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On Contra-$(\tau_1, \tau_2)p$-continuity and Almost Contra-$(\tau_1, \tau_2)p$-continuity for Multifunctions

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Abstract. This paper presents four new classes of multifunctions called upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions, lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions, upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions. Furthermore, several characterizations and some properties concerning upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions, lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions, upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions are established.

2020 Mathematics Subject Classifications: 54C08, 54C60

Key Words and Phrases: upper contra-$(\tau_1, \tau_2)p$-continuous multifunction, lower contra-$(\tau_1, \tau_2)p$-continuous multifunction, upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunction, lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunction

1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. In topology, there has been recently significant interest in characterizing and investigating the characterizations of some weak forms of continuity for functions and multifunctions. Weaker and stronger forms of open sets play an important role in the generalization of different forms of continuity. Using different forms of open sets, several authors have introduced and studied various types of continuity. The concepts of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets and $\beta(\Lambda, sp)$-open sets were studied in [1]. Viriyapong and Boonpok [2] investigated several characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets.

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Dungthaisong et al. [3] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [4] introduced and investigated the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions and slightly $(\tau_1, \tau_2)s$-continuous functions were presented in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] and [19], respectively. Kong-ied at al. [20] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [21] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions. Thongmoon et al. [22] introduced and studied the notion of rarely (τ_1, τ_2)-continuous functions. Srisarakham et al. [23] introduced and investigated the concept of faintly (τ_1, τ_2)-continuous functions. On the other hand, the present authors introduced and studied the notions of $\delta(\tau_1, \tau_2)$-continuous functions [24], quasi $\theta(\tau_1, \tau_2)$-continuous functions [25], almost weakly (τ_1, τ_2)-continuous functions [26] and almost nearly (τ_1, τ_2)-continuous functions [27]. In 1966, Dontchev [28] introduced the concepts of contra-continuity and strong S-closedness in topological spaces. Moreover, Dontchev [28] obtained very interesting and important results concerning contra-continuity, compactness, S-closedness and strong S-closedness. Dontchev et al. [29] defined a new class of functions called regular set-connected functions. Dontchev and Noiri [30] introduced and studied the concept of RC-continuity between topological spaces which is weaker than contra-continuity. Jafari and Noiri [31] introduced a new class of functions called contra-precontinuous functions which is weaker than contra-continuous functions and studied several basic properties of contra-precontinuous functions. In 2004, Ekici [32] introduced and studied a new class of functions called almost contra-precontinuous functions which generalize classes of regular set-connected functions [29], contra-precontinuous functions [31], contra-continuous functions [28], almost s-continuous functions [33] and perfectly continuous functions [34].

In 2008, Ekici et al. [35] extended the notion of contra-continuous functions to the setting of multifunctions. Noiri and Popa [36] introduced the notion of weakly precontinuous multifunctions. Ekici et al. [37] introduced and studied two new concepts namely contra-precontinuous multifunctions and almost contra-precontinuous multifunctions which are containing the class of contra-continuous multifunctions [35] and contained in the class of weakly precontinuous multifunctions. Laprom et al. [38] introduced and investigated the notion of almost $\beta(\tau_1, \tau_2)$-continuous multifunctions. Moreover, some characterizations of $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, almost quasi (τ_1, τ_2)-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions, c-quasi (τ_1, τ_2)-continuous multifunctions, s-$(\tau_1, \tau_2)p$-continuous multifunctions, slightly $\alpha(\tau_1, \tau_2)$-continuous multifunctions and slightly $(\tau_1, \tau_2)p$-continuous multifunctions were established in [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63] and [64], respectively. On the other hand, the present authors introduced and investigated the notions of rarely s-$(\tau_1, \tau_2)p$-continuous multifunctions [65], almost nearly (τ_1, τ_2)-continuous multifunctions [66], s-(τ_1, τ_2)-continuous multifunctions [67], quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions [68], almost nearly quasi (τ_1, τ_2)-continuous multifunctions [69], weakly s-(τ_1, τ_2)-continuous multifunctions [70], nearly (τ_1, τ_2)-continuous multifunctions [71] and almost quasi (τ_1, τ_2)-continuous multifunctions [72]. In 2023, the present authors [73] introduced and investigated the notion of almost contra-(Λ, sp)-continuous multifunctions. Pue-on et al. [74] introduced and studied the notions of upper (τ_1, τ_2)-continuous multifunctions and lower (τ_1, τ_2)-continuous multifunctions. Klanarong et al. [75] introduced and investigated the concepts of upper almost (τ_1, τ_2)-continuous multifunctions and lower almost (τ_1, τ_2)-continuous multifunctions. Thongmoon et al. [76] introduced and studied the notions of upper weakly (τ_1, τ_2)-continuous multifunctions and lower weakly (τ_1, τ_2)-continuous multifunctions. In this paper, we introduce the concepts of upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions, lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions, upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions. We also investigate several characterizations of upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions, lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions, upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called *$\tau_1\tau_2$-closed* [77] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called *$\tau_1\tau_2$-open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the *$\tau_1\tau_2$-closure* [77] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the *$\tau_1\tau_2$-interior* [77] of A and is denoted by $\tau_1\tau_2$-Int(A). Let A be a subset of a bitopological space (X, τ_1, τ_2). The set $\cap\{G \mid A \subseteq G$ and G is $\tau_1\tau_2$-open$\}$ is called the *$\tau_1\tau_2$-kernel* [77] of A and is denoted by $\tau_1\tau_2$-$ker(A)$.

