and*

$$G : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is upper contra-(σ_1, σ_2)*-continuous, then* $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ *is upper contra-*(τ_1, τ_2)*-continuous.*

Proof. Let K be any $\rho_1\rho_2$-closed set of Z. Since G is upper contra-(σ_1, σ_2)-continuous, by Theorem 1 we have $F^+(K)$ is $\sigma_1\sigma_2$-open in Y. Since F is upper (τ_1, τ_2)-continuous, by Lemma 5 we have $(G \circ F)^+(K) = F^+(G^+(K))$ is $\tau_1\tau_2$-open in X. Thus by Theorem 1, $G \circ F$ is upper contra-(τ_1, τ_2)-continuous.

Lemma 6. [56] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower* (τ_1, τ_2)*-continuous;*

(2) $F^-(V)$ *is* $\tau_1\tau_2$*-open in* X *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(3) $F^+(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $\tau_1\tau_2\text{-}Cl(F^+(B)) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(5) $F(\tau_1\tau_2\text{-}Cl(A)) \subseteq \sigma_1\sigma_2\text{-}Cl(F(A))$ *for every subset* A *of* X*;*

(6) $F^-(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(F^-(B))$ *for every subset* B *of* Y.

Theorem 11. *If* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is lower* (τ_1, τ_2)*-continuous and*

$$G : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$$

is lower contra-(σ_1, σ_2)*-continuous, then* $G \circ F : (X, \tau_1, \tau_2) \to (Z, \rho_1, \rho_2)$ *is lower contra-*(τ_1, τ_2)*-continuous.*

Proof. The proof is similar to that of Theorem 10.

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, a multifunction

$$\mathrm{Cl}F_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$$

is defined in [77] as follows: $\mathrm{Cl}F_{\circledast}(x) = \sigma_1\sigma_2\text{-Cl}(F(x))$ for each $x \in X$.

Definition 7. [77] *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is said to be:*

(1) $\tau_1\tau_2$*-paracompact if every cover of* A *by* $\tau_1\tau_2$*-open sets of* X *is refined by a cover of* A *which consists of* $\tau_1\tau_2$*-open sets of* X *and is* $\tau_1\tau_2$*-locally finite in* X*;*

(2) $\tau_1\tau_2$-regular if for each $x \in A$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V of X such that $x \in V \subseteq \tau_1\tau_2\text{-}Cl(V) \subseteq U$.

Lemma 7. [77] *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-regular and $\sigma_1\sigma_2$-paracompact for each $x \in X$, then $ClF^+_{\circledast}(V) = F^+(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Lemma 8. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-regular and $\sigma_1\sigma_2$-paracompact for each $x \in X$, then $ClF^-_{\circledast}(K) = F^-(K)$ for each $\sigma_1\sigma_2$-closed set K of Y.*

Proof. It follows from Lemma 7.

Lemma 9. [77] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, $ClF^-_{\circledast}(V) = F^-(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Lemma 10. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, $ClF^+_{\circledast}(K) = F^+(K)$ for each $\sigma_1\sigma_2$-closed set K of Y.*

Proof. It follows from Lemma 9.

Theorem 12. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper contra-(τ_1, τ_2)-continuous if and only if $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper contra-(τ_1, τ_2)-continuous.*

Proof. Suppose that F is upper contra-(τ_1, τ_2)-continuous. Let K be any $\sigma_1\sigma_2$-closed set of Y. It follows from Lemma 9, Lemma 10 and Theorem 1, $\text{Cl}F^+_{\circledast}(K) = F^+(K)$ is $\tau_1\tau_2$-open in X. Thus, $\text{Cl}F_{\circledast}$ is upper contra-(τ_1, τ_2)-continuous.

Conversely, suppose that $\text{Cl}F_{\circledast}$ is upper contra-(τ_1, τ_2)-continuous. Let K be any $\sigma_1\sigma_2$-closed set of Y. By Lemma 9, Lemma 10 and Theorem 1, $F^+(K) = \text{Cl}F^+_{\circledast}(K)$ is $\tau_1\tau_2$-open in X. Thus, F is upper contra-(τ_1, τ_2)-continuous.

Theorem 13. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-paracompact and $\sigma_1\sigma_2$-regular for each $x \in X$. Then, F is lower contra-(τ_1, τ_2)-continuous if and only if $ClF_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower contra-(τ_1, τ_2)-continuous.*

Proof. Suppose that F is lower contra-(τ_1, τ_2)-continuous. Let K be any $\sigma_1\sigma_2$-closed set of Y. It follows from Lemma 7, Lemma 8 and Theorem 2 that $\text{Cl}F^-_{\circledast}(K) = F^-(K)$ is $\tau_1\tau_2$-open in X. This shows that $\text{Cl}F_{\circledast}$ is lower contra-(τ_1, τ_2)-continuous.

Conversely, suppose that $\text{Cl}F_{\circledast}$ is lower contra-(τ_1, τ_2)-continuous. Let K be any $\sigma_1\sigma_2$-closed set of Y. By Lemma 7, Lemma 8 and Theorem 2, $F^-(K) = \text{Cl}F^-_{\circledast}(K)$ is $\tau_1\tau_2$-open in X. This shows that F is lower contra-(τ_1, τ_2)-continuous.

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