Lemma 1. [77] *For subsets A, B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*ker*(A).

(2) If $A \subseteq B$, *then* $\tau_1\tau_2$-*ker*$(A) \subseteq \tau_1\tau_2$-*ker*$(B)$.

(3) If A *is* $\tau_1\tau_2$-*open, then* $\tau_1\tau_2$-*ker*$(A) = A$.

(4) $x \in \tau_1\tau_2$-*ker*(A) *if and only if* $A \cap H \neq \emptyset$ *for every* $\tau_1\tau_2$-*closed set* H *containing* x.

Lemma 2. [77] *Let* A *and* B *be subsets of a bitopological space* (X, τ_1, τ_2). *For the* $\tau_1\tau_2$-*closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).

(2) If $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.

(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*

(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).

(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [78] (resp. $(\tau_1, \tau_2)s$-*open* [39], $(\tau_1, \tau_2)p$-*open* [39], $(\tau_1, \tau_2)\beta$-*open* [39]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))))$. The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [79] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is called $\alpha(\tau_1, \tau_2)$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-δ-*open* [17] if A is the union of $(\tau_1, \tau_2)r$-open sets of X. The complement of a $\tau_1\tau_2$-δ-open set is called $\tau_1\tau_2$-δ-*closed* [17]. Let A be a subset of a bitopological space (X, τ_1, τ_2). The union of all $\tau_1\tau_2$-δ-open sets of X contained in A is called the $\tau_1\tau_2$-δ-*interior* [17] of A and is denoted by $\tau_1\tau_2$-δ-Int(A). The intersection of all $\tau_1\tau_2$-δ-closed sets of X containing A is called the $\tau_1\tau_2$-δ-*closure* [17] of A and is denoted by $\tau_1\tau_2$-δ-Cl(A). The intersection of all $(\tau_1, \tau_2)p$-closed (resp. $(\tau_1, \tau_2)s$-closed, $\alpha(\tau_1, \tau_2)$-closed) sets of X containing A is called the $(\tau_1, \tau_2)p$-*closure* [62] (resp. $(\tau_1, \tau_2)s$-*closure* [39], $\alpha(\tau_1, \tau_2)$-*closure* [63]) of A and is denoted by (τ_1, τ_2)-pCl(A) (resp. (τ_1, τ_2)-sCl(A), $\alpha(\tau_1, \tau_2)$-Cl(A)). The union of all $(\tau_1, \tau_2)p$-open (resp. $(\tau_1, \tau_2)s$-open, $\alpha(\tau_1, \tau_2)$-open) sets of X contained in A is called the $(\tau_1, \tau_2)p$-*interior* [62] (resp. $(\tau_1, \tau_2)s$-*interior* [39], $\alpha(\tau_1, \tau_2)$-*interior* [63]) of A and is denoted by (τ_1, τ_2)-pInt(A) (resp. (τ_1, τ_2)-sInt(A), $\alpha(\tau_1, \tau_2)$-Int(A)).

Lemma 3. *For a subset* A *of a bitopological space* (X, τ_1, τ_2), *the following properties hold:*

(1) (τ_1, τ_2)-*pCl*$(A) = \tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Int*$(A)) \cup A$ [62];

(2) (τ_1, τ_2)-*pInt*$(A) = \tau_1\tau_2$-*Int*$(\tau_1\tau_2$-*Cl*$(A)) \cap A$ [26];

(3) (τ_1, τ_2)*-sCl*$(A) = \tau_1\tau_2$*-Int*$(\tau_1\tau_2$*-Cl*$(A)) \cup A$ [39]*;*

(4) (τ_1, τ_2)*-sInt*$(A) = \tau_1\tau_2$*-Cl*$(\tau_1\tau_2$*-Int*$(A)) \cap A$ [59].

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $s(\tau_1, \tau_2)\theta$*-cluster point* [80] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $(\tau_1, \tau_2)s$-open set U containing x. The set of all $s(\tau_1, \tau_2)\theta$-cluster points of A is called the $s(\tau_1, \tau_2)\theta$*-closure* [80] of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is called $s(\tau_1, \tau_2)\theta$*-closed* [80] if $s(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $s(\tau_1, \tau_2)\theta$-closed set is said to be $s(\tau_1, \tau_2)\theta$*-open* [80]. The union of all $s(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $s(\tau_1, \tau_2)\theta$*-interior* [80] of A and is denoted by $s(\tau_1, \tau_2)\theta\text{-Int}(A)$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x\in A}F(x)$.

3. Upper and lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions

In this section, we introduce the concepts of upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions. Furthermore, several characterizations of upper contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower contra-$(\tau_1, \tau_2)p$-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-$(\tau_1, \tau_2)p$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-closed set K of Y with $x \in F^+(K)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper contra-$(\tau_1, \tau_2)p$-continuous if F is upper contra-$(\tau_1, \tau_2)p$-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper contra-$(\tau_1, \tau_2)p$-continuous;

(2) $F^+(K)$ is $(\tau_1, \tau_2)p$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(3) $F^-(V)$ is $(\tau_1, \tau_2)p$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-closed set K of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that if $y \in U$, then $F(y) \subseteq K$.

Proof. (1) $\Leftrightarrow$ (2): Let K be any $\sigma_1\sigma_2$-closed set of Y and $x \in F^+(K)$. Since F is upper contra-(τ_1, τ_2)-continuous, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \subseteq F^+(K)$. Thus, $F^+(K)$ is $(\tau_1, \tau_2)p$-open in X. The converse of the proof is similar.

(2) ⇔ (3): This follows from the fact that $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(1) ⇔ (4): Obvious.

Definition 2. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra-(τ_1, τ_2)p-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-closed set K of Y with $x \in F^-(K)$, there exists a (τ_1, τ_2)p-open set U of X containing x such that $U \subseteq F^-(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called lower contra-(τ_1, τ_2)p-continuous if F is lower contra-(τ_1, τ_2)p-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower contra-(τ_1, τ_2)p-continuous;

(2) $F^-(K)$ is (τ_1, τ_2)p-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(3) $F^+(V)$ is (τ_1, τ_2)p-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(4) for each $x \in X$ and each $\sigma_1\sigma_2$-closed set K of Y such that $F(x) \cap K \neq \emptyset$, there exists a (τ_1, τ_2)p-open set U of X containing x such that if $y \in U$, then $F(y) \cap K \neq \emptyset$.

Proof. The proof is similar to that of Theorem 1.

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)*s-regular* [6] if for each $(\tau_1, \tau_2)s$-closed set F and each $x \notin F$, there exist disjoint $(\tau_1, \tau_2)s$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Lemma 4. [81] *Let (X, τ_1, τ_2) be a $(\tau_1, \tau_2)s$-regular space. Then, the following properties hold:*

(1) $\tau_1\tau_2$-Cl$(A) = \tau_1\tau_2$-δ-Cl(A) for every subset A of X;

(2) every $\tau_1\tau_2$-open set is $\tau_1\tau_2$-δ-open.

Theorem 3. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, where (Y, σ_1, σ_2) is $(\sigma_1, \sigma_2)s$-regular, the following properties are equivalent:*

(1) F is upper contra-(τ_1, τ_2)p-continuous;

(2) $F^+(\sigma_1\sigma_2$-δ-Cl$(B))$ is (τ_1, τ_2)p-open in X for every subset B of Y;

(3) $F^+(K)$ is (τ_1, τ_2)p-open in X for every $\sigma_1\sigma_2$-δ-closed set K of Y;

(4) $F^-(V)$ is (τ_1, τ_2)p-closed in X for every $\sigma_1\sigma_2$-δ-open set V of Y.

Proof. (1) ⇒ (2): Let B be any subset of Y. Then, $\sigma_1\sigma_2$-δ-Cl(B) is a $\sigma_1\sigma_2$-closed set of Y and by Theorem 1, $F^+(\sigma_1\sigma_2\text{-}\delta\text{-Cl}(B))$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (3): Let K be any $\sigma_1\sigma_2$-δ-closed set of Y. Then, $\sigma_1\sigma_2\text{-}\delta\text{-Cl}(K) = K$. By (2), $F^+(K)$ is $(\tau_1, \tau_2)p$-open in X.

(3) ⇒ (4): Let V be any $\sigma_1\sigma_2$-δ-open set of Y. Then, $Y - V$ is $\sigma_1\sigma_2$-δ-closed in Y. By (3), $F^+(Y - V) = X - F^-(V)$ is $(\tau_1, \tau_2)p$-open in X. Thus, $F^-(V)$ is $(\tau_1, \tau_2)p$-closed in X.

(4) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Since (Y, σ_1, σ_2) is $(\sigma_1, \sigma_2)s$-regular, by Lemma 4 we have V is $\sigma_1\sigma_2$-δ-open in Y. By (4), $F^-(V)$ is $(\tau_1, \tau_2)p$-closed in X and by Theorem 1, F is upper contra-$(\tau_1, \tau_2)p$-continuous.

Theorem 4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, where (Y, σ_1, σ_2) is $(\sigma_1, \sigma_2)s$-regular, the following properties are equivalent:*

(1) F is lower contra-$(\tau_1, \tau_2)p$-continuous;

(2) $F^-(\sigma_1\sigma_2\text{-}\delta\text{-}Cl(B))$ is $(\tau_1, \tau_2)p$-open in X for every subset B of Y;

(3) $F^-(K)$ is $(\tau_1, \tau_2)p$-open in X for every $\sigma_1\sigma_2$-δ-closed set K of Y;

(4) $F^+(V)$ is $(\tau_1, \tau_2)p$-closed in X for every $\sigma_1\sigma_2$-δ-open set V of Y.

Proof. The proof is similar to that of Theorem 3.

4. Upper and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions

In this section, we introduce the concepts of upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions. Moreover, some characterizations of upper almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions and lower almost contra-$(\tau_1, \tau_2)p$-continuous multifunctions are considered.

Definition 3. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost contra-$(\tau_1, \tau_2)p$-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)r$-closed set K of Y with $x \in F^+(K)$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \subseteq F^+(K)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost contra-$(\tau_1, \tau_2)p$-continuous if F is upper almost contra-$(\tau_1, \tau_2)p$-continuous at each point x of X.*

Remark 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following implication holds:*

upper contra-$(\tau_1, \tau_2)p$-continuity ⇒ upper almost contra-$(\tau_1, \tau_2)p$-continuity.

The converse of the implication is not true in general. We give an example for the implication as follows.

Example 1. *Let* $X = \{a, b, c, d\}$ *with topologies*

$$\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$$

and $\tau_2 = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, X\}$. *Let* $Y = \{1, 2, 3, 4\}$ *with topologies* $\sigma_1 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, Y\}$ *and*

$$\sigma_2 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, Y\}.$$

A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is defined as follows:* $F(a) = \{4\}$, $F(b) = \{3\}$, $F(c) = \{1\}$ *and* $F(d) = \{2\}$. *Then,* F *is upper almost contra-*$(\tau_1, \tau_2)p$*-continuous but* F *is not upper contra-*$(\tau_1, \tau_2)p$*-continuous.*

Theorem 5. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper almost contra-*$(\tau_1, \tau_2)p$*-continuous;*

(2) $F^+(K)$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*;*

(3) $F^-(V)$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $(\sigma_1, \sigma_2)r$*-open set* V *of* Y*;*

(4) $F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(5) $F^+(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(K)))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(6) for each $x \in X$ *and each* $(\sigma_1, \sigma_2)s$*-open set* V *of* Y *containing* $F(x)$*, there exists a* $(\tau_1, \tau_2)p$*-open set* U *of* X *containing* x *such that* $F(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$*;*

(7) $F^+(V) \subseteq (\tau_1, \tau_2)\text{-}pInt(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ *for every* $(\sigma_1, \sigma_2)s$*-open set* V *of* Y*.*

Proof. (1) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y and $x \in F^+(K)$. Since F is upper almost contra-$(\tau_1, \tau_2)p$-continuous, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \subseteq F^+(K)$. Thus, $F^+(K)$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (1): The proof is obvious.

(2) ⇔ (3) and (4) ⇔ (5): It follows from the fact that $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(3) ⇔ (4): Let V be any $\sigma_1\sigma_2$-open set of Y. Since $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-open in Y, by (3) we have $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X. The converse is obvious.

(5) ⇔ (2): It is similar to that of (3) ⇔ (4).

(6) ⇒ (7): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y and $x \in F^+(V)$. Then, $F(x) \subseteq V$. By (6), there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. This implies that $x \in U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$. Thus, $x \in (\tau_1, \tau_2)\text{-pInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$ and hence $F^+(V) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$.

(7) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Then, K is $(\sigma_1, \sigma_2)s$-open in Y. By (7), we have $F^+(K) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^+(\sigma_1\sigma_2\text{-Cl}(K)))$ and hence $F^+(K)$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (6): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)s$-open set of Y with $F(x) \subseteq V$. Since $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1, \sigma_2)r$-closed, by (2) we have $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\tau_1, \tau_2)p$-open in X. Then, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$. Thus, $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$.

Theorem 6. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $F^-(K)$ is $(\tau_1, \tau_2)p$-open in X for every $(\sigma_1, \sigma_2)r$-closed set K of Y;

(3) $F^+(V)$ is $(\tau_1, \tau_2)p$-closed in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;

(4) $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X for every $\sigma_1\sigma_2$-open set V of Y;

(5) $F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $(\tau_1, \tau_2)p$-open in X for every $\sigma_1\sigma_2$-closed set K of Y;

(6) for each $x \in X$ and each $(\sigma_1, \sigma_2)s$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)p$-open set U of X containing x such that $F(z) \cap \sigma_1\sigma_2\text{-Cl}(V) \neq \emptyset$ for each $z \in U$;

(7) $F^-(V) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^-(\sigma_1\sigma_2\text{-Cl}(V)))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y.

Proof. The proof is similar to that of Theorem 5.

Theorem 7. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $(\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(K)$ for every $(\sigma_1, \sigma_2)s$-closed set K of Y;

(3) $(\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\text{-sCl}(B)))) \subseteq F^-((\sigma_1, \sigma_2)\text{-sCl}(B))$ for every subset B of Y;

(4) $F^+((\sigma_1, \sigma_2)\text{-sInt}(B)) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B))))$ for every subset B of Y.

Proof. (1) ⇒ (2): Let K be any $(\sigma_1, \sigma_2)s$-closed set of Y. Then, $Y - K$ is $(\sigma_1, \sigma_2)s$-open in Y. By Theorem 5, $F^+(Y - K) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^+(Y - \sigma_1\sigma_2\text{-Int}(K)))$. Thus,

$$\begin{aligned} X - F^-(K) &\subseteq (\tau_1, \tau_2)\text{-pInt}(F^+(Y - \sigma_1\sigma_2\text{-Int}(K))) \\ &= (\tau_1, \tau_2)\text{-pInt}(X - F^-(\sigma_1\sigma_2\text{-Int}(K))) \\ &= X - (\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \end{aligned}$$

and hence $(\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \subseteq F^-(K)$.

(2) ⇒ (3): Let B be any subset of Y. Then, (σ_1, σ_2)-sCl$(B))$ is $(\sigma_1, \sigma_2)s$-closed in Y, by (2) we have (τ_1, τ_2)-pCl$(F^-(\sigma_1\sigma_2$-Int$((\sigma_1, \sigma_2)$-sCl$(B)))) \subseteq F^-((\sigma_1, \sigma_2)$-sCl$(B))$.

(3) ⇒ (4): Let B be any subset of Y. By (3), we have

$$\begin{aligned}
X - F^+((\sigma_1, \sigma_2)\text{-sInt}(B)) &= F^-((\sigma_1, \sigma_2)\text{-sCl}(Y - B)) \\
&\supseteq (\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\text{-sCl}(Y - B)))) \\
&= (\tau_1, \tau_2)\text{-pCl}(F^-(\sigma_1\sigma_2\text{-Int}(Y - (\sigma_1, \sigma_2)\text{-sInt}(B)))) \\
&= (\tau_1, \tau_2)\text{-pCl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B)))) \\
&= (\tau_1, \tau_2)\text{-pCl}(X - F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B)))) \\
&= X - (\tau_1, \tau_2)\text{-pInt}(F^+(\sigma_1\sigma_2\text{-Cl}((\sigma_1, \sigma_2)\text{-sInt}(B))))
\end{aligned}$$

and hence $F^+((\sigma_1, \sigma_2)$-sInt$(B)) \subseteq (\tau_1, \tau_2)$-pInt$(F^+(\sigma_1\sigma_2$-Cl$((\sigma_1, \sigma_2)$-sInt$(B))))$.

(4) ⇒ (1): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y. Then, $V = (\sigma_1, \sigma_2)$-sInt(V) and by (4), $F^+(V) \subseteq (\tau_1, \tau_2)$-pInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$. By Theorem 5, F is upper almost contra-$(\tau_1, \tau_2)p$-continuous.

Theorem 8. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) (τ_1, τ_2)-pCl$(F^+(\sigma_1\sigma_2$-Int$(K))) \subseteq F^+(K)$ for every $(\sigma_1, \sigma_2)s$-closed set K of Y;

(3) (τ_1, τ_2)-pCl$(F^+(\sigma_1\sigma_2$-Int$((\sigma_1, \sigma_2)$-sCl$(B)))) \subseteq F^+((\sigma_1, \sigma_2)$-sCl$(B))$ for every subset B of Y;

(4) $F^-((\sigma_1, \sigma_2)$-sInt$(B)) \subseteq (\tau_1, \tau_2)$-pInt$(F^-(\sigma_1\sigma_2$-Cl$((\sigma_1, \sigma_2)$-sInt$(B))))$ for every subset B of Y.

Proof. The proof is similar to that of Theorem 7.

Lemma 5. [80] *For a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $\alpha(\tau_1, \tau_2)$-Cl$(V) = \tau_1\tau_2$-Cl(V) for every $(\tau_1, \tau_2)\beta$-open set V of X;

(2) (τ_1, τ_2)-pCl$(V) = \tau_1\tau_2$-Cl(V) for every $(\tau_1, \tau_2)s$-open set V of X;

(3) (τ_1, τ_2)-sCl$(V) = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(V))$ for every $(\tau_1, \tau_2)p$-open set V of X.

Theorem 9. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) $F^-(V)$ is $(\tau_1, \tau_2)p$-open in X for every $s(\sigma_1, \sigma_2)\theta$-open set V of Y;

(3) $F^+(K)$ is $(\tau_1, \tau_2)p$-closed in X for every $s(\sigma_1, \sigma_2)\theta$-closed set K of Y;

(4) (τ_1, τ_2)*-pCl*$(F^+(\sigma_1\sigma_2$*-Int*$(\sigma_1\sigma_2$*-Cl*$(B)))) \subseteq F^+((\sigma_1, \sigma_2)$*-sCl*$(B))$ *for every subset* B *of* Y*;*

(5) (τ_1, τ_2)*-pCl*$(F^+(B)) \subseteq F^+(s(\sigma_1, \sigma_2)\theta$*-Cl*$(B))$ *for every subset* B *of* Y*;*

(6) $F((\tau_1, \tau_2)$*-pCl*$(A)) \subseteq s(\sigma_1, \sigma_2)\theta$*-Cl*$(F(A))$ *for every subset* A *of* X*.*

Proof. (1) ⇒ (2): Let V be any $s(\sigma_1, \sigma_2)\theta$-open set of Y. There exists a family of $(\sigma_1, \sigma_2)r$-closed sets $\{K_\gamma \mid \gamma \in \nabla\}$ such that $V = \cup\{K_\gamma \mid \gamma \in \nabla\}$. It follows from Theorem 5 that $F^-(V) = \cup\{F^-(K_\gamma) \mid \gamma \in \nabla\}$ is $(\tau_1, \tau_2)p$-open in X.

(2) ⇒ (3): The proof is obvious.

(3) ⇒ (4): Let B be any subset of Y. Then, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))$ is $(\sigma_1, \sigma_2)r$-open and hence $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))$ is $s(\sigma_1, \sigma_2)\theta$-closed in Y. By (3), $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))$ is $(\tau_1, \tau_2)p$-closed in X. Thus,

$$\begin{aligned}(\tau_1, \tau_2)\text{-pCl}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) &= F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))) \\ &\subseteq F^+((\sigma_1, \sigma_2)\text{-sCl}(B)).\end{aligned}$$

(4) ⇒ (5): Let B be any subset of Y. For any $(\sigma_1, \sigma_2)r$-open set V of Y with $B \subseteq V$, by (4) and Lemma 5 we have

$$\begin{aligned}(\tau_1, \tau_2)\text{-pCl}(F^+(B)) &\subseteq (\tau_1, \tau_2)\text{-pCl}(F^+(V)) \\ &= (\tau_1, \tau_2)\text{-pCl}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq F^+((\sigma_1, \sigma_2)\text{-sCl}(V)) \\ &= F^+(V).\end{aligned}$$

Thus,

$$\begin{aligned}(\tau_1, \tau_2)\text{-pCl}(F^+(B)) &\subseteq F^+(\cap\{V \mid V \text{ is } (\sigma_1, \sigma_2)r\text{-open in } Y \text{ and } B \subseteq V\}) \\ &= F^+(s(\sigma_1, \sigma_2)\theta\text{-Cl}(B)).\end{aligned}$$

(5) ⇒ (1): Let V be any $(\sigma_1, \sigma_2)s$-open set of Y. By (5), we have

$$\begin{aligned}X - (\tau_1, \tau_2)\text{-pInt}(F^-(\sigma_1\sigma_2\text{-Cl}(V))) &= (\tau_1, \tau_2)\text{-pCl}(F^+(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^+((\sigma_1, \sigma_2)\text{-sCl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &= F^+(Y - \sigma_1\sigma_2\text{-Cl}(V)) \\ &= X - F^-(\sigma_1\sigma_2\text{-Cl}(V))\end{aligned}$$

and hence $F^-(V) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)) \subseteq (\tau_1, \tau_2)\text{-pInt}(F^-(\sigma_1\sigma_2\text{-Cl}(V)))$. By Theorem 6, F is lower almost contra-$(\tau_1, \tau_2)p$-continuous.

(5) ⇒ (6): Let A be any subset of X and $B = F(A)$. Then, $A \subseteq F^+(B)$ and by (5), $(\tau_1, \tau_2)\text{-pCl}(A) \subseteq (\tau_1, \tau_2)\text{-pCl}(F^+(B)) \subseteq F^+(s(\sigma_1, \sigma_2)\theta\text{-Cl}(B))$. Thus,

$$F((\tau_1, \tau_2)\text{-pCl}(A)) \subseteq F(F^+(s(\sigma_1, \sigma_2)\theta\text{-Cl}(B)))$$

$$\begin{aligned}&\subseteq s(\sigma_1,\sigma_2)\theta\text{-Cl}(B)\\&= s(\sigma_1,\sigma_2)\theta\text{-Cl}(F(A)).\end{aligned}$$

(6) ⇒ (5): Let B be any subset of Y. By (6), we have

$$F((\tau_1,\tau_2)\text{-pCl}(F^+(B))) \subseteq s(\sigma_1,\sigma_2)\theta\text{-Cl}(F(F^+(B))) \subseteq s(\sigma_1,\sigma_2)\theta\text{-Cl}(B)$$

and hence $(\tau_1,\tau_2)\text{-pCl}(F^+(B)) \subseteq F^+(s(\sigma_1,\sigma_2)\theta\text{-Cl}(B))$.

Theorem 10. *For a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is upper almost contra-$(\tau_1,\tau_2)p$-continuous;

(2) $F^+(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1,\tau_2)p$-open in X for every $(\sigma_1,\sigma_2)\beta$-open set V of Y;

(3) $F^+(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1,\tau_2)p$-open in X for every $(\sigma_1,\sigma_2)s$-open set V of Y;

(4) $F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ is $(\tau_1,\tau_2)p$-closed in X for every $(\sigma_1,\sigma_2)p$-open set V of Y.

Proof. (1) ⇒ (2): Let V be any $(\sigma_1,\sigma_2)\beta$-open set of Y. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1,\sigma_2)r$-closed in Y, by Theorem 5 we have $F^+(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\tau_1,\tau_2)p$-open in X.

(2) ⇒ (3): This is obvious since every $(\sigma_1,\sigma_2)s$-open set is $(\sigma_1,\sigma_2)\beta$-open.

(3) ⇒ (4): Let V be any $(\sigma_1,\sigma_2)p$-open set of Y. This implies that

$$Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$$

is $(\sigma_1,\sigma_2)r$-closed and $(\sigma_1,\sigma_2)s$-open in Y. By (3), we have

$$\begin{aligned}X - F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) &= F^+(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))\\&= F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))\end{aligned}$$

is $(\tau_1,\tau_2)p$-open in X. Thus, $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1,\tau_2)p$-closed in X.

(4) ⇒ (1): Let V be any $(\sigma_1,\sigma_2)r$-open set of Y. Then, V is $(\sigma_1,\sigma_2)p$-open in Y and by (4), $F^-(V) = F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1,\tau_2)p$-closed in X. Thus by Theorem 5, F is upper almost contra-$(\tau_1,\tau_2)p$-continuous.

Theorem 11. *For a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-$(\tau_1,\tau_2)p$-continuous;

(2) $F^-(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1,\tau_2)p$-open in X for every $(\sigma_1,\sigma_2)\beta$-open set V of Y;

(3) $F^-(\sigma_1\sigma_2\text{-}Cl(V))$ is $(\tau_1,\tau_2)p$-open in X for every $(\sigma_1,\sigma_2)s$-open set V of Y;

(4) $F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $(\sigma_1, \sigma_2)p$*-open set* V *of* Y*.*

Proof. The proof is similar to that of Theorem 10.

Corollary 1. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper almost contra-*$(\tau_1, \tau_2)p$*-continuous;*

(2) $F^+(\alpha(\sigma_1, \sigma_2)\text{-}Cl(V))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $(\sigma_1, \sigma_2)\beta$*-open set* V *of* Y*;*

(3) $F^+((\sigma_1, \sigma_2)\text{-}pCl(V))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $(\sigma_1, \sigma_2)s$*-open set* V *of* Y*;*

(4) $F^-((\sigma_1, \sigma_2)\text{-}sCl(V))$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $(\sigma_1, \sigma_2)p$*-open set* V *of* Y*.*

Corollary 2. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is lower almost contra-*$(\tau_1, \tau_2)p$*-continuous;*

(2) $F^-(\alpha(\sigma_1, \sigma_2)\text{-}Cl(V))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $(\sigma_1, \sigma_2)\beta$*-open set* V *of* Y*;*

(3) $F^-((\sigma_1, \sigma_2)\text{-}pCl(V))$ *is* $(\tau_1, \tau_2)p$*-open in* X *for every* $(\sigma_1, \sigma_2)s$*-open set* V *of* Y*;*

(4) $F^+((\sigma_1, \sigma_2)\text{-}sCl(V))$ *is* $(\tau_1, \tau_2)p$*-closed in* X *for every* $(\sigma_1, \sigma_2)p$*-open set* V *of* Y*.*

Theorem 12. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper almost contra-*$(\tau_1, \tau_2)p$*-continuous;*

(2) $(\tau_1, \tau_2)\text{-}pCl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(3) $(\tau_1, \tau_2)\text{-}pCl(F^-(V)) \subseteq F^-((\sigma_1, \sigma_2)\text{-}sCl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is $(\sigma_1, \sigma_2)r$-open in Y. Thus by (1), $F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $(\tau_1, \tau_2)p$-closed in X. Since $V \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$, we have $F^-(V) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ and hence $(\tau_1, \tau_2)\text{-pCl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$.

(2) $\Rightarrow$ (1): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. By (2), we have

$$(\tau_1, \tau_2)\text{-pCl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) = F^-(V)$$

and hence $F^-(V)$ is $(\tau_1, \tau_2)p$-closed in X, by Theorem 5 we have F is upper almost contra-$(\tau_1, \tau_2)p$-continuous.

(2) $\Leftrightarrow$ (3): It follows from Lemma 5.

Theorem 13. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is lower almost contra-$(\tau_1, \tau_2)p$-continuous;

(2) (τ_1, τ_2)-pCl$(F^+(V)) \subseteq F^+(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;

(3) (τ_1, τ_2)-pCl$(F^+(V)) \subseteq F^+((\sigma_1, \sigma_2)$-sCl$(V))$ for every $\sigma_1\sigma_2$-open set V of Y.

Proof. The proof is similar to that of Theorem 12.

Theorem 14. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If (τ_1, τ_2)-pCl$(F^-(B)) \subseteq F^-(\sigma_1\sigma_2$-ker$(B))$ for every subset B of Y, then F is upper almost contra-$(\tau_1, \tau_2)p$-continuous.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. By Lemma 1, we have

$$(\tau_1, \tau_2)\text{-pCl}(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}ker(V)) = F^-(V)$$

and hence $F^-(V)$ is $(\tau_1, \tau_2)p$-closed in X. Thus by Theorem 5, F is upper almost contra-$(\tau_1, \tau_2)p$-continuous.

Theorem 15. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction. If $F((\tau_1, \tau_2)$-pCl$(A)) \subseteq \sigma_1\sigma_2$-ker$(F(A))$ for every subset A of X, then F is lower almost contra-$(\tau_1, \tau_2)p$-continuous.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y. This implies that

$$F((\tau_1, \tau_2)\text{-pCl}(F^+(V))) \subseteq \sigma_1\sigma_2\text{-}ker(V)$$

and hence (τ_1, τ_2)-pCl$(F^+(V)) \subseteq F^+(\sigma_1\sigma_2$-$ker(V))$. By Lemma 1, we have

$$(\tau_1, \tau_2)\text{-pCl}(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}ker(V)) = F^+(V)$$

and so $F^+(V)$ is $\tau_1\tau_2$-closed in X. By Theorem 6, F is lower almost contra-$(\tau_1, \tau_2)p$-continuous.

